

# Real-time multiple model joint estimation for an urban traffic junction subject to jump dynamics

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**Abstract:** Traffic conditions in signalized junctions are highly dynamic and may be subject to abrupt changes due to unanticipated traffic incidents or network obstructions. These abrupt changing conditions are represented as different regimes or modes where each mode is represented by its own distinct model, forming a set of multiple models. At any instance in time, only one model of the set has the potential of representing the physical system dynamics at that time. However the dynamics may arbitrarily jump over to a different regime when an abnormal condition arises. Furthermore, it might be impossible to identify these models *a priori*. Hence, a multiple model approach is developed to self-detect these abrupt changes, identify which member of the set best represents the actual system and automatically self-configure and add a new model to the set when a previously unmodelled regime arises. This approach makes use of a real-time joint (dual) estimation algorithm to estimate traffic state variables such as queue lengths and traffic flow, as well as model parameters such as turning ratios, saturation flow values and noise covariance resulting from unmodelled dynamics and measurement errors. The proposed algorithm is validated through simulations on signalized 3-arm and 4-arm junctions with typical day-to-day traffic conditions including several network irregularities occurring at different times of the day such as arm closures as a result of traffic incidents. This work is aimed to form part of adaptive control loops for traffic light systems that are able to autonomously adjust to changing traffic conditions so as to ensure efficient vehicle flows.

*Keywords:* intelligent traffic systems, multiple model estimation, jump dynamics, online estimation, self-detection, self-configuration

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## 1. INTRODUCTION

Traffic conditions of signalized junctions may be subject to several potential dynamical regimes that change over abruptly in time due to traffic incidents or unanticipated network obstructions. Under such conditions a model structure and its parameters are not constant or varying slowly, but switch value abruptly, a phenomenon known as jump dynamics (Fabri and Kadiramanathan, 2001). Hence the dynamics of the traffic system are not represented by one model, but by a set of different models each corresponding to a given condition. Furthermore, it might be impossible to identify these models *a priori*. Consequently, a system that is able to detect in real-time any switching among the multiple regimes as well as learning and modelling the dynamics of each of these regimes, will be of great benefit. This is called multiple model estimation (Soken and Sakai, 2016) and it lends itself very well to traffic junction scenarios.

Several research efforts have been directed to multiple model approaches in different applications (Baldi et al., 2011; Hespanha et al., 2003). Mode switching detection has been tackled using two predominant techniques: deterministic approaches (Baldi et al., 2011) or stochastic (Hespanha et al., 2003). The essence of both techniques

is based on choosing the model which yields some best defined estimation accuracy out of all the models available in a set of candidate models at any point in time.

Multiple model adaptive estimation has been previously applied for traffic incident detections for freeway conditions. For example, a multiple model extended Kalman filter was used by Willsky et al. (1980), where known changing conditions and known models parameters are assumed. A limited number of known changing conditions are assumed, hence resulting in a finite set of models. An extended Kalman filter is proposed for each model to sequentially estimate the traffic states. The residual data is produced by comparing measured data with the estimated states, and probabilities are assigned to each model based on how closely the residual characteristics match their respective anticipated values.

Similarly, Wang and Work (2014) applied an interactive multiple model ensemble Kalman filter, with a known number of changing conditions and known models. An ensemble Kalman filter is proposed to solve the sequential state estimation problem and to accommodate the switching dynamics and nonlinearity of the traffic incident model. Unlike the work of Willsky et al. (1980), the Interactive Multiple Model algorithm (IMM) makes use of Markov

chains to model the evolution of the transitions from one model to another. Furthermore, Wang et al. (2016b) applied multiple model particle filtering for the traffic state estimation to improve the accuracy of the estimate when data is limited. Wang et al. (2016a) extends previous works by proposing an Efficient Multiple Model Particle Filter (EMMPF) that uses a single sample in each particle filter to infer the correct model and then all model particles are evolved forward in time using the most likely model determined in the selection step.

