Robot Calibration combining Kinematic Model and Neural Network for enhanced Positioning and Orientation Accuracy *

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Abstract: Traditionally, the calibration of robots is pursued either using model-based or model-free methods. Only a few attempts to combine both approaches were reported, particularly the combination of geometric calibration and artificial neural network (ANN). The latter was mostly used to compensate the positioning error, however. This paper introduces an ANN for compensation of residual positioning as well as orientation error. Moreover, the ANN compensation can be applied with or without prior geometric calibration. An automatic measurement procedure was developed and nearly 14,000 robot poses were measured using a laser tracker. Five-fold cross validation on the training data was applied to find the best parameters of the ANN. These tests indicate that better accuracy is achievable by combining geometric calibration and ANN. Applying this combination on the test data reduced the maximum/average position error to 6.28%/4.26% and the maximum/average orientation error to 7.41%/3.34% of the original values (obtained without calibration).

Keywords: Robot calibration, model identification, neural networks, positioning accuracy, orientation accuracy, robot kinematics, industrial robots

1. INTRODUCTION

Modern industrial robots are more and more used for automated manufacturing tasks like grinding, milling, or measuring. Thus, high positioning and orientation accuracy is essential and can only be achieved by robot calibration. According to Elatta et al. (2004), calibration methods are typically classified as model-based or model-free approaches.

Model-based approaches take error sources in the robot model into account. As mentioned in Roth et al. (1987), many researcher considered deviations in the geometric parameters of the kinematic model, which is also known as geometric calibration. Neubauer et al. (2015) additionally compensated non-geometric error sources like joint and drive stiffness to achieve an improved robot accuracy. While this parametric calibration procedure works very well, at the same time, requires some effort in robot modelling.

Model-free, or rather non-parametric approaches do not rely on modelling of error sources. Instead, error compensation is pursued directly on measurement. Bai (2007) used a 3D grid to divide the robot workspace in small discrete areas and measured the end effector error at all grid points. The error compensation of a distinct robot pose was achieved by fuzzy interpolation of the measured errors of the surrounding grid points. Meggiolaro et al. (2005) used polynomial approximations to consider errors due to geometric and elastic deformation of a patient positioning system.

Some preliminary work of combining the advantages of parametric and non-parametric approaches was carried out several years ago by Zhong et al. (1996) who applied an artificial neural network (ANN) for inverse calibration compensation. However, the training data was inaccurate due to the usage of nominal inverse kinematics. Nguyen et al. (2015) geometrically calibrated a robot and further used an ANN with the robot joint angles as inputs to compensate residual positioning errors due to unmodelled error sources, which represents a forward calibration compensation procedure. Zhao et al. (2019) revised this approach and used many more measurements to compensate the non-linear residual positioning errors even better. However, the orientation error was not considered and just a limited workspace was used.

The aim of this paper is to amend the approach of Zhao et al. (2019) and extend it with orientation information for combined compensation of positioning and orientation error within the whole workspace. This paper also addresses the question as to what extent geometric calibration prior
to ANN error compensation further improves robot accuracy.

2. ROBOT KINEMATICS

2.1 Forward Kinematics

The relative configuration of frames \( F_i \) and \( F_j \) is described by the homogeneous transformation matrix

\[
T_{ij} = \begin{bmatrix} R_{ij} & t_{ij} \\ 0 & 1 \end{bmatrix},
\]

where \( t_{ij} \in \mathbb{R}^3 \) is the coordinate vector of the origin of \( F_j \), measured in \( F_i \) and \( R_{ij} \in SO(3) \) represents the orientation of frame \( F_j \) in \( F_i \). Rotation matrices are parameterized in terms of Cardan angles \((x−y−z)\) rotations, denoted as \( R(\alpha, \beta, \gamma) \).

Table 1 summarizes the nominal translation and orientation of the zero reference configuration of the COMAU Racer5-0.80 with joint angles \( q_i \) to \( q_6 \) and corresponding body-fixed frames \( F_i \) to \( F_6 \). The \( z \)-axes of frames \( F_i \) to \( F_6 \) are defined by the rotation axes, respectively. The inertial frame \( F_6 \) serves as a point of reference for the application. The frame at the end effector (calibration tool) is denoted by \( F_E \). The position of the end effector is measured with a Leica laser tracker with reference frame \( F_6 \).

