Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Flatness-Based Active Disturbance Rejection Control For a Wheeled Mobile Robot Subject To Slips and External Environmental Disturbances ⋆

Amine Abadi * **Adnen El Amraoui** ** **Hassen Mekki** ***
Nacim Ramdani *

* *University of Orléans, INSA CVL, PRISME EA 4229, F45072 Orléans, France (e-mail: amine.abadi@etu.univ-orleans.fr).*
** *University of Artois, LGI2A, EA 3926, F62400, Béthune, France (e-mail: adnen.elamraoui@univ-artois.fr)*
*** *University of Sousse, NOCCS Labortory , Tunisia (e-mail: mekki.hassen@gmail.com)*

Abstract: This work suggests flatness-based Active Disturbance Rejection Control (ADRC) to deal with the problem of trajectory tracking for Wheeled Mobile Robot (WMR). Based on the differential flatness theory, the nonlinear WMR system is transformed into two integral-chains, which makes the creation of a state feedback controller easier. In order to improve the WMR tracking, slip and external environmental disturbance must be considered in the controller design. Therefore, an Extended State Observer (ESO) is created to estimate the obtained linearized system state and the extended state known as lumped uncertainties. The latter represent the total effects of slip and the external environmental disturbances to WMR. After that, according to the ESO results, a complementary element is added to the state feedback controller to compensate the effects of lumped disturbances. Simulation results are introduced to demonstrate the advantages of combining ADRC with flatness control.

Keywords: Wheeled mobile robot, robust tracking, extended state observer, active disturbance rejection control, differential flatness.

1. INTRODUCTION

A Wheeled Mobile Robot (WMR) is a classic kind of nonholonomic systems, which is extensively used in many applications, such as national defense, industrial production, home robotics and other areas. Moreover, trajectory tracking control is the key to develop autonomous motion for the WMR. Although the WMR is a nonlinear system with multivariate and strong coupling, it is hard to obtain good tracking control performance. In the last decade, the differential flatness theory presented by (Fliess, 1995) has proven to be a good tool to improve the trajectory planning and to create tracking controllers for linear and nonlinear systems. With flatness property, the state and input variables of the system can be expressed as a function of the flat outputs and their derivatives. Then the integration of differential equations is not needed. In addition, the flatness has specific advantages when used in nonlinear control systems. Indeed, by permitting an accurate linearization of the system's dynamical model, it can be possible to avoid using linear models with limited validity in the controller design. This characteristic makes flatness a good tool for solving various problems in many areas such as robotics (Abadi, 2019; Markus, 2017) and electric domains (García, 2019) because the mathematical models of these systems are strongly nonlinear. Usually, the control method applied to a WMR enables obtaining asymptotic tracking of the desired trajectory without considering the existence of slips and disturbances in the model, which is not true in practical applications. Thus, a control law that compensates all perturbation effects is needed to improve the WMR tracking performance. This new control requires the measurement of the system state and the lumped disturbance effect. In practice, direct measurements of disturbances is not available, an estimation algorithm is needed to estimate the lumped disturbance effect. Recently, the Active Disturbance Rejection Control (ADRC), developed by (Han, 2009), appeared as a good solution to deal with nonlinear systems containing uncertainties and external disturbances together. The Extended State Observer (ESO) is an important element of ADRC, which estimates not only system states but also an extended state known as the lumped disturbance. Thus, the internal disturbances caused by un-modeled dynamics and the parameter uncertainties are lumped together as an extended state, which can be estimated by the ESO and eliminated in the control inputs. This observer does not depend on the mathematical model of disturbances. The prominent feature of the ADRC approach is that it requires only a little knowledge of the controlled process. The ESO advantages have been used to develop different

⋆ This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 823887.

robust tracking controllers, which can be applied strongly in many applications such as nonlinear adaptive control of hypersonic vehicle system (Pu, 2015), sliding mode control of electric multiple units (LIU, 2017) and feedback linearization of a rotor-active magnetic-bearing model (Ren, 2019).

In this paper, the contribution consists in the combination of the ADRC strategies and the simplicity introduced by the flatness control concept to design a robust tracking controller for the WMR in the presence of unmeasurable states, slips and external environmental disturbances. First, it is proven that flatness control permits transforming the nonlinear WMR system into a linearized system known as the canonical Brunovsky Form. Hence, for the obtained linearized model, a feedback controller that enables accurate tracking is created. However, since measurements are available just for some elements of the transformed state vector and because the WMR system can be affected by slips and external environmental disturbances, an ESO is suggested to estimate every state of the extended system for each channel of the linearized system and the total uncertainties affecting it. Finally, robust feedback control considering the ESO outputs can be designed to compensate the effect of the lumped perturbation on the WMR.

The structure of the paper is as follows: In section II, we present the robot mobile model. In section III, we define the non-linear feedback control using the differential flatness property for the WMR without considering uncertainties. Section IV is devoted to the robust tracking controller design based on flatness and the ADRC, and section V deals with the simulation results.

