Estimation Oriented Co-design of Sensor Scheduling over Stochastic Delayed Channel under Power Constraints ⋆

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Abstract: This paper investigates the scheduling problem over a delayed channel. Different from most existing documents, a novel hybrid model is proposed which combines both delay and packet-loss to minimize the error covariance update at the estimator side. With the help of this setup, a co-design problem between power consumption and estimation performance is considered. We first derive out a globally optimal off-line schedule. Moreover, an on-line schedule based on a designed threshold is proposed to further enhance the performance, which is aided by feedback information. Comparisons between on-line and off-line strategies are illustrated by numerical simulations, which has shown the superiority of on-line one to the off-line one.

Keywords: state estimation, delayed sensor networks, co-design problem.

1. INTRODUCTION

The past few decades have witnessed the significant improvement in communication technologies, which leads to a wide utilization for wireless sensor networks in control systems. However, due to the limited wireless resources of sensors, i.e., power or bandwidth Lyu et al. (2018), Chen et al. (2015), Zhu et al. (2018), and etc, the quality of communication can be seriously affected. This may result in time-delay or even packet-loss, which indirectly degrades the performance of the entire system. Consequently, it is a key and urgent task to design a suitable transmission strategy to ensure the performance of the system, especially for resource-limited cases.

Numbers of works have been accomplished on the trade-off between estimation performance and communication resources in time or state domains. For example, in Shi and Zhang (2012), an optimal time-triggered transmission schedule of two Markovian systems is proposed under bandwidth constraint. Scheduling problem of an energy harvesting sensor is illustrated in Li et al. (2017), where an approximately analytical form of optimal solution is presented using dynamic programming algorithm. Qi et al. (2016) have focused on the case over a time-varying channel, where an optimal scheme is presented in a closed-form. Li et al. (2018b) have discussed the scheduling problem over relay-assisted wireless control systems, where two different on-line strategies are designed with satisfying performance. He also investigates the scheduling problem over multiple power levels in Li et al. (2018a), where a hierarchical event-triggered on-line scheduling scheme is proposed. The above literatures mainly emphasize the issues of possible packet-loss over data transmissions. Other kinds of trade-off problems over lossy channels are formulated via co-design manners in Gatsis et al. (2014), Dey et al. (2017) and etc.

In the practical scenes, transmission delays often occur in wireless communications, which is a non-negligible factor existing especially in resource-limited scenarios. Some literatures have explored the scheduling problem under delayed channel setup. For example, in Li et al. (2019), a latency-aware virtual network embedding method is proposed for IWNs, which guarantees the deadlines for various industrial networks. In Shi et al. (2011), scheduling problem over a delayed channel with one time-step delay is investigated. In Ren et al. (2018), multi-hop scheduling based on delayed network is investigated with the upper and lower bounds of cost provided. However, how to obtain and analyze the cost with a higher accuracy is an intractable task under both delayed and power limitations. Therefore, in this paper, we accomplish a wide research on delayed networks for the power-constrained cases Our contributions can be summarized as the following two folds:

- **A power-constrained delayed transmission model:** We build up a more general model that combines both time delay and packet drop-out, which minimizes the update of covariance at the same time.
- **Off-line and on-line strategies design:** Under our proposed delayed model, we design two different kinds of scheduling schemes in infinite-time horizon, i.e., off-line scheduling and on-line scheduling. Especially, for the on-line scheduling scheme, we give the determina-

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tion process of proposed selection probabilities. Based on these conclusions, we figure out the the optimal solution of our co-designed problem in a closed-form, where how the weighed parameter influences the final results is analyzed.

The rest of this paper is organized as follows. In section 2, the proposed delayed models and main interests of this paper are presented. Design of both off-line and on-line scheduling schemes are shown in section 3, based on which, the optimal solutions are derived out for the off-line and on-line cases, respectively. Numerical simulations and comparisons are accomplished in Section 4.

2. PROBLEM FORMULATION

In this paper, we consider a standard linear time-invariant system with the dynamics described as:

$$x_{k+1} = Ax_k + w_k, \tag{1}$$

where A represents the state transfer matrix of system, x_k is the state of the system at time k, w_k is a standard white Gaussian noise with a known distribution $w_k \sim \mathcal{N}(0, Q)$.

The observation equation of system is formulate by:

$$y_k = Cx_k + \upsilon_k, \tag{2}$$

where C is the measurement matrix, y_k is the measurement at time k, υ_k is another standard white Gaussian process with a known distribution $\upsilon_k \sim \mathcal{N}(0, R)$.