All the above works on traffic estimation and incident detection which made use of the extended Kalman filter and particle filter have been applied and performed well in freeway traffic conditions. In this work, a novel stochastic multiple model adaptive estimation algorithm is developed and applied for urban signalized traffic junctions, where, similar to previously cited literature, the algorithm needs to learn the traffic regimes and determine which mode is active at any given time. However, whereas the reviewed literature assumed that the monitoring data contained a limited number of previously modelled regimes, this work will not be limited to previously known modes. As proposed by Fabri and Kadirkamanathan (2001) for general systems, the algorithm is adapted for urban traffic junctions and is able to learn the various modes in real-time and hence automatically configure and grow its model set as new modes are detected. Furthermore, an online joint state and parameter estimation technique is applied to estimate the traffic states, apart from the model parameters and the noise parameters that are not assumed known *a priori*, and will be estimated online. This avoids the need to calibrate such noise parameters based on the type of sensors present in the junction and the noise they may produce.

The proposed multiple model adaptive estimation algorithm makes use of an online dual estimation algorithm to jointly estimate in real-time traffic states such as queue lengths, occupancies and flows, as well as the model parameters such as turning ratios, saturation flows and noise parameters. The approach uses an EM algorithm, modified for real-time estimation, with a Kalman filter implementing the expectation step and a multivariate gradient-based approach for the maximisation step, as proposed by Zammit et al. (2019). The algorithm computes residual data by comparing the measured information with the estimated states and assigns probabilities to each model in the set. A new mode is learnt if the residual characteristics do not match any of the previously learnt modes in the set, as will be discussed in the next section.

## 2. MULTIPLE MODEL ESTIMATION

Suppose that  $H$  distinct modes of operation are postulated initially, each representing one specific regime. Following the generic notation of Fabri and Kadirkamanathan (2001), only one particular mode from these  $H$  could be active at any given time  $t$ . Denote this as mode  $f$ , where  $f \in [1, \dots, H]$ . The dynamics of each mode  $f$  can be represented by model  $M^f$  in discrete-time stochastic state-space form, (where in this case, time  $t$  denotes the traffic light cycle index) as the one presented by Zammit et al. (2019), with state space matrices  $\mathbf{A}^f$ ,  $\mathbf{B}^f$ ,  $\mathbf{C}^f$  and

$\mathbf{D}^f$ , process and sensor noise covariances  $\mathbf{Q}^f$  and  $\mathbf{R}^f$ , and model parameters  $\Theta^f$  which could either be known *a priori* or else estimated online as presented by Zammit et al. (2019). Thus for model  $M^f$ ,

$$\begin{aligned} \mathbf{x}_{t+1}^f &= \mathbf{A}^f \mathbf{x}_t^f + \mathbf{B}^f \mathbf{z}_t + \mathbf{w}_t \\ \mathbf{y}_t^f &= \mathbf{C}^f \mathbf{x}_t^f + \mathbf{D}^f \mathbf{z}_t + \mathbf{v}_t \end{aligned} \quad (1)$$

where  $\mathbf{z}_t$  is the control input, representing the proportion of green traffic signal in a cycle  $t$ ,  $\mathbf{w}_t$  and  $\mathbf{v}_t$  are zero mean, Gaussian distributed process and measurement noise. The state variables in  $M^f$  comprise the queue length in arm  $i$ , denoted as  $\zeta_i(t)$ , the inflow in arm  $i$ , denoted as  $\gamma_{I_i}(t)$  and the occupancy  $\phi_i(t)$ . The vector of model parameters  $\Theta^f$  is given by  $\Theta^f = [\alpha_{12}, \alpha_{13}, \dots, \alpha_{1i}, \alpha_{21}, \alpha_{23}, \dots, \alpha_{2i}, \dots, \alpha_{i1}, \alpha_{i2}, \alpha_{i3}, \dots, \alpha_{i(i-1)}, \kappa_1, \kappa_2, \kappa_3, \dots, \kappa_i, \beta_1, \beta_2, \beta_3, \dots, \beta_i, S_1, S_2, S_3, \dots, S_i, ]$  as presented by Zammit et al. (2019), where  $\alpha_{ij}$  represents the ratio of cars turning from arm  $i$  to arm  $j$ ,  $\kappa_i$  and  $\beta_i$  characterize  $\phi_i(t)$  and  $S_i$  represents the saturation value for arm  $i$ .

Let  $E^f(t)$  denote the event that the system dynamics at time  $t$  correspond to model  $M^f$  with parameters  $\Theta^f$ . Also let  $S_j(t)$  represent one specific sequence of such events from start up to time  $t$  (e.g.  $S_1(t) = \{E^1(1), E^2(2), E^2(3), \dots, E^1(t)\}$  denotes an example of one possible sequence). Since every element of this sequence has  $H$  possibilities, there exist  $H^t$  different possible sequences at time  $t$ , whereby only one of these  $H^t$  has actually taken place.