2. WHEELED MOBILE ROBOT MODEL

The considered system in our application is a WMR. It is equipped with two independently driven wheels (right and left) and a front wheel to ensure the equilibrium of the robot movement. The generalized configuration of the driven mobile robot is given by $q = [x, y, \theta]$, where x and y are the center position coordinates of the mobile robot in the fixed frame (O, X, Y), and θ represents the robot orientation angle with respect to the X axis. The kinematic model of the robot mobile without any slip is defined as follows:

$$\begin{aligned} \dot{x} &= cos(\theta)v \\ \dot{y} &= sin(\theta)v \\ \dot{\theta} &= w \end{aligned} \tag{1}$$

where v and w are the transnational and rotational velocity of the robot, respectively. These latter can be written as a function of right and left angular speeds of the wheels (w_r and w_l) as follows:

$$v = (\frac{w_r + w_l}{2})r \tag{2}$$

$$w = (\frac{w_r - w_l}{2b})r \tag{3}$$

where r is the radius of the wheel, and $2b$ is the distance between the wheels. According to the non-slip condition, the non-holonomic constraint is defined as follows:

$$\dot{x}sin\theta - \dot{y}cos\theta = 0 \tag{4}$$

3. NON LINEAR FEEDBACK CONTROL USING DIFFERENTIAL FLATNESS

3.1 Differential flatness theory

Let us consider the following nonlinear system:

$$\dot{x} = f(x, u) \tag{5}$$

where x and u represent the state and the input vectors. The nonlinear system (5) is differentially flat if there exists an output α in the following form:

$$\alpha = \Xi_1(x, u, \dot{u}, ..., u^{(r)}) \tag{6}$$

such that the state and the input can be expressed as follows:

$$x = \Xi_2(\alpha, \dot{\alpha}, \ddot{\alpha}...., \alpha^{(\beta)}) \tag{7}$$

$$u = \Xi_3(\alpha, \dot{\alpha}, \ddot{\alpha}..., \alpha^{(\beta+1)}) \tag{8}$$

where β and r are finite multi-indices, and Ξ_1, Ξ_2 and Ξ_3 are smooth vector functions of the output vector α and its derivatives.

3.2 Differential flatness of wheeled mobile robots

The kinematic model of the WMR is known as a differentially flat system, with flat outputs defined as the Cartesian coordinates of the robot center:

$$\alpha = [\alpha_{11}, \alpha_{21}]^T = [x, y]^T \tag{9}$$

All the states and control of the WMR system can be calculated from the flat output α and their derivatives as follows:

$$\theta = arctan\frac{\dot{\alpha}_{21}}{\dot{\alpha}_{11}} \tag{10}$$

$$v = \sqrt{\dot{\alpha}_{11}^2 + \dot{\alpha}_{21}^2} \tag{11}$$

$$w = \frac{\dot{\alpha}_{11}\ddot{\alpha}_{21} - \ddot{\alpha}_{11}\dot{\alpha}_{21}}{\dot{\alpha}_{11}^2 + \dot{\alpha}_{21}^2} \tag{12}$$

3.3 Flatness-based tracking control

The flatness property permits us to calculate a feedback linearization, which transforms the nonlinear system into a controllable linear system where the flat outputs depict the state vector. The relationship between the control input vector, w and v, and the flat output's highest derivatives is not invertible. This problem obviously exposes an obstacle to realize static feedback linearization. To overcome this fact, the control input v is considered as an additional state for the kinematics model (1). Consequently, the new system is defined as follows:

$$\begin{aligned} \dot{x} &= cos(\theta)v \\ \dot{y} &= sin(\theta)v \\ \dot{v} &= u_1 \\ \dot{\theta} &= u_2 \end{aligned} \tag{13}$$

where the state system of the mobile robot is $X = [x, y, v, \theta]^T$ and the new control input is defined by $u_1 = \dot{v}$ and $u_2 = w$. To obtain the invertible relation between the inputs u_1 and u_2 and the higher derivatives of the flat outputs $\alpha_{11} = x$ and $\alpha_{21} = y$, we differentiate the flat outputs until an input appears as follows:

$$\begin{bmatrix} \ddot{\alpha}_{11} \\ \ddot{\alpha}_{21} \end{bmatrix} = B_r \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{14}$$

where B_r is the decoupling matrix defined as follows:

$$B_r = \begin{bmatrix} cos(\theta) & -vsin(\theta) \\ sin(\theta) & vcos(\theta) \end{bmatrix} \quad (15)$$

Matrix B_r is not singular if $v \neq 0$. Under this assumption, the control can be defined as follows:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = B_r^{-1} \begin{bmatrix} \ddot{\alpha}_{11} \\ \ddot{\alpha}_{21} \end{bmatrix} \quad (16)$$

