Mixed Stochastic Process Modelling for Accelerated Degradation Testing

Yang Li*, Yue Liu*** Enrico Zio**,**** Ningyun Lu* Xiuli Wang* Bin Jiang***

**Automation Engineering Department, Nanjing University of Aeronautics and Astronautics Nanjing 211106, China (e-mail: yongerli@nuaa.edu.cn, xiuliwang@nuaa.edu.cn, luningyun@nuaa.edu.cn, binjiang@nuaa.edu.cn).*
***Energy Department, Politechnico di Milano, Milan 20156, Italy (e-mail: enrico.zio@polimi.it).*
****China North Vehicle Research Institute, Beijing, 100072 (yushang917@sina.com);*
*****MINES ParisTech, PSL Research University, CRC, Sophia Antipolis, France;*
Aramis Srl, Via pergolesi 5, Milano, Italy;
Department of Nuclear Engineering, College of Engineering, Kyung Hee University, Republic of Korea

Abstract: Accelerated degradation testing (ADT) is used to efficiently assess the reliability and lifetime of a high reliable products under normal stress. In general, it is common in practice to build stochastic models of degradation under a single failure mechanism based on the ADT data. However, in real applications, multi-failure mechanisms may influence the degradation process. Motivated by this, a mixed stochastic process model for ADT is proposed in this paper. The mixed stochastic process combines two single-stochastic processes with weights determined by a quantitative method that establishes the relationship with accelerated stress. After the unknown parameter estimation, the proposed model under normal stress level can be obtained. The results show that the proposed model can be used for ADT modeling under multi-failure mechanisms.

Keywords: Accelerated degradation testing (ADT), Multi-failure mechanisms, Degradation, Mixed stochastic process.

1. INTRODUCTION

Unexpected failures in engineering systems can cause serious accidents (E. Zio, 2009, Y. Li et al., 2018, D. W. Coit et al., 2018, X. L. Wang et al. 2018). For example, the Japanese “Fukushima nuclear power plant accident” in 2011 caused serious casualties, environmental pollution and economic losses. The Chinese “7.23 Yong-Wen line major railway accident” in 2011 was caused by a design flaw in the signal control subsystem and resulted in 40 deaths and more than 200 injuries. As a result, the safety and reliability of complex systems increasingly appeals to the concern of the worldwide researchers in recent years (S. Woo et al., 2020, Y. Li et al., 2019).

With the development of reliability technology, a large number of long-life, high-reliability products have been equipped in various systems. The degradation process for such products is slow, so it will cost a long time and money to obtain sufficient run-to-failure degradation data; traditional reliability testing techniques under normal stress conditions are not adequate. Therefore, accelerated degradation testing (ADT) was proposed and has attracted much attentions (Z. S. Ye, 2015, C. Park et al., 2005).

ADT can obtain sufficient degradation data in a short time period by accelerating the degradation process. According to these data, the reliability and lifetime can predicted under normal conditions. When one considers to construct the degradation model, it is necessary to analyze the essence of the failure mechanism and the ADT data. Failure mechanism models obtained from physics and chemical reaction laws are widely used to design ADT. For example, based on analysis of the failure mechanism of indium tin oxide film, Yun et al. (2006) used the diffusion mechanism of oxide to model the degradation of product performance and to predict the product lifetime by accelerating degradation experiments. J. G. Surles et al. (2001) studied the stress-intensity model based on the scale-type Weibull distribution. C. Park et al. (2005) established a generalized cumulative damage model using a stochastic process and used this model to analyze the failure of a carbon fiber product.