The question that follows is how to identify which of the modes is active at time  $t$ . This can be addressed by finding the probability that a model in the set  $\{M^1, M^2, \dots, M^H\}$  is best representing the current observations. A Kalman filter is matched to each mode and a probabilistic framework is formulated to identify the posterior probability of the event that the actual model sequence conditioned on the observation set,  $\mathbf{Y}^t$ , is  $S_f$ . This approach is faced with evaluation complications and high computation and storage requirements (Fabri and Kadirkamanathan, 2001), as the number of possible sequences to be considered increases exponentially with time, hence becoming impractical to implement. Three main possible sub-optimal solutions have been proposed to mitigate such complications: the *Generalised Pseudo-Bayes (GPB) method*, the *Interacting Multiple Model (IMM) method* and the *lower bounding approach*. The GPB method (Ackerson and Fu, 1970) and the IMM approach (Blom and Bar-Shalom, 1988) introduce the concept of *pruning* to limit the increase in the number of Kalman filters and *merging* for the state estimation information to be propagated to the filters. These approaches offer a reliable suboptimal solution to the problem of switching dynamics but computational complexity could still be rather high.

To reduce the computational demand even further, the lower bounding approach can be considered. To explain this approach, the problem of identifying the most probable model from a given set of possible modes in a non-jump dynamics scenario is first considered. Bayes' rule is applied to infer the posterior probability of model  $M^f$  conditioned on  $\mathbf{Y}^t$ :

$$Pr(M^f | \mathbf{Y}^t) = \frac{p(\mathbf{y}_t | M^f, \mathbf{Y}^{t-1}) Pr(M^f | \mathbf{Y}^{t-1})}{\sum_{j=1}^H p(\mathbf{y}_t | M^j, \mathbf{Y}^{t-1}) Pr(M^j | \mathbf{Y}^{t-1})} \quad (2)$$

where  $Pr()$  denotes probability and  $p()$  denotes a probability distribution.

Since the models are linear with Gaussian noise, a Kalman filter matched to each mode can be used to calculate the mean square estimate of state  $\mathbf{x}_t$  according to each model  $M^f$ , denoted as  $\hat{\mathbf{x}}_t^f$  and the corresponding covariance of the estimation error,  $\mathbf{P}_{t|t-1}^f$ . The likelihood function can then be evaluated by the normal distribution function:

$$p(\mathbf{y}_t|M^f, \mathbf{Y}^{t-1}) = -\frac{1}{(2\pi)^{\frac{1}{2}}|\mathbf{Z}_t^f|^{\frac{1}{2}}} \exp^{-\frac{1}{2}(\mathbf{y}_t - \hat{\mathbf{y}}_t^f)'(\mathbf{Z}_t^f)^{-1}(\mathbf{y}_t - \hat{\mathbf{y}}_t^f)} \quad (3)$$

where the corresponding variance  $\mathbf{Z}_t^f$  attributed to model  $M^f$  is estimated as:

$$\mathbf{Z}_t^f = \mathbf{C}^f \mathbf{P}_{t|t-1}^f \mathbf{C}^{f'} + \mathbf{R}^f \quad (4)$$

$\hat{\mathbf{y}}_t^f$  denotes the estimate of the observation  $\mathbf{y}_t$ , where:

$$\hat{\mathbf{y}}_t^f = \mathbf{C}^f \hat{\mathbf{x}}_t^f + \mathbf{D}^f \mathbf{z}_t \quad (5)$$

Thus  $(\mathbf{y}_t - \hat{\mathbf{y}}_t^f)$  in Equation (3) represents the difference between the observation  $\mathbf{y}_t$  and its estimate  $\hat{\mathbf{y}}_t^f$ , referred to as the residual.

In a jump dynamics scenario, where switching between different modes could occur at any time  $t$ , Equation (2) is used together with a small lower bound placed on the computed candidate probabilities, to prevent a mode from being *locked-out* (Pogoda and Maybeck, 1989) because its associated probability has gone to zero. Hence the lower bounding approach requires only  $H$  Kalman filters at every time instant, one for each possible mode, as opposed to GBP which requires a maximum of  $H^k$  Kalman filters pruned down to  $H$  after  $d$  time steps.