When substituting the control input defined by equations (16) in equation (14), we obtain the state-space Burnovsky form (BF) as follows:

$$(BF_1) \begin{cases} \dot{\alpha}_{11} = \alpha_{12} \\ \dot{\alpha}_{12} = v_1 \\ Y_1 = \alpha_{11} = x \end{cases} \quad (BF_2) \begin{cases} \dot{\alpha}_{21} = \alpha_{22} \\ \dot{\alpha}_{22} = v_2 \\ Y_2 = \alpha_{21} = y \end{cases} \quad (17)$$

where v_1 and v_2 are an appropriate feedback controller defined as follows:

$$v_1 = \ddot{\alpha}_{xd} + \lambda_{x2}(\dot{\alpha}_{xd} - \alpha_{12}) + \lambda_{x1}(\alpha_{xd} - \alpha_{11}) \quad (18)$$

$$v_2 = \ddot{\alpha}_{yd} + \lambda_{y2}(\dot{\alpha}_{yd} - \alpha_{22}) + \lambda_{y1}(\alpha_{yd} - \alpha_{21}) \quad (19)$$

where α_{xd} and α_{yd} represent the reference trajectories for the flat outputs α_{11} and α_{21}, and λ_{x1}, λ_{x2}, λ_{y1} and λ_{y2} are the controller gains. The characteristic polynomials of the BF (17) are defined as follows:

$$s^2 + \lambda_{x2}s + \lambda_{x1} = s^2 + 2\xi_x\gamma_{xc}s + \gamma_{xc}^2 \quad (20)$$

$$s^2 + \lambda_{y2}s + \lambda_{y1} = s^2 + 2\xi_y\gamma_{yc}s + \gamma_{yc}^2 \quad (21)$$

where parameters ξ_x and ξ_y are the damping coefficient, and γ_{xc} and γ_{yc} are the bandwidths of the controller. Based on equations (20) and (21), the controller gain can be calculated as follows:

$$\lambda_{x1} = \gamma_{xc}^2, \lambda_{x2} = 2\xi_x\gamma_{xc}, \lambda_{y1} = \gamma_{yc}^2, \lambda_{y2} = 2\xi_y\gamma_{yc} \quad (22)$$

Plugging the feedback law given by equations (18-19) in the flatness-based open loop control (16), we obtain the Flatness-Based Tracking Control (FBTC) applied to the WMR as follows:

$$\begin{bmatrix} u_{F1} \\ u_{F2} \end{bmatrix} = B_r^{-1} \begin{bmatrix} \ddot{\alpha}_{xd} + \lambda_{x2}\dot{e}_{rx1} + \lambda_{x1}e_{rx1} \\ \ddot{\alpha}_{yd} + \lambda_{y2}\dot{e}_{ry1} + \lambda_{y1}e_{ry1} \end{bmatrix} \quad (23)$$

where $e_{rx1} = \alpha_{xd} - \alpha_{11}$ and $e_{ry1} = \alpha_{yd} - \alpha_{21}$.

4. ROBUST CONTROLLER VIA FLATNESS AND ADRC

4.1 Uncertain kinematic model

We consider that the WMR is subject to slips and external environmental disturbance. In this case, the uncertain kinematic model will be defined as follows:

$$\begin{cases} \dot{x} = cos(\theta)v + v_t cos(\theta) + v_s sin(\theta) + \Delta_x \\ \dot{y} = sin(\theta)v + v_t sin(\theta) - v_s cos(\theta) + \Delta_y \\ \dot{\theta} = w + w_s \end{cases} \quad (24)$$

where Δ_x and Δ_y represent the external environmental disturbances, v_t and v_s represent the slip velocities in the forward direction and normal to the forward direction, respectively, and w_s represents the angular slip velocity component. Based on (Ryu, 2011), we assume that the slip components are defined as follows:

$$v_t(t) = v_s(t) = w_s(t) = \mu_1 v(t) \quad (25)$$

where μ_1 is a positive constant. We assume that the component velocity and the external disturbance and their derivatives are bounded as follows:

$$||v_t|| \leq \mu_1||v||, \; ||v_s|| \leq \mu_2||v||, ||w_s|| \leq \mu_3 \quad (26)$$

$$||\dot{v}_t|| \leq \mu_4, \; ||\dot{v}_s|| \leq \mu_5, ||\dot{w}_s|| \leq \mu_6 \quad (27)$$

$$\underline{\Delta}_1 \leq \Delta_x \leq \overline{\Delta}_1, \; \underline{\Delta}_2 \leq \Delta_y \leq \overline{\Delta}_2, \quad (28)$$

$$\underline{\Delta}_3 \leq \dot{\Delta}_x \leq \overline{\Delta}_3, \; \underline{\Delta}_4 \leq \dot{\Delta}_y \leq \overline{\Delta}_4 \quad (29)$$

where μ_i, $\underline{\Delta}_i$ and $\overline{\Delta}_i$ $i = 1,..,4$ are known values. When considering the uncertain kinematic model (24) and differentiating α_{11} and α_{21} until the input terms u_1 and u_2 appear, we can write the following relationship:

$$\begin{bmatrix} \ddot{\alpha}_{11} \\ \ddot{\alpha}_{21} \end{bmatrix} = B_r \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + C_r + D_r \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (30)$$

where C_r and D_r are two matrices defined as follows:

$$C_r = \begin{pmatrix} (v_s w_s + \dot{v}_t)\cos\theta - (v_l w_s + v_t w_s - \dot{v}_s)\sin\theta + \dot{\Delta}_x \\ (v_s w_s + \dot{v}_t)\sin\theta + (v_l w_s + v_t w_s - \dot{v}_s)\cos\theta + \dot{\Delta}_y \end{pmatrix} \quad (31)$$

$$D_r = \begin{pmatrix} 0 & -v_t\sin\theta + v_s\cos\theta \\ 0 & v_t\cos\theta + v_s\sin\theta \end{pmatrix} \quad (32)$$