Although ADT based on failure mechanism model has good credibility, it is difficult to achieve the models. A common methodology is to assume the failure process subjecting to a known and deterministic stochastic model and then to identify the model parameters. General references for this model are M. A. Freitas et al. (2009), X. L. Wang et al. (2018) and (2019), N. Gebraeel (2006), X. X. Yuan et al. (2009), M. Marseguerra et al. (2003). However, when using a deterministic stochastic model, it is difficult to get the closed expression of the first hitting time (FHT). In addition, one must assume the failure process subjecting to a certain time-invariant probability distribution model. it may be applicable to products with single structure. It is not adequate for complex systems such as the complex systems in modern industry, energy, aerospace and transportation fields. For these complex systems, the degradation behaviors often change with time, operation conditions and environments (Q. Sun et al., 2012; W. Huang et al., 2005; R. Jiang et al., 2008). Time-varying stochastic models or mixed stochastic models can be two possible solutions to have better performance in describing the degradation process.

In the existing literatures, Wiener process, Gamma process, and Inverse Gaussian process are widely used for degradation modeling in recent years (AD Kalafatis et al., 1997). To solve the small sample problem, Wiener process was used to construct a degradation model based on the fuzzy theory (X. Y. Li et al., 2018). Bayesian model averaging was adopted to solve model uncertainties of the Wiener process, Gamma process, and Inverse Gaussian process (L. Liu et al., 2017). The effects of model mis-specification for predicting the product's mean-time-to-failure (MTTF) were considered in (C. Y. Peng, 2009). Multiple degradation paths were also considered in (P. Wang, 2004; W. W. Peng, 2016), and the variance-covariance matrix and copula functions were used to address the dependent or independent problems, respectively. However, all these methods need to assume the degradation data obeying a certain single stochastic process.

In reality, performance degradation of a system may be caused by multiple failure mechanisms. Single stochastic process model cannot describe the degradation behaviors exactly for complex systems. Aiming at this issue, a novel ADT model based on finite mixed stochastic processes strategy under constant-step ADT (CSADT) is studied in this paper. The degradation data is no longer assumed to be single form of stochastic process, but a mixture of finite stochastics processes. On the basis of this strategy, the main contributions of this work are summarized as follows: (1) multiple failure mechanisms are considered in a complex system; (2) it can solve the model mis-specification problem. (3) a quantitative method is developed to determine the weights of different stochastic process models to achieve a more accurate ADT strategy.

The paper is organized as follows. In Section 2, mixed stochastic modeling method is proposed for ADT. Estimation of the unknown parameters is given in Section 3 by MLE, using a quantitative weighting method. Section 4 shows the performance of the proposed method. And Section 5 draws some conclusions.

2. FINITE MIXED STOCHASTIC PROCESS MODEL FOR ADT

2.1 Mixed stochastic process model

The conventional ADT model assumes the degradation path $X(t)$ following a single stochastic process, such as Wiener process, Gamma process, etc. Here, one define $X(t)$ following a mixed stochastic process, and has the formulation as follows:

$$\begin{aligned} X(t) &= \lambda_1 X_1(t) + \lambda_2 X_2(t) + \cdots + \lambda_Q X_Q(t) \\ &= \sum_{q=1}^{Q} \lambda_q X_q(t). \end{aligned} \tag{1}$$

where $\lambda_q \in \{\lambda_1, \lambda_2, \ldots, \lambda_Q\}$ contains nonnegative, weights of $X_q(t)$ (q=1, 2,…, Q), subjecting to $\sum_{q=1}^{Q} \lambda_q = 1$. If one of weights λ_q equals to 1, the mixed stochastic process becomes a single stochastic process.

Considering that Wiener process and Gamma process are widely used in practice. For simplicity, the mixed stochastic process in this paper considers these two processes, which can be expressed as:

$$X(t) = \lambda_1 X_1(t) + (1-\lambda_1) X_2(t) \tag{2}$$

where $X_1(t)$ and $X_2(t)$ represent Wiener process and Gamma process respectively.

