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# Arbitrary Polynomial Chaos Based Simulation of Probabilistic Power Flow Including Renewable Energies

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**Abstract:** In this paper, a method is introduced for probabilistic power flow calculations based on arbitrary polynomial chaos. For the polynomial chaos, orthogonal polynomial sets are used to represent the uncertainties of renewable power generation, and these orthogonal polynomials are generated from actual data. The aforementioned method is applied to probabilistic power flow calculations, and its applicability is confirmed in application to an actual transmission network. The calculation time and accuracy achieved using the arbitrary polynomial-chaos method are compared with those achieved using the popular Monte Carlo method. The results show that the calculation speed is 246–680 times greater than that with the direct Monte Carlo method, while the accuracy is almost same.

*Keywords:* Arbitrary polynomial chaos, Orthogonal polynomial, Uncertainty, Renewable power generation, Probabilistic power flow, Algorithms, Transmission network, Accuracy, Calculation time

## 1. INTRODUCTION

In this paper, a method is introduced for representing probabilistic uncertainties by means of orthogonal polynomials based on arbitrary polynomial chaos (APC). The proposed method is then applied to probabilistic power flows (PFs) in actual transmission networks that involve multiple renewable power sources such as wind power generation.

To date, many methods have been devised for calculating probabilistic PFs. There are three types of methods, numerical, analytical, and polynomial-chaos method. The representative of first one is Monte Carlo (MC) method. Dopazo (1975) introduced power distribution functions into PF calculations. In MC methods, after selecting probabilistic variables according to probability distribution functions (PDFs), these variables are input into the PF equation, and the output of a specified power line is obtained iteratively. For faster calculation, point estimation technique (Morales 2007) and Gram–Charlier expansion technique (Yuan 2011) are applied. For analytical method, the combination technique of convolution and linearization (Allan 1981) is proposed. The convolution method uses Tayler expansion and obtained linearized equations are solved.

For polynomial-chaos method, a relationship is obtained between probabilistic distribution functions and polynomial chaos represented by a class of orthogonal polynomials, and this relationship can be extended to any probability distribution function that appears in engineering. Polynomial chaos is an uncertainty representation for PDFs by means of orthogonal polynomials. There are two methods for implementing the polynomial-chaos approach, namely general polynomial chaos (GPC) and APC. GPC represents PDFs that correspond to the Wiener–Askey scheme (WAS) (Xiu, 2012), which is a natural expansion that arises from a class of hypergeometric functions. Nguyen (2016) applied adaptive sparse polynomial chaos based on GPC to a probabilistic PF simulation of a test network. By contrast, APC represents PDFs that are based on actual data, but in engineering many PDFs are not concerned with distributions in the WAS; for example, the Weibull distribution, which is said to approximate wind power generation, is not included in the WAS. In wind power, a probabilistic distribution is usually approximated by the Weibull distribution, and in solar power any distributions represented by the Gaussian mixture model are used. Therefore, the APC method would also be adequate for solving actual engineering problems. Further, Laowantitwattana (2018) applied APC to PDFs and showed aspects of APC.

In the present paper, because probabilistic PFs with wind power generation are considered, the APC method is used. Our paper describes aspects of the calculation algorithm of APC in more detail. The APC method is outlined in Section 2. Section 3 presents the algorithm that is used to solve probabilistic PFs. Moments to obtain orthogonal polynomials in APC are calculated. Furthermore, the collocation method, which is a calculation-point selection method that is often used in numerical simulations, is used to obtain the probabilistic response equation instead of using the PF equation iteratively. In Sections 4 and 5, this algorithm is applied to an actual power transmission network, and the APC method is compared with the popular MC method regarding the calculation time and accuracy. In Section 6, Sensitivity application by an outage of a transmission line is explained. Finally, an application is presented in Section 6 and conclusions are drawn.

