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# Co-design of Sampling Pattern and Control in Self-triggered Model Predictive Control for Sampled-data Systems

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**Abstract:** This paper studies the event-triggered model predictive control (MPC) problem for networked control systems with input constraints, where the control is of the sampled-data form. A novel self-triggered MPC (STMPC) method which enables the optimal design of sampling pattern and control law is proposed to reduce the conservatism of separate design of trigger and control law in existing approaches. The conditions on ensuring the algorithm feasibility and the closed-loop system stability are developed. In addition, an upper bound of the closed-loop system performance is derived which provides performance guarantee for the designed STMPC. Finally, simulation results are presented to verify the effectiveness of the proposed STMPC method.

*Keywords:* Event-triggered control, networked control, sampled-data systems, model predictive control.

## 1. INTRODUCTION

Recently, the development of network and computation technologies make it possible to build and implement networked control systems, in which the plant and the controller are connected via a shared communication network. Such a control framework enables the remote control and enhances the extendibility of control systems, see Asif and Webb (2015). Therefore, it is of great importance to develop an optimal control algorithm to improve the control performance of the networked control. Considering the optimization-based nature and the extraordinary ability in handling systems with constraints, model predictive control (MPC) is an inspiring choice. However, traditional manner to transmit the control input periodically may cause excessive communication burden, which renders networked systems infeasible, as the communication resource is quite scar in networked systems. To address this issue, event-triggered MPC provides a promising scheme to transmit the control input aperiodically and efficiently.

According to different triggering mechanisms, existing results on the event-triggerd MPC mainly fall into two types, the event-based MPC (EMPC) and the self-triggered MPC (STMPC). The quintessence of the EMPC lies in that the strategy will not update the control law, unless the error between the real and the optimal predicted system states violates a designed threshold. In the framework of the EMPC, Yoo et al. (2017) studied discrete-time systems with large disturbances, in which the machine learning is introduced to attenuate the model uncertainty. Li and Shi (2014a) proposed a continuous-time EMPC algorithm to control nonlinear systems with bounded external disturbances, where a shrunken constraint is added to the predicted state trajectory of the nominal system. Liu et al. (2018) extended this work to a more general one, which provides a larger feasible state space and a more relaxed requirement on disturbances. Note that implementing the EMPC strategy demands an additional monitoring to continuously measure the real system state, which is impractical.

The STMPC gets rid of the persistent monitoring by specifying the next triggering instant in advance. For discrete-time systems, Gommans and Heemels (2015) first constituted the predicted control input trajectory with different triggering intervals, then the proposed STMPC strategy jointly designed the control law and the maximum triggering interval with the desired performance guarantee. Henriksson et al. (2015) proposed a STMPC algorithm to schedule multiple linear systems linked by a common communication network. Li et al. (2018) developed a new performance index considering the communication cost, then the co-design of the control law and the triggering interval was realized in the receding optimization. For continuous-time systems, in order to release the communication load of transmitting a continuous-time control input trajectory, Hashimoto et al. (2017) developed a STMPC algorithm to transport the control input with a sample-and-hold fashion, and they employed the Lyapunov stability conditions to determine triggering interval. In the same framework, He et al. (2018) approximated the continuous-time control input with a first-order-holder, which further prolonged the triggering interval.

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Note that practical networked control systems are realized in the digital or the computer control architecture, where continuous-time systems are usually controlled by the piecewise-constant control inputs. Thus, it is necessary to study the event-triggered MPC for sampled-data systems. In this framework, a continuous-time performance index is minimized with respect to a control sequence whose elements are predicted control input values holding during each sampling interval. Then, the triggering mechanism specifies the closed-loop control input by choosing suitable amount of the predicted control input values, and the triggering interval is set as the summation of the corresponding inter-sampling intervals. In the existing results on event-based sampled-data MPC, He and Shi (2015) developed an EMPC method to determine the number of control elements to be transmitted at each triggering instant, where all inter-sampling intervals equal to a fixed time span. It should be noted that the triggering interval is prolonged, while the amount of control elements to be transmitted is increased. Therefore, the communication burden might not be reduced. The STMPC strategy proposed by Hashimoto et al. (2016) implemented one control element at each triggering instant, but each inter-sampling interval can only be chosen from an integer multiple of some fixed time span, which is quite conservative.Yang and Wang (2018) presented a STMPC strategy transmitting the first element of the control sequence, in which the triggering condition is designed from the decrease of the optimal performance index, but the triggering condition is related with the Lipschitz constant of the stage cost, which made the strategy rather conservative, since the Lipschitz constant is over-estimated.