The lower bounding approach that was discussed in this section will now be applied to signalized traffic junctions. As noted previously, the model development for the 3-arm and 4-arm junctions and the online joint estimation algorithm for the model states and parameters, all discussed by Zammit et al. (2019) will be used.

### 3. MULTIPLE MODEL APPROACH FOR SIGNALIZED JUNCTIONS

Suppose that at some cycle  $t$ , a set of  $H$  candidate models for different regimes of operation are known, each corresponding to a mode that already occurred. A self-organized model allocation approach (Fabri and Kadiramanathan, 2001) is applied, whereby any new modes are learnt in real-time, hence growing the model set if a new mode occurs. Just after the  $H^{th}$  model has been allocated by the mode estimation algorithm, as a result of a new mode being detected active, a freshly initialized model is introduced by adding a randomly initialized parameter vector for this model. The probability density function of the freshly initialized model is initially made wide because it is not yet tuned to any mode. This narrows as the variance decreases when it starts learning a new mode. This procedure of adding a fresh local model once the previous ones have been allocated is repeated continuously, hence growing the set of candidate models in the multiple model set. Thus there is always one spare model ready to accept a new regime that has not appeared before.

Table 1. Estimation Algorithm

<p>Initialise <math>\hat{\Theta}^f</math>, <math>\hat{\mathbf{Q}}^f</math> and <math>\hat{\mathbf{R}}^f</math> estimates for <math>H</math> local models.                  Step 1:                  Measure <math>\mathbf{y}_t</math>.  <b>E-step</b>                  Run Kalman-filter recursions for each local candidate model <math>f</math> to compute <math>\hat{\mathbf{x}}_t^f</math>, where <math>f = 1 \dots H</math>.  <b>M-step</b>                  For each candidate model <math>f</math>, (<math>f = 1 \dots H</math>), minimise <math>-2E\{G(\Theta^f, \hat{\Theta}_t^f)\}</math> over <math>\Theta^f</math> for dynamic traffic conditions as presented by Zammit et al. (2019)                  Minimise <math>-2E\{G(\Theta^f, \hat{\Theta}_t^f)\}</math> over <math>\hat{\mathbf{Q}}^f</math> and similarly for <math>\hat{\mathbf{R}}^f</math> as presented by Zammit et al. (2019).                  Calculate the posterior probability distribution for such candidate model <math>f</math> given in Equation (2).                  Just after the <math>H^{th}</math> model has been selected, as a result of a new mode being detected active, freshly initialize a new model with randomly selected <math>\hat{\Theta}^f</math>, <math>\hat{\mathbf{Q}}^f</math> and <math>\hat{\mathbf{R}}^f</math>, and Let <math>H \rightarrow H + 1</math>.                  Calculate <math>\hat{\mathbf{x}}_t</math> using Equation (6) or by <math>\hat{\mathbf{x}}_{t_{max:Pr(M^j \mathbf{Y}^t)}}</math> representation.                  For the current active model, that is the model which corresponds to the <math>max:Pr(M^j \mathbf{Y}^t)</math> update <math>\hat{\mathbf{A}}^f</math>, <math>\hat{\mathbf{B}}^f</math>, <math>\hat{\mathbf{C}}^f</math>, <math>\hat{\mathbf{D}}^f</math>, with <math>\hat{\Theta}_t^f</math> to reflect the traffic conditions per arm.                  Increment <math>t</math> and repeat from Step 1.</p>
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Individual state estimates  $\hat{\mathbf{x}}_{t|t}^f$  are obtained by running a Kalman filter for each candidate model. The lower bounding approach together with Bayes' rule Equation (2) are applied to estimate which mode is active at a given time instant. The active mode is the one that gives the highest posterior probability in Equation (2) from all the other models. The resultant state estimate,  $\hat{\mathbf{x}}_t$ , is either calculated as a combination of the individual state estimates from the set of Kalman filters given by:

$$\hat{\mathbf{x}}_t = \sum_{f=1}^H \hat{\mathbf{x}}_t^f Pr(M^f|\mathbf{Y}^t) \quad (6)$$

or else as the state estimate from the Kalman filter corresponding to the mode exhibiting maximum posterior probability in Equation (2), denoted by  $\hat{\mathbf{x}}_{t_{max:Pr(M^j|\mathbf{Y}^t)}}$

The model parameters  $\Theta^f$  for the active mode and the noise covariances matrices  $\hat{\mathbf{Q}}$  and  $\hat{\mathbf{R}}$  are also updated using the multivariate stochastic approximation method presented by Zammit et al. (2019). Table 1 shows the complete model allocation, growth and state/parameter estimation algorithm that is proposed in this work.

