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Transforming Time-Delay System Observers to Adaptive Observers

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Abstract
For joint estimation of states and parameters in time varying time-delay systems (TDS) involving both distributed and lumped time-delays, a general approach is proposed in this paper to transforming existing (non adaptive) observers to adaptive observers. In addition to the convergence conditions of the considered existing observers, a persistent excitation condition is introduced in order to ensure the convergence of parameter estimation. In contrast to implicitly formulated convergence conditions, which are usually assumed jointly for both state and parameter estimations in most TDS adaptive observers, the persistent excitation condition in the proposed approach is explicitly formulated and decoupled from the conditions initially assumed for state estimation.

Keywords: Time-delay systems, Adaptive observers

1. INTRODUCTION

Due to the increasing importance of time-delays in control systems equipped with more and more digital components, a great deal of interest has been given to state observer design for *time-delay systems* (TDS). In this respect, systems with time-delays involved in state equations have been studied in e.g. Germani et al. [2001], Xia et al. [2002], Yeganefar et al. [2008]. Similar interests have been given to systems with delayed output measurements Kazantzis and Wright [2005], Besançon et al. [2007], Cacace et al. [2010], Cacace et al. [2014], Kahelras et al. [2018].

For TDS involving unknown parameters, the problem of adaptive observer design is still not sufficiently investigated. In Mondal and Chung [2013], an adaptive observer design, following an approach similar to nonlinear adaptive observer design Cho and Rajamani [1997], has been proposed for a class of time-invariant TDS. In Sassi et al. [2016], a functional adaptive observer has been proposed for a bilinear TDS. In each of these recalled results, the asymptotical convergence of the designed adaptive observer is ensured by some *implicitly formulated condition*: the existence of a solution to some inequalities, which are specific to each design method.

This paper proposes a general approach to *transforming (non adaptive) observers to adaptive observers* for joint state-parameter estimation *in linear time varying (LTV) TDS*. In the case of classical (delay-free) LTV state-space systems, a general method has been proposed in Zhang [2002] for such a transformation. The results of this paper are essentially an extension of this method to TDS, but some *non trivial difficulties* have to be overcome to achieve this extension, as highlighted in the following sections.

Basically, it is assumed that

- if the parameters involved in the considered TDS *were* all known, then an exponentially convergent (non adaptive) observer *would* be available for state estimation;
- an explicitly formulated persistent excitation (PE) condition is satisfied.

It is well known in classical adaptive estimation problems that PE is essential for parameter estimation Narendra and Annaswamy [1987], Shi, Nar, Karl and Bjorn [1994]. This paper proposes a natural extension of the classical PE condition to the design of TDS adaptive observers. The considered existing observers may have been designed with different methods, yet their transformations to adaptive observers follow the same general approach. This generality covering existing observers designed with different methods, and the decoupling of the parameter estimation condition (the PE condition) from the state estimation condition, are important particularities *which distinguish the presented results from existing results.*

The results of this paper have been inspired by the preliminary idea summarized in the working note Zhang [2017].

The paper is organized as follows: the problem under study is formulated in Section II; the observer design and analysis are respectively dealt with in Sections III and IV. The conclusion in section VI.

2. PROBLEM STATEMENT

Consider the class of *time varying* linear differential *time-delay systems* (TDS) in the form of

$$\dot{x}(t) = A_0(t)x(t) + A_1(t)x(t-h) + \Phi(t)\theta \quad (1a)$$
$$y = C_0(t)x(t) + C_1(t)x(t-h) \quad (1b)$$

where $x(t) \in \mathbb{R}^n$ denotes the states, $y \in \mathbb{R}^m$ the outputs, $\theta \in \mathbb{R}^p$ the unknown parameters, h a positive constant specifying the time delay, $A_0(t), A_1(t), C_0(t), C_1(t), \Phi(t)$ are matrices of appropriate sizes filled with real-valued functions which are bounded and piecewise continuous in t.

All quantities in this model are known or accessible from sensor measurements, except the state vector $x(t)$ and the parameter vector θ.