In this paper we propose a novel STMPC approach to design the triggering interval via optimization from an arbitrary sampling pattern to reduce the conservatism, and the controller only transmits one control element to the actuator at each triggering instant. The main contributions of this study are two-fold:

- A novel STMPC strategy is proposed for constrained sampled-data systems. In the proposed strategy, the sampling pattern and the system control input are jointly regarded as the optimization variables of the performance index in the receding-horizon optimization. In this way, the proposed scheme enables the co-design of the sampling instants and the MPC control input without utilizing the Lipschitz constant, which is different from Yang and Wang (2018). In addition, the inter-sampling intervals are designed online and can be any positive number, so the conservativeness in He and Shi (2015) and Hashimoto et al. (2016) is reduced. Besides, this work captures the work in Li et al. (2018) as a special case, where all inter-sampling intervals in the prediction horizon are set the same.
- The theoretical conditions of ensuring the algorithm feasibility and the closed-loop system stability are developed. In addition, we show the proposed strategy is free of the Zeno behavior and provides the performance guarantee.

The rest of this paper is organized as follows. Section 2 presents the description of the problem formulation, and Section 3 designs the detailed STMPC algorithm. In Section 4, the main theoretical results by implementing the proposed algorithm are provided. Then, a simulation example is introduced in Section 5. Finally, Section 6 gives the conclusion.

Notations: Let $\mathbb{R}^n$ denote the Euclidian space of all real n-dimensional column vectors. $\mathbb{I}_a^b$ represents the set of all integers from $a$ to $b$, and $\mathbb{R}^+$ denotes the set of all positive real numbers. For a given matrix $A$, its transpose is denoted by $A^T$. We use the $P$-weighted norm of the column vector $x \in \mathbb{R}^n$ to indicate the operation $\|x\|_P \triangleq \sqrt{x^T P x}$, where $P \in \mathbb{R}^{n\times n}$.

## 2. PROBLEM FORMULATION

Consider a networked continuous-time linear system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

with $x(0) = x_0$, where $x(t) \in \mathbb{R}^n$ is the system state and $u(t) \in \mathbb{R}^m$ denotes the control input. The pair $(A, B)$ is controllable and $u(t)$ is constrained by a compact set $\mathcal{U}$ with the origin in its interior, i.e., $u(t) \in \mathcal{U}$.

The structure of the networked control system is shown in Fig. 1. From the perspective of the networked control with the control law $u(t)$ designed by the MPC algorithm, the system in (1) controlled by a continuous-time control input trajectory suffers from two disadvantages:

i) Transmitting such a control input via the communication network requires infinite bandwidth.
ii) The computer has to solve an optimal control problem (OCP) with infinite-dimensional optimization variables, which is computationally intractable.

To this end, in this paper, we study the event-based MPC for sampled-data systems.

For the system in (1), by applying a constant control input $u(t_k)$ from the time instant $t_k$ to $t$, the system state emanating from $x(t_k)$ can be formulated as

$$x(t) = A(h_t)x(t_k) + B(h_t)u(t_k), \tag{2}$$

where $h_t = t - t_k$, and

$$A(h_t) = e^{Ah_t} \quad B(h_t) = \int_0^{h_t} e^{A(\tau)} d\tau B.$$

In the framework of the standard MPC algorithm for sampled-data systems, at each time instant $t_k$, an OCP is solved to obtain an optimal piecewise-constant control input trajectory $u^*(\tau; t_k)$. To be more specific, a performance index $J(\bar{\boldsymbol{u}}_{t_k}; x(t_k))$ with respect to a feasible control sequence $\bar{\boldsymbol{u}}_{t_k} \triangleq \{\bar{u}(t_k^0), \cdots, \bar{u}(t_k^{N-1})\}$ is first defined as

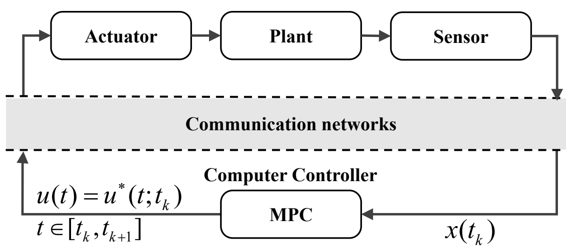


Fig. 1. Networked control with digital controller.

