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Adaptive neural network control for nonlinear output-feedback systems under disturbances with unknown bounds ⋆

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Abstract: This paper is concerned with the problem of adaptive backstepping neural network tracking control for a class of output feedback systems with unknown functions under bounded disturbances whose boundaries is unknown. Unknown functions are approximated via online radial basis function (RBF) neural network, high order continuous differentiable functions are introduced into Lyapunov function to realize the estimation of unknown parameters and unknown boundary, and a new dead zone function is designed to replace symbolic function to realize the continuity of virtual control. During the design process, the backstepping design method is applied to deal with the cross terms generated by the tuning function. Barbalat's lemma proves that all the signals of closed-loop system are bounded and the output tracking error converges to an arbitrarily small neighborhood of the origin. A simulation example are given to illustrate the effectiveness of the control scheme.

Keywords: Adaptive backstepping design, radial basis function (RBF) neural network (NN), dead zone function, Barbalat's Lemma.

1. INTRODUCTION

In recent decades, the adaptive control design of nonlinear systems has made remarkable achievements, unknown parameters in the control system were estimated by the combination of the adaptive methodology and the backstepping technique, which has been solved the control problem of a linearly parameterized output-feedback system, see Krstic (1995). Backstepping design as a kind of constructive and effective design tool for the control of nonlinear systems, in which all the virtual functions are continuously differentiable, in order to satisfy this requirement, Ding (2000) introduced a flat zone around the neighbourhood of the origin into the Lyapunov function, nonetheless dead zones appear in the bound estimation and parameter adaptation because of the flat zone, as a result, high-order terms are introduced in the Lyapunov function such that the virtual functions remain smooth enough, which has ensured that output tracking error converge to the any small region of origin.

In practice, uncertainty is one of the important factors that affect the control performance and the closed-loop stability of the whole system, for cases where unknown functions exist in the system, the method of online function approximation is used to deal with these unknown functions, see Sanner (1992). Adaptive backstepping NN control methods which combine neural network models with adaptive backstepping technology have proposed for the lower-triangular nonlinear systems, see Zhang (2000), Li (2004), Du (2008), Stoev (2002), Liu (2011). In the last few years, in order to ensure that the tracking error ultimately converges to the predetermined tracking accuracy, Ren (2010) has developed a control method based on Barrier Lyapunov function for nonlinear output-feedback systems with unknown functions, the tracking error can eventually reach to the predetermined accuracy.

In this paper, for output feedback system with unknown functions and bounded disturbances whose boundaries is unknown, under the assumption of the the tracking error accuracy can be known beforehead, we put forward an adaptive neural network tracking control. First, unknown functions are approximated via online radial basis function (RBF) neural network. Second, high order continuous differentiable functions are introduced into Lyapunov function to realize the estimation of unknown parameters and unknown boundary, and a new dead zone function is designed to replace symbolic function to realize the continuity of virtual control. Finally, the backstepping design method is applied to deal with the cross terms generated by the tuning function in the control design and Barbalat's Lemma is adopting to analyze the convergence of the tracking error during the procedure of stability analysis.

2. PRELIMINARIES AND SYSTEM DESCRIPTION

2.1 Barbalat's Lemma

Lemma 1 (Barbalat's Lemma, Tao (2003)): If a scalar function $f(t)$ is uniformly continuous such that $\lim_{t\to\infty} \int_0^t f(\tau)d\tau$ exists and is finite, then $\lim_{t\to\infty} f(t) = 0$.

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Corollary 1 (Tao (2003)): If $f(t) \in L^2 \cap L^\infty$, $\dot{f}(t) \in L^\infty$, then $\lim_{t\to\infty} f(t) = 0$.