Two pairs $\left(A, \sqrt{Q}\right)$ and (A, C) are assumed to be controllable and observable, respectively. The conventional Kalman algorithm is run to obtain the optimal estimate $\hat{x}_k^s$, i.e.,

$$\begin{aligned}
\hat{x}_{k+1}^{s-} &= A\hat{x}_k^s, \\
P_{k+1}^{s-} &= AP_k^s A^T + Q, \\
K_{k+1} &= CP_{k+1}^{s-}\left(CP_{k+1}^{s-}C^T + R\right)^{-1}, \\
P_{k+1}^s &= (I - K_{k+1}C)\, P_{k+1}^{s-}, \\
\hat{x}_{k+1}^s &= \hat{x}_{k+1}^{s-} + K_{k+1}\left(y_{k+1} - C\hat{x}_{k+1}^{s-}\right).
\end{aligned}$$

The following two operators are defined on $\mathbb{S}_n^+ \to \mathbb{S}_n^+$ to simply denote the whole Kalman iterations:

$$h(X) \triangleq AXA^T + Q,$$

$$g(X) \triangleq h(X) - h(X)C^T\left[Ch(X)C^T + R\right]^{-1} Ch(X).$$

$h(X)$ can be viewed as the prediction phase of Kalman filter, and $g(X)$ can be viewed as whole phase of Kalman filter. In other words, when y_k is unavailable, $g(X)$ is iterated; when the y_k is unavailable, $h(X)$ is iterated. The steady-state error covariance of Kalman filter, denoted as $\overline{P}$, can be obtained by the equation $g(X) = X$.

After the sensor obtains $\hat{x}_k^s$, it will send the estimate over a delayed network through multiple power levels. For the convenience of presentation, we denote d_k^s as the sensor transmission delay at time k. It is assumed that there are two optional power levels: the higher level Δ, and the lower level δ. When Δ is used, the perfect channel is used to transmit thus with no time delay, i.e., $d_k^s = 0$; however, when δ is used, in order to enhance the arrival probability, the time varying packet-delay channel is used with stochastic delays, where numbers of relays

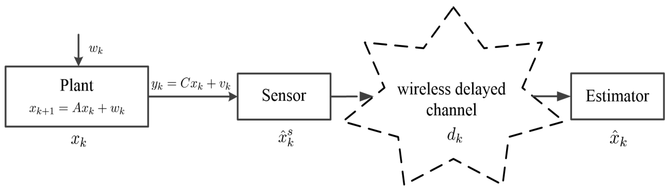

Fig. 1. The basic block diagram of state estimation over a time-delaying network.

or gateways are deployed as described in Shi and Zhang (2012), i.e.,

$$d_k^s = \begin{cases} 0, & w.p. \quad \lambda_0, \\ 1, & w.p. \quad \lambda_1, \\ \vdots & \quad \vdots \\ \infty, & w.p. \quad \lambda_\infty, \end{cases} \tag{3}$$

where $w.p.$ means "with the probability of", d_k^s denotes the transmission delay. However, at the estimator side, in order to define the delay d_k, we propose the following principle.

The Minimum Principle: In order to minimize the update, at the estimator side, when there are more than one packets arrived simultaneously at time k, d_k can be determined by

$$d_k \triangleq \arg\min_{d_i^s} \left\{ i + d_i^s = k | k - d_{k-1} \le i \le k \right\}. \tag{4}$$

We always regard the remarkably large delays as packet-loss in the practical scenes. Thus we introduce a parameter d to represent the upper-bound of time delay. We consequently denote the packet-loss at the estimator side as $d_k = \infty$.

At each time of transmission, the sensor will decide on which power level is used to transmit packet. Denote $\theta = \{\omega_k\}_{k=0}^{T-1}$ as the transmission power sequence used at each time. We have the state iteration as

$$\hat{x}_k = \begin{cases} A^i \hat{x}_{k-i}^s, & d_k = i, \\ A\hat{x}_{k-1}, & d_k = \infty. \end{cases} \tag{5}$$

with the covariance update

$$P_k = \begin{cases} h^i\left(\overline{P}\right), & d_k = i, \\ h\left(P_{k-1}\right), & d_k = \infty. \end{cases} \tag{6}$$

The main goal of our paper is to design a proper transmission policy to optimize the following co-design problem in a infinite-time scale, i.e.,

$$J(\theta) = \lim_{T\to\infty} \sum_{k=0}^{T-1} \mathrm{Tr}\left\{\mathbb{E}\left[P_k\right]\right\} + \mu\omega_k,\ \omega_k \in \{\delta, \Delta\}. \tag{7}$$

where $\mu \ge 0$ is the introduced factor which balances the performance of estimation and the cost of power consumption.

3. STRATEGIES DESIGN

In this section, we will design our proposed optimal strategies from two different perspectives, i.e., the off-line and on-line schedules, where arriving feedback information is available for sensor to make decisions or not, respectively.

It is noticed that original problem (7) can be converted into (8) as follows:

$$J(\theta) = \min_{\overline{\omega}} \lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} \mathrm{Tr}\left\{\mathbb{E}\left[P_k\right]\right\} + \mu\overline{\omega},$$
$$s.t. \quad \lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} \omega_k = \overline{\omega}, \quad \overline{\omega} \in [\delta, \Delta]. \tag{8}$$

where the parameter $\overline{\omega}$ is defined as an average power budget over an infinite-time horizon. Besides, the following definitions are given.