For the modelling of the kinematics, homogeneous transformation matrices are used and the notation is as follows. The relative configuration of frames \( F_6 \) and \( F_i \) is described by the homogeneous transformation matrix

\[
T_{6i} = \begin{bmatrix} R_{6i} & t_{6i} \\ 0 & 1 \end{bmatrix},
\]

where \( t_{6i} \in \mathbb{R}^3 \) is the coordinate vector of the origin of \( F_i \), measured in \( F_6 \) and \( R_{6i} \in SO(3) \) represents the orientation of frame \( F_i \) in \( F_6 \). Rotation matrices are parameterized in terms of Cardan angles \((x−y−z)\) rotations, denoted as \( R(\alpha, \beta, \gamma) \).

Table 1: Transformations with its nominal geometric parameters and joint coordinates

<table>
<thead>
<tr>
<th>T</th>
<th>( P_{geo} )</th>
<th>t</th>
<th>q</th>
<th>T</th>
<th>( P_{geo} )</th>
<th>t</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \varphi_{i,j} )</td>
<td>( t )</td>
<td>( q )</td>
<td></td>
<td>( \varphi_{i,j} )</td>
<td>( t )</td>
<td>( q )</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>( \pi/2 )</td>
<td>( L_{010} )</td>
<td>( 0 )</td>
<td>( \pi/2 )</td>
<td>( L_{34} )</td>
<td>( 0 )</td>
<td>( q_4 )</td>
</tr>
<tr>
<td>( T_{01} )</td>
<td>( L_{010} )</td>
<td>( L_{010} )</td>
<td>( L_{010} )</td>
<td>( 0 )</td>
<td>( L_{34} )</td>
<td>( 0 )</td>
<td>( q_4 )</td>
</tr>
<tr>
<td>( T_{12} )</td>
<td>( -\pi/2 )</td>
<td>( L_{12} )</td>
<td>( q_1 )</td>
<td>( -\pi/2 )</td>
<td>( L_{23} )</td>
<td>( q_5 )</td>
<td></td>
</tr>
<tr>
<td>( T_{23} )</td>
<td>( 0 )</td>
<td>( L_{23} )</td>
<td>( q_2 )</td>
<td>( \pi/2 )</td>
<td>( L_{6E} )</td>
<td>( 0 )</td>
<td>( q_6 )</td>
</tr>
</tbody>
</table>

The transformation matrix for adjacent frames is

\[
T_{i−1,i} = \begin{bmatrix} R_{i−1,i} & (q_i − i−1,i) \\ 0 & 1 \end{bmatrix}
\]

with \( i = 1...6 \) and the one for the end effector is

\[
T_{6E} = \begin{bmatrix} R_{6E} & \hat{v}_{6E} \\ 0 & 1 \end{bmatrix}
\]

Finally, this leads to the overall transformation matrix from \( F_E \) to \( F_i \)

\[
T_{BE} = T_{00} T_{12} T_{23} T_{34} T_{45} T_{56} T_{6E}.
\]

2.2 Geometric Error Modelling

The kinematic model with geometric errors accounts for \( n_c = 48 \) error parameters, as shown in Table 2. The vector of geometric error parameters \( p_e \in \mathbb{R}^{n_e} \) consists of the parameter vector \( p_v \) due to length deviations as well as the parameter vector \( p_h \) due to inaccurate zero positions and axes misalignment.

Table 2: Geometric error parameters

<table>
<thead>
<tr>
<th>T</th>
<th>( p_v )</th>
<th>( p_h )</th>
<th>T</th>
<th>( p_v )</th>
<th>( p_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{00} )</td>
<td>( p_{0x} )</td>
<td>( p_{0y} )</td>
<td>( T_{34} )</td>
<td>( p_{34x} )</td>
<td>( p_{34y} )</td>
</tr>
<tr>
<td>( T_{01} )</td>
<td>( p_{01x} )</td>
<td>( p_{01y} )</td>
<td>( T_{45} )</td>
<td>( p_{45x} )</td>
<td>( p_{45y} )</td>
</tr>
<tr>
<td>( T_{12} )</td>
<td>( p_{12x} )</td>
<td>( p_{12y} )</td>
<td>( T_{56} )</td>
<td>( p_{56x} )</td>
<td>( p_{56y} )</td>
</tr>
<tr>
<td>( T_{23} )</td>
<td>( p_{23x} )</td>
<td>( p_{23y} )</td>
<td>( T_{6E} )</td>
<td>( p_{6Ex} )</td>
<td>( p_{6Ey} )</td>
</tr>
</tbody>
</table>