2.2 Higher order terms in Lyapunov functions

A number of differentiable functions are needed to carry out the control design and they are introduced here. The first λ_i-neighborhood sign function as follows

$$s(i, \lambda_i, \upsilon) = \begin{cases} -1 & , \quad \upsilon \leq -\lambda_i \\ F(i, \upsilon) & , \quad |\upsilon| < \lambda_i \\ 1 & , \quad \upsilon \geq \lambda_i \end{cases} \tag{1}$$

where $F(i, \upsilon) = 1 - 2\cos^i(\frac{\pi}{2}\sin^i(\frac{\pi}{4\lambda_i}(\upsilon + \lambda_i)))$. Further, we introduce notations $f(\lambda_i, \upsilon), i = 1, 2, \ldots, \rho$ as

$$f(\lambda_i, \upsilon) = \begin{cases} 0 & , \quad |\upsilon| < \lambda_i \\ 1 & , \quad |\upsilon| \geq \lambda_i \end{cases} \tag{2}$$

where λ_i are positive real design parameters. For simplicity, define

$$s_i = s(i, \lambda_i, z_i) \tag{3}$$

$$f_i = f(\lambda_i, z_i) \tag{4}$$

The second λ_i-neighborhood nonnegative function is $(|z_i| - \lambda_i)^{\iota(i)} f_i, i = 1, 2, \ldots, \rho$, which is introduced in the Lyapunov function, with the series of integers $\iota(i)$ defined as: $\iota(i) = 2\text{int}[(\rho - i + 3)/2]$, where $\text{int}(\cdot)$ returns the integer part of the operand Ding (2000), and satisfies $\iota(i) \geq 2$.

Lemma 2.2 The function $s(\cdot)$ and $f(\cdot)$ have the following properties:

(i) $s(i, \lambda_i, \cdot) \in C^{i-1} : R \to R, i = 1, \ldots, \rho$

(ii) $f_i s_i = \begin{cases} -1 & , \quad z_i \leq -\lambda_i \\ 0 & , \quad -\lambda_i < z_i < \lambda_i \\ 1 & , \quad z_i \geq \lambda_i \end{cases}$

(iii)$f_i^j = f_i$, $f_i s_i^2 = f_i$ and $f_i^2 s_i^2 = f_i$

(iv)$(|z_i| - \lambda_i)^j f_i \geq 0, \forall z_i \in R, j = 1, 2, \ldots$

(v)$\frac{d}{dz_i}[\frac{1}{j}(|z_i| - \lambda_i)^j f_i] = (|z_i| - \lambda_i)^{j-1} f_i s_i$ and $(|z_i| - \lambda_i)^j f_i \in C^{j-1}, j = 1, 2, \ldots$

2.3 RBF NN Approximation

It has been proven in Sanner (1992) that network can approximate any smooth function over a compact set to arbitrarily any accuracy. In this paper, we will employ a RBF NN over a compact set $\Omega_y \in R$ to approximate an unknown and smooth function $f(y) : R \to R$ as

$$f(y, w) = h^{\mathrm{T}}(y)w + \varepsilon(y) \tag{5}$$

where $\varepsilon(y)$ denotes the NN inherent approximation error; and $h(y) = [h_1(y), \ldots, h_m(y)]^{\mathrm{T}}$ is a known smooth vector-valued function and m is the neural node number, the smooth basis functions $h_i(y)(1 \leq i \leq m)$ being chosen as the Gaussian functions:

$$h_i(y) = \exp[-\frac{(y - \mu_i)^{\mathrm{T}}(y - \mu_i)}{\eta_i^2}] \tag{6}$$

where μ_i denotes the center of the ith basis function and η_i determines the width of the ith basis function.

In general the optimal weight vector $w = [w_1, w_2, \ldots, w_m]^{\mathrm{T}}$, is chosen that minimizes the network approximation error $\varepsilon(y)$ for all $y \in \Omega_y$, where $\Omega_y \in R$ is a suffciently large compact region, i.e.

$$w = \arg\min_{\hat{w} \in R^m} \sup_{y \in \Omega_y} |f(y) - h^{\mathrm{T}}(y)\hat{w}| \tag{7}$$

which is unknown and need to be estimated. Let $\hat{w}$ be estimate of the ideal w and the estimation errors as $\tilde{w} = w - \hat{w}$.