Several scenarios that represent different operating regimes (modes) were considered to be tested by simulation of signalized traffic junctions, with normal day-to-day traffic conditions and arm closures representing the different modes, as presented in the next section.

### 4. SIMULATION EXPERIMENTS

The proposed algorithm for the multiple model estimation approach were tested and validated by simulating in Aimsun (Transport Simulation Systems, 2019), both signalized 3-arm and 4-arm junctions, with geometry represented in Figure 1 and 2 respectively and with model parameters presented in Tables 2 and 3. The saturation parameters of the junction were determined from Aimsun, as these

correspond to the maximum number of vehicles flowing through the intersection when subject to a high inflow of vehicles. For the 4-arm junction, the saturation parameters were found to be equal to  $S_1=120$  uv/cycle,  $S_2=127$  uv/cycle,  $S_3=80$  uv/cycle and  $S_4=120$  uv/cycle, while for the 3-arm junction these were found to be equal to  $S_1=118$  uv/cycle,  $S_2=120$  uv/cycle and  $S_3=80$  uv/cycle.

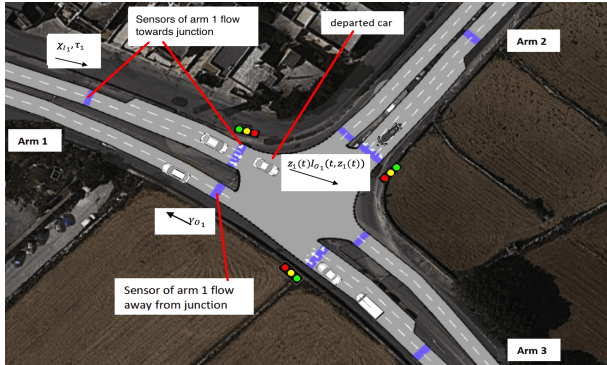


Fig. 1. 3-arm signalized junction

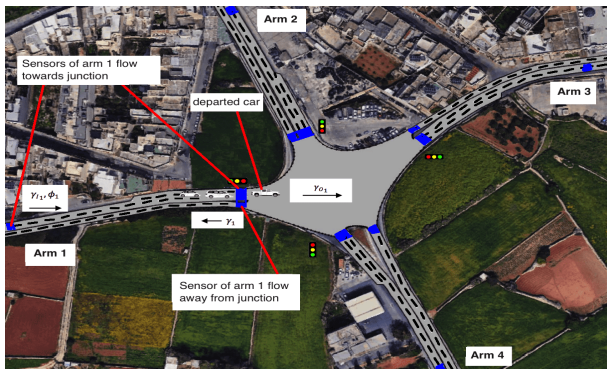


Fig. 2. 4-arm signalized junction

Table 2. Parameters for the 3-arm junction

Model parameters	$\alpha_{12}$	$\alpha_{13}$	$\alpha_{21}$	$\alpha_{23}$	$\alpha_{31}$	$\alpha_{32}$	Cycle time
Actual mean	0.504	0.496	0.845	0.155	0.805	0.195	104 seconds

Table 3. Parameters for the 4-arm junction

Model parameter	$\alpha_{12}$	$\alpha_{13}$	$\alpha_{14}$	$\alpha_{21}$	$\alpha_{23}$	$\alpha_{24}$	$\alpha_{31}$
Value	0.5	0.2	0.3	0.2	0.6	0.2	0.2
Model Parameter	$\alpha_{32}$	$\alpha_{34}$	$\alpha_{41}$	$\alpha_{42}$	$\alpha_{43}$	Cycle time	
Value	0.3	0.5	0.2	0.3	0.5	110 seconds	

Aimsun micro traffic simulation software was used to generate traffic data at one second intervals for a typical working day. This data which includes the inflow in arm  $i$ ,  $\gamma_{I_i}$ , the occupancy  $\phi_i$  and the outflow from arm  $i$ ,  $\gamma_{O_i}$ , for  $i = 1, 2, 3$ , with each having  $N = 830$  cycles at 104 seconds per cycle (equivalent to 24 hours), was used to form the sensor measurement vector  $y$  input to the proposed

algorithm for the 3-arm junction. Similarly for the 4-arm junction. Figure 3 shows the flow of vehicles away from the