Considering these error parameters, the corresponding parameter dependent rotation matrices are

\[
\hat{R}_{i0} = R_{i0} R(\hat{p}_{010}, \hat{p}_{010}, \hat{p}_{010}),
\]

\[
\hat{R}_{i−1,i} = R_{i−1,i} R(\hat{p}_{i−1,i}, \hat{p}_{i−1,i}, q_i + \hat{p}_{q_i}),
\]

\[
R_{6E} = R_{6E} R(\hat{p}_{6Ex}, \hat{p}_{6Ey}, \hat{p}_{6Ez}),
\]

and the translation vectors are
\[ r_i = r_i + (p_{i_0}, p_{i_1}, p_{i_2})^T, \]
\[ \delta r_i = \delta r_i + (\delta p_{i_0}, \delta p_{i_1}, \delta p_{i_2})^T, \]
\[ \delta r_i = \delta r_i + (\delta p_{i_0}, \delta p_{i_1}, \delta p_{i_2})^T, \]
where \( i = 1 \ldots 6 \). Hence, (5) changes to
\[ T_{IE} = T_{I0} T_{01} T_{12} T_{23} T_{34} T_{45} T_{56} T_{6E} \]
\[ = \begin{bmatrix} R_{IE} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}, \]
where \( r_{IE} \) is the position of the end effector, represented in \( F_i \). The rotation matrix \( R_{IE} \) describes the orientation of \( F_i \) relative to \( F_j \).

### 2.3 Elimination of Linear Dependent Parameters

Considering position and orientation measurement, the end effector error \( \Delta z \in \mathbb{R}^6 \) is defined as
\[ \Delta z = \begin{bmatrix} \Delta r_{IE} \\ \Delta \phi_{IE} \end{bmatrix} \]
with positioning error \( \Delta r_{IE} \in \mathbb{R}^3 \) and orientation error \( \Delta \phi_{IE} \in \mathbb{R}^3 \). Therein \( \Delta \phi_{IE} \) is proportional to the rotation axis that can be associated to the relative rotation (see Axis–Angle representation of Stuler (1993)).

Setting (13) to zero, i.e. \( \Delta z = 0 \), provides six independent equations and leads to a non-linear optimization problem (root-finding) to find the geometric error parameters \( p_e \). This problem can be solved iteratively using a Taylor series of \( \Delta z = \Delta z(z_{\text{meas}}, q, p_e) \) at the start parameters \( p_e^{(0)} \), i.e.
\[ \Delta z(z_{\text{meas}}, q, p_e^{(0)}) + \frac{\partial \Delta z}{\partial p_e} \bigg|_{p_e^{(0)}} \Delta p_e + \ldots = 0 \]

or after evaluating (14) with \( j = 1 \ldots m \) measurements
\[ \begin{bmatrix} \Delta z(z_{\text{meas},1}, q, p_e^{(0)}) \\ \vdots \\ \Delta z(z_{\text{meas},m}, q, p_e^{(0)}) \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_m \end{bmatrix} \Delta p_e = 0. \]