When applying the control input defined by equation (23) in system (30), we obtain two linear integrator plants with uncertainties as follows:

$$MBF_1 \begin{cases} \dot{\alpha}_{11} = \alpha_{12} \\ \dot{\alpha}_{12} = v_1 + \tau_1 \\ Y_1 = \alpha_{11} \end{cases} \quad MBF_2 \begin{cases} \dot{\alpha}_{21} = \alpha_{22} \\ \dot{\alpha}_{22} = v_2 + \tau_2 \\ Y_2 = \alpha_{21} \end{cases} \quad (33)$$

with

$$\tau = [\tau_1, \tau_2]^T = D_r B_r^{-1} v_{ro} + C_r \quad (34)$$

and where $v_{ro} = [v_1, v_2]^T$, and τ_1 and τ_2 are two bounded additive functions that collect slips and external environmental disturbances acting on the system. Let ρ_1 and ρ_2 indicate the differential of τ_1 and τ_2 about time t. Both τ_i and ρ_i $i = 1, 2$ are assumed to be bounded. In a real application, calculating the expression of the lumped disturbances τ_1 and τ_2 represents a big challenge. As a result, an observer to estimate them is necessary.

4.2 ESO design

An ESO is an observer that can estimate the uncertainties together with the states of the system permitting disturbance rejection or compensation. The ESO considers all the elements affecting the system and integrating the parameter uncertainties, the nonlinear dynamics and the external disturbances as a lumped disturbance to be observed. Since the observer estimates the uncertainties as an extended state, it is called the ESO. Its advantage consists in the independence of the system's mathematical model, the good performance, and the simplicity of implantation. Consider $\alpha_{13} = \tau_1$, $\alpha_{23} = \tau_2$ as an extended state for systems (33). These latter takes the following form:

$$\begin{cases} \dot{\alpha}_{11} = \alpha_{12} \\ \dot{\alpha}_{12} = \alpha_{13} + v_1 \\ \dot{\alpha}_{13} = \rho_1 \\ Y_1 = \alpha_{11} \end{cases} \quad \begin{cases} \dot{\alpha}_{21} = \alpha_{22} \\ \dot{\alpha}_{22} = \alpha_{23} + v_2 \\ \dot{\alpha}_{23} = \rho_2 \\ Y_1 = \alpha_{21} \end{cases} \quad (35)$$

Systems (35) can be written in a matrix form as follows:

$$\begin{cases} \dot{\alpha}_1 = A_x\alpha_1 + B_x v_1 + E_x\rho_1 \\ Y_1 = C_x\alpha_1 \end{cases} \begin{cases} \dot{\alpha}_2 = A_y\alpha_2 + B_y v_2 + E_y\rho_2 \\ Y_2 = C_y\alpha_2 \end{cases} \quad (36)$$

with $\alpha_1 = [\alpha_{11}, \alpha_{12}, \alpha_{13}]^T$, $\alpha_2 = [\alpha_{21}, \alpha_{22}, \alpha_{23}]^T$,

$$A_x = A_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_x = B_y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$C_x = C_y = [1\ 0\ 0]$, $E_x = E_y = [0\ 0\ 1]^T$

The ESO of each extended system (36-37) can be written as follows:

$$\dot{\hat{\alpha}}_1 = A_x\hat{\alpha}_1 + B_x v_x + L_{gx}C_x(\alpha_1 - \hat{\alpha}_1) \quad (37)$$

$$\dot{\hat{\alpha}}_2 = A_y\hat{\alpha}_2 + B_y v_y + L_{gy}C_y(\alpha_2 - \hat{\alpha}_2) \quad (38)$$

with $L_{gx} = [l_{11}, l_{12}, l_{13}]^T$, $L_{gy} = [l_{21}, l_{22}, l_{23}]^T$.

The observer gains l_{ij} $(i = 1, 2, 3)$, $(j = 1, 2, 3)$ can be chosen according to (Gao, 2003) as follows:

$$s^3 + l_{11}s^2 + l_{12}s + l_{13} = (s + \Gamma_{xo})^3 \quad (39)$$

$$s^3 + l_{21}s^2 + l_{22}s + l_{23} = (s + \Gamma_{yo})^3 \quad (40)$$

where Γ_{xo} and Γ_{yo} are the bandwidth of ESOs (38-39), which can be selected so that equations (40-41) are Hurwitz polynomial regarding the complex variable. Indeed, according to equations (40-41), the observer gain can be written as a function of the bandwidth of the ESO as follows:

$$\begin{aligned} l_{11} &= 3\Gamma_{xo}, l_{12} = 3\Gamma_{xo}^2, l_{13} = \Gamma_{xo}^3 \\ l_{21} &= 3\Gamma_{yo}, l_{22} = 3\Gamma_{yo}^2, l_{23} = \Gamma_{yo}^3. \end{aligned} \quad (41)$$