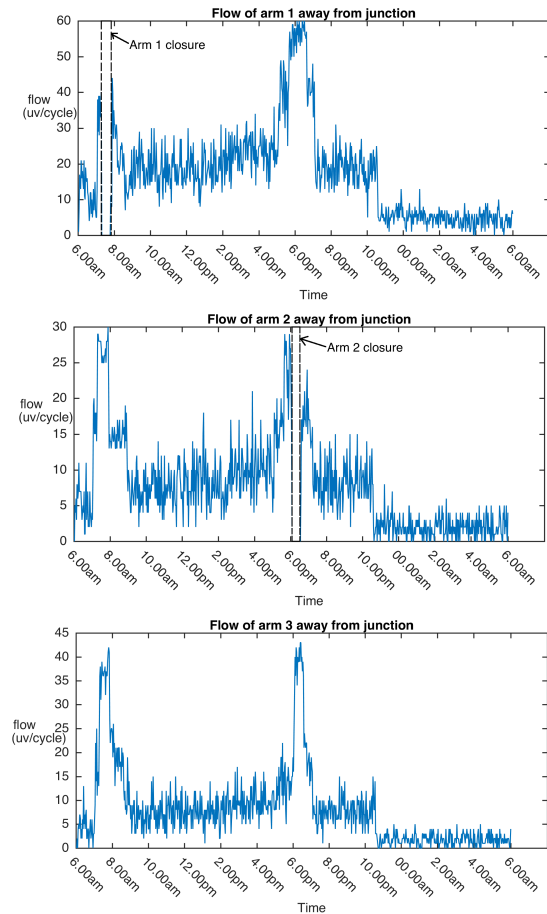


Fig. 3. Flow away from junction

3-arm junction for one arm for a typical working day from 6:00 am of one day to 6:00 am of the next day. The morning peak period is between the 37<sup>th</sup> and 68<sup>th</sup> cycle (between 7.00am till 8.00am) whilst the evening peak period is between the 405<sup>th</sup> and 427<sup>th</sup> cycle (between 5.45pm till 6.20pm) for the first arm. Similarly for the other arms and for the 4-arm junction. Furthermore, the dotted windows represented in Figure 3, show the occurrence and duration of an abrupt jump change due to closures in arm 1 and arm 2. Arm 1 closure is shown in Figure 4, marked with a dark red cross, where all outflow lanes in arm 1 are blocked. Similarly for arm 2 closure.

Arm 1 closure occurs between the 43<sup>rd</sup> and 63<sup>rd</sup> cycle (between 7.15am till 7.50am) whilst arm 2 closure occurs between the 416<sup>th</sup> and 433<sup>rd</sup> cycle (between 6.00pm till 6.30pm) during the morning and evening peak periods respectively. Similar arm closure scenarios were simulated on the 4-arm junction, where arm 1 closure occurs between the 43<sup>rd</sup> and 63<sup>rd</sup> cycle (between 7.18am till 7.55am) whilst arm 2 closure occurs between the 416<sup>th</sup> and 433<sup>rd</sup> cycle (between 6.42pm till 7.13pm).

A Root Mean Square Error (RMSE) measure is defined to determine the accuracy of the estimation results given by

$$J \triangleq \sqrt{\frac{\sum_{N} (p(t) - \hat{p}(t))^2}{N}}$$

where  $p$  is the actual value and  $\hat{p}$  the

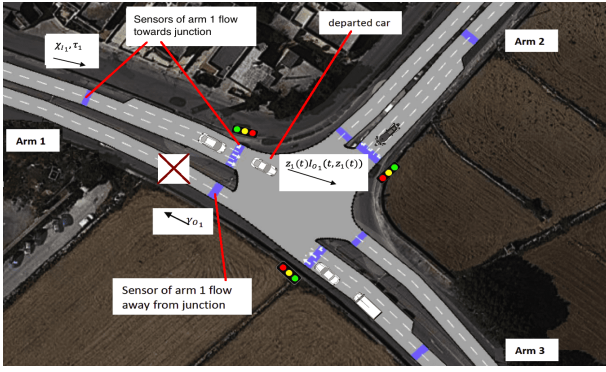


Fig. 4. 3-arm signalized junction with arm 1 closure

estimated value. The RMSE is expressed as a percentage of  $\sqrt{\frac{1}{N} \sum p^2(t)}$  to yield a normalized measurement.