An additional term $B(t)u(t)$ could be inserted into the TDS (1) in order to represent (control) inputs. Because such a term would imply trivial modifications of the results of this paper, it is omitted for lighter presentations.

As mentioned in the introduction, such a TDS is an infinite dimensional system. *By abuse of terminology*, the term "state" is still used to refer to the finite dimensional vector $x(t)$, by analogy to classical state-space systems, in order to ease the presentation. The "state equation" (1a) is sometimes called a *delay differential equation* (DDE).

The problem considered in this paper is to design an adaptive observer for joint estimation of $x(t)$ and θ under two assumptions:

- the existence of an exponentially convergent (non adaptive) *state observer* for the TDS (1) in the case where θ is known, and
- a persistent excitation condition, as typically required in parameter estimation problems.

The designed adaptive observer will be in the form of a DDE, so that it can be numerically solved with efficient DDE solvers.

The first assumption is more accurately formulated below, whereas the second one will be completed later.

Assumption 1. There exists a (time varying) bounded matrix gain $L(t) \in \mathbb{R}^{n \times m}$ such that the following DDE :

$$\dot{\xi}(t) = (A_0(t) - L(t)C_0(t))\xi(t)$$
$$+(A_1(t) - L(t)C_1(t))\xi(t-h) \quad (2)$$

is exponentially stable in the sense that (Yegafner et al. [2008]) :

$$|\xi(t)| \leq \sup_{t_0 \leq s \leq t_0+h} |\xi(s)| M_0 e^{-\sigma t}, \quad \forall t > t_0 + h \quad (3)$$

for some positive scalars M_0 and σ. □

Remark 1. This assumption simply means that, if the true parameter vector θ was known, then an exponentially convergent (non adaptive) observer of system (1) (for state estimation only) would be available in the form of

$$\dot{\hat{x}}(t) = A_0(t)\hat{x}(t) + A_1(t)\hat{x}(t-h) + \Phi(t)\theta - L(t)\tilde{y}(t) \quad (4a)$$
$$\tilde{y}(t) = C_0(t)\hat{x}(t) + C_1(t)\hat{x}(t-h) - y(t), \quad (4b)$$

where $L(t)$ is the same gain matrix as in Assumption 1. □

Lemma 1. Add an additive input term $d(t)$ into the homogeneous system (2), then Assumption 1 implies that the resulting TDS

$$\dot{\xi}(t) = (A_0(t) - L(t)C_0(t))\xi(t)$$
$$+(A_1(t) - L(t)C_1(t))\xi(t-h) + d(t) \quad (5)$$

is *input-to-state stable*, *i.e.*, $\forall t > t_0 + h$,

$$|\xi(t)| \leq \beta(\sup_{t_0 \leq s \leq t_0+h} |\xi(s)|, t) + \gamma(ess. \sup_{t_0+h \leq s < t} |d(s)|) \quad (6)$$

where $s \to \gamma(s)$ is a K function (γ is strictly increasing and $\gamma(0) = 0$) and $(t, s) \to \beta(t, s)$ is a KL function (for each fixed t, the function $s \to \beta(s, t)$ is a K function, and for each fixed s the function $t \to \beta(s, t)$ is non increasing and goes to zero as $t \to \infty$) . □

This result is a simple consequence of the Theorem 3.2 in (Yegafner et al. [2008]).

3. ADAPTIVE OBSERVER DESIGN

The proposed method for adaptive observer design will be based on the state-parameter decoupling transformation approach, initially introduced in the case of classical (delay-free) LTV systems Zhang [2002].