The Wiener process $\{X_1(t), t\geq 0\}$ with mean value $\mu_1\Lambda(t)$ and standard deviation $\sigma_1\sqrt{\Lambda(t)}$ is denoted as $X_1(t) \sim N(\mu\Lambda(t), \sigma^2\Lambda(t))$, where $\Lambda(t)$ is a nonnegative increasing function and also an approximate description of time t. The probability density function (PDF) of an Gaussian distribution for $X_1(t) \sim N(\mu\Lambda(t), \sigma^2\Lambda(t))$ is

$$f_{Gau}(x_1) = \frac{1}{\sqrt{2\pi\sigma^2\Lambda(t)}} \exp\left[-\frac{(x_1-\mu\Lambda(t))^2}{2\sigma^2\Lambda(t)}\right] \tag{3}$$

Similarly, the Gamma process $\{X_2(t), t\geq 0\}$ is denoted as $X_2(t) \sim Ga(\alpha\Lambda(t), \beta)$ where α is the shape parameter, β is the scale parameter. The PDF of $X_2(t)$ is

$$f_{Gam}(x_2) = \frac{\beta^{-\alpha\Lambda(t)}}{\Gamma(\alpha\Lambda(t))} x_2^{\alpha\Lambda(t)-1} \exp(-\frac{x_2}{\beta}) \tag{4}$$

where $\Gamma(\cdot)$ represents the Gamma function.

Through the above definitions, the PDF of the mixed stochastic process $X(t)$ can be formulated as:

$$f_{Mix}(x) = \frac{\lambda_1}{\sqrt{2\pi\sigma^2\Lambda(t)}} \exp\left[-\frac{(x-\mu\Lambda(t))^2}{2\sigma^2\Lambda(t)}\right] + \frac{(1-\lambda_1)\beta^{-\alpha\Lambda(t)}}{\Gamma(\alpha\Lambda(t))} x^{\alpha\Lambda(t)-1} \exp(-\frac{x}{\beta}) \tag{5}$$

Remark 1. The Wiener process and Gamma process both have the property of independent increments, then the mixed process has the same property.

2.2 Accelerated model for ADT

In ADT modeling, the accelerated model is used to describe the relationship between the degradation rate R and the accelerated stress S. Usually, the three main used accelerated models are Arrhenius model, power law model, and exponential model. The formulation between the degradation rate R and S are shown as follows (X. Y. Li et al., 2018):

$$R_1 = \delta_0 \cdot \exp(-\frac{\delta_1}{S}) \tag{6}$$

$$R_2 = \delta_0 \cdot S^{\delta_1} \tag{7}$$

$$R_3 = \delta_0 \cdot \exp(\delta_1 S) \tag{8}$$

where δ_0 and δ_1 are both constant parameters; R_1, R_2, R_3 denote the outputs of Arrhenius model, power law model, and exponential model respectively. Through log-transformation, an unified expression can be formulated as :

$$\ln R = \eta_0 + \eta_1 L(S) \tag{9}$$

when $\eta_0 = \ln(\delta_0)$, $\eta_1 = -\delta_1$, $L(S) = 1/S$, Eq. (9) becomes the Arrhenius model;

when $\eta_0 = \ln(\delta_0)$, $\eta_1 = \delta_1$, $L(S) = \ln(S)$, Eq. (9) becomes the power law model;

when $\eta_0 = \ln(\delta_0)$, $\eta_1 = \delta_1$, $L(S) = S$, Eq. (9) becomes the exponential model.

Accordingly, the linear hypothesis can be applicable for the three widely used models. μ, σ^2 and α vary with the change of stress. Then, the following linear hypothesis for μ, σ^2 and α are formulated as

$$\ln \mu = a_1 + b_1 L(S) \tag{10}$$

$$\ln \sigma = a_2 - 0.5 b_1 L(S) \tag{11}$$

$$\ln \alpha = u_1 + v_1 L(S) \tag{12}$$

β in equation (5) does not vary with the change of stress. Then, a_1, b_1, a_2, u_1, v_1, β are the parameters to be estimated.

Remark 2. The equations (10), (11), (12) are derived based on the equation (9) and the "acceleration factor constant" principle (H. W. Wang et al., 2016).