According to equations (36-37) and (38-39), the observer error of each ESO is defined as follows:

$$\dot{\hat{e}}_x = \dot{\alpha}_1 - \dot{\hat{\alpha}}_1 = (A_x - L_{gx}C_x)\hat{e}_x + E_x\rho_1 \quad (42)$$

$$\dot{\hat{e}}_y = \dot{\alpha}_2 - \dot{\hat{\alpha}}_2 = (A_y - L_{gy}C_y)\hat{e}_y + E_y\rho_2 \quad (43)$$

Equations (43-44) can be represented in a matrix form as follows:

$$\dot{\hat{e}} = \hat{H}\hat{e} + E_d \quad (44)$$

with $\hat{e} = [\hat{e}_x, \dot{\hat{e}}_x, \ddot{\hat{e}}_x, \hat{e}_y, \dot{\hat{e}}_y, \ddot{\hat{e}}_y]$, $E_d = [0\ 0\ \rho_1\ 0\ 0\ \rho_2]$,

$$\hat{H} = \begin{bmatrix} \hat{H}_1 & 0_{33} \\ 0_{33} & \hat{H}_2 \end{bmatrix}, \hat{H}_1 = \begin{bmatrix} -l_{11} & 1 & 0 \\ -l_{12} & 0 & 1 \\ -l_{13} & 0 & 0 \end{bmatrix}, \hat{H}_2 = \begin{bmatrix} -l_{21} & 1 & 0 \\ -l_{22} & 0 & 1 \\ -l_{23} & 0 & 0 \end{bmatrix}$$

Since bandwidths Γ_{xo} and Γ_{yo} are nonnegative, the roots of the matrix of $\hat{H}$ in equation (45) are all in the left half plane. Hence, all the estimated error dynamics defined by equations (45) are stable provided Lemma 1 (Zhang, 2015).

4.3 Robust control

Except for α_{11} and α_{21}, the states used in the feedback controller given by equations (18-19) cannot be accurately measured. Thus, they are replaced by their estimations given by the two ESOs defined in equations (38-39). In addition, the lumped disturbances τ_1 and τ_2 are replaced by their estimations $\hat{\alpha}_{13}$ and $\hat{\alpha}_{23}$ in order to facilitate their compensation. Therefore, a robust feedback controller can be designed based on the results of ESOs as follows:

$$v_{r1} = \ddot{\alpha}_{xd} + \lambda_{x2}(\dot{\alpha}_{xd} - \hat{\alpha}_{12}) + \lambda_{x1}(\alpha_{xd} - \hat{\alpha}_{11}) - \hat{\alpha}_{13} \quad (45)$$

$$v_{r2} = \ddot{\alpha}_{yd} + \lambda_{y2}(\dot{\alpha}_{yd} - \hat{\alpha}_{22}) + \lambda_{y1}(\alpha_{yd} - \hat{\alpha}_{21}) - \hat{\alpha}_{23} \quad (46)$$

Based on the robust feedback controller given in equations (46-47), the Flatness-Based-Active-Disturbance-Rejection Control (FBADRC) can be proposed as follows:

$$\begin{bmatrix} u_{r1} \\ u_{r2} \end{bmatrix} = B_r^{-1} \begin{bmatrix} \ddot{\alpha}_{xd} + \lambda_{x2}\dot{\hat{e}}_{rx1} + \lambda_{x1}\hat{e}_{rx1} - \hat{\alpha}_{13} \\ \ddot{\alpha}_{yd} + \lambda_{y2}\dot{\hat{e}}_{ry1} + \lambda_{y1}\hat{e}_{ry1} - \hat{\alpha}_{23} \end{bmatrix} \quad (47)$$

with $\hat{e}_{r1} = \alpha_{xd} - \hat{\alpha}_{11}$ and $\hat{e}_{r2} = \alpha_{yd} - \hat{\alpha}_{21}$.

4.4 Stability of closed loop

The stability analysis of the closed-loop control system with the ESO will be discussed in this part. The state tracking error vector of position x is defined as follows:

$$e_{rx} = \alpha_{d1} - X_1 \quad (48)$$

Note that $\alpha_{d1} = [\alpha_{xd}, \dot{\alpha}_{xd}]^T$ and $X = [\alpha_{11}, \alpha_{12}]^T$. It is evident that the following equation holds:

$$\dot{\alpha}_{d1} = A_1\alpha_{d1} + B_1\ddot{\alpha}_{xd} \quad (49)$$

where $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

When using the new robsut feedback, system (MBF1) can be written in a matrix form as follows:

$$\dot{X}_1 = A_1X_1 + B_1(v_{r1} + \tau_1) \quad (50)$$

Using the following relation :

$$\hat{e}_{r1} = \alpha_{xd} - \hat{\alpha}_{11} = \alpha_{xd} - \alpha_{11} + \alpha_{11} - \hat{\alpha}_{11} \quad (51)$$

feedback controller given in equation (46) can be written as follows:

$$v_{r1} = \lambda_x(e_{rx} + \hat{e}_x) + \ddot{\alpha}_{dx} - \hat{\tau}_1 \quad (52)$$

with $\lambda_x = [\lambda_{x1}, \lambda_{x2}]$. Using equations (50), (51) and (53), the state tracking error dynamics can be expressed as:

$$\dot{e}_{rx} = (A_1 - B_1\lambda_x)(e_{rx}) - B_1\lambda_x\hat{e}_x - B_1\hat{e}_x \quad (53)$$