- State: We define i as the state $P_k = h^i\left(\overline{P}\right), i = 0, 1, 2, \cdots$.
- Steady-state probability: Denote π_i as the steady-state probability of state $h^i\left(\overline{P}\right)$; denote $\pi(\cdot)$ as the steady-state probability of a specific state $(\cdot)$
- Transition probability: Denote $\pi(m, n)$ as the transition probability from the state n to m.
- Equivalent drop-out length: Define τ_k as the equivalent drop-out length, i.e.,

$$\tau_k \triangleq \begin{cases} d_k, & d_k < \infty, \\ \tau_{k-1} + 1, & d_k = \infty. \end{cases}$$

3.1 Model of Delay in Time Domain

In order to derive out the delayed model. We first provide the following lemma.

Lemma 1. For the proposed minimum principle, we have the transition relationship with respect to different values of delay are

- For $i \leq d$ and $j \leq d$, we have
$$\pi\left(d_{k+1} = j, d_k = i\right) = \lambda_j,$$
$$\pi\left(d_{k+1} = i+1, d_k = i\right) = 1 - \sum\nolimits_{j=0}^{i} \lambda_j. \tag{9}$$
- For $d_k = \infty$ and $j \leq d$, we have
$$\pi\left(d_{k+1} = j, d_k = \infty\right) = \lambda_j,$$
$$\pi\left(d_{k+1} = \infty, d_k = \infty\right) = 1 - \sum\nolimits_{j=0}^{d} \lambda_j. \tag{10}$$

where $\lambda_0, \lambda_1, \cdots, \lambda_d$ is defined in (3).

Proof. According to the *minimum principle*, we have

$$\begin{aligned}&\pi\left(d_{k+1} = j, d_k = i\right)\\ =&\frac{\mathbf{Pr}\left(d^s_{k-j+1} = j\right)}{\mathbf{Pr}\left(d^s_{k-j+1} \geq j\right)} * \frac{\mathbf{Pr}\left(d^s_{k-j+2} \geq j\right)}{\mathbf{Pr}\left(d^s_{k-j+2} \geq j-1\right)} * \cdots * \frac{\mathbf{Pr}\left(d^s_k \geq 1\right)}{\mathbf{Pr}\left(d^s_k \geq 0\right)}\\ =&\mathbf{Pr}\left(d^s_{k-j+1} = j\right) = \lambda_j,\end{aligned}$$

where the second equality is due to the independence of distribution d^s_k for different instants, thus the first case is proved. Similarly, the second case follows as long as we take the upper-bound into account, which completes the proof.

Lemma 2. For the sequence $\left\{H^i\left(\overline{P}\right)\right\}_{i=1}^{\infty}$, for $\forall i \leq j$, it always holds that

$$\mathrm{Tr}\left[H^i\left(\overline{P}\right)\right] \leq \mathrm{Tr}\left[H^j\left(\overline{P}\right)\right]. \tag{11}$$

where $H^i\left(\overline{P}\right) \triangleq \mathbb{E}\left[P_i | \omega_0 = \Delta, \omega_1 = \omega_2 = \cdots = \omega_i = \delta\right]$.

Proof. Rewrite $H^i\left(\overline{P}\right)$ as

$$H^i\left(\overline{P}\right) = c_{i,0}\overline{P} + c_{i,1}h\left(\overline{P}\right) + \cdots + c_{i,i}h^i\left(\overline{P}\right),$$

where $c_{i,j}$ denotes the relative coefficient. According to Lemma 1, it holds that $c_{1,0} < c_{0,0}$, $c_{1,1} > c_{0,1} = 0$, and $c_{i,j} = c_{i-1,j}$, $c_{i,i-1} < c_{i-1,i-1}$, $c_{i,i} > c_{i-1,i} = 0$ $j = 0, 1, \cdots, i-2$. Therefore, it follows that

$$\mathrm{Tr}\left[H^i\left(\overline{P}\right) - H^{i-1}\left(\overline{P}\right)\right] = c_{i,i}\mathrm{Tr}\left[h^{i+1}\left(\overline{P}\right) - h^i\left(\overline{P}\right)\right] \geq 0,$$

which completes the proof.

3.2 Optimal Off-line Strategy

Lemma 3. An optimal off-line schedule θ^*_{off} under power constraint $\overline{\omega}$ over infinite-time horizon can be periodically presented as follows:

$$\theta^*_{\text{off}} = \begin{cases} \theta^{1*}_{\text{off}}, & w.p.\ p^*, \\ \theta^{2*}_{\text{off}}, & w.p.\ 1-p^* \end{cases} \tag{12}$$

where θ^{1*}_{off} and θ^{2*}_{off} are defined as θ^*_{off}: $\theta^{1*}_{\text{off}} \triangleq (\Delta, \boldsymbol{\delta}_l)$, and $\theta^{2*}_{\text{off}} \triangleq (\Delta, \boldsymbol{\delta}_{l+1})$. $\boldsymbol{\delta}_x \triangleq (\delta, \cdots, \delta)\,(x \text{ times})$, $l = \left\lfloor \frac{\Delta - \overline{\omega}}{\overline{\omega} - \delta} \right\rfloor$, $p^* = l + 1 - \frac{\Delta - \overline{\omega}}{\overline{\omega} - \delta}$.