## 2. MODELING BASED ON APC

### *2.1 Polynomial-Chaos Method*

The probabilistic response equation based on polynomial chaos is introduced, where $\xi_1, \xi_2, \dots, \xi_N$ are probabilistic variables, $\Phi_i$ is a basis of orthogonal polynomials, *Ci* is a coefficient and constant, and *K* and *N* are the numbers of polynomials and probabilistic variables, respectively. The response $R(\xi_1, \xi_2, \dots, \xi_N)$ is

$$R(\xi_1, \xi_2, \dots, \xi_N) = \sum_{i=0}^{K-1} C_i \Phi_i(\xi_1, \xi_2, \dots, \xi_N). \quad (1)$$

There is a relationship between *K* and *N*. If *d* is the maximum order of the orthogonal polynomials and ${}_aC_b$ is a combination operator, then

$$K = {}_{N+d}C_d = (N+d)!/N!\,d!. \quad (2)$$

If $N = 1$ (single probabilistic variable), then $K = d + 1$. In this manner, the order expansion of the orthogonal polynomials is restricted. $\Phi_i$ is divided into multiple one-variable functions as follows:

$$\Phi_i(\xi_1, \xi_2, \dots, \xi_N) = \Phi_{i_1}(\xi_1) \otimes \Phi_{i_2}(\xi_2) \otimes \cdots \Phi_{i_N}(\xi_N)$$
$$= \prod_{j=1}^{N} \phi_j^{(k_j)}(\xi_j), \quad (3)$$

$I = 0, 1, 2, \dots, K-1, k_j = 0, 1, \dots, d,$

$$\sum_{j=1}^{N} k_j \leq d. (4)$$

Equation (4) means that the sum of the orders of the polynomials for a single probabilistic variable must be less than *d*.

In (3), $\phi_j^{(k_j)}(\xi_j)$ are single-probabilistic-variable orthogonal polynomials, with constant $h_{j,i}^{(p)}$:

$$\phi_j^{(k_j)}(\xi_j) = \sum_{i=0}^{p} h_{j,i}^{(p)} \xi_j^i, (5)$$

where $p$ is selected in $\{0, 1, \dots, d\}$. Thus, the basis polynomials are organized by powers of the single probabilistic variable.

### *2.2 Orthogonalization*

By orthogonalizing the polynomials, the PDFs can be represented by linear combinations of orthogonal polynomial bases. Orthogonalization is represented by the inner-product formula. $\{P_n\}\ n = 0, 1, \dots, d$(where *d* is the maximum order of the polynomials) is a sequence of orthogonal polynomials, $G_n$ is a constant value, and $\delta_{mn}$ is the Kronecker delta defined as

$$\delta_{mn} = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$$

$$\langle P_m(x), P_n(x) \rangle = \int_S P_m(x) P_n(x) d\mu = G_n \delta_{mn} \quad (6)$$

μ is called a measure on Domain S of functions, *Pi(x)*. $d\mu(x)$ is represented by the PDF function *w*(*x*), called the weight:

$$d\mu(x) = w(x)dx. \quad (7)$$

A coefficient $h_{j,i}^{(p)}$ is calculated based on moment analysis. Because $\phi_j^{(k_j)}(\xi_1)$ are orthogonal, the following is satisfied:

$$\langle \phi_m^{(k_j)}(\xi), \phi_n^{(k_j)}(\xi) \rangle = G\delta_{mn}, \quad (8)$$

from which the following are obtained,

$$\int_S h_0^{(0)} \left[ \sum_{i=0}^{k} h_{j,i}^{(p)} \xi_j^i \right] d\mu = 0,$$
$$\int_S \left[ \sum_{i=0}^{1} h_{j,i}^{(1)} \xi_j^i \right] \left[ \sum_{i=0}^{k} h_{j,i}^{(p)} \xi_j^i \right] d\mu = 0, \quad (9)$$
$$\dots\dots$$
$$\int_S \left[ \sum_{i=0}^{k-1} h_{j,i}^{(k-1)} \xi_j^i \right] \left[ \sum_{i=0}^{k} h_{j,i}^{(p)} \xi_j^i \right] d\mu = 0$$

When *j* is omitted and (7) is used, the above equations are converted into

$$\int_{\xi\in\Omega} \sum_{i=0}^{k} h_i^{(k)} \xi^i w(\xi) d\xi = 0,$$
$$\int_{\xi\in\Omega} \sum_{i=0}^{k} h_i^{(k)} \xi^{i+1} w(\xi) d\xi = 0, \quad (10)$$
$$\int_{\xi\in\Omega} \sum_{i=0}^{k} h_i^{(k)} \xi^{i+k-1} w(\xi) d\xi = 0,$$
$$\dots\dots$$
$$h_k^{(k)} = 1$$