2.4 System Description

Consider a class of nonlinear systems with the following output feedback form

$$\begin{cases} \dot{x} = A_c x + F(y) + \Psi(y)d + b\sigma(y)u \\ y = e_1^{\mathrm{T}} x \end{cases} \tag{8}$$

where $x = [x_1, \ldots, x_n]^{\mathrm{T}} \in R^n$, $y \in R$ and $u \in R$, are the system state vector, output and control input, respectively; $F(y) = [f_1(y), \ldots, f_n(y)]^{\mathrm{T}} : R \to R^n$, $f_i(y)$ is unknown smooth function; $b = [b_1, b_2, \ldots, b_n]^{\mathrm{T}} \in R^n$ is the uncertain nonzero parameter vector; $d = [d_1(t), d_2(t), \ldots, d_n(t)] \in R^n$ is a bounded time-varying disturbance vector; the components of $\Psi(y)$, denoted by $\psi_i(y), 1, \ldots, n$, which is smooth vector field in R^n; $\sigma(y) \neq 0, \forall\, y \in R$. Only the output y is available for measurement, and

$$A_c = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix} \in R^{n\times n}$$

Our control objectives are outlined below. For a given reference signal $y_r(t)$ that is bounded and continuously differentiable to order ρ, design an adaptive NN control $u(t)$ such that the signals of closed-loop control system are bounded and the convergence of the tracking error to a pre-specified neighbourhood of the origin.

In order to achieve the above goals, we give the following assumptions.

Assumption 1: The relative degree ρ is a known constant.

Remark 1: Form Assumption 1, we can know that $b_i = 0, i = 1, \ldots, \rho - 1$,and $b_\rho \neq 0$ according to the definition of the relative degree, where b_ρ is also referred to the high-frequency gain of the system, which sign is known.

Assumption 2: The system is of minimum phase, i.e., the polynomial $B(s) = b_\rho s^{n-\rho} + \cdots + b_{n-1}s + b_n$ is Hurwitz.

3. STATE ESTIMATION FILTER AND OBSERVER DESIGN

By using RBF NN (5) to approximate the unknown and smooth function $f_i(y)$, system (8) can be rewritten as follows

$$\begin{cases} \dot{x} = A_c x + H^{\mathrm{T}}(y)W + \delta(y) + \Psi(y)d + b\sigma(y)u \\ y = e_1^{\mathrm{T}} x \end{cases} \tag{9}$$

where

$$W = [w_1, \cdots, w_n]^{\mathrm{T}} \in R^{mn\times 1}, w_i = [w_{i1}, \cdots, w_{im}]^{\mathrm{T}}$$

$$H^{\mathrm{T}}(y)=\begin{bmatrix} h_1^{\mathrm{T}}(y) & 0 & \cdots & 0 \\ 0 & h_2^{\mathrm{T}}(y) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_n^{\mathrm{T}}(y) \end{bmatrix} \in R^{n\times mn}$$

$$h_i^{\mathrm{T}}(y)=[h_{i1}(y),\cdots,h_{im}(y)]$$

$$\Psi(y)=\begin{bmatrix} \psi_1(y) & 0 & \cdots & 0 \\ 0 & \psi_2(y) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \psi_n(y) \end{bmatrix} \in R^{n\times n}$$

$$\delta(y)=[\varepsilon_1(y),\cdots,\varepsilon_n(y)]^{\mathrm{T}}$$

Then, in order to the design of the controller and the adaptive laws, we make the following assumption

Assumption 3: The disturbance d and the approximation error $\delta(y)$ are bounded, i.e., there exist two constant positive reals D and A so that $\max_{t\geq 0}\|d\|\leq D$, $\max_{t\geq 0}\|\delta(y)\|\leq A$.