Proof. Rewrite $\theta^*_{\text{off}} = \oplus_{i=1}^{\infty}\left(\Delta, \boldsymbol{\delta}_{l_i}\right)$. For $\forall l_i, l_j \geq 0\,(i \neq j)$ and $l_j \geq l_i + 2$, it follows that

$$\begin{aligned}&J\left(\Delta, \boldsymbol{\delta}_{l_i}\right) + J\left(\Delta, \boldsymbol{\delta}_{l_j}\right) - J\left(\Delta, \boldsymbol{\delta}_{l_i+1}\right) - J\left(\Delta, \boldsymbol{\delta}_{l_j-1}\right)\\ =&H^j\left(\overline{P}\right) - H^i\left(\overline{P}\right) \geq 0,\end{aligned}$$

which implies the property $\max_{i\neq j}|l_i - l_j| \leq 1$, and the form of θ^*_{off} must satisfy (13). The proof is now completed.

Theorem 4. (Optimal off-line schedule). For different values of μ, the optimal θ^*_{off} over infinite-time horizon can be explicitly given as

- For $\mu \leq \frac{\mathrm{Tr}\left[h\left(\overline{P}\right)\right] - \mathrm{Tr}\left(\overline{P}\right)}{\Delta - \delta}$,
$$\theta^*_{\text{off}} = \left(\Delta, \Delta, \Delta, \Delta, \cdots\right),$$
with $\overline{\omega}^* = \Delta$.
- For $\mu \in (\mu_i, \mu_{i+1}]$,
$$\theta^*_{\text{off}} = \left(\Delta, \boldsymbol{\delta}_i, \Delta, \boldsymbol{\delta}_i, \cdots\right),$$
with $\overline{\omega}^* = \frac{1}{i+1}(\Delta + i * \delta)$, where l is the same as that denoted in Lemma 3 and the sequence $\{\mu_i\}_{i=0}^{\infty}$ is defined as
$$\mu_i \triangleq \frac{(i+1)\,\mathrm{Tr}\left[H^{i+1}\left(\overline{P}\right)\right] - \mathrm{Tr}\left[\sum_{j=0}^{i} H^j\left(\overline{P}\right)\right]}{\Delta - \delta}.$$

Proof. The proof is omitted because of the limited space.

3.3 On-line Strategy Design

In this subsection, we will discuss a different scheduling framework, i.e., the on-line policy. For the on-line case, at each instant, the feedback information can be used to make decisions, which is shown in Fig. 2.

Aided by this feedback framework, how to design a proper schedule to switch between different power levels is a challenging task because of the intractability brought by the stochastic delay characteristics. Inspired by the idea of event-triggered scheme illustrated in Demirel et al. (2019) and Demirel et al. (2018), considering the power consumption for feedback is regarded to be relative small Ren et al. (2018), we propose an on-line scheduling scheme based on a designed threshold, which additionally cut the communication cost.

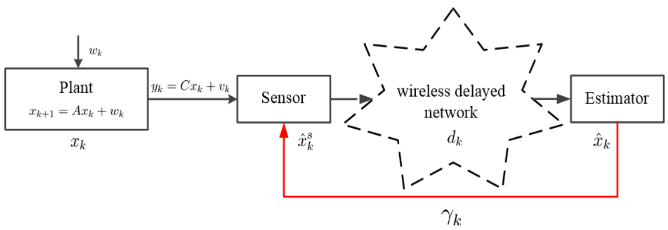

Fig. 2. The framework of on-line scheduling.

Algorithm 1 On-line Scheduling Scheme

Parameters: $\lambda_1, \lambda_2, \cdots, \lambda_d, \tau_k, d, \alpha, \beta$.
Initiation: $\omega_0 = \delta$, $k = 0$.