When $\xi$ is defined in the Domain D, the moments $m_p$ for order *p* are calculated by

$$m_p = \mathrm{L}(\xi^p) = \int_{\xi\in\mathrm{D}} \xi^p w(\xi) d\xi. \quad (11)$$

In actual engineering, PDFs are defined as step functions for $\xi$. Therefore, the discrete version of (10) is obtained as follows.

$$m_p = \sum_{i=1}^{m} \xi_i^p w(\xi_i)(\xi_i - \xi_{i-1}). \quad (12)$$

Using these moments, (10) are written as

$$\begin{bmatrix} m_0 & m_1 & \cdots & m_k \\ \vdots & \vdots & \ddots & \vdots \\ m_{k-1} & m_k & \cdots & m_{2k-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_0^{(k)} \\ \vdots \\ h_{k-1}^{(k)} \\ h_k^{(k)} \end{bmatrix} = M \begin{bmatrix} h_0^{(k)} \\ \vdots \\ h_{k-1}^{(k)} \\ h_k^{(k)} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (13)$$

where $M$ is a Hankel matrix. Because a Hankel matrix is always non-singular, the inverse matrix of $M$ always exists and the coefficients $\{h_0^{(k)}, \ldots, h_{k-1}^{(k)}, h_k^{(k)}\}$ are sure to obtained.

### 2.3 Structure of Probabilistic Response Equations

The probabilistic PF is obtained as a linear combination of orthogonal polynomials that represent renewable powers. This is a probabilistic response and is represented by (1) and (3).

For $\zeta^m = \{\xi_1^m, \xi_2^m, \ldots, \xi_N^m\}$,

$$\begin{bmatrix} \Phi_0(\zeta^1) & \Phi_1(\zeta^1) & \cdots & \Phi_k(\zeta^1) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_0(\zeta^{m-1}) & \Phi_1(\zeta^{m-1}) & \cdots & \Phi_k(\zeta^{m-1}) \\ \Phi_0(\zeta^m) & \Phi_1(\zeta^m) & \cdots & \Phi_k(\zeta^m) \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_k \end{bmatrix} = \begin{bmatrix} R(\zeta^1) \\ R(\zeta^2) \\ \vdots \\ R(\zeta^m) \end{bmatrix} \quad (14)$$

The matrix (14) is of order $k + 1 \times m$ ($k + 1 \leq$ m), therefore the coefficient vector $\{C_0, C_1, \ldots, C_k\}$ is not obtained. From the group of multiple vectors $\{\Phi_0(\zeta_i), \Phi_1(\zeta_i), \ldots, \Phi_k(\zeta_i)\}$ in the matrix, a non-singular matrix of order $k + 1 \times k + 1$ is selected by Gauss elimination. Then, the inverse matrix is calculated and, consequently, a new coefficient vector $\{C_0, C_1, \ldots, C_k\}$ is obtained. After $\{C_0, C_1, \ldots, C_k\}$ is obtained, the active and reactive powers $Q$ of the target branches in the transmission network are calculated without using alternative current (AC) PF equations. When the popular MC method is used, $\xi$ is selected iteratively according to the PDF, and the AC PF is calculated by a convergent process using the Newton–Raphson method based on iterative calculations; in that case, the iterative calculation must be performed twice. By contrast, APC uses iteration only in the selection of the probabilistic variables $\xi = \{\xi_1, \xi_2, \ldots, \xi_N\}$.