$$\begin{aligned} &J(\bar{\boldsymbol{u}}_{t_k}; x(t_k)) \\ &\triangleq \sum_{i=0}^{N-1} \int_{t_k^i}^{t_k^{i+1}} \left\| \bar{x}(\tau; t_k^i) \right\|_Q^2 + \left\| \bar{u}(t_k^i) \right\|_R^2 d\tau \\ &\quad + \left\| \bar{x}(t_k^N) \right\|_P^2, \end{aligned} \tag{3}$$

where the weighting matrixes $Q$, $R$ and $P$ are positive definite; $t_k^i$ with $i \in \mathbb{I}_0^N$ is the sampling instant, and $t_k^0 = t_k$; each inter-sampling interval $h(t_k^i) \triangleq t_k^{i+1} - t_k^i, i \in \mathbb{I}_0^{N-1}$ equals to a predefined positive constant $h$; $\bar{x}(\tau; t_k^i)$ denotes the feasible state trajectory, which is formulated as

$$\bar{x}(\tau; t_k^i) = A(\tau - t_k^i)x(t_k^i) + B(\tau - t_k^i)\bar{u}(t_k^i),$$

for all $i \in \mathbb{I}_0^{N-1}$, and $x(t_k^{i+1}) = A(h(t_k^i))x(t_k^i) + B(h(t_k^i))\bar{u}(t_k^i)$. Next, by minimizing the performance index in (3) with the satisfaction of constraints, an optimal control sequence $\boldsymbol{u}_{t_k}^*$ is calculated. Finally, the optimal piecewise-constant control input trajectory is

$$u^*(\tau; t_k) = u^*(t_k^i), \tag{4}$$

for all $\tau \in [t_k^i, t_k^{i+1})$ with $i \in \mathbb{I}_0^{N-1}$. The closed-loop system control input is then set as $u(t) = u^*(t; t_k)$, $t \in [t_k^0, t_k^1)$ and is updated at the next optimization instant or triggering instant

$$t_{k+1} = t_k + h(t_k^0). \tag{5}$$

However, in the standard MPC algorithm, it should be noticed that the sampling pattern determined by the inter-sampling interval sequence $\boldsymbol{h}_{t_k} \triangleq \{h(t_k^0), \cdots, h(t_k^{N-1})\}$ is pre-specified, which makes the obtained optimal solution be conservative. In addition, in order to reduce the communication load of networked systems, a longer triggering interval $h(t_k^0)$ is of great importance. To that end, we aim at designing a STMPC strategy with the following two remarkable features:

i) Co-design the inter-sampling interval sequence $\boldsymbol{h}_{t_k}$ and the control sequence $\bar{\boldsymbol{u}}_{t_k}$ to prolong $h(t_k^0)$.
ii) Design the performance index to incorporate the sampling interval (i.e., communication cost) and the control performance, simutaneously.

## 3. STMPC ALGORITHM

In this subsection, we first present a new performance index to be minimized as follows:

$$\begin{aligned} &J(\bar{\boldsymbol{u}}_{t_k}, \bar{\boldsymbol{h}}_{t_k}; x(t_k)) \\ &\triangleq \sum_{i=0}^{N-1} \frac{1}{e^{\gamma \bar{h}(t_k^i)} - 1} \int_{t_k^i}^{t_k^{i+1}} \left\| \bar{x}(\tau; t_k^i) \right\|_Q^2 + \left\| \bar{u}(t_k^i) \right\|_R^2 d\tau \\ &\quad + \left\| \bar{x}(t_k^N) \right\|_P^2, \end{aligned} \tag{6}$$

where $1/(e^{\gamma \bar{h}(t_k^i)} - 1)$ is used to address the effect of the communication cost in the corresponding inter-sampling interval $\bar{h}(t_k^i)$; $\gamma$ is the factor determined by a specific communication network.

**Remark 1.** For a given $\gamma$, it can be seen that: (i) The communication cost $1/(e^{\gamma \bar{h}(t_k^i)} - 1) \to \infty$ as $\bar{h}(t_k^i) \to 0$, which is consistent with the fact that transmitting a continuous signal requires infinite bandwidth. (ii) There is a tradeoff between the stage cost and the communication cost in each inter-sampling interval. More specifically, a longer inter-sampling interval $\bar{h}(t_k^i)$ will result in a larger stage cost, while the communication cost will decrease. (iii) The designed performance falls into the one defined in the standard MPC if $\bar{h}(t_k^i) = h$ for all $i \in \mathbb{I}_0^{N-1}$.