Using position and orientation measurement for the geometric calibration leads to the regressor matrix \( \Theta = \mathbb{Q} \mathbf{R} \) with \( n = 6m \) rows for \( m \) measurements. Not all of the \( n_e \) parameters deviations \( \Delta p_e \in \mathbb{R}^{n_e} \) are independent. Therefore, \( s \) linear dependent columns of \( \Theta \) are eliminated via a QR Decomposition \( \Theta = Q \mathbf{R} \). This determines the identifiable geometric error parameter vector \( \mathbf{p}_e \in \mathbb{R}^{n_e = n-8} \), which is also called base parameter vector. Using nominal forward kinematics with random joint angles instead of real measurements and following the procedure of Khalil and Gautier (1991), yields
\[ \mathbf{Q} + \mathbf{Q} \Delta \mathbf{p}_e = 0 \]
with regressor matrix \( \mathbf{Q} \in \mathbb{R}^{6m, n_e} \), parameters deviation vector \( \Delta \mathbf{p}_e \in \mathbb{R}^{n_e} \) and \( \mathbf{Q} \in \mathbb{R}^{6m, 1} \), which is a reordered version of \( \mathbf{Q} \) after doing the same permutation as used for \( \Theta \). The \( n_e = 30 \) determined base parameters are given in Table 3.

### 3. MEASUREMENT PROCEDURE

As stated in Zhao et al. (2019), an ANN needs a large amount of training poses to guarantee successful modelling of the residual end effector error. The developed automatic measurement procedure consists of four stages: robot pose selection, suitability check, trajectory planning between poses and automatic measurement using laser tracker.

The used laser tracker LTD 800 from Leica Geosystems has a resolution of 1 mm and an accuracy of ±25 μm. To be able to measure position and orientation at the same time, three spherically mounted retroreflectors (SMR) are used.

Figure 1 shows the setup involving two red ring reflectors (RRR) and one break resistant reflector (BRR). The whole calibration setup is shown in Fig. 2. Using the SMRs for the measurement of the centre and respective points on \( x \) and \( y \)-axes of \( F_i \), allows the determination of \( T_{IF} \), which is constant for a rigid calibration setup. However, it should be noted that the robot setup stands on slightly elastic ground, i.e. compliant wooden structure.

![Fig. 2. Calibration setup with robot and laser tracker](image-url)
The first stage of the measurement procedure requires to appropriately discretize the range of joint variables of the COMAU Racer5-0.80 with respective limits
\[-168^\circ \leq q_i \leq 168^\circ, -198^\circ \leq q_4 \leq 198^\circ, -83^\circ \leq q_2 \leq 133^\circ, -98^\circ \leq q_3 \leq 98^\circ, -178^\circ \leq q_5 \leq 63^\circ, -180^\circ \leq q_6 \leq 180^\circ.\] (17)
For example the first axis should be discretized with a higher resolution than the last one because a movement of \(q_1\) has a higher impact on the end effector translation. Following this idea, the chosen discretization delivers joints values \(q_i\) to \(q_6\) every 19.80\(^\circ\), 21.60\(^\circ\), 24.10\(^\circ\), 28.30\(^\circ\), 32.70\(^\circ\) and 36.00\(^\circ\), respectively, and results in 1,428,000 possible poses. These poses need to be checked for validity and suitability.

The suitability check involves three criteria. Firstly, the robot for selected joint coordinates must be outside the collision area, i.e. the minimal spatial distance between robot and collision objects must be greater than 3.50 cm.

Secondly, the calibration tool must be visible for the laser tracker, i.e. the calibration tool has to be above the wall as shown in Fig. 2. Finally, the opening angle \(\alpha\) between laser and all three reflectors must be \(|\alpha| \leq 30^\circ\). To consider possible deviations of the reflector orientation and the shape of BRR, we decided to limit the allowed cone angles even more, i.e. \(|\alpha_{\text{exn}}| \leq 26.50^\circ\) and \(|\alpha_{\text{inn}}| \leq 20^\circ\). This results in 13,919 suitable poses for measurement of all three reflectors for position and orientation information.

The third stage of the measurement procedure is the trajectory planning between suitable poses. After separating the robot poses into poses in front of the wall and behind the wall, the trajectory is planned in joint coordinates. Simulating this trajectory and continuously calculating the minimal distance shows at which poses a collision between robot and environment would happen. After a few iterations of changing the order of critical poses, a valid trajectory without collision can be found.

The fourth stage is the automatic measurement itself. The laser tracker and the robot controller (B & R Automation PC) are able to communicate with each other using TCP/IP and allow to automatically measure the position of all three SMRs. At first the robot moves to a pose and gives a signal to the tracker after reach. Then the tracker measures all three reflectors and returns a signal after finish. This procedure is repeated until all poses are measured in the experiment. The measurement of all 13,919 poses took about 3.5 days and needed no supervision at all. Assuming known homogeneous transformation matrix \(T_{L1}\), the measurement of all three SMRs defines position \(p_{\text{ref}}\) and orientation via rotation matrix \(R_{L1}\).