### 2.4 Collocation-Point Method

When $C_i$ in (14) is obtained, the PFs can be calculated and adequate probabilistic variables $\{\xi_1^m, \xi_2^m, \ldots, \xi_N^m\}$ are selected. For this purpose, the collocation-point method (Judd, 2014) is applied. The collocation-point method designates the calculation points of (13) for the zeros of polynomials (5). For example, if the order of (5) is two and the number of probabilistic variables $\xi$ is three, then

$$\begin{aligned} &\varphi_{1,0} = a_{1,0}, \varphi_{1,1} = \xi_1 + b_{1,,0}, \varphi_{1,2} = \xi_1^2 + c_{1,1}\xi_1 + c_{1,0} \\ &\varphi_{2,0} = a_{2,0}, \varphi_{1,1} = \xi_2 + b_{2,0}, \varphi_{1,2} = \xi_2^2 + c_{2,1}\xi_2 + c_{2,0} \quad (15) \\ &\varphi_{3,0} = a_{3,0}, \varphi_{3,1} = \xi_3 + b_{3,0}, \varphi_{3,2} = \xi_3^2 + c_{3,1}\xi_3 + c_{3,0} \end{aligned}$$

Because $N = 3$ and $d = 2$, $K = (3 + 2)!/3!2! = 10$ in (2), therefore, the generated polynomials are

$$\begin{aligned} &a_{1,0}a_{2,0}a_{3,0} \\ &a_{1,0}a_{2,0}(\xi_1 + b_{3,0}) \\ &a_{1,0}a_{2,0}(\xi_3^2 + c_{3,1}\xi_3 + c_{3,0}) \\ &a_{1,0}(\xi_2 + b_{2,0})a_{3,0} \\ &a_{1,0}(\xi_2 + b_{2,0})(\xi_3 + b_{3,0}) \\ &a_{1,0}(\xi_2^2 + c_{2,1}\xi_2 + c_{2,0})a_{3,0} \quad (16) \\ &(\xi_1 + b_{1,,0})a_{2,0}a_{3,0} \\ &(\xi_1 + b_{1,,0})a_{2,0}(\xi_3 + b_{3,0}) \\ &(\xi_1 + b_{1,,0})(\xi_2 + b_{2,0})a_{3,0} \\ &(\xi_1^2 + c_{1,1}\xi_1 + c_{1,0})a_{2,0}a_{3,0} \end{aligned}$$

Because $d + 1 = 3$, zeros of order-3 polynomials are selected, the zeros of the polynomials

$$\begin{aligned} &\xi_1^3 + d_{1,2}\xi_2^2 + d_{1,1}\xi_2 + d_{1,0} = 0 \\ &\xi_2^3 + d_{2,2}\xi_2^2 + d_{2,1}\xi_2 + d_{2,0} = 0 \quad (17) \\ &\xi_3^3 + d_{3,2}\xi_3^2 + d_{3,1}\xi_3 + d_{3,0} = 0, \end{aligned}$$

are $\xi_{10} = \{\xi_{110}, \xi_{120}, \xi_{130}\}$, $\xi_{20} = \{\xi_{210}, \xi_{220}, \xi_{230}\}$, $\xi_{30} = \{\xi_{310}, \xi_{320}, \xi_{330}\}$. All these variables are always real roots (Chihara, 1978; p.27), and variables in equations whose order is more than two include no equal roots. The total number of combinations is $(d + 1)^N = 3^3 (=27)$, and the combinations are

$$(\xi_{110}, \xi_{210}, \xi_{310}), (\xi_{110}, \xi_{210}, \xi_{320}), \ldots. (\xi_{130}, \xi_{230}, \xi_{330}).$$

## 3. ALGORITHM OF PROBABILISTIC POWER FLOW

### 3.1 Algorithm for Probabilistic Power Flow Calculation

The algorithm for calculating probabilistic PFs is shown in Fig.1.

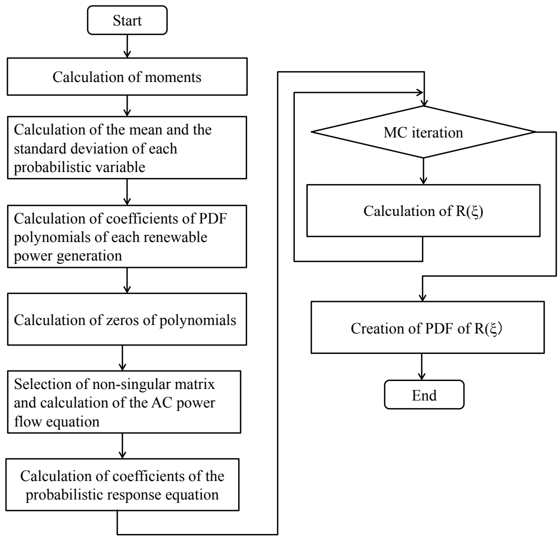



Fig. 1. Flow chart of output PDF calculation algorithm of probabilistic power flow

Each steps is explained below.