Combining equations (43) and (54), we obtain:

$$\begin{bmatrix} \dot{e}_{rx} \\ \dot{\hat{e}}_x \end{bmatrix} = \begin{bmatrix} A_1 - B_1\lambda_x & -[B_1\lambda_x\ B_1] \\ 0 & A_x - L_{gx}C_x \end{bmatrix} \begin{bmatrix} e_{rx} \\ \hat{e}_x \end{bmatrix} + \begin{bmatrix} 0 \\ E_x \end{bmatrix}\rho_1 \quad (54)$$

The stability of system x can be verified by checking the eigenvalues of the error dynamics (55), which are given by the eigenvalues of $A_1 - B_1\lambda_x$ and $A_x - L_{gx}C_x$. Since couple (A_1, B_1) is controllable and couple $(A_x,\ C_x)$ is observable, the stability of the error dynamics of position x is ensured by appropriately selecting the controller and observer poles. Based on the observer error dynamics of ESOs (39) and the robust feedback controllers (47), the error dynamics of the closed-loop of y can be defined by the following system matrix:

$$\begin{bmatrix} \dot{e}_{ry} \\ \dot{\hat{e}}_y \end{bmatrix} = \begin{bmatrix} A_2 - B_2\lambda_y & -[B_2\lambda_y\ B_2] \\ 0 & A_y - L_{gy}C_y \end{bmatrix} \begin{bmatrix} e_{ry} \\ \hat{e}_y \end{bmatrix} + \begin{bmatrix} 0 \\ E_y \end{bmatrix}\rho_2 \quad (55)$$

where $A_1 = A_2$,$B_1 = B_2$, $\lambda_y = [\lambda_{y1}, \lambda_{y2}]$

Similar to position x, the same results about the stability of the closed-loop system y can be concluded.

5. SIMULATION AND RESULTS

Simulation is carried out to check the performance of the ESO in the estimation of uncertainties and to prove the tracking performance of FBADRC. The parameters of the considered WMR are given by: $r = 0.1\ m$, $b = 0.15\ m$. It is desired to generate the trajectory which enables the robot to move from an initial state $q(0) = [0, 0, 45]^T$ to a final one $q(10) = [3.5, 5, -90]^T$ in a room full of obstacles. The obtained trajectory should be minimal in energy, able to avoid static obstacles and respect some state constraints defined as follows:

$$0\ m \leq \sigma_{xd} \leq\ 4\ m, 0\ m \leq \sigma_{yd} \leq\ 6\ m \quad (56)$$

The desired reference trajectory can be obtained by solving a constrained optimization problem. To resolve this latter, we use the optimal trajectory generation method (Abadi,

2017) based on flatness, the collocation method and the B-spline function. The controller design parameters are chosen as $\xi_x = \xi_y = 1$ and $\gamma_{xc} = \gamma_{yc} = 2\ rad/s$. According to (Gao, 2003), the observer bandwidth must be chosen at least three times higher than the controller bandwidth to guarantee that the observer dynamics are still faster than those of the system. Then the observer bandwidths are chosen as $\Gamma_{xo} = \Gamma_{yo} = 6\ rad/s$. We consider in the simulation two other controllers for comparison as follows:
- FBTC defined by equation (23).
-Flatness Sliding Mode Control FSMC defined by equation (60).

The sliding mode control enhances the tracking performance by sliding surfaces. As it is still applicable for the WMR (Mu, 2017) with good robustness, then SMC is utilized for comparison. For more details about the principle of FSMC, the reader can refer to (Abadi, 2018). The design of the sliding mode control needs two steps: The choice of the sliding surface and the design of the control law. In tracking example, the sliding variable $\sigma = [\sigma_x, \sigma_y]^T$ is taken as the tracking error. Therefore, the sliding surface for the WMR can be chosen as follows:

$$\sigma_x = \dot{e}_{rx1} + \beta_1 e_{rx1} \tag{57}$$

$$\sigma_y = \dot{e}_{rx2} + \beta_2 e_{rx2} \tag{58}$$

where gains β_1 and β_2, can be determined by using pole-placement techniques.

The FSMC applied for WMR is defined as follows:

$$\begin{bmatrix} u_{s1} \\ u_{s2} \end{bmatrix} = B_r^{-1} \begin{bmatrix} \ddot{\alpha}_{xd} + \beta_1 \dot{e}_{rx1} + k_1 sign(\sigma_x) \\ \ddot{\alpha}_{yd} + \beta_2 \dot{e}_{rx2} + k_2 sign(\sigma_y) \end{bmatrix} \tag{59}$$

where the controller gain of FSMC used in simulation as chosen as: $\beta_1 = \beta_2 = 5$, $k_1 = k_2 = 10$. Indeed, FSMC is discontinuous control due to the existence of function $sign(\sigma)$, which provokes the chattering of the control inputs. Thereby, to reduce this problem, function $sign(\sigma)$ can be replaced by $\frac{\sigma}{\parallel \sigma \parallel + \gamma_s}$, where γ_s is a tuning parameter utilized to reduce the chattering effect.