At each triggering instant $t_k$, we solve the following OCP to co-calculate the optimal control sequence $\boldsymbol{u}_{t_k}^*$ and the optimal inter-sampling interval sequence $\boldsymbol{h}_{t_k}^*$.

**Problem $\mathcal{P}$ :**

$$\begin{aligned} \min_{\bar{\boldsymbol{u}}_{t_k}, \bar{\boldsymbol{h}}_{t_k}} \; & J(\bar{\boldsymbol{u}}_{t_k}, \bar{\boldsymbol{h}}_{t_k}; x(t_k)), \text{ subject to} \\ & \bar{u}(t_k^i) \in \mathcal{U} \\ & \delta \leqslant \bar{h}(t_k^i) \leqslant h_{max} \\ & \left\| \bar{x}(t_k^N) \right\|_P^2 \in \Omega(\varepsilon) \end{aligned} \tag{7}$$

where $\delta \in \mathbb{R}^+$ and $h_{max} \in \mathbb{R}^+$ are the lower bound and the upper bound of the inter-sampling intervals, respectively, and $\Omega(\varepsilon) \triangleq \{x \in \mathbb{R}^n : \|x\|_P^2 \leqslant \varepsilon\}$ denotes the terminal region, which satisfies the following assumption.

**Assumption 1.** For the sampled-data system in (2), there exists a local linear control law $u(t) = Kx(t) \in \mathcal{U}$ such that

$$\begin{aligned} &\|x(t+\delta)\|_P^2 - \|x(t)\|_P^2 \\ &\leqslant -\frac{1}{e^{\gamma\delta} - 1} \int_t^{t+\delta} \|x(\tau; t)\|_Q^2 + \|u(t)\|_R^2 d\tau, \end{aligned} \tag{8}$$

for all $x(t) \in \Omega(\varepsilon)$.

**Remark 2.** In practical applications, communication networks usually suffer from the bandwidth limitation, which makes the requirement on a minimum inter-sampling interval $\delta$ for sampled-data systems. Besides, from the point view of the computational feasibility and the practical implementation, the existence of $h_{max}$ is also reasonable.

**Remark 3.** Considering that the pair (A,B) is controllable, an optional choice of $K$ is to employ the linear quadratic regulator (LQR) of the continuous-time system in (1). Next, we set the terminal weighting matrix $P$ as the solution to the following Lyapunov function.

$${A_{cl}^\delta}^T P A_{cl}^\delta - P = -\frac{1}{e^{\gamma\delta} - 1} \int_0^\delta ({A_{cl}^\tau}^T Q A_{cl}^\tau + K^T R K) d\tau - \epsilon I, \tag{9}$$

where $A_{cl}^\delta = A(\delta) + B(\delta)K$ and $A_{cl}^\tau = A(\tau) + B(\tau)K$, respectively, and $\epsilon$ is a positive constant, which is small enough. Then, Assumption 1 holds by choosing suitable value of $\varepsilon$ such that $Kx \in \mathcal{U}$ for all $x \in \Omega(\varepsilon)$.

Now, the detailed description of the proposed STMPC algorithm is shown in Algorithm 1.

**Algorithm 1** STMPC Algorithm

**Require:** Initial the triggering instant $t_0$ and set $k = 0$.
1 : Sample the system state to obtain $x(t_k)$.
2 : Solve Problem $\mathcal{P}$ to obtain $\{\boldsymbol{u}_{t_k}^*, \boldsymbol{h}_{t_k}^*\}$.
3 : Apply $u^*(t_k^1)$ to the sampled-data system in (2) for $t \in [t_k, t_k + h^*(t_k^0))$.
4 : Set the next triggering instant $t_{k+1} = t_k + h^*(t_k^0)$ and make $t_k \leftarrow t_{k+1}$
5 : Return to step 1.

## 4. MAIN THEORETICAL RESULTS

### 4.1 Feasibility Analysis

The theoretical results of the algorithm feasibility is summarized as follows.

*Theorem 1.* For the sampled-data system in (2), suppose that Assumption 1 holds, and Problem $\mathcal{P}$ has a solution for the initial system state $x(0)$. Then the proposed STMPC algorithm is always feasible.