1: $\tilde{\tau}_{\text{th}} = \begin{cases} \tau_{\text{th}}, & \mathbf{Pr}\left(\tilde{\tau}_{\text{th}} = \tau_{\text{th}}\right) = \alpha, \\ \tau_{\text{th}} + 1, & \mathbf{Pr}\left(\tilde{\tau}_{\text{th}} = \tau_{\text{th}} + 1\right) = \beta. \end{cases}$
2: **while** $\tau_k \neq \tilde{\tau}_{\text{th}}$ **do**
3:     **if** $d_k = \infty$ **then**
4:         $\tau_{k+1} \leftarrow \tau_k + 1$, $\omega_{k+1} = \delta$;
5:     **else**
6:         $\tau_{k+1} \leftarrow d_k$, $\omega_{k+1} = \delta$;
7:     **end if**
8:     $k = k + 1$, jump to Line 2;
9: **end while**
10: $\omega_{k+1} = \Delta$, $k = k + 1$, jump to Line 1;

Algorithm 1 has clearly shown the main concept of our proposed scheme, where the parameters τ_{th} and $\tau_{\text{th}} + 1$ represent the thresholds. Moreover, α and β are introduced triggering probabilities for τ_{th} and $\tau_{\text{th}}+1$, respectively. The sensor will use δ level to transmit until the value of equivalent length is equal to τ_{th} or $\tau_{\text{th}} + 1$. At the same time, and feedback signal is transmitted from the estimator to sensor for the change to Δ. Based on Algorithm 1, the expect cost can be accurately calculated by the following analysis.

The whole process can be formulated as a Markovian chain model. Two different sets of states are defined as follows

$$\epsilon_i \triangleq \{P_k = h^i\left(\overline{P}\right), \tilde{\tau}_{\text{th}} = \tau_{\text{th}}\}, \qquad i \in [0, \tau_{\text{th}}],$$
$$\psi_i \triangleq \{P_k = h^i\left(\overline{P}\right), \tilde{\tau}_{\text{th}} = \tau_{\text{th}+1}\}, \qquad i \in [0, \tau_{\text{th}+1}].$$

Based on this definition, we first derive out an important proposition that is fundamental in our following analysis.

Proposition 5. For any feasible scheduling scheme that shares the same power constraint ϖ, it always holds that

$$\pi_0 = \frac{1}{\Delta - \delta}\left[\lambda_0 \Delta + (1 - \lambda_0)\,\varpi - \delta\right].$$

Proof. Consider a feasible schedule $\theta_{\text{on}}^* = (\omega_0, \omega_1, \omega_2, \cdots)$ with the constraint $\sum_{i=0}^{\infty} \pi_i = 1$, due to the transition relations, π_0 can be determined by

$$\begin{aligned}
\pi_0 &= \frac{1}{T}\lim_{T\to\infty}\sum_{i=0}^{T-1}\mathbf{Pr}\left(P_i = \overline{P}\right) \\
&= \mathbf{Pr}\left(P_i = \overline{P}|\omega_i = \Delta\right)\frac{\lim_{T\to\infty}\sum_{i=0}^{T-1}\mathbf{Pr}\left(\omega_i = \Delta\right)}{T} \\
&\quad + \mathbf{Pr}\left(P_i = \overline{P}|\omega_i = \delta\right)\frac{\lim_{T\to\infty}\sum_{i=0}^{T-1}\mathbf{Pr}\left(\omega_i = \delta\right)}{T} \\
&= \frac{1}{\Delta - \delta}\left[\lambda_0 \Delta + (1 - \lambda_0)\,\varpi - \delta\right],
\end{aligned}$$

which completes the proof.

Proposition 5 reveals the fact that π_0 is an constant independent of the specific schedules under an given power budget, which provides an accessible way to compare performance between different proposed schedules.

Based on the above proposition, we accomplish our derivations of $J\left(\theta_{\text{on}}\right)$ by the following two different cases. For the explicit presentation, we introduce the notation $\lambda_{\infty,i} \triangleq 1 - \sum_{j=0}^{i}\lambda_j$, and π_T as the probability of triggering.

Case 1($\tau_{\text{th}} < d$): Because of the transition relations, we have the equations

$$\pi\left(\epsilon_k\right) = \begin{cases} \alpha\pi_0, & k = 0, \\ \lambda_k \sum_{i=k}^{\tau_{\text{th}}-1}\pi\left(\epsilon_i\right) + \lambda_{\infty,k-1}\pi\left(\epsilon_{k-1}\right), & k = 1, 2, \cdots, \tau_{\text{th}} - 1, \\ \lambda_k \pi\left(\epsilon_{\tau_{\text{th}}-1}\right), & k = \tau_{\text{th}}, \end{cases}$$

and

$$\pi\left(\psi_k\right) = \begin{cases} \beta\pi_0, & k = 0, \\ \lambda_k \sum_{i=k}^{\tau_{\text{th}}}\pi\left(\psi_i\right) + \lambda_{\infty,k-1}\pi\left(\psi_{k-1}\right), & k = 1, 2, \cdots, \tau_{\text{th}}, \\ \lambda_k \pi\left(\psi_{\tau_{\text{th}}}\right), & k = \tau_{\text{th}} + 1. \end{cases}$$

with the solutions provided in the following cases.