Choose the K-filters Krstic (1995) as follows

$$\dot{\zeta}=A_0\zeta+ky \tag{10}$$
$$\dot{\xi}=A_0\xi+e_n\sigma(y)u \tag{11}$$
$$v_j=A_0^{n-j}\xi,\rho\leq j\leq n \tag{12}$$
$$\dot{\Xi}=A_0\Xi+H^{\mathrm{T}}(y) \tag{13}$$

where the vector $k=[k_1,\ldots,k_n]^{\mathrm{T}}$ is chosen so that the matrix $A_0=A_c-ke_1^{\mathrm{T}}$ is Hurwitz, that is, there exists a positive definite matrix P such that $PA_0+A_0^{\mathrm{T}}P=-I,P=P^{\mathrm{T}}>0$. Then, we construct the state estimates

$$\hat{x}=\zeta+\sum_{j=\rho}^{n}b_jv_j+\Xi W \tag{14}$$

where $\hat{x}$ is the estimate of x. And, from (10)–(14), we have

$$\dot{\hat{x}}=A_0\hat{x}+ky+b\sigma(y)u+H^{\mathrm{T}}(y)W \tag{15}$$

Define the observer error as $\epsilon=x-\hat{x}$, from (9) and (15), we can obtain

$$\dot{\epsilon}=A_0\epsilon+\delta(y)+\Psi(y)d \tag{16}$$

Lemma 3.1 The state error ϵ satisfies

(i) $\epsilon=\epsilon_a+\epsilon_{b_1}+\epsilon_{b_2}$

(ii) Choose positive real numbers λ and μ ,such that $\mu e^{-\lambda t}\geq\|e_2^{\mathrm{T}}e^{A_0t}\|$, and design

$$\dot{g}_1=-\lambda g_1+\mu S(\|\Psi(y)\|),g_1(0)=0 \tag{17}$$
$$\dot{g}_2=-\lambda g_2+\mu,g_2(0)=0 \tag{18}$$

we have $\|\epsilon_{b_1,2}\|\leq Dg_1,\|\epsilon_{b_2,2}\|\leq Ag_2$.

In order to complete the design, next we calculate the derivative of $v_j(\rho\leq j\leq n)$ as follows

$$\begin{cases}\dot{v}_{\rho,i}=-k_iv_{j,1}+v_{\rho,i+1},i=1,\ldots,\rho-1\\ \dot{v}_{\rho,\rho}=-k_\rho v_{\rho,1}+v_{\rho,\rho+1}\end{cases} \tag{19}$$

Rewrite the unavailable state x_2 as follows

$$x_2=\zeta_2+b_\rho v_{\rho,2}+\sum_{j=\rho+1}^{n}b_jv_{j,2}+\Xi_{(2)}W+\epsilon_2 \tag{20}$$

By replacing the unavailable state x_2 with (20), the first row of (9) is expressed as

$$\dot{y}=\epsilon_2+\zeta_2+b_\rho v_{\rho,2}+\bar{\omega}^{\mathrm{T}}\theta+\delta_{(1)}(y)+\Psi_{(1)}(y)d \tag{21}$$

where

$$\bar{\omega}^{\mathrm{T}}=[0,v_{\rho+1,2},\ldots,v_{n,2},H_{(1)}^{\mathrm{T}}+\Xi_{(2)}]$$
$$\omega^{\mathrm{T}}=[v_{\rho,2},v_{\rho+1,2},\ldots,v_{n,2},H_{(1)}^{\mathrm{T}}+\Xi_{(2)}]$$
$$\theta=[b_\rho,\ldots,b_n,W^{\mathrm{T}}]^{\mathrm{T}}$$

4. ADAPTIVE BACKSTEPPING NN CONTROL DESIGN AND STABILITY ANALYSIS

In this section, we employ the backstepping technique similar with design procedure in Krstic (1995). Define coordinate transformation

$$z_1=y-y_r \tag{22}$$
$$z_i=v_{\rho,i}-\hat{\varrho}y_r^{(i-1)}-\alpha_{i-1},i=2,\ldots,\rho$$
$$z_{\rho+1}=0 \tag{23}$$

where α_i, $i=1,\ldots,\rho$, are stabilizing functions to be decided in then adaptive control design, $\varrho=\frac{1}{b_\rho}$, $\hat{\varrho}$ is the estimate of ϱ.