**Proof.** We prove the results by using the mathematical induction.

At a triggering instant $t_k$, assume that Problem $\mathcal{P}$ admits an optimal control sequence $\boldsymbol{u}^*_{t_k}$ and an optimal inter-sampling interval sequence $\boldsymbol{h}^*_{t_k}$. Then, according to Algorithm 1, the next triggering instant $t_{k+1} = t_k + h^*(t_k^0)$.

At $t_{k+1}$, by choosing the inter-sampling interval sequence

$$\bar{\boldsymbol{h}}_{t_{k+1}} \triangleq \{\bar{h}(t^0_{k+1}), \cdots, \bar{h}(t^{N-1}_{k+1})\}$$

as

$$\bar{\boldsymbol{h}}_{t_{k+1}} = \{h^*(t^1_k), \cdots, h^*(t^{N-1}_k), \delta\}, \tag{10}$$

we have that the corresponding sampling instants are

$$t^i_{k+1} = \begin{cases} t^{i+1}_k & i \in \mathbb{I}^{N-1}_0, \\ t^N_k + \delta & i = N. \end{cases} \tag{11}$$

If we design the control sequence

$$\bar{\boldsymbol{u}}_{t_{k+1}} \triangleq \{\bar{u}(t^0_{k+1}), \cdots, \bar{u}(t^{N-1}_{k+1})\}$$

as

$$\bar{\boldsymbol{u}}_{t_{k+1}} = \{u^*(t^1_k), \cdots, u^*(t^{N-1}_k), Kx^*(t^N_k)\}, \tag{12}$$

it is straightforward to get that the feasible state trajectory

$$\bar{x}(\tau; t^i_{k+1}) = x^*(\tau; t^{i+1}_k),$$

for all $\tau \in [t^i_{k+1}, t^{i+1}_{k+1})$ with $i \in \mathbb{I}^{N-2}_0$, and one can obtain

$$\bar{x}(\tau; t^{N-1}_{k+1}) = (A(\tau - t^N_k) + BK)x^*(t^N_k), \tag{13}$$

for all $\tau \in [t^{N-1}_{k+1}, t^N_{k+1})$. In particular, $\bar{x}(t^N_{k+1}) = (A(\delta) + BK)x^*(t^N_k)$. By using the similar argument in Chen and Allgöwer (1998), we show that $\bar{\boldsymbol{u}}_{t_{k+1}}$ and $\bar{\boldsymbol{h}}_{t_{k+1}}$ make all the constraints in Problem $\mathcal{P}$ be satisfied, i.e., the two sequences $\bar{\boldsymbol{u}}_{t_{k+1}}$ and $\bar{\boldsymbol{h}}_{t_{k+1}}$ constitute a feasible solution to Problem $\mathcal{P}$ at $t_{k+1}$.

The proof is completed.

□

### 4.2 Stability Analysis

Considering that the system to be controlled is executed in a continuous-time manner, in this subsection, we develop the closed-loop system stability results by showing the monotonic decrease of a continuous-time Lyapunov function.

Before proceeding on the stability results, we first claim that the proposed algorithm will not result in the Zeno behavior, i.e, triggering Problem $\mathcal{P}$ infinite times in finite timespan. In Algorithm 1, noticing that the triggering interval $t_{k+1} - t_k$ uniquely depends on the first element of the inter-sampling interval sequence $\bar{\boldsymbol{h}}_{t_{k+1}}$, we can conclude that the inter-triggering time interval is lower bounded by $\delta$.

Then the detailed results on the closed-loop stability are summarized in the following theorem.

*Theorem 2.* For the sampled-data system in (2), suppose that Assumption 1 holds. Then the closed-loop system is asymptotically stable.

**Proof.** At each triggering time instant $t_k$, we define the Lyapunov function $V(x(t_k))$ as the optimal performance index of Problem $\mathcal{P}$, that is

$$V(x(t_k)) \triangleq J(\boldsymbol{u}^*_{t_k}, \boldsymbol{h}^*_{t_k}; x(t_k)). \tag{14}$$

For all $t \in (t_k, t_{k+1})$, the $V(x(t))$ employed is

$$\begin{aligned} V(x(t)) \triangleq & J(\boldsymbol{u}^*_{t_k}, \boldsymbol{h}^*_{t_k}; x(t_k)) - \frac{1}{e^{\gamma h^*(t^0_k)} - 1} \\ & \int_{t^0_k}^{t} (\left\|x^*(\tau; t^0_k)\right\|^2_Q + \left\|u^*(t^0_k)\right\|^2_R d\tau. \end{aligned} \tag{15}$$

Next we prove the results from the following two facts:

i) $V(x(t_k))$ is monotonically decreasing at each triggering instant $t_k$.
ii) $V(x(t))$ is monotonically decreasing within two successive triggering instants.