3. LIFETIME DISTRIBUTION FOR ADT MODEL

3.1 Lifetime distribution of proposed model

Let γ represent the critical failure threshold for degradation path of the proposed mixed model. Then the lifetime T can be defined as the time when the degradation process $X(t)$ first crosses the failure threshold γ, i.e., the first hitting time (FHT), that is,

$$T = \inf\{t : t \ge 0 \mid X(t) = \gamma\} \tag{13}$$

Therefore, as the $\Lambda(t)$ is monotone increasing function, the PDF and cumulative distribution function (CDF) of FHT for the proposed model are formulated as follows:

$$\begin{aligned} f_T(t) = {} & \frac{\lambda_1 \gamma}{\sqrt{2\pi\sigma^2\Lambda^3(t)}} \exp[-\frac{(\gamma - \mu\Lambda(t))^2}{2\sigma^2\Lambda(t)}] \\ & + \frac{(1-\lambda_1)[(\frac{n}{\Lambda(t)})^{\frac{1}{2}} + (\frac{n}{\Lambda(t)})^{\frac{3}{2}}]}{2\sqrt{2\pi} mn} \exp(-\frac{(n-\Lambda(t))^2}{2m^2 n\Lambda(t)}) \end{aligned} \tag{14}$$

$$\begin{aligned} F_T(t) = {} & \lambda_1[\Phi(\frac{\mu\Lambda(t) - \gamma}{\sigma\sqrt{\Lambda(t)}}) + \exp(\frac{2\mu\gamma}{\sigma^2})\Phi(-\frac{\mu\Lambda(t) + \gamma}{\sigma\sqrt{\Lambda(t)}})] \\ & + (1-\lambda_1)\Phi[\frac{1}{m}(\sqrt{\frac{\Lambda(t)}{n}} - \sqrt{\frac{n}{\Lambda(t)}})] \end{aligned} \tag{15}$$

where $\Phi(\cdot)$ denotes the standard normal distribution; $m = \sqrt{\beta/\gamma}$, and $n = \gamma/\alpha\beta$. In addition, the exponential form of $\Lambda(t) = t^c$ can be used for time-scale transformation. Then, the reliability function for the proposed degradation model is given by

$$R_T(t) = 1 - F_T(t) \tag{16}$$

As is well known, MTTF is another important reliability index, and can be expressed as follows:

$$\mathrm{MTTF} = \mathrm{E}[T] = \int_0^\infty R(t)dt \tag{17}$$

Remark 3. The PDF and CDF of sub-Gamma process are difficult to compute. In order to address this problem, the Birnbaum-Saunders (BS) distribution is used to approximate the distribution of sub-Gamma process.

3.2 Unknown parameters estimation

In CSADT, there are n samples and L accelerated stress levels. We assume that $X(t_{kij})$ denotes the jth degradation value of unit i under the kth stress level, and t_{kij} is the corresponding measurement time, where $k = 1,2, \ldots, L$, $i = 1, 2, \ldots, n_k$, $j = 1, 2, \ldots, m_{ki}$. Let $\tau_{kij} = \Lambda(t_{kij}) - \Lambda(t_{ki(j-1)})$ be the degradation measurement and $x_{kij} = X(t_{kij}) - X(t_{ki(j-1)})$ is the corresponding degradation increment.

From the definitions of equation (5), the unknow parameter vector is $\theta = [\lambda_1, a_1, b_1, a_2, u_1, v_1, \beta]$, then, the log-likelihood function of x is given as

$$\begin{aligned} l(\theta \mid x) = \sum_{k=1}^{L}\sum_{i=1}^{n_k}\sum_{j=1}^{m_{ki}} \ln \Bigg\{ & \frac{\lambda_1}{\sqrt{2\pi\sigma^2\Lambda(t)}} \exp\left[-\frac{(x-\mu\Lambda(t))^2}{2\sigma^2\Lambda(t)}\right] \\ & + \frac{(1-\lambda_1)\beta^{-\alpha\Lambda(t)}}{\Gamma(\alpha\Lambda(t))} x^{\alpha\Lambda(t)-1} \exp(-\frac{x}{\beta}) \Bigg\} \end{aligned} \tag{18}$$