3.1 Adaptive observer structure

In order to transform the (non adaptive) state observer (4) to an adaptive observer for joint estimation of $x(t)$ and θ, a natural idea is to replace in (4) the unknown parameter vector θ by its estimate $\hat{\theta}(t)$, which is computed by a somehow designed adaptive law. The following design will essentially follow this idea, except an extra term $v_0(t)$ in the state estimation equation, yielding an estimator in the form of

$$\dot{\hat{x}}(t) = A_0(t)\hat{x}(t) + A_1(t)\hat{x}(t-h)$$
$$+ \Phi(t)\hat{\theta}(t) - L(t)\tilde{y}(t) + v_0(t) \quad (7a)$$
$$\tilde{y}(t) = C_0(t)\hat{x}(t) + C_1(t)\hat{x}(t-h) - y(t). \quad (7b)$$

The extra term $v_0(t)$ is added in order to *compensate the parameter estimation error* introduced in the state estimation equation (7a), due to the fact that the true θ has been replaced by its estimate $\hat{\theta}(t)$.

Before designing an adaptive law computing the parameter estimate $\hat{\theta}(t)$, it will be first shown that an appropriately chosen $v_0(t)$ will greatly help the convergence analysis of the designed algorithm.

3.2 Decoupling transformation

Define the state and parameter estimation errors

$$\tilde{x}(t) \triangleq \hat{x}(t) - x(t) \quad (8)$$
$$\tilde{\theta}(t) \triangleq \hat{\theta}(t) - \theta, \quad (9)$$

then it is straightforward to derive the state estimation error dynamics equation

$$\dot{\tilde{x}}(t) = (A_0(t) - L(t)C_0(t))\tilde{x}(t)$$
$$+ (A_1(t) - L(t)C_1(t))\tilde{x}(t-h) + \Phi(t)\tilde{\theta}(t) + v_0(t). \quad (10)$$

The key step of the error analysis is the decoupling transformation, which has been proposed originally for the design of adaptive observers of classical (delay-free) systems Zhang [2002], in the form of

$$\eta(t) = \tilde{x}(t) - \Upsilon(t)\tilde{\theta}(t), \tag{11}$$

where $\Upsilon(t) \in \mathbb{R}^{n\times p}$ is a time varying matrix to be specified so that the transformed error variable $\eta(t)$ is decoupled from $\tilde{\theta}$.

Some simple computations then lead to

$$\begin{aligned}\dot{\eta}(t) &= [A_0(t) - L(t)C_0(t)][\eta(t) + \Upsilon(t)\tilde{\theta}(t)] \\ &\quad + [A_1(t) - L(t)C_1(t)][\eta(t-h) + \Upsilon(t-h)\tilde{\theta}(t-h)] \\ &\quad + \Phi(t)\tilde{\theta}(t) - \dot{\Upsilon}(t)\tilde{\theta}(t) - \Upsilon(t)\dot{\tilde{\theta}}(t) + v_0(t) \\ &= \omega_1(t) + \omega_2(t) + \omega_3(t), \end{aligned} \tag{12}$$

with

$$\omega_1(t) = [A_0(t) - L(t)C_0(t)]\eta(t) + [A_1(t) - L(t)C_1(t)]\eta(t-h)$$
$$\omega_2(t) = \left[(A_0(t) - L(t)C_0(t))\Upsilon(t) + \Phi(t) - \dot{\Upsilon}(t)\right]\tilde{\theta}(t)$$
$$\omega_3(t) = (A_1(t) - L(t)C_1(t))\Upsilon(t-h)\tilde{\theta}(t-h) - \Upsilon(t)\dot{\tilde{\theta}}(t) + v_0(t).$$

At this point, if the decoupling technique proposed in Zhang [2002] was naively followed, then one would choose $\Upsilon(t)$ ensuring $\omega_2(t) = 0$ for all $t \geq t_0$, as a solution of the differential equation

$$\dot{\Upsilon}(t) = (A_0(t) - L(t)C_0(t))\Upsilon(t) + \Phi(t); \tag{13}$$

and then choose $v_0(t)$ so that $\omega_3(t) = 0$ for all t, yielding

$$v_0(t) = \Upsilon(t)\dot{\tilde{\theta}}(t) - (A_1(t) - L(t)C_1(t))\Upsilon(t-h)\tilde{\theta}(t-h). \tag{14}$$