Step 1: We start with the equation for the tracking error z_1 , noticing $v_{\rho,2}=z_2+\hat{\varrho}\dot{y}_r+\alpha_1=z_2+(\varrho-\tilde{\varrho})\dot{y}_r+\alpha_1$, and define the first stable function $\alpha_1=\hat{\varrho}\bar{\alpha}_1$, according from (21)–(23) we can obtain

$$\begin{aligned}\dot{z}_1=&\bar{\alpha}_1+\bar{\omega}^{\mathrm{T}}\hat{\theta}+(\omega-\hat{\varrho}(\dot{y}_r+\bar{\alpha}_1)e_1)^{\mathrm{T}}\tilde{\theta}\\&+\epsilon_2+\zeta_2+\delta_{(1)}(y)+\Psi_{(1)}(y)d\\&+\hat{b}_\rho z_2-b_\rho\tilde{\varrho}(\dot{y}_r+\bar{\alpha}_1)\end{aligned} \tag{24}$$

Choose stabilizing function $\bar{\alpha}_1$ as

$$\begin{aligned}\bar{\alpha}_1=&-[(c_{1,1}+c_{1,2}+\hat{b}_\rho^2)(|z_1|-\lambda_1)^{\iota(1)-1}f_1+\hat{D}(g_1\\&+S(\|\Psi_{(1)}(y)\|))+\hat{A}(g_2+1)+\lambda_2^{'}S(|\hat{b}_\rho|)]s_1\\&-\bar{\omega}^{\mathrm{T}}\hat{\theta}-\zeta_2\end{aligned} \tag{25}$$

where $\lambda_2^{'}$ is positive real design parameters satisfying $\lambda_2^{'}>\lambda_2$. Estimate the unknown parameters $\hat{\varrho}$ and select the tuning functions as

$$\dot{\hat{\varrho}}=-\mathrm{sign}(b_\rho)(|z_1|-\lambda_1)^{\iota(1)-1}f_1s_1\gamma_b(\dot{y}_r+\bar{\alpha}_1) \tag{26}$$

$$\tau_{\theta,1}=(|z_1|-\lambda_1)^{\iota(1)-1}f_1s_1(\omega-\hat{\varrho}(\dot{y}_r+\bar{\alpha}_1)e_1) \tag{27}$$
$$\tau_{d,1}=(|z_1|-\lambda_1)^{\iota(1)-1}f_1(g_1+S(\|\Psi_{(1)}(y)\|)) \tag{28}$$
$$\tau_{a,1}=(|z_1|-\lambda_1)^{\iota(1)-1}f_1(g_2+1) \tag{29}$$

Regard $\dot{\hat{\varrho}}$ as the actual update law for $\hat{\varrho}$, retain $\tau_{\theta,1}$, $\tau_{d,1}$, $\tau_{a,1}$ as first tuning function and α_1 as first stabilizing function.

Step $i(i=2,\ldots,\rho)$: According to the expression of α_1 derived in the previous step, we have

$$\alpha_{i-1} = \alpha_{i-1}(X_{i-1}, \hat{\theta}, \hat{D}, \hat{A}, y), i = 2, \ldots, \rho \tag{30}$$

where $X_{i-1} = X_{i-1}(\zeta^{\mathrm{T}}, \mathrm{vec}(\Xi)^{\mathrm{T}}, \bar{\xi}_{\mathrm{n}-\rho+\mathrm{i}}, \bar{\mathrm{y}}_{\mathrm{r}}^{(\mathrm{i}-1)}, \mathrm{g}_1, \mathrm{g}_2, \hat{\varrho})$. Differentiating (23), with the help of (19), the derivative of z_i is expressed as

$$\begin{aligned}\dot{z}_i = &\alpha_i - \beta_i - \frac{\partial \alpha_{i-1}}{\partial y}(\epsilon_2 + \omega^{\mathrm{T}}\tilde{\theta} + \delta_{(1)}(y) + \Psi_{(1)}(y)d) \\ &- \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \frac{\partial \alpha_{i-1}}{\partial \hat{D}}\dot{\hat{D}} - \frac{\partial \alpha_{i-1}}{\partial \hat{A}}\dot{\hat{A}} + z_{i+1}\end{aligned} \tag{31}$$

where β_i is a function of available signals

$$\beta_i = k_i v_{\rho,1} + \dot{\hat{\varrho}} y_r^{(i-1)} + \frac{\partial \alpha_{i-1}}{\partial X_{i-1}}\dot{X}_{i-1} + \frac{\partial \alpha_{i-1}}{\partial y}(\zeta_2 + \omega^{\mathrm{T}}\hat{\theta}) \tag{32}$$