The unknow parameters in the proposed model cannot be directly estimated by the maximum likelihood method (MLE) due to the complexity of its log-likelihood function. In addition, accuracy of the weight can improve the performance of the proposed model. Therefore, the Gradient Descent (GD) algorithm is used to estimate the unknown parameters. Then, the weight estimation in GD is regarded as the initial value to calculate the accuracy weight under real condition. The key equation of GD is shown as follows,

$$\theta^{next} = \theta^{now} - \iota \nabla l(\theta^{now}) \tag{19}$$

where θ^{next} and θ^{now} represent the parameters value in the next and current time respectively; ι denotes the learning rate; $\nabla l(\theta^{now})$ means the derivative of $l(\theta \mid x)$ at θ^{now}.

3.3 Weight estimation

In the real condition, the weight estimation should consider the stress influence since the accelerated stress can impact the physicochemical process of a product. One can define R_{qk} is the reaction speed of the qth failure mechanisms under the kth stress level. After the exponential transformation, R_{qk} can be defined by equation (9) as

$$R_{qk} = \exp(\omega_q + \xi_q L(S_k)) \tag{20}$$

Equation (20) keeps consistency with equation (9). In our work, $q \in \{1,2\}$. Then, an assumption can be described as follows (W. Gao et al., 2006),

Assumption. The mixing ratio of each sub distribution in the proposed model to the ratio of reaction speed of its corresponding failure mechanism is consistent, and can be expressed as

$$\frac{R_{1k}}{R_{2k}} = d\frac{\lambda_{1k}}{\lambda_{2k}} \tag{21}$$

where d denotes the proportional constant, and λ_{1k}, λ_{2k} represent the mixing ratio of the first and second failure mechanisms under kth stress level.

Theorem. Let λ_{10} denote the mixing ratio of the first failure mechanism under normal stress level. The relation between λ_{1k} and λ_{10} can be formulated as,

$$\lambda_{10} = \frac{\lambda_{1k}}{\kappa_k + (1-\kappa_k)\lambda_{1k}} \tag{22}$$

where $\kappa_k = \exp\left[(L(S_0) - L(S_k))(\xi_1 - \xi_2)\right]$.

Proof. From equation (21), one can obtain,

$$\begin{aligned} &\frac{R_{1k}}{R_{2k}} \bigg/ \frac{\lambda_{1k}}{\lambda_{2k}} = \frac{R_{10}}{R_{20}} \bigg/ \frac{\lambda_{10}}{\lambda_{20}} \\ &\Rightarrow \frac{R_{1k}}{R_{2k}} \bigg/ \frac{R_{10}}{R_{20}} = \frac{\lambda_{1k}}{\lambda_{2k}} \bigg/ \frac{\lambda_{10}}{\lambda_{20}} \\ &\Rightarrow \frac{R_{1k}}{R_{10}} \bigg/ \frac{R_{2k}}{R_{20}} = \frac{\lambda_{1k}}{\lambda_{10}} \bigg/ \frac{\lambda_{2k}}{\lambda_{20}} \end{aligned} \tag{23}$$

The following formulation can be obtained from equation (20),

$$\begin{cases} \dfrac{R_{1k}}{R_{10}} = \dfrac{\exp(\omega_1 + \xi_1 L(S_k))}{\exp(\omega_1 + \xi_1 L(S_0))} = \exp\left[\xi_1 \left(L(S_k) - L(S_0)\right)\right] \\ \dfrac{R_{2k}}{R_{20}} = \dfrac{\exp(\omega_2 + \xi_2 L(S_k))}{\exp(\omega_2 + \xi_2 L(S_0))} = \exp\left[\xi_2 \left(L(S_k) - L(S_0)\right)\right] \end{cases} \tag{24}$$