We denote the difference of the designed Lyapunov functions between the triggering instants $t_{k+1}$ and $t_k$ as

$$\triangle V_{t_k} \triangleq V(x(t_{k+1})) - V(x(t_k)). \tag{16}$$

Considering the optimality of the performance index at $t_{k+1}$, an upper bound of $\triangle V_{t_k}$ is given as follows.

$$\begin{aligned} & \triangle V_{t_k} \\ \leqslant & J(\bar{\boldsymbol{u}}_{t_{k+1}}, \bar{\boldsymbol{h}}_{t_{k+1}}; x(t_{k+1})) - J(\boldsymbol{u}^*_{t_k}, \boldsymbol{h}^*_{t_k}; x(t_k)) \\ = & \sum_{i=0}^{N-2} \frac{1}{e^{\gamma \bar{h}(t^i_{k+1})} - 1} \int_{t^i_{k+1}}^{t^{i+1}_{k+1}} \left\|x^*(\tau; t^i_{k+1})\right\|^2_Q + \left\|u^*(t^i_{k+1})\right\|^2_R d\tau \\ & - \sum_{i=1}^{N-1} \frac{1}{e^{\gamma \bar{h}(t^i_k)} - 1} \int_{t^i_k}^{t^{i+1}_k} \left\|x^*(\tau; t^i_k)\right\|^2_Q + \left\|u^*(t^i_k)\right\|^2_R d\tau \\ & - \frac{1}{e^{\gamma h^*(t^0_k)} - 1} \int_{t^0_k}^{t^1_k} \left\|x^*(\tau; t^0_k)\right\|^2_Q + \left\|u^*(t^0_k)\right\|^2_R d\tau \\ & + \frac{1}{e^{\gamma\delta} - 1} \int_{t^{N-1}_{k+1}}^{t^N_{k+1}} \left\|\bar{x}(\tau; t^{N-1}_{k+1})\right\|^2_Q + \left\|\bar{u}(t^{N-1}_{k+1})\right\|^2_R d\tau \\ & + \left\|\bar{x}(t^N_{k+1})\right\|^2_P - \left\|x^*(t^N_k)\right\|^2_P . \end{aligned} \tag{17}$$

Substituting the feasible sampling interval sequence $\bar{\boldsymbol{h}}_{t_{k+1}}$ in (10) and the feasible control sequence $\bar{\boldsymbol{u}}_{t_{k+1}}$ in (12) into (17), it yields

$$\begin{aligned} \triangle V_{t_k} \leqslant & - \frac{1}{e^{\gamma h^*(t^0_k)} - 1} \int_{t^0_k}^{t^1_k} \left\|x^*(\tau; t^0_k)\right\|^2_Q + \left\|u^*(t^0_k)\right\|^2_R d\tau \\ & + \frac{1}{e^{\gamma\delta} - 1} \int_{t^N_k}^{t^N_k + \delta} \left\|\bar{x}(\tau; t^{N-1}_{k+1})\right\|^2_Q + \left\|Kx^*(t^N_k)\right\|^2_R d\tau \\ & + \left\|\bar{x}(t^N_{k+1})\right\|^2_P - \left\|x^*(t^N_k)\right\|^2_P . \end{aligned} \tag{18}$$

Following from the state trajectory defined in (13) and using Assumption 1, we acquire

$$\triangle V_{t_k} \leqslant -\frac{1}{e^{\gamma h^*(t_k^0)}-1}\int_{t_k^0}^{t_k^1} \left\|x^*(\tau;t_k^0)\right\|_Q^2 + \left\|u^*(t_k^0)\right\|_R^2 d\tau \tag{19}$$

When $t \in (t_k, t_{k+1})$, taking the designed Lyapunov function in (15) into consideration, it can be shown that $\triangle V_t \triangleq V(x(t)) - V(x(t_k))$ satisfies

$$\triangle V_t = -\frac{1}{e^{\gamma h^*(t_k^0)}-1}\int_{t_k^0}^{t}(\left\|x^*(\tau;t_k^0)\right\|_Q^2 + \left\|u^*(t_k^0)\right\|_R^2) d\tau. \tag{20}$$

Because $h^*(t_k^0) \geqslant \delta$ holds at any triggering instant, it follows that $\triangle V_t$ is monotonically decreasing with respect to the inter-triggering instant $t$. In addition, $\triangle V_{t_k}$ is also guaranteed to monotonically decreasing with respect to the triggering instant $t_k$.