Step 1: Calculation of moments. According to (11), the moments $m_p$ ($p = 0, \ldots, 2k - 1$) are calculated.

Step 2: Calculation of mean and standard deviation of each probabilistic variable. The mean and standard deviation are calculated for normalization of each probabilistic variable.

Step 3: Calculation of coefficients of polynomials for each renewable power generation. For each probability variable, the coefficients $h_{j,i}^{(p)}$ of the orthogonal polynomials are calculated by inverting the Hankel matrix. Thus, any polynomials can be represented by linear combinations of the basis orthogonal polynomials. (Chihara, 1972; p.9).

Step 4: Calculation of zeros of polynomials. For orthogonal polynomials for wind power generation PDFs, the zeros of the polynomials are calculated using Durand–Kerner–Aberth method. These zeros are collocation points and are all real values.

Step 5: Selection of a non-singular matrix and calculation of AC PF. At the obtained zeros. The specifications of the transmission network topology and attributes such as the nodes at which wind power is injected are shown in Section 4. Then, because the matrix (13) is of order $k + 1 \times m$ ($k + 1 \leq m$), a non-singular $k + 1 \times k + 1$ matrix is selected by Gauss elimination. Successively, AC PFs are calculated.

Step 6: Calculation of coefficients of the probabilistic response equation. The coefficients $C_i$ of the probabilistic response equation are calculated by inverting the matrix $M$ in (13).

Step 7: Execution of MC iteration process. Up to Step 6, the probabilistic response equations are obtained, Then, MC iteration processing is executed by selecting the probabilistic variables.

Step 8: Creation of PDF of $R(\xi)$. By generating combinations of probabilistic variables, the PDF of $R(\xi)$ is created.

*3.2 Normalization of Probabilistic Variables*

The ranges of the effective probabilistic variables are different from each other. To use these variables equally, all the variables are normalized. When $a_i$ the appearance frequency and N is the total number of range sets of the probabilistic variables, then the new normalized probabilistic value $\xi$ is

$$\xi \leftarrow \xi - E[\xi]/\sigma$$

$$Mean\ value;\ E[\xi] = \left(\sum_{i=0}^{N} \xi_i a_i\right)/N \quad (18)$$

$$\text{Standard deviation value; } \sigma = \sqrt{\mathrm{E}[\xi^2] - \mathrm{E}[\xi]^2}$$

*3.3 Modeling of Power Flow Equations*

The PF is represented as

$$P_{ij} = Y_{ij} ViVj \cos(\delta i - \delta j + \theta ij), \quad (19)$$
$$Q_{ij} = Y_{ij} ViVj \sin(\delta i - \delta j + \theta ij),$$

where $P_{ij}$ is the active power and $Q_{ij}$ is the reactive power. This equation is based on the PQ parameter, which means that at the power generation buses, parameters $P_{ij}$ and $Q_{ij}$ are specified. Above, $i$ and $j$ are node numbers. Therefore, Vi is the voltage of node $i$. Here $X_{ij}$ is the reactance of transmission line $(i, j)$, $Y_{ij}$ is the admittance of transmission line $(i, j)$, $\delta_i$ is the voltage phase angle of node $i$, and $\theta ij$ is as follows:

$$\theta_{ij} = \tan^{-1} X_{ij}/R_{ij} \quad (20)$$

## 4. APPLICATION TO TRANSMISSION NETWORK WITH WIND POWER SITES

The proposed method is applied to an actual transmission network topology as shown in Fig. 2.

The voltage level is 500 kV and 275 kV. This network was available in 2018. The impedances of the transmission branches are calculated using a geographic information system called EARDAS, which is a platform for environment assessment. All branches are double lines. The impedance calculation is executed based on the p.u. unit system whose reference capacity is 1,000 MVA.