### 4. GEOMETRIC CALIBRATION

Considering (16) and using well-known method of least squares, the base parameters vector results in
\[
\Delta \mathbf{p}_b = -[\mathbf{Q}^\top \mathbf{Q}]^{-1} \mathbf{Q}^\top \mathbf{q}_{\text{base}} \]
assuming enough measurements so that regressor matrix \(\mathbf{Q}\) has a full rank, i.e. \(\text{rank}(\mathbf{Q}) = n_b\). However, the full rank of \(\mathbf{Q}\) should be guaranteed due to the usage of regularization and randomly selected calibration poses.

The usage of Equ. (18) leads to the first solution \(\mathbf{p}^{(1)}_b = \mathbf{p}^{(0)}_b + \Delta \mathbf{p}_b\) of the geometric error parameters. For a more accurate solution of the non-linear model, more iterations are necessary, i.e.
\[
\mathbf{p}^{(n+1)}_b = \mathbf{p}^{(n)}_b - [\mathbf{G}^\top \mathbf{G}]^{-1} \mathbf{G}^\top \mathbf{q}_{\text{base}}\]
(19)
The parameters \(\mathbf{p}^{(0)}_b = \mathbf{0}\) can serve as the start parameters and the nominal parameters \(\mathbf{p}_{\text{nom}}\) can be taken from publicly available CAD data. Table 4 summarizes the determined base parameters \(\mathbf{p}_{\text{nom}} = \mathbf{p}^{(0)}_b\) using 220 random and independent from the previously derived 13,919 poses due to measurements for another research project.

<table>
<thead>
<tr>
<th>#</th>
<th>(p_{\text{base}}) (m)</th>
<th>Unit</th>
<th>#</th>
<th>(p_{\text{base}}) (m)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.752 × 10^{-4}</td>
<td>m</td>
<td>16</td>
<td>8.150 × 10^{-4}</td>
<td>rad</td>
</tr>
<tr>
<td>2</td>
<td>2.343 × 10^{-4}</td>
<td>m</td>
<td>17</td>
<td>-1.654 × 10^{-4}</td>
<td>rad</td>
</tr>
<tr>
<td>3</td>
<td>-2.720 × 10^{-4}</td>
<td>m</td>
<td>18</td>
<td>3.050 × 10^{-3}</td>
<td>rad</td>
</tr>
<tr>
<td>4</td>
<td>-2.567 × 10^{-4}</td>
<td>m</td>
<td>19</td>
<td>-2.759 × 10^{-3}</td>
<td>rad</td>
</tr>
<tr>
<td>5</td>
<td>6.334 × 10^{-5}</td>
<td>m</td>
<td>20</td>
<td>1.279 × 10^{-2}</td>
<td>rad</td>
</tr>
<tr>
<td>6</td>
<td>1.676 × 10^{-3}</td>
<td>m</td>
<td>21</td>
<td>-8.877 × 10^{-4}</td>
<td>rad</td>
</tr>
<tr>
<td>7</td>
<td>5.611 × 10^{-4}</td>
<td>m</td>
<td>22</td>
<td>2.778 × 10^{-3}</td>
<td>rad</td>
</tr>
<tr>
<td>8</td>
<td>8.479 × 10^{-4}</td>
<td>m</td>
<td>23</td>
<td>-5.821 × 10^{-5}</td>
<td>rad</td>
</tr>
<tr>
<td>9</td>
<td>-3.293 × 10^{-4}</td>
<td>m</td>
<td>24</td>
<td>-1.393 × 10^{-4}</td>
<td>rad</td>
</tr>
<tr>
<td>10</td>
<td>8.669 × 10^{-5}</td>
<td>m</td>
<td>25</td>
<td>-2.455 × 10^{-4}</td>
<td>rad</td>
</tr>
<tr>
<td>11</td>
<td>-7.575 × 10^{-4}</td>
<td>m</td>
<td>26</td>
<td>2.380 × 10^{-4}</td>
<td>rad</td>
</tr>
<tr>
<td>12</td>
<td>-6.430 × 10^{-4}</td>
<td>m</td>
<td>27</td>
<td>-1.934 × 10^{-2}</td>
<td>rad</td>
</tr>
<tr>
<td>13</td>
<td>-2.026 × 10^{-5}</td>
<td>m</td>
<td>28</td>
<td>7.956 × 10^{-3}</td>
<td>rad</td>
</tr>
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<td>29</td>
<td>2.588 × 10^{-4}</td>
<td>rad</td>
</tr>
<tr>
<td>15</td>
<td>-1.639 × 10^{-3}</td>
<td>m</td>
<td>30</td>
<td>1.134 × 10^{-3}</td>
<td>rad</td>
</tr>
</tbody>
</table>