With $\omega_2(t)$ and $\omega_3(t)$ annihilated, equation (12) would become simply

$$\begin{aligned}\dot{\eta}(t) &= [A_0(t) - L(t)C_0(t)]\eta(t) \\ &\quad + [A_1(t) - L(t)C_1(t)]\eta(t-h),\end{aligned} \tag{15}$$

and $\eta(t)$ governed by this equation would converge exponentially to zero, according to Assumption 1. This decoupling technique worked well for classical (delay-free) systems, as in Zhang [2002], Li et al. [2011], where the variable $v_0(t)$ as chosen in (14) was simply

$$v_0(t) = \Upsilon(t)\dot{\tilde{\theta}}(t), \tag{16}$$

because the terms in (14) involving delays did not exist. Moreover, $\dot{\tilde{\theta}}(t) = \dot{\hat{\theta}}(t) - \dot{\theta}$ and $\dot{\theta} = 0$, therefore $v_0(t)$ was simply written, in the classical case, as

$$v_0(t) = \Upsilon(t)\dot{\hat{\theta}}(t), \tag{17}$$

which could be readily computed with an appropriately designed parameter adaptation law.

However, for the systems considered in this paper, the situation is more complicated due to the presence of the terms involving time-delays. The trouble is that the variable $v_0(t)$ chosen in (14) cannot be computed in practice, since $\tilde{\theta}(t-h) = \hat{\theta}(t-h) - \theta$ would require the true value of θ, which is of course unknown in the considered estimation problem! Apparently, this is a non trivial difficulty that did not exist in the classical case as considered in Zhang [2002] and Li et al. [2011].

Fortunately, this difficulty can be addressed as follows.

Add the term $(A_1(t)-L(t)C_1(t))\Upsilon(t-h)\tilde{\theta}(t)$ to $\omega_2(t)$ and subtract the same term from $\omega_3(t)$, so that the sum in (12) remains unchanged. Accordingly, the second term of $v_0(t)$ in (14)

$$-(A_1(t) - L(t)C_1(t))\Upsilon(t-h)\tilde{\theta}(t-h), \tag{18}$$

which could not be computed due to the dependence of $\tilde{\theta}(t-h)$ on the unknown true parameter vector θ, then becomes

$$\begin{aligned} &(A_1(t) - L(t)C_1(t))\Upsilon(t-h)[\tilde{\theta}(t) - \tilde{\theta}(t-h)] \\ &= (A_1(t) - L(t)C_1(t))\Upsilon(t-h)[\hat{\theta}(t) - \theta - \hat{\theta}(t-h) + \theta] \qquad (19) \\ &= (A_1(t) - L(t)C_1(t))\Upsilon(t-h)[\hat{\theta}(t) - \hat{\theta}(t-h)], \qquad (20)\end{aligned}$$

which no longer involves the unknown θ, and therefore can be computed with a parameter adaptation law to be designed.

Then $\omega_2(t)$ and $\omega_3(t)$ become

$$\begin{aligned}\omega_2(t) &= \Big[(A_0(t) - L(t)C_0(t))\Upsilon(t) + \Phi(t) - \dot{\Upsilon}(t) \\ &\qquad + (A_1(t) - L(t)C_1(t))\Upsilon(t-h)\Big]\tilde{\theta}(t)\end{aligned} \tag{21}$$

$$\begin{aligned}\omega_3(t) &= [A_1(t) - L(t)C_1(t)]\Upsilon(t-h)[\tilde{\theta}(t-h) - \tilde{\theta}(t)] \\ &\qquad - \Upsilon(t)\dot{\tilde{\theta}}(t) + v_0(t).\end{aligned} \tag{22}$$