### 5. RESULTS

Tests were carried out where the number of modes  $H$  is unknown and hence modes and their models are learnt as they occur in real-time, while estimating the time-variant system parameters  $\Theta$  and the process and measurement noise covariances  $Q$  and  $R$  as presented by Zammit et al. (2019). Only one local model is assumed known initially and this denotes ‘normal’ traffic conditions without arm closures. Thus initially  $H = 2$ , one model for the normal conditions and a freshly initialised ‘spare’ second model prepared to capture a new mode when it arises. Figure 5 shows the estimated switching conditions for this test for the 3-arm junction with the actual switching conditions denoting arm closures for such junctions as described in Section 4.

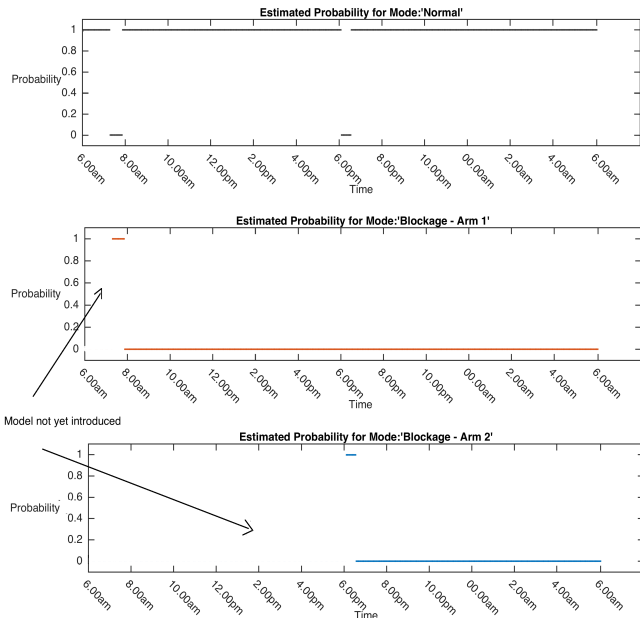


Fig. 5. Estimated switching conditions for the 3-arm junction

As shown in Figure 5 spare models are introduced at the 44<sup>th</sup> cycle (at 7.16am) and 417<sup>th</sup> cycles (at 6.01pm)

respectively when closures of the arms are detected for the 3-arm junction. Before the 44<sup>th</sup> cycle, there is no trace in the figure for the probability for ‘blockage - arm 1’, because this model is introduced at 7.16am. Similarly, before the 417<sup>th</sup> cycle, there is no trace for the probability for ‘blockage - arm 2’, because this model is introduced at 6.01pm. The system switches back to normal conditions at the 65<sup>th</sup> cycle (at 7.53am) and at the 435<sup>th</sup> cycle (at 6.32pm) respectively. This means that the mode detection algorithm required only a one cycle delay to detect arm 1 and arm 2 closures, and another cycle delay to switch back to normal conditions following the respective arm closures. Similar performance was observed when testing the algorithm on a 4-arm junction. This very minor delay is attributed to the Kalman filter state estimations that depend on the estimated states from the previous cycle and the current measurements.

Table 4 compares the % RMSE of every individual state variable estimate obtained when applying Equation (6) with those obtained when applying  $\hat{x}_{t_{max:Pr}(M^f|Y^t)}$  respectively for the 3-arm junction. For comparison reasons, one figure of merit was computed for both cases, consisting of the mean % RMSE. This was computed by calculating the mean value of the % RMSE over all 9 state variables. A mean % RMSE of 0.687 was obtained for the first case and 0.686 for the second case. Hence from the estimated results, there is no significant difference observed between the maximum a posteriori estimates and the estimates resulting from Equation (2) based upon the combination of Kalman filters. This occurs because the probabilities of the inactive models dropped to 0.01%, hence resulting in no significant differences between the state estimations of the two cases. Similar results were obtained for the 4-arm junction.