In order to show the robustness of the proposed control, FBTC, FSMC and FBADRC are applied to the uncertain WMR system under the same conditions. As a consequence, first, we consider that the WMR starts from an uncertain initial condition $\hat{x}(0) = 1$ and $\hat{y}(0) = 1$. Second, we select $\mu_i = 0.5\ i = 1, 2, 3, 4$; i.e, the slip velocities v_t and v_s can be up to 50% of the forward speed. Finally, we assume that the WMR is subjected to an external disturbance wind defined according to (Fethalla, 2018) as follows:

$$\Delta_x = \Delta_y = 1.5 + 2.5 sin(4t) \tag{60}$$

Fig. 2 depicts the tracking results under the application of FBTC, FSMC and FBADRC to the uncertain WMR systems. Fig. 3 illustrates the estimates of the lumped disturbances by the ESO. From Fig. 2, it can be observed that when the slips and external environmental disturbances are integrated into the WMR system, FBTC is a poor controller for the WMR because the latter diverges strongly from the reference trajectory, whereas FSMC and FBADRC are robust enough against the lumped disturbances affecting the WMR model. As a result, the controllers that are not based on an uncertain model, even if they are feedback controllers, may not work correctly.

The main difference between both FSMC and FBADRC controllers is that FBADRC integrates the performance of the ESO, which is based on the estimation result of the lumped disturbances depicted in Fig. 3. These latter are compensated with relatively low gains, while FSMC utilizes just tuned gains that should be relatively important to carry out the disturbance rejection. Generally, the suggested ESO-based design offers some distinct advantages, because it does not need to know all the states and lumped disturbances since the ESO estimates the states as well as uncertainties in an integrated manner. This makes the tracking controller for the WMR based on flatness and the ESO computationally efficient compared to many other formulations presented in the literature

6. CONCLUSION

We have studied in this paper the problem of tracking trajectories for a WMR system subject to slips and disturbances. The differential flatness theory has been used to transform the nonlinear WMR system into an equivalent linearized form that depends on the flat output and its derivatives. To improve the robustness of the control scheme against slip and disturbances, an ESO has been designed to estimate the state vector of the linearized system as well as the lumped disturbances in an integrated manner. Therefore, by identifying the perturbation variables, their compensation is possible with the inclusion of an additional term in the control input. The simulation results have demonstrated the robustness of the proposed FBARDC method compared to other tracking controllers.

REFERENCES

Abadi, A., Mekki, H., Brahim, A. B. H., El Amraoui, A., & Ramdani, N. (2017, December). Optimal trajectory generation and flatness tracking control for a mobile robot. In 2017 18th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA) (pp. 223-228). IEEE

Abadi, A., Brahim, A. B. H., Mekki, H., El Amraoui, A., & Ramdani, N. (2018, March). Sliding Mode Control of Quadrotor based on Differential Flatness. In 2018 International Conference on Control, (ICCAD) (pp. 1-6). IEEE.

Abadi, A., El Amraoui, A., Mekki, H., & Ramdani, N. (2019). Optimal trajectory generation and robust flatness–based tracking control of quadrotors. Optimal Control Applications and Methods,40(4), 728-749.

Fethalla, N., Saad, M., Michalska, H., & Ghommam, J. (2018). Robust observer-based dynamic sliding mode controller for a quadrotor UAV. IEEE Access, 6, 45846-45859.

Fliess M, Lévine J., Martin P, Rouchon P. Flatness and defect of non-linear systems: introductory theory and examples. Int J Control, 1995;61(6):1327-61.

Gao Z.:'Scaling and bandwidth-parameterization based controller tuning', American Control Conference, 2003, pp. 4989-4996.

García-Sánchez, José Rafael, et al. "A Robust Differential Flatness-Based Tracking Control for the "MIMO DC/DC Boost Converter–Inverter–DC Motor" System: Experimental Results." IEEE Access 7 (2019): 84497-84505.

HAN, Jingqing. From PID to active disturbance rejection control. IEEE transactions on Industrial Electronics, 2009, vol. 56, no 3, p. 900-906.