Accordingly, let $\Upsilon(t)$ be generated through the TDS

$$\begin{aligned}\dot{\Upsilon}(t) &= (A_0(t) - L(t)C_0(t))\Upsilon(t) \\ &\quad + (A_1(t) - L(t)C_1(t))\Upsilon(t-h) + \Phi(t)\end{aligned} \tag{23}$$

with an arbitrary initial condition, so that $\omega_2(t) = 0$ for all $t > t_0 + h$. Moreover, choose $v_0(t)$ as

$$\begin{aligned} v_0(t) &= \Upsilon(t)\dot{\tilde{\theta}}(t) - [A_1(t) - L(t)C_1(t)]\Upsilon(t-h)[\tilde{\theta}(t-h) - \tilde{\theta}(t)] \\ &= \Upsilon(t)\dot{\hat{\theta}}(t) - [A_1(t) - L(t)C_1(t)]\Upsilon(t-h)[\hat{\theta}(t-h) - \hat{\theta}(t)]\end{aligned}$$

so that $\omega_3(t) = 0$ for all $t > t_0 + h$, by noticing that $\dot{\tilde{\theta}}(t) = \dot{\hat{\theta}}(t) - \dot{\theta} = \dot{\hat{\theta}}(t)$ and $\tilde{\theta}(t-h) - \tilde{\theta}(t) = \hat{\theta}(t-h) - \hat{\theta}(t)$.

Now in (12), $\omega_2(t) = \omega_3(t) = 0$, therefore $\eta(t)$ is indeed governed by the TDS (15), which is exponentially stable according to Assumption 1.

At this point, it is only shown that the decoupled estimation error $\eta(t)$ as defined in (11) converges to zero exponentially. Based on this result, the convergences of the state estimation error $\tilde{x}(t)$ and of the parameter estimation error $\tilde{\theta}(t)$ will be analyzed. For this purpose, the adaptive law for parameter estimation should designed appropriately.

3.3 Parameter estimation law design

If the output error $\tilde{y}(t)$ *was* available, then a linear regression parameter estimation algorithm inspired by the classical least squares estimator *would* be in the form of

$$\begin{aligned}\dot{\hat{\theta}}(t) &= -R(t)\Lambda(t)^T \\ &\qquad \cdot \left[\tilde{y}(t) - C_1(t)\Upsilon(t-h)[\hat{\theta}(t-h) - \hat{\theta}(t)]\right]\end{aligned} \tag{24a}$$

$$\dot{R}(t) = \lambda R(t) - R(t)\Lambda(t)^T\Lambda(t)R(t) \tag{24b}$$

$$\Lambda(t) = C_0(t)\Upsilon(t) + C_1(t)\Upsilon(t-h) \tag{24c}$$

where the scalar $\lambda > 0$ is an exponential forgetting factor.

Of course, the output error $\tilde{y}(t)$ as defined in (7b) cannot be obtained without computing $\hat{x}(t)$ and $\hat{\theta}(t)$. Therefore, parameter estimation should be jointly performed with state estimation, by combining (24a) with (7).

The whole adaptive observer for joint state-parameter estimation is then expressed in the following algorithm, where $x_0(t) : [t_0, t_0 + h] \to \mathbb{R}^n$ is the initial state estimate, $\theta_0 \in \mathbb{R}^p$ the initial parameter estimate, $0_{n\times p}$ the $n \times p$ zero matrix, I_p the $p \times p$ identity matrix, $r_0 > 0$ a scalar value, and $\lambda > 0$ a forgetting factor.

Adaptive observer algorithm

For $t \in [t_0, t_0 + h]$:

$$\hat{x}(t) = x_0(t),\ \hat{\theta}(t) = \theta_0,\ \Upsilon(t) = 0_{n\times p}. \tag{25}$$

For $t = t_0 + h$:

$$R(t) = r_0 I_p \tag{26}$$

For $t > t_0 + h$:

$$\tilde{y}(t) = C_0(t)\hat{x}(t) + C_1(t)\hat{x}(t-h) - y(t) \tag{27a}$$

$$\Lambda(t) = C_0(t)\Upsilon(t) + C_1(t)\Upsilon(t-h) \tag{27b}$$

$$\begin{aligned} v_0(t) = &-\Upsilon(t)R(t)\Lambda(t)^T\Big[\tilde{y}(t) \\ &- C_1(t)\Upsilon(t-h)[\hat{\theta}(t-h) - \hat{\theta}(t)]\Big] \\ &- [A_1(t) - L(t)C_1(t)]\Upsilon(t-h)[\hat{\theta}(t-h) - \hat{\theta}(t)] \end{aligned} \tag{27c}$$