Finally, taking the limit of $\triangle V_t$ as $t$ approaches $t_{k+1}$, we have

$$\triangle V_{t_{k+1}}^- = -\frac{1}{e^{\gamma h^*(t_k^0)}-1}\int_{t_k^0}^{t_k^{1-}} \left\|x^*(\tau;t_k^0)\right\|_Q^2 + \left\|u^*(t_k^0)\right\|_R^2 d\tau, \tag{21}$$

which indicates that $\triangle V_{t_k} \leqslant \triangle V_{t_{k+1}}^-$. Then a simple induction gives the conclusion that the Lyapunov function $V(x(t))$ is monotonically decreasing for all $t \geqslant t_0$.

Following the theorem 1 in Chen and Allgöwer (1998), the asymptotic stability results of the closed-loop system is proved.

□

*4.3 Performance Guarantee*

In this subsection, we provide the performance guarantee of the closed-loop system.

We first show that $V(x(t)) \to 0$ as $t \to \infty$. Using the inequality in (19) and the inequality in (20), we get

$$\begin{aligned} &V(x(t)) - V(x(t_0)) \\ &\leqslant -\sum_{i=0}^{k}\frac{1}{e^{\gamma h^*(t_i^0)}-1}\int_{t_i^0}^{t_i^1} \left\|x^*(\tau;t_k^0)\right\|_Q^2 + \left\|u^*(t_k^0)\right\|_R^2 d\tau \\ &\quad -\frac{1}{e^{\gamma h^*(t_k^0)}-1}\int_{t_k^0}^{t} \left\|x^*(\tau;t_k^0)\right\|_Q^2 + \left\|u^*(t_k^0)\right\|_R^2 d\tau, \end{aligned} \tag{22}$$

where $t_k^0$ is the closest triggering instant before $t$. Noting that both $V(x(t_0))$ and the right hand side of the inequality (22) are bounded, and by using the nonnegative property of the $V(x(\infty))$, we can obtain that

$$V(x(\infty)) = 0. \tag{23}$$

Then, if we plug (23) back into (22), it follows

$$\begin{aligned} &\sum_{i=0}^{\infty}\frac{1}{e^{\gamma h^*(t_i^0)}-1}\int_{t_i^0}^{t_i^1} \left\|x^*(\tau;t_k^0)\right\|_Q^2 d\tau \\ &+\frac{1}{e^{\gamma h^*(t_k^0)}-1}\int_{t_k^0}^{t} \left\|x^*(\tau;t_k^0)\right\|_Q^2 d\tau \leqslant V(x(t_0)). \end{aligned} \tag{24}$$

In particular, when $t = t_k$ and $k = \infty$, one has

$$\sum_{i=0}^{\infty}\frac{1}{e^{\gamma h^*(t_i^0)}-1}\int_{t_i^0}^{t_i^1} \left\|x^*(\tau;t_k^0)\right\|_Q^2 d\tau \leqslant V(x(t_0)). \tag{25}$$

## 5. SIMULATION

This section implements the proposed STMPC algorithm on a cart-spring-damper system. By linearizing the cart-spring-damper system in Li and Shi (2014b) at the equilibrium point, we obtain the following system dynamics:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -k/m & -h/m\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1/m\end{bmatrix}u, \tag{26}$$

where the position $x_1$ and the velocity $x_2$ compose the system state variable $x = [x_1, x_2]^T$; $k = 0.25N/s$ is the spring factor, and $h = 1.2N/s$ represents the damping coefficient; $m = 1.5kg$ denotes the mass. The control input is constrained by $\|u\| \leqslant 1$.