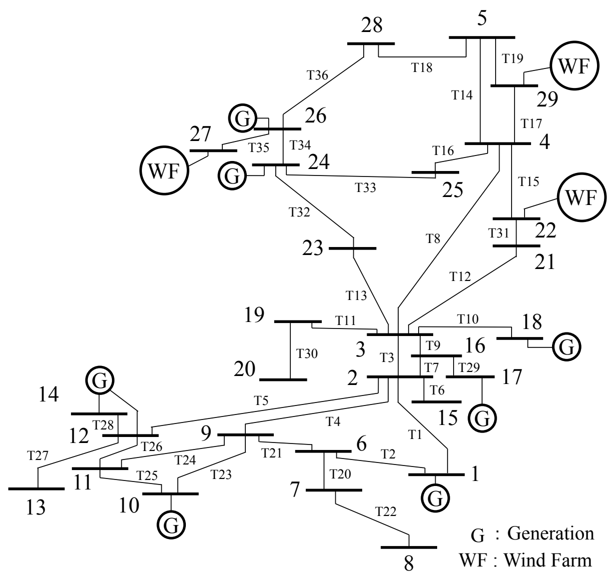



Fig. 2. Structure of transmission network topology

In Fig. 2, for renewable power generations, wind powers with uncertainty properties are injected at nodes, 22, 27, and 29. Nodes, 1, 10, 12, 17, 18, 24, and 26 are generation sites. Other nodes are demand sites (Fig. 2). The power factor is 0.9. The PDFs of the wind power generations are shown in Section 5.

## 5. RESULTS AND DISCUSSION

*5.1 Orthogonal Polynomials of Wind Power PDF*

The three PDFs of wind power generation at nodes 22, 27, and 29, wind farm (WF), are shown in Fig. 3. The orthogonal

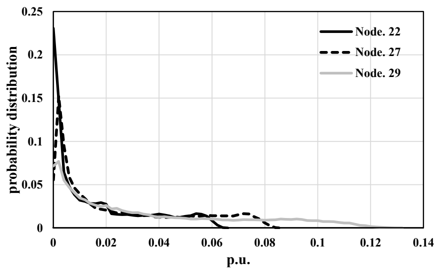



Fig. 3. PDFs of real wind power generation

polynomial representations with orders from zero to four of the wind power generation PDF are given in Table 1.

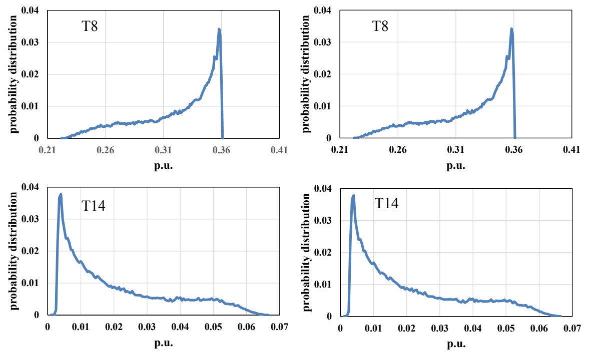
**Table 1. Orthogonal Polynomial Representation of Wind Power Generation PDF**

| WF | Order | Orthogonal Polynomial |
|---|---|---|
| WF at Node 22 | 0 | 1 |
| | 1 | $\xi - 0.065$ |
| | 2 | $\xi^2 - 0.13\,\xi - 0.002729$ |
| | 3 | $\xi^3 - 0.195\,\xi^2 + 0.009983\,\xi - 0.0001$ |
| | 4 | $\xi^4 - 0.026\,\xi^3 + 0.021506\,\xi^2 - 0.000599\,\xi + 0.000003$ |
| WF at Node 27 | 0 | 1 |
| | 1 | $\xi - 0.0424$ |
| | 2 | $\xi^2 - 0.0848\,\xi + 0.001153$ |
| | 3 | $\xi^3 - 0.1272\,\xi^2 + 0.004233\,\xi - 0.000027$ |
| | 4 | $\xi^4 - 0.1696\,\xi^3 + 0.009131\,\xi^2 - 0.000164\,\xi + 0.000001$ |
| WF at Node 29 | 0 | 1 |
| | 1 | $\xi - 0.0322$ |
| | 2 | $\xi^2 - 0.0644\,\xi + 0.000652$ |
| | 3 | $\xi^3 - 0.0966\,\xi^2 + 0.002418\,\xi - 0.000011$ |
| | 4 | $\xi^4 - 0.1288\,\xi^3 + 0.005234\,\xi^2 - 0.00007\,\xi + 0.0000001$ |