$$\begin{aligned} \dot{\Upsilon}(t) = &(A_0(t) - L(t)C_0(t))\Upsilon(t) \\ &+ (A_1(t) - L(t)C_1(t))\Upsilon(t-h) + \Phi(t) \end{aligned} \tag{27d}$$

$$\dot{R}(t) = \lambda R(t) - R(t)\Lambda(t)^T\Lambda(t)R(t) \tag{27e}$$

$$\begin{aligned} \dot{\hat{\theta}}(t) = &-R(t)\Lambda(t)^T \\ &\cdot \Big[\tilde{y}(t) - C_1(t)\Upsilon(t-h)[\hat{\theta}(t-h) - \hat{\theta}(t)]\Big] \end{aligned} \tag{27f}$$

$$\begin{aligned} \dot{\hat{x}}(t) = &A_0(t)\hat{x}(t) + A_1(t)\hat{x}(t-h) \\ &+ \Phi(t)\hat{\theta}(t) - L(t)\tilde{y}(t) + v_0(t). \end{aligned} \tag{27g}$$

4. ADAPTIVE OBSERVER CONVERGENCE ANALYSIS

Before analyzing the convergence of the state and parameter estimation errors, it is important to ensure that the auxiliary variables computed in the adaptive observer (27) are all bounded.

4.1 Boundedness of auxiliary variables

Let us start with the recursively computed $\Upsilon(t)$.

Proposition 1. Under Assumptions 1, the recursively generated matrix $\Upsilon(t)$ through the DDE (27d) from bounded matrices $A_0(t), A_1(t),\ C_0(t), C_1(t), \Phi(t), L(t)$ is bounded. □

Further boundedness results will require a persistent excitation assumption, as typically required in parameter estimation problems.

Assumption 2. There exist positive scalar constants T and δ, such that for all $t > t_0 + h$,

$$\int_t^{t+T} \Lambda(s)^T\Lambda(s)ds \geq \delta I_p, \tag{28}$$

where the inequality $X \geq Y$ means that $X - Y$ is a positive semidefinite matrix. □

Remark 2. The symmetric integral in (28) is by construction a positive semidefinite matrix. Assumption 2 states that it is positive definite with a strictly positive lower bound. This integral is also *upper bounded* due to the already established boundedness of $\Lambda(t)$. □

Remark 3. The matrix $\Lambda(t)$ is generated from $\Phi(t)$ through (27d) and (27b). For more clarity, let us group these two equations as

$$\begin{aligned} \dot{\Upsilon}(t) = &(A_0(t) - L(t)C_0(t))\Upsilon(t) \\ &+ (A_1(t) - L(t)C_1(t))\Upsilon(t-h) + \Phi(t) \end{aligned} \tag{29a}$$

$$\Lambda(t) = C_0(t)\Upsilon(t) + C_1(t)\Upsilon(t-h). \tag{29b}$$

It is then clear that $\Lambda(t)$ is the output of a linear TDS driven by the input $\Phi(t)$. In other words, $\Lambda(t)$ is obtained by *linearly filtering* $\Phi(t)$ through a TDS. Therefore, the persistent excitation condition stated in Assumption 2 is indeed an assumed property of $\Phi(t)$. In practice, inequality (28) can be monitored by numerically solving the TDS (29) with a DDE solver. □

Proposition 2. Under Assumptions 1 and 2, the recursively generated matrix $R(t)$ through the ODE (27e) from $\Lambda(t)$ is bounded for all $t > t_0 + h$. Moreover, $R(t)$ is an invertible matrix and its inverse is also bounded for all $t > t_0 + h$. □

4.2 Convergence of estimation errors

The convergence of $\tilde{\theta}(t)$ and $\tilde{x}(t)$ will be analyzed, based on the already established convergence of $\eta(t)$.