Then, from equation (24), $\dfrac{R_{1k}}{R_{10}} \bigg/ \dfrac{R_{2k}}{R_{20}}$ can be expressed as,

$$\frac{R_{1k}}{R_{10}} \bigg/ \frac{R_{2k}}{R_{20}} = \exp\left[\left((L(S_k) - L(S_0))\right)(\xi_1 - \xi_2)\right] = \kappa_k$$

The following equation can be formulated from equation (23),

$$\frac{\lambda_{1k}}{\lambda_{10}} \bigg/ \frac{\lambda_{2k}}{\lambda_{20}} = \kappa_k$$

Due to $\sum_{q=1}^{Q} \lambda_q = 1$, in each stress level, one has $\lambda_{1k} + \lambda_{2k} = 1$.

Then,

$$\frac{\lambda_{1k}}{\lambda_{10}} = \kappa_k \frac{\lambda_{2k}}{\lambda_{20}} = \kappa_k \frac{(1-\lambda_{1k})}{(1-\lambda_{10})},$$

and equation (22) can be derived.

For each stress level, λ_{1k} can be estimated by the estimated parameters $\hat{\theta}$, and $\hat{\lambda}_1$ is regarded as the initial value for each stress level. As a result, we can obtain a corresponding λ_{10}, and can be marked as λ_{10}^k. Then, the terminal estimation of the weight can be obtained from the following equation,

$$\lambda_1 = \frac{\sum_{k=1}^{L} n_k \lambda_{10}^k}{\sum_{k=1}^{L} n_k} \tag{25}$$

4. THE PERFORMANCE OF THE PROPOSED METHOD

4.1 Stress relaxation data

The stress relaxation is the stress loss of a component under a constant strain over time. For example, the excessive stress relaxation can cause a failure for the contacts of electrical connectors. The stress relaxation data are originated from (G. Yang, 2007) and used in (Z. S. Ye, et al., 2014) and (L. Liu, et al., 2017). The degradation data are shown in Figure 1. The CSADT strategy are used to generate these data. Three constant accelerated temperature stress values include 65ºC, 85ºC, 100ºC have been considered in ADT, and normal temperature stress is 40ºC. When the stress relaxation exceeds 30%, i.e., $\gamma = 30$, the electrical connector can be said to have failed.

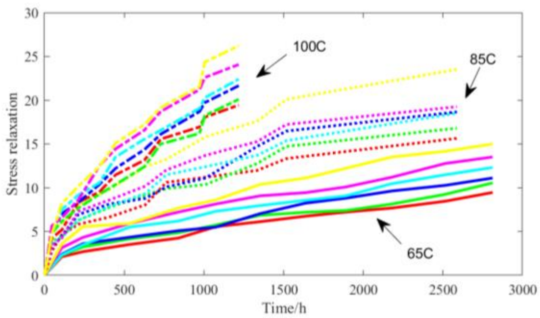

Fig. 1. Stress relaxation data under accelerated stresses

4.2 Results analysis

The unknown parameters estimated in Section 3.2 and 3.3 are shown in Table 1.

Considering the mixed stochastic process, the PDF and CDF of the FHT for the proposed model are shown in Figure 2, and

Figure 3 respectively. Through the estimation of the unknown parameters, the parameters μ, σ, α in normal stress level can be formulated in equation (26).

Table 1. Unknown parameters estimation

Model parameters	Models (parameters estimation) MSPM	WPM	GPM
λ_1	0.462	*	*
a_1	-2.7743	-2.1031	*
b_1	0.3630	1.2553	*
a_2	-2.4258	-0.8981	*
u_1	-2.6465	*	-2.6954
v_1	1.5807	*	1.6968
β	1.6310	*	0.4328
c	0.4915	0.4561	0.5682

$$\begin{cases} \mu_0 = \exp(a_1 + b_1 L(S_0)) \\ \sigma_0 = \exp(a_2 - 0.5 b_1 L(S_0)) \\ \alpha_0 = \exp(u_1 + v_1 L(S_0)) \end{cases} \tag{26}$$

Substituting equation (26) into equations (14) and (15), the PDF and CDF of FHT for the proposed model under normal stress level can be obtained directly.