- For $\tau_{\text{th}} = 0$, $\pi_1 = 1 - \pi_0$.
- For $\tau_{\text{th}} \geq 1$, we can obtain the solutions of π_1 to $\pi_{\tau_{\text{th}}+1}$ recursively, i.e.,

$$\begin{aligned}
\pi_1 &= \lambda_{\infty,0}\pi_0 + \left(1 - \pi_0 - \pi_T\right)\lambda_1, \\
\pi_2 &= \lambda_{\infty,1}\pi_1 + \left(1 - \pi_0 - \pi_1 - \pi_T\right)\lambda_2, \\
&\vdots \qquad\qquad\qquad \vdots \\
\pi_{\tau_{\text{th}}-1} &= \lambda_{\infty,\tau_{\text{th}}-2}\pi_{\tau_{\text{th}}-2} + \left(1 - \sum_{i=0}^{\tau_{\text{th}}-2}\pi_i - \pi_T\right)\lambda_{\tau_{\text{th}-1}}, \\
\pi_{\tau_{\text{th}}} &= \lambda_{\infty,\tau_{\text{th}}-1}\pi_{\tau_{\text{th}}-1} + \left(1 - \sum_{i=0}^{\tau_{\text{th}}-1}\pi_i - \pi_T\right)\lambda_{\tau_{\text{th}}}, \\
\pi_{\tau_{\text{th}}+1} &= 1 - \sum_{i=0}^{\tau_{\text{th}}}\pi_i.
\end{aligned}$$

Case 2($\tau_{\text{th}} \geq d$): Similarly,

$$\pi\left(\epsilon_k\right) = \begin{cases} \alpha\pi_0, & k = 0, \\ \lambda_k \sum_{i=k}^{\tau_{\text{th}}-1}\pi\left(\epsilon_i\right) + \lambda_{\infty,k-1}\pi\left(\epsilon_{k-1}\right), & k = 1, 2, \cdots, d, \\ \lambda_{\infty,d}\pi\left(\epsilon_{k-1}\right), & k = d + 1, \cdots, \tau_{\text{th}} \end{cases}$$

and

$$\pi\left(\psi_k\right) = \begin{cases} \beta\pi_0, & k = 0, \\ \lambda_k \sum_{i=k}^{\tau_{\text{th}}-1}\pi\left(\psi_i\right) + \lambda_{\infty,k-1}\pi\left(\psi_{k-1}\right), & k = 1, 2, \cdots, d, \\ \lambda_{\infty,d}\pi\left(\psi_{k-1}\right), & k = d + 1, \cdots, \tau_{\text{th}} + 1. \end{cases}$$

The solution is similar to that of case 1 and thus omitted. Combining the above two cases, the overall cost is calculated as

$$J\left(\theta_{\text{on}}\right) = \sum_{i=0}^{\tau_{\text{th}}+1}\pi_i \text{Tr}\left[h^i\left(\overline{P}\right)\right] + \mu\varpi.$$

3.4 Parameters Determination

In this subsection, we make discussions of the parameters determination of τ_{th} defined in the last subsection. We have

$$\pi_T = \frac{\overline{\omega} - \delta}{\Delta - \delta}.$$

Based on the definitions, we first calculate the value of τ_{th} by the following statement

$$\begin{aligned}\tau_{\text{th}} &\triangleq \arg\max_i \left\{ \sum_{k=0}^{i} \pi_k \leq 1 - \pi_T \right\} + 1 \\ &= \arg\max_i \left\{ \sum_{k=0}^{i} \pi_k \leq \frac{\Delta - \overline{\omega}}{\Delta - \delta} \right\} + 1.\end{aligned}$$

Then we can determine the values of α and β by the following cases.

Case 1($\tau_{\text{th}} = 0$): It is obvious that π_1 is only related with π_0, we have

$$\beta = \frac{\pi_1}{\pi_0}, \ \ \alpha = 1 - \beta.$$

Case 2($\tau_{\text{th}} = 1$):

- For $\tau_{\text{th}} < d$, solve the equation of $\pi(\epsilon_1) + \pi(\psi_2) = \pi_T$, i.e.,
$$\alpha \lambda_{\infty,0} \pi_0 + (1 - \pi_0 - \pi_T) \lambda_{\infty,1} = \pi_T,$$
and obtain the solutions
$$\alpha = \frac{(1 + \lambda_1) \pi_T - (1 - \pi_0) \lambda_{\infty,1}}{\lambda_{\infty,0} \pi_0}, \ \ \beta = 1 - \alpha.$$
- For $\tau_{\text{th}} \geq d$, as the value of d is either 0 or 1, thus it follows that
$$\beta = \frac{\pi_2}{\pi_2|_{\beta=1}} = \frac{1 - \pi_0 - \pi_1}{\lambda_{\infty,1} \pi_1}, \ \ \alpha = 1 - \beta.$$

Case 3($\tau_{\text{th}} \geq 2$):