Figures 3 and 4 show the achieved reduction of positioning as well as orientation error. Yet unmodelled error sources, in particular the elastic ground of the robot setup, have a major negative influence on the robot accuracy.

### 5. NEURAL NETWORK ERROR COMPENSATION

#### 5.1 ANN Structure and Parameters

To further improve the positioning and orientation accuracy, an ANN with two hidden layers with respective hidden neurons \(N_1\) and \(N_2\) as well as an output layer with six neurons is considered. As common for regression,
Fig. 4. Orientation error for the whole data
the two hidden layers use a hyperbolic tangent and the
output layer a linear activation function. According to
Zhao et al. (2019) and our own tests, more layers do not
further improve the accuracy.

Fig. 5. Geometric error compensation with ANN

The networks are trained using batch learning and
Bayesian regularization backpropagation, which is known
to generate a network that generalizes well (see MacKay
(1992) and Foresee and Hagan (1997)). This is achieved
by minimization of squared errors and network weights
instead of just the errors. So Bayesian regularization back-
propagation successfully manages to avoid the overfitting
problem even in the case when many hidden neurons or
many training epochs are used.

To be able to verify the performance of a trained ANN,
the recorded pose data is split into a training set with
11 136 poses (80%) and a test set with 2783 poses (20%).
Applying five-fold cross validation on the training set helps
to find the best number of hidden neurons (N1, N2). It
should be noted, that mapping of inputs and outputs was
applied separately to each training set.

Table 5 shows the cross validation results on the training
set for the considered numbers of hidden neurons (16,
32 or 64 neurons per layer) using Bayesian regularization
backpropagation (see MacKay (1992)) and 2048 training
epochs. These results indicate, that the usage of geomet-
ric calibration indeed helps to improve the positioning
and orientation accuracy even in the case of ANN error
compensation. Furthermore, a higher number of hidden
neurons might lead to even better results to the point
where the ANN starts overfitting due to the usage of too
many neurons. However, an ANN with more than (64, 32)
hidden neurons was not tested due to limited time or
rather computational power for training.

Table 5. Cross validation results on training
data of tested ANNs

<table>
<thead>
<tr>
<th>Hidden neurons</th>
<th>RMSE</th>
<th>Max ∥Δr_{τKE}∥</th>
<th>Max ∥Δφ_{τKE}∥</th>
<th>P_e</th>
</tr>
</thead>
<tbody>
<tr>
<td>(16, 16)</td>
<td>2.436</td>
<td>4.173</td>
<td>19.277</td>
<td>0</td>
</tr>
<tr>
<td>(32, 16)</td>
<td>1.226</td>
<td>2.905</td>
<td>12.840</td>
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</tr>
<tr>
<td>(32, 32)</td>
<td>0.799</td>
<td>2.086</td>
<td>7.928</td>
<td>0</td>
</tr>
<tr>
<td>(64, 32)</td>
<td>0.520</td>
<td>1.362</td>
<td>5.418</td>
<td>0</td>
</tr>
<tr>
<td>(16, 16)</td>
<td>1.779</td>
<td>2.328</td>
<td>17.398</td>
<td>0</td>
</tr>
<tr>
<td>(32, 16)</td>
<td>1.130</td>
<td>1.550</td>
<td>9.782</td>
<td>0</td>
</tr>
<tr>
<td>(32, 32)</td>
<td>0.721</td>
<td>1.112</td>
<td>7.447</td>
<td>0</td>
</tr>
<tr>
<td>(64, 32)</td>
<td>0.462</td>
<td>0.812</td>
<td>5.338</td>
<td>0</td>
</tr>
</tbody>
</table>

The best results could be achieved with (64, 32) hidden
neurons. Considering these parameters, we train two new
ANNs (with and without prior geometric calibration) with
new corresponding mappings using the whole training set
to get the finally trained ANNs.