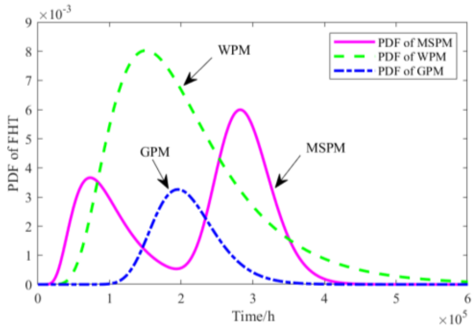

Fig. 2. PDF of FHT under MSPM, WPM, and GPM

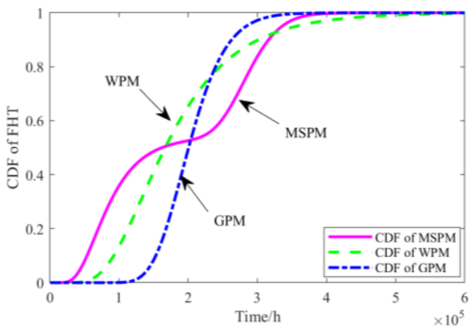

Fig. 3. CDF of FHT under MSPM, WPM, and GPM

The MSPM, WPM and GPM represent the mixed stochastic process model, Wiener process model and Gamma process model respectively. The reliability curves of MSPM, WPM and GPM are shown in Figure 4.

The WPM and GPM both obey a single distribution, in real condition, considering the influence of multi-failure mechanisms, the degradation data assumed can be fitted by both WPM and GPM. Then, the mixed stochastic process obeying a bimodal distribution is reasonable.

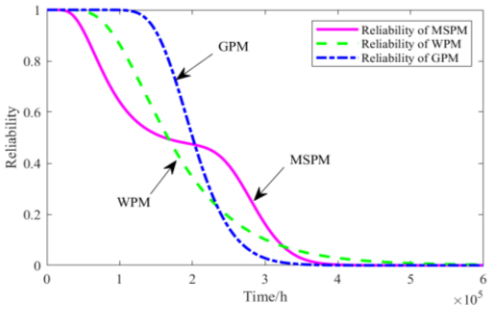

Fig. 4. Reliability curves of MSPM, WPM and GPM

According to equation (18), the MTTF of the proposed model can be obtained, and compared to the WPM and GPM, the performances are listed in Table 2.

Table 2. MTTF of MSP, WPM and GPM

	MSPM	WPM	GPM
MTTF ($\times 10^5$)	1.8147	1.8512	2.0399

A like-transient process can be found in the CDF curves, as well as in the reliability curves of Figures 3 and 4. With the emergence of the degradation processes, the reliability of MSPM begins to decrease. Both WPM and GPM have the similar initial decreasing trends, although the trend of MSPM starts earlier. A possible reason of this phenomenon is that, the multi-failure mechanisms considered result in a combined impact on MSPM that accelerates the occurrence of degradation. As the degradation continues, the reliability of WPM and GPM are gradually reduced. However, there is a like-transient process during the decrease of the reliability of MSPM. It may better reflect an impact of the multi-failure mechanisms on reliability of a product, as which has been listed in Table 3, the MTTF of the MSPM is less than that of WPM and GPM. Probably, to model the ADT, MSPM is a better choice at the used condition by considering the multi-failure mechanisms.

5. CONCLUSION

Considering the multi-failure mechanisms, a mixed stochastic process is proposed to model the ADT in this paper. Two widely used stochastic processes (Wiener process and Gamma process) are fused by the weights in the proposed model. Therefore, it can avoid the mis-specification problem of the two models in some real application. As the weights take an important role during the fused process, to determine the weights, a quantitative method is proposed to construct a relationship with the accelerated stress. The reliability and MTTF results show that, the proposed method can combine the characteristics of the two processes under multi-failure mechanisms in some real conditions.

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