*5.2 Results of Output Branch Powers*

The power PDFs of T8 and T14 in Fig. 2 by the MC method and APC are shown in Fig. 4. The reasons two lines are selected, are that these lines are close to buses where WF are connected, and are affected by uncertainty of power output of WF. The iteration number is 105,120. The results are graph representations that are obtained by connecting among tops of histograms.

The results by MC and APC PDFs seem to be almost the same. Then, the calculation time and accuracy of the results are compared. For the calculation time, the calculation time ratio, namely rate = (calculation time of MC)/(calculation time APC) = t-MC/t - APC, is compared. The calculation time of MC is 584.6 s. The two ratios are compared. One is that t-APC is for Steps 5–8 in Fig. 1 and the other is that t-APC is for whole steps (Step 1–8) in Fig.1 according to the order 1 to 4 of orthogonal polynomials. Both rates are shown in Table 2.




(a) Power PDF based on MC (b) Power PDF based on APC (order four)

Fig. 4. Power PDFs by MC and APC

For accuracy, the next formula, namely the mean square error is used. $APC_{val}[i]$ is the power value by APC in the *i*th combination of $\xi$. $MC_{val}[i]$ is the power value by MC. Then, the RMSE is

$$\mathrm{RMSE} = \left(\frac{1}{n}\sum_{i=1}^{n}(\mathrm{APC}_{val}[i] - \mathrm{MC}_{val}[i])^2\right)^{1/2} \quad (21)$$

**Table 2. Comparison of Calculation Time**

| Order | Calculation time | | | |
|---|---|---|---|---|
| | t-APC is for Steps 5–8 | | t-APC is for Steps 1–8 | |
| | t-APC[s] | rate | t-APC[s] | rate |
| 1 | 0.78 | 756.6 | 0.86 | 680.6 |
| 2 | 1.16 | 507.0 | 1.28 | 460.0 |
| 3 | 1.60 | 366.7 | 1.77 | 331.0 |
| 4 | 2.13 | 275.3 | 2.38 | 246.3 |

In Table 3, the accuracy comparison is shown according to the order of the polynomials for T8, T14 and T16 (T16 is also close to a WF bus).

**Table 3. Accuracy Comparison**

| Order | T8 | T14 | T16 |
|---|---|---|---|
| 1 | 4.88E-05 | 5.25E-05 | 1.98E-06 |
| 2 | 7.00E-07 | 7.68E-07 | 2.61E-07 |
| 3 | 7.79E-08 | 7.87E-08 | 4.77E-08 |
| 4 | 3.49E-08 | 3.41E-08 | 4.35E-08 |

When the order of the polynomials increases, the errors between APC and MCS become very small.

## 6. APPLICATIN TO SENCITIVITY ANALYSIS

The proposed APC method can be applied to a lot of applications such as voltage control facility planning and power transfer change estimation by outage accidents. In this paper, power change by an outage accident in facility planning is introduced. When more than one lines are disconnected by serious disasters, power transport on each transmission line will change. Thus, in order to introduce a lot of wind powers, line outage affections to all transmission network are to be estimated rapidly to detect serious power changes.

The grid condition is same as the real grid shown in Fig. 2. Then, it is supposed that transmission line T34 of near the WF bus 27 is outage. The result of power distribution change on T8 which is the composition element of a main transmission network, is show in Fig. 5.

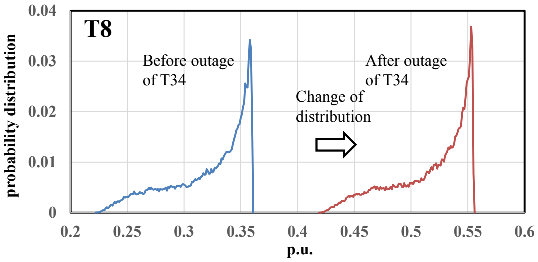



Fig.5. Power distribution change in T8 by an outage of T34

The result suggests that power distribution changes by serious accidents are obtained by fast calculations. In fact, all power changes in T1 to T36 are calculated within only 83.3 [sec].