5.3 Test

The test set is used to check the performance of the two
chosen ANNs on novel data. Figures 6 and 7 show the
results, obtained with the proposed method. Compared
to the uncalibrated robot, both ANNs compensate the
positioning and orientation error very well.

Table 6 provides the statistical analysis of positioning
and orientation error. The first column corresponds to
the original results using kinematic model with nominal
parameters. Column two shows the errors using geometric
Model with nominal parameters (uncalibrated robot)
Model with base parameters (geo. calibrated robot)
Model with ... ∥ ∆ItIE ∥ in mm
Neural network (64, 32) - Position error

<table>
<thead>
<tr>
<th>Pose number</th>
<th>500</th>
<th>1 000</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
</tr>
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<tbody>
<tr>
<td>Error (mm)</td>
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<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Error (mrad)</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>

Fig. 6. Positioning error for the test data

Model with nominal parameters (uncalibrated robot)
Model with base parameters (geo. calibrated robot)
Model with base parameters and ANN compensation

ANN p no ANN

Fig. 7. Orientation error for the test data
calibration. The third column is associated with ANN (64,32) without prior geometric calibration. The best results are shown in the fourth column belonging to the combination of geometric error and ANN (64,32). For 2783 poses, the absolute positioning/orientation error is less than 0.605 mm/3.753 mrad and for 90% of the poses (0.9-quantile $Q_{0.9}$) it is less than 0.258 mm/1.604 mrad.

Table 6. Positioning and orientation error for the test data

<table>
<thead>
<tr>
<th>Error in mm or mrad</th>
<th>$\bar{F}_p = 0$ ANN</th>
<th>$\bar{F}<em>p = P</em>{max}$ ANN</th>
<th>$\bar{F}_p = 0$ ANN</th>
<th>$\bar{F}<em>p = P</em>{max}$ ANN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>9.636</td>
<td>6.351</td>
<td>1.405</td>
<td>0.605</td>
</tr>
<tr>
<td>Mean</td>
<td>3.522</td>
<td>1.844</td>
<td>0.192</td>
<td>0.150</td>
</tr>
<tr>
<td>$Q_{0.9}$</td>
<td>5.377</td>
<td>3.060</td>
<td>0.336</td>
<td>0.258</td>
</tr>
<tr>
<td>Max</td>
<td>50.617</td>
<td>28.699</td>
<td>5.969</td>
<td>3.753</td>
</tr>
<tr>
<td>Mean</td>
<td>28.063</td>
<td>15.191</td>
<td>1.076</td>
<td>0.938</td>
</tr>
<tr>
<td>$Q_{0.9}$</td>
<td>43.874</td>
<td>23.350</td>
<td>1.868</td>
<td>1.604</td>
</tr>
</tbody>
</table>

6. CONCLUSION

We proposed a combined model-based/model-free calibration method for an industrial robot and showed how we could use this model together with an ANN to achieve improved positioning and orientation accuracy in the whole workspace. The developed measurement procedure guaranteed the automatic measurement of 13 919 suitable robot poses without the requirement of human supervision. Combining geometric calibration and ANN led to a maximal positioning/orientation error of 0.605 mm/3.753 mrad, which is a reduction to 6.28%/7.41% of the error, obtained by the uncalibrated robot. Without prior geometric calibration, the ANN compensation resulted in similar but slightly higher errors. So the usage of a calibrated kinematic model indeed improved the accuracy. However, compared to the accuracy without any calibration, both ANN achieve a major improvement, which confirms the usefulness of our approach. Future work will involve tests of the developed calibration procedure also on other industrial robots to further prove the effectiveness. The usage of two separate networks for positioning and orientation error compensation might be another research topic.

REFERENCES