Table 4. % RMSE of estimates for the maximum a posteriori or for the combination of Kalman filters

Estimates	% RMSE on the state estimates from maximum a posteriori	% RMSE on the state estimates from the combination of Kalman filters
$\hat{\zeta}_1$	1.564	1.565
$\hat{\zeta}_2$	0.947	0.945
$\hat{\zeta}_3$	1.171	1.171
$\hat{\gamma}_{I_1}$	0.510	0.510
$\hat{\gamma}_{I_2}$	0.265	0.266
$\hat{\gamma}_{I_3}$	0.310	0.311
$\hat{\phi}_1$	0.510	0.509
$\hat{\phi}_2$	0.443	0.443
$\hat{\phi}_3$	0.459	0.458
Average	0.687	0.686

Figure 6 shows the estimation results for turning ratio  $\alpha_{12}$ , one of the parameters selected at random, to show the performance of the estimation. The estimated values compare well with the true values. In fact, the estimation of  $\alpha_{12}$  resulted in a mean of 0.483, for the last 10 cycles during normal traffic conditions and 0.481 for the last 10 cycles during arm 1 closure, where the expected value for  $\alpha_{12}$  for both conditions was 0.504. During arm 2 closure the estimated value was found to be equal to 0.014, when the expected value was 0. Similarly for the other model



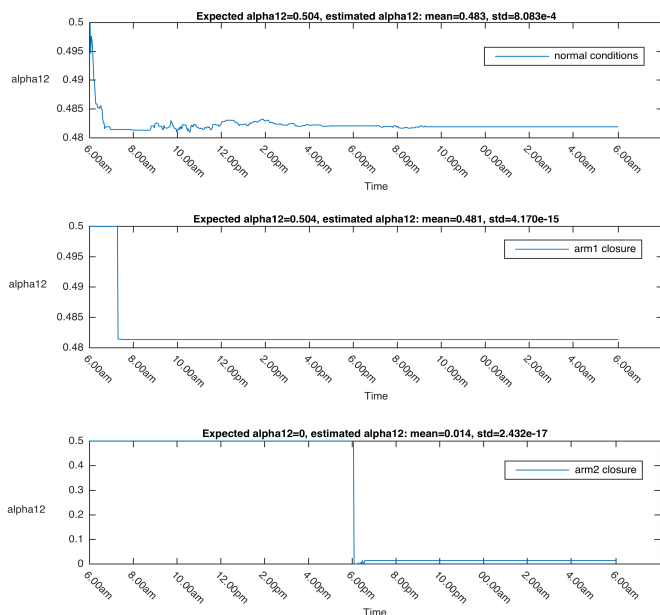


Fig. 6. Estimated turning ratios

parameters. These results are superior to the estimation results obtained when executing the same online state and parameter estimation algorithm but without multiple model estimation, using the same initial and simulation conditions as before. For example, the estimation of  $\alpha_{12}$  resulted in a mean of 0.481, for the last 10 cycles during normal traffic conditions and a mean of 0.481 for the last 10 cycles during arm 1 closure, similar to the previous case, where the expected value for  $\alpha_{12}$  for both conditions was 0.504. However, during arm 2 closure the estimated value was found to be equal to 0.473, when the expected value was 0. Similar performance was exhibited for the other model parameters. Hence, these results elucidate the advantages of using a multiple model estimation approach as proposed in this work for junctions subject to jump dynamic scenarios.

## 6. CONCLUSION

This work proposed a stochastic multiple model joint estimation approach for representing jump dynamics within an urban signalized traffic junction. The proposed algorithm learns the mode dynamics in real-time and determines which mode is active at a given time. An online dual estimation algorithm is adopted as presented by Zammit et al. (2019) to jointly estimate traffic states in real-time, as well as the model and noise covariance parameters. The algorithm is also able to learn an unlimited number of potential dynamical regimes not anticipated *a priori* and will automatically configure and grow its model set if new modes appear in the scene.

The developed algorithm was tested by simulation experiments of 3-arm and 4-arm signalized traffic junctions, with several arm closures. The results highlight the accuracy in estimating and detecting the switching conditions, with a very minor delay of just one cycle. The results also highlight the accuracy of the estimates when executing the online estimation algorithm with multiple model estimation as proposed in this paper compared to the the online

estimation algorithm, that is not based on a multiple model estimation formulation.

Further work is directed to integrate the proposed stochastic multiple model joint estimation approach with adaptive control methods that are able to autonomously adjust to jump changing traffic conditions so as to ensure efficient vehicle flows. This work will be useful in detecting abnormal traffic conditions, such as arm closures, and will select controllers that are tuned to such structural abnormalities.

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