- For $\tau_{\text{th}} < d$,
$$\begin{aligned}\beta &= \frac{\pi_{\tau_{\text{th}}+1}}{\pi_{\tau_{\text{th}}+1}|_{\beta=1}} \\ &= \frac{1 - \sum_{i=0}^{\tau_{\text{th}}} \pi_i}{\lambda_{\infty,\tau_{\text{th}}} \pi_{\tau_{\text{th}}} + (1 - \sum_{i=0}^{\tau_{\text{th}}} \pi_i - \pi_T) \lambda_{\tau_{\text{th}}+1}}.\end{aligned}$$
- For $\tau_{\text{th}} \geq d$,
$$\beta = \frac{\pi_{\tau_{\text{th}}+1}}{\pi_{\tau_{\text{th}}+1}|_{\beta=1}} = \frac{1 - \sum_{i=0}^{\tau_{\text{th}}} \pi_i}{\lambda_{\infty,d}^{\tau_{\text{th}}-d} \pi_{\tau_d}}.$$

Based on these analysis, we present the optimal on-line schedule in a closed form as follows.

Theorem 6. (Optimal on-line schedule). The optimal on-line schedule θ_{on}^* over infinite-time horizon with respect to μ is explicitly given as follows:

- For $\mu \in (0, \mu_0]$, $\overline{\omega}^* \triangleq \Delta$.
- For $\mu \in (\mu_i, \mu_{i+1}]$ $(i \geq 0)$,
$$\overline{\omega}^* \triangleq \arg\max_{\overline{\omega}} \{\tau_{\text{th}} = i + 1\},$$

where $\{\mu_i\}_{i=0}^{\infty}$ is determined by

$$\mu_i \triangleq -\sum_{k=0}^{i+1} \frac{d\pi_k}{d\overline{\omega}} \text{Tr}\left[h^k\left(\overline{P}\right)\right].$$

Proof. The proof is omitted because of the limited space.

Based on the above conclusions, we will present the convergence condition shown as the following corollary.

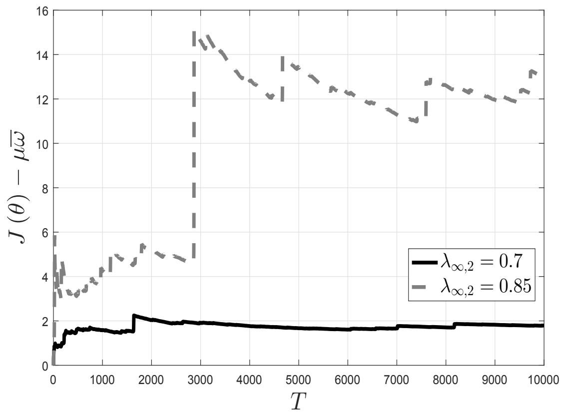

Fig. 3. Stability analysis with $\lambda_{\infty,2} = 0.7$ and 0.85.

Corollary 7. (Convergence analysis). For $\overline{\omega} = \delta$, the estimator converges if $\lambda_{\infty,d} < \frac{1}{\rho^2(A)}$, where $\rho(A)$ represents the spectral radius of matrix A.

Proof. According to Lemma 2, $H^i\left(\overline{P}\right)$ can be rewritten as

$$\begin{aligned}H^i\left(\overline{P}\right) &= \sum_{i=1}^{\infty}\left(H^i\left(\overline{P}\right) - H^{i-1}\left(\overline{P}\right)\right) \\ &= \sum_{i=1}^{\infty} c_{i,i}\left(h^i\left(\overline{P}\right) - h^{i-1}\left(\overline{P}\right)\right),\end{aligned}$$

where $c_{i,i}$ equals to

$$c_{i,i} = \begin{cases} \prod_{j=0}^{i-1} \lambda_{\infty,j}, & i \leq d, \\ \lambda_{\infty,d}^{i-d} \prod_{j=0}^{d-1} \lambda_{\infty,j}, & i > d. \end{cases}$$

Thus we consider the convergence of $\left\{H^i\left(\overline{P}\right)\right\}_{i=0}^{\infty}$, it is noticed that for $\rho(A) > 1$, we have to ensure

$$\begin{aligned}&\lim_{i\to\infty} \frac{\text{Tr}\left[H^{i+1}\left(\overline{P}\right) - H^i\left(\overline{P}\right)\right]}{\text{Tr}\left[H^i\left(\overline{P}\right) - H^{i-1}\left(\overline{P}\right)\right]} \\ &= \lim_{i\to\infty} \lambda_{\infty,d} * \frac{\text{Tr}\left[h^{i+1}\left(\overline{P}\right) - h^i\left(\overline{P}\right)\right]}{\text{Tr}\left[h^i\left(\overline{P}\right) - h^{i-1}\left(\overline{P}\right)\right]} \\ &= \lambda_{\infty,d}\rho^2(A) < 1 \ \ \Rightarrow \lambda_{\infty,d} < \frac{1}{\rho^2(A)},\end{aligned}$$

which derives out the property.