Proposition 3. Under Assumptions 1 and 2, both the state estimation error $\tilde{x}(t) = x - \hat{x}(t)$ and the parameter estimation error $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$ of the adaptive observer (27) converge to zero exponentially when $t \to +\infty$. □

Proof. Let us first analyze $\tilde{\theta}(t)$. Notice that $\dot{\tilde{\theta}} = \dot{\hat{\theta}} - \dot{\theta} = \dot{\hat{\theta}}$, then

$$\begin{aligned} \dot{\tilde{\theta}}(t) = &-R(t)\Lambda(t)^T[C_0(t)\Upsilon(t) + C_1(t)\Upsilon(t-h)]\tilde{\theta}(t) \\ &+ C_0(t)\eta(t) + C_1(t)\eta(t-h)] \end{aligned} \tag{30}$$

$$\begin{aligned} = &-R(t)\Lambda(t)^T\Lambda(t)\tilde{\theta}(t) \\ &- R(t)\Lambda(t)^T\left[C_0(t)\eta(t) + C_1(t)\eta(t-h)\right] \end{aligned} \tag{31}$$

where $\Lambda(t)$ is as defined in (27b).

By viewing $R(t)\Lambda(t)^T\Lambda(t)$ as a (bounded) time varying matrix, $\tilde{\theta}(t)$ is the state of a linear time varying (LTV) system, driven by the inputs $\eta(t)$ and $\eta(t-h)$, which tend exponentially to zero. Before studying the behavior of this LTV system, let us first consider its homogeneous part, namely

$$\dot{\vartheta} = -R(t)\Lambda(t)^T\Lambda(t)\vartheta(t). \tag{32}$$

In order to study the stability of this homogeneous LTV system, define a Lyapunov function candidate

$$V(\vartheta(t), t) = \vartheta^T(t)M(t)\vartheta(t) \tag{33}$$

with $M(t) = R^{-1}(t)$, which is positive definite, has a finite upper bound and a strictly positive lower bound, according to Proposition 2. Then

$$\begin{aligned} \frac{d}{dt}V(\vartheta(t), t) = &-\vartheta^T(t)M(t)R(t)\Lambda(t)^T\Lambda(t)\vartheta(t) \\ &- \vartheta^T(t)\Lambda(t)^T\Lambda(t)R(t)M(t)\vartheta(t) \\ &+ \vartheta^T(t)[-\lambda M(t) + \Lambda(t)^T\Lambda(t)]\vartheta(t) \end{aligned} \tag{34}$$

$$\begin{aligned} = &-\vartheta^T(t)\Lambda(t)^T\Lambda(t)\vartheta(t) - \vartheta^T(t)\Lambda(t)^T\Lambda(t)\vartheta(t) \\ &+ \vartheta^T(t)[-\lambda M(t) + \Lambda(t)^T\Lambda(t)]\vartheta(t) \end{aligned} \tag{35}$$

$$= -\lambda\vartheta^T(t)M(t)\vartheta(t) - \vartheta^T(t)\Lambda(t)^T\Lambda(t)\vartheta(t) \tag{36}$$

$$\leq -\lambda\vartheta^T(t)M(t)\vartheta(t) \tag{37}$$

$$\leq -\lambda V(\vartheta(t), t). \tag{38}$$

This result then implies that the homogeneous part of the LTV system (31), as expressed in (32), is exponentially stable. Therefore, the state $\tilde{\theta}(t)$ of the LTV system (31) driven by exponentially vanishing $\eta(t)$ and $\eta(t-h)$ converges exponentially to zero.

Finally, it follows from (11), the exponential convergences to zero of $\eta(t)$ and of $\tilde{\theta}(t)$, and the boundedness of $\Upsilon(t)$ that

$$\tilde{x}(t) = \eta(t) + \Upsilon(t)\tilde{\theta}(t) \tag{39}$$

converges exponentially to zero. □

5. CONCLUSION

A general approach has been proposed in this paper to transforming existing (non adaptive) state observers to adaptive observers, for linear time varying TDS. This approach greatly simplifies the design of adaptive observers for joint state-parameter estimation, under an explicitly formulated persistent excitation condition for parameter estimation, which is decoupled from the conditions initially assumed by existing state observers.

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