## 7. DISCUSSION

The characteristics of the APC are as follows:

- Data-oriented orthogonal polynomial sets based on APC represent real PDFs.

- Actual distribution of wind power generation includes few bumps. Orthogonal polynomial representation is able to approximate these phenomena with high accuracy.

The calculation time and accuracy of the results are key issues for the comparison between APC and MC.

- The calculations are 246–680 times faster than MC. Even though MC and APC use the same number of iteration calculations, there are big differences between them. This fast processing is caused by the content of the processing. The MC includes other iteration steps using the Newton–Raphson method. On the other hand, for APC, these other iterations are substituted by algebraic calculations.

- The preparation Steps 1–4 in Fig.2 do not dominate a large part of the calculation time. When a number of renewable power generations increase, because the combination of probabilistic values increases, APC is more effective than MC.

- For accuracy, the APC method approximates with higher accuracy as shown in Table 3. When the polynomial order increases, the accuracy increases. In actual use of APC, order three would be adequate.

## 8. CONCLUSIONS

In this paper, fast probabilistic PF calculation based on APC is introduced. APC is a data-oriented method for which a probabilistic distribution is represented by orthogonal polynomials. This method is applied to probabilistic PFs that include multiple wind power generations. The algorithm for probabilistic PF calculation is shown and is confirmed as effective using actual transmission networks. The characteristics of the proposed method are as follows:

- Real distribution is more complicated. Data-oriented method approximates this complexity well.

- Fast and high accuracy simulations are available. Comparing MC and APC, APC is 246–680 times faster than direct MC.

- MC is the standard to compare accuracy. The accuracy of APC is almost the same as that of MC.

For future work, not only wind power generation but also other types of renewable energy such as solar power generations and demand should be included.

## REFERENCES

Allan, R.N., Leite da Silva, A.M. and Burchet, R.C. (1981). Evaluation method and accuracy in probabilistic load flow simulation, *IEEE Transaction on Power Apparatus and Systems*, vol.PAS-100(5), 2539-2546

Chihara, T.S. (1978). *An Introduction to Orthogonal Polynomials*, p.9, p.27. Dover Publications, Mineola, New York.

Dopazo, J.F., Klitin, O.A., and Sasson, A.M. (1975). Stochastic load flow. *IEEE Transaction on Power Apparatus and Systems*, PAS-94(2), 299–309.

Judd, K.L., Maliar, L., Maliar, S., and Valero, R. (2014). Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain. *Journal of Economic Dynamics & Control*, 44, 92–123.

Laowantitwattana, J. and Uatrongjit, S. (2018). Probabilistic power flow analysis based on arbitrary polynomial chaos expansion for networks with uncertain renewable sources. *IEEJ Transaction on Electrical and Electronic Engineering*, 13, 1754–1759.

Morales, J.M and Perez-Ruiz (2007). Point Estimate schemes to.Solve the probabilistic power flow, *IEEE Transaction on Power Systems*, 22(4), 1594–1601.

Nguyen, F.N., Nguyen, P., and Cobben, F.G. (2017). Basis-Adaptive Sparse Polynomial chaos expansion for probabilistic power flow. *IEEE Transaction on Power Systems*, 32(1), 694–704.

Oladyshkin, S. and Nowak, W. (2012). Data-Driven uncertainty Quantification using Arbitrary Polynomial Chaos Expansion, *Reliability engineering and System Safety*, 106, 179–190.

Xiu, D. and Karniadakis, G. E. (2002). The Wiener–Askey polynomial chaos for stochastic differential equations. *SIAM Journal of Scientific Computation*, 24(2), 619–644.

Yuan, Y., D. Zhou P and Feuchtwang, J. (2011). Probabilistic Load Flow computation of a power system containing wind farms using the method of combined cumulants and Gram-Charlier expansion, IET Renewable Power Generation, 5(6), 448‑454