4. NUMERICAL SIMULATIONS

Parameter setup: $A = \begin{pmatrix} 1 & 0.4 \\ 0.2 & 0.5 \end{pmatrix}$, $C = (0.5\ 0.8)$, $Q = \begin{pmatrix} 0.15 & 0 \\ 0 & 0.1 \end{pmatrix}$, $R = 0.1$, $d = 2$. The higher level $\Delta = 4$ and the lower level $\delta = 1$.

We first verify the convergence condition presented in Remark 7. It is obvious that the convergence can be satisfied if $\lambda_{\infty,2} < \frac{1}{\rho^2(A)} \approx 0.78$. We choose $\lambda_{\infty,2} = 0.7$ and 0.85 as examples for simulations shown in Fig. 3. From the result, when $\lambda_{\infty,2} = 0.7$, the estimator converges; when $\lambda_{\infty,2} = 0.85$, the estimator diverges.

Next, we compare the performance of different proposed schedules under the same power budget. Let the param-

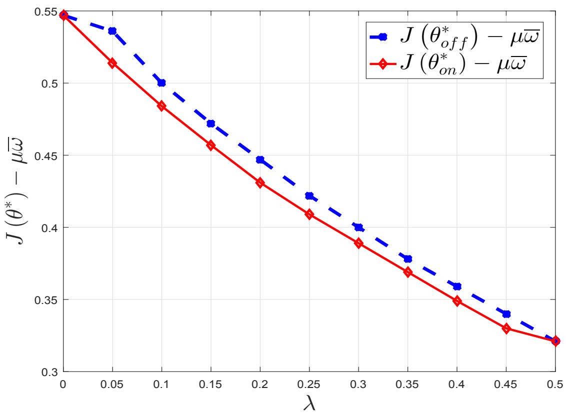

Fig. 4. Stability analysis with $\lambda_{\infty,2} = 0.7$ and 0.85.

eters $\lambda_0 = 0.2$, $\lambda_1 = 0.3$ and $\lambda_2 = 0.3$. We plot $J(\theta^*_{\text{off}})$ and $J(\theta^*_{\text{on}})$ versus different π_T with the step size 0.05 from range 0-1. The simulation results are shown in Fig. 3. When $\pi_T = 0$, as $\lambda_{\infty,2} = 0.2$, the convergence condition in Corollary 7 is satisfied, thus two schedules are equivalent and it holds that $J(\theta^*_{\text{off}}) = J(\theta^*_{\text{on}}) < +\infty$. The gap between l and τ_{th} is relatively high when π_T is small, and they tend to the same when π_T approaches 1, i.e., $J(\theta^*_{\text{off}}) - \mu\overline{\omega} = J(\theta^*_{\text{on}}) - \mu\overline{\omega} \to \text{Tr}\left(\overline{P}\right)$. It should be pointed out that when $\pi_T \geq 0.5$, we have the state domain including only 0 and 1, which leads to $J(\theta^*_{\text{off}}) = J(\theta^*_{\text{on}})$.

At the same time, the relative parameters, i.e., π_T, l, τ_{th} and β are listed in Tab. 1. From the table it can be seen that, both l and τ_{th} decrease as the power budget π_T increases. Moreover, τ_{th} is smaller than l for all range of π_T, which is consistent with our former analysis.

Based on the above results, we finally validate the global optimality with respect to different values of μ. Let $\mu = 0.05, 0.1$, 0.15 and 0.2. We plot the global optimal $J(\theta^*_{\text{off}})$ and $J(\theta^*_{\text{on}})$ with $\overline{\omega}^*$ as shown in Tab. 2. It is conspicuously seen that $J(\theta^*_{\text{on}})$ is superior to that of $J(\theta^*_{\text{off}})$ with the smaller cost.

Table 1. Parameters with respect to different π_T

π_T	0	0.05	0.1	0.15	0.2	0.25	0.3
l	$+\infty$	19	9	5	4	3	2
τ_{th}	$+\infty$	3	2	2	2	1	1
β	-	0.187	0.784	0.475	0.093	0.824	0.605
π_T	0.35	0.4	0.45	0.5	0.55	0.6	0.65
l	1	1	1	1	0	0	0
τ_{th}	1	1	0	0	0	0	0
β	0.391	0.182	0.982	0.833	0.703	0.588	0.486
π_T	0.7	0.75	0.8	0.85	0.9	0.95	1
l	0	0	0	0	0	0	0
τ_{th}	0	0	0	0	0	0	0
β	0.395	0.313	0.238	0.170	0.109	0.052	0

Table 2. Comparisons of $J(\theta^*_{\text{off}})$ and $J(\theta^*_{\text{on}})$ with respect to different μ

μ	0.05	0.1	0.15	0.2
$\overline{\omega}^*_{\text{off}}$	4	2.5	2	1
$J(\theta^*_{\text{off}})$	0.439	0.571	0.684	0.747
$\overline{\omega}^*_{\text{on}}$	4	2.33	1.63	1.17
$J(\theta^*_{\text{on}})$	0.439	0.563	0.669	0.740

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