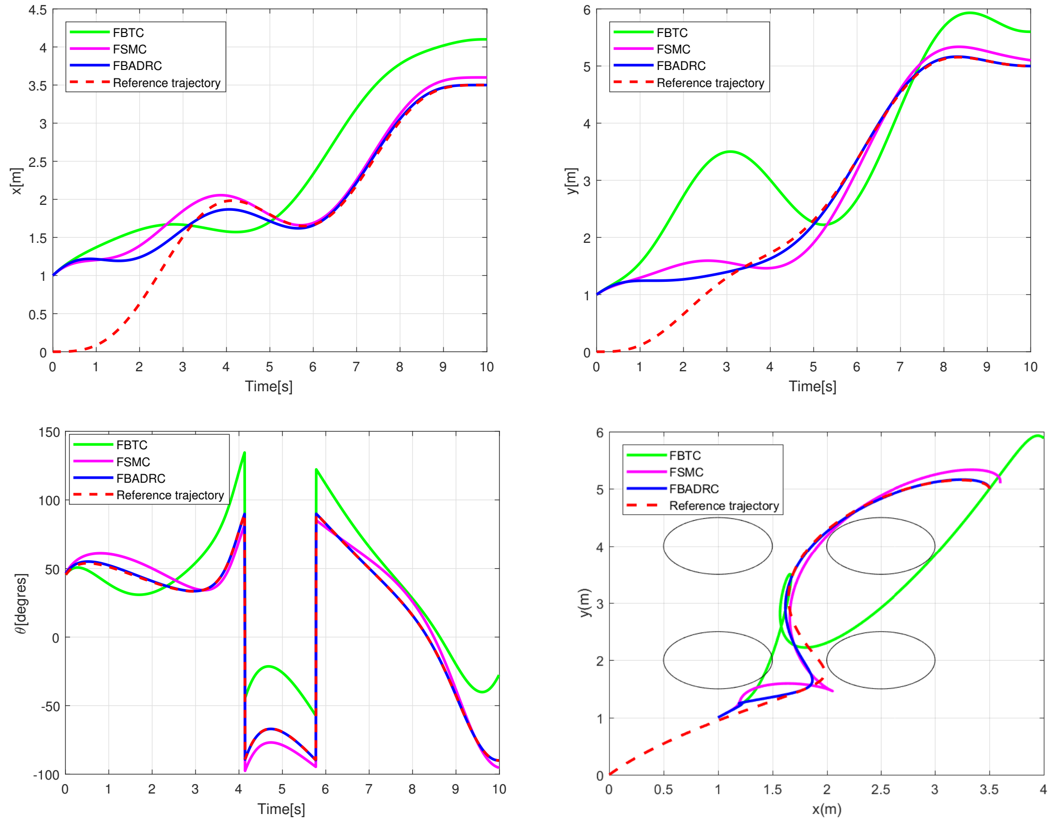

Fig. 1. Tracking simulation results of WMR subject to slips and external environmental disturbances

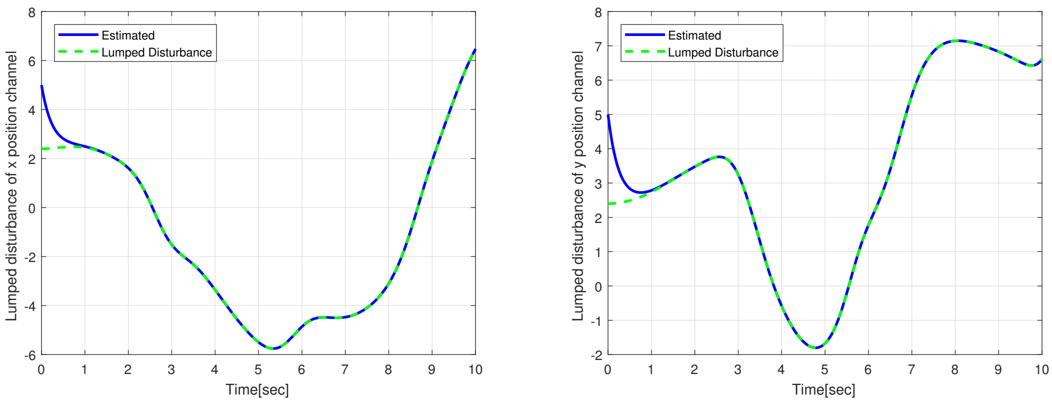

Fig. 2. Estimation of lumped disturbances

LIU, Chao, LIU, Gang, et FANG, Jiancheng. Feedback Linearization and Extended State Observer-Based Control for Rotor-AMBs System With Mismatched Uncertainties. IEEE Trans. Industrial Electronics, 2017, vol. 64, no 2, p. 1313-1322.

Markus, E. D., Agee, J. T., & Jimoh, A. A. (2017). Flat control of industrial robotic manipulators. Robotics and Autonomous systems, 87, 226-236.

Mu, J., Yan, X. G., Spurgeon, S. K., & Mao, Z. (2017). Generalized regular form based SMC for nonlinear systems with application to a WMR. IEEE Transactions on Industrial Electronics, 64(8), 6714-6723.

Pu, Z., Yuan, R., Yi, J., & Tan, X. (2015). A class of adaptive extended state observers for nonlinear disturbed systems. IEEE Transactions on Industrial Electronics, 62(9), 5858-5869

Ren, C., Li, X., Yang, X., & Ma, S. (2019). Extended State Observer based Sliding Mode Control of an Omnidirectional Mobile Robot with Friction Compensation. IEEE Transactions on Industrial Electronics.

Ryu J. C., & Agrawal S. K., "Differential flatness-based robust control of mobile robots in the presence of slip". The International Journal of Robotics Research, 2011, 30(4), 463-475.

Zhang, Y., Jiang, Z., Yang, H., Cheng, J., & Zhang, W. (2015). High-order extended state observer-enhanced control for a hypersonic flight vehicle with parameter uncertainty and external disturbance. Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, 229(13), 2481-2496.