The state weighting matrix $Q = 0.5I_2$, and the input weighting matrix is designed as $R = 0.1$; the number of the control elements in the prediction horizon is $N = 5$. Here, we choose the minimal and the maximal inter-sampling intervals as $\delta = 0.1s$ and $h_{max} = 1s$, respectively, and the parameter $\gamma$ in the communication cost is set to be 0.1. In order to make Assumption 1 hold, we calculate the terminal weighting matrix $P = \begin{bmatrix}12.4745 & -4.0757\\ -4.0757 & 5.1220\end{bmatrix}$ according to remark 3, and we set the terminal region parameter $\varepsilon$ as 0.32.

In experiment, we operate the simulation on the MATLAB package, and the *fmincon* function is employed to solve the optimization problem. By selecting the initial system state $x = [-2.5, 2]^T$, the results are shown in Figs. 2-5. Fig. 2 and Fig. 3 are the evolution of the system state and the control input trajectories, respectively. These two figures show that the system state eventually converge to the origin and the control input satisfies the constraints. Fig. 4 verifies that the proposed algorithm results in an aperiodic update of the OCP, which is the so-called event-triggered mechanism. In Fig. 5, we find that the minimal triggering interval is around 0.55$s$, which is much longer than the triggering lower bound, i.e., 0.1s.

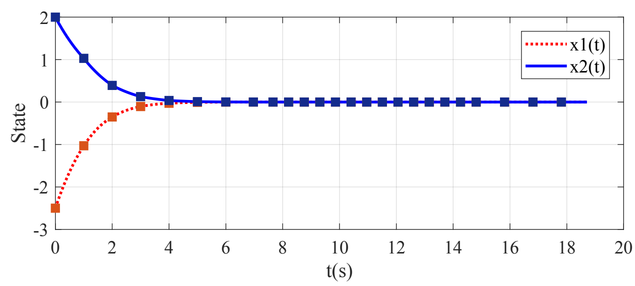


Fig. 2. The system state trajectories.

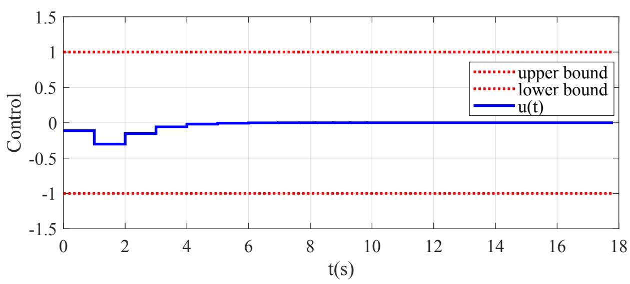


Fig. 3. The control input trajectory and its bounds.

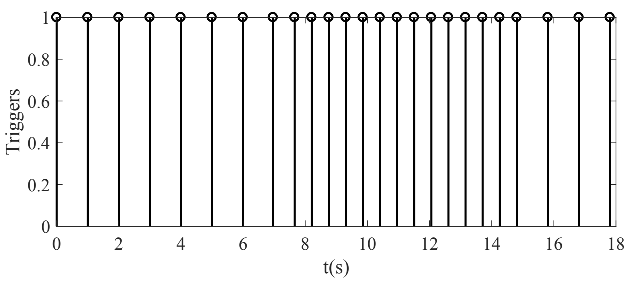


Fig. 4. The distribution of the triggering instants.

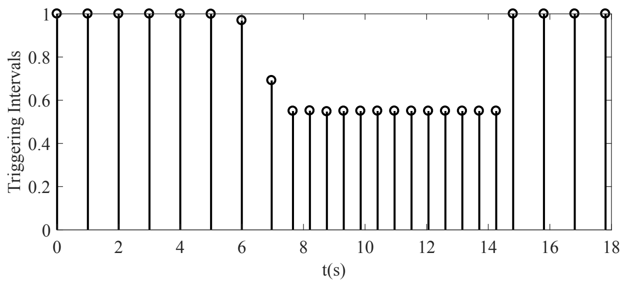


Fig. 5. The triggering interval at each triggering instant.

## 6. CONCLUSION

We have designed a new STMPC scheme for linear sampled-data systems with input constraints. In the new scheme, we have added the inter-sampling intervals as the decision variables in the receding horizon optimization, with this, we have realized the co-design of the inter-sampling interval and the control sequence. This reduces the conservatism of the existing results. In addition, we have shown that under mild assumption, the proposed algorithm is feasible and the closed-loop system is asymptotically stable, and the designed algorithm provides a ensured performance index.

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