Choose the stabilizing function α_i and tuning function as

$$\begin{aligned}\alpha_i = &-[(c_{i,1} + c_{i,2}(\frac{\partial \alpha_{i-1}}{\partial y})^2 + c_{i,3} + 1) \\ &\times(|z_i| - \lambda_i)^{\iota(i)-1} f_i + \hat{D}S(|\frac{\partial \alpha_{i-1}}{\partial y}|)(g_1 \\ &+S(\|\Psi_{(1)}(y)\|)) + \hat{A}S(|\frac{\partial \alpha_{i-1}}{\partial y}|)(g_2 + 1) \\ &+\lambda_{i+1}^{'}]s_i + \beta_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\Gamma\tau_{\theta,i} + \frac{\partial \alpha_{i-1}}{\partial \hat{D}}\gamma_d\tau_{d,i} \\ &+\frac{\partial \alpha_{i-1}}{\partial \hat{A}}\gamma_a\tau_{a,i} \\ &-[\sum_{k=2}^{i-1}(|z_k| - \lambda_k)^{\iota(k)-1} f_k s_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}]\Gamma\frac{\partial \alpha_{i-1}}{\partial y}\omega \\ &+[\sum_{k=2}^{i-1}(|z_k| - \lambda_k)^{\iota(k)-1} f_k s_k \frac{\partial \alpha_{k-1}}{\partial \hat{D}}] \\ &\times\gamma_d S(|\frac{\partial \alpha_{i-1}}{\partial y}|)(g_1 + S(\|\Psi_{(1)}(y)\|)) \\ &+[\sum_{k=2}^{i-1}(|z_k| - \lambda_k)^{\iota(k)-1} f_k s_k \frac{\partial \alpha_{k-1}}{\partial \hat{A}}] \\ &\times\gamma_a S(|\frac{\partial \alpha_{i-1}}{\partial y}|)(g_2 + 1)\end{aligned} \tag{33}$$

where $\lambda_{i+1}^{'}$ is positive real design parameters satisfying $\lambda_{i+1}^{'} > \lambda_{i+1}$.

$$\tau_{\theta,i} = \tau_{\theta,i-1} - \frac{\partial \alpha_{i-1}}{\partial y}\omega(|z_i| - \lambda_i)^{\iota(i)-1} f_i s_i \tag{34}$$

$$\begin{aligned}\tau_{d,i} = &\tau_{d,i-1} + (|z_i| - \lambda_i)^{\iota(i)-1} f_i S(|\frac{\partial \alpha_{i-1}}{\partial y}|) \\ &\times(g_1 + S(\|\Psi_{(1)}(y)\|))\end{aligned} \tag{35}$$

$$\begin{aligned}\tau_{a,i} = &\tau_{a,i-1} + (|z_i| - \lambda_i)^{\iota(i)-1} f_i S(|\frac{\partial \alpha_{i-1}}{\partial y}|) \\ &\times(g_2 + 1)\end{aligned} \tag{36}$$

we retain $\tau_{\theta,i}$, $\tau_{d,i}$, $\tau_{a,i}$ and α_i as the ith tuning function and stabilizing function, respectively. At the end of the recursive procedure, the last stabilizing function α_ρ is used in the actual control law, because of $z_{\rho+1} = 0$ and $\lambda_{\rho+1}^{'} = 0$, so the term $-(z_\rho - \lambda_\rho)^{\iota(\rho)-1} f_\rho$ of (33) can be omitted in α_ρ, then the control law u can be denoted as

$$u = \frac{1}{\sigma(y)}(\alpha_\rho - v_{\rho,\rho+1} + \hat{\varrho} y_r^{(\rho)}) \tag{37}$$

and the last tuning functions as the actual update laws

$$\dot{\hat{\theta}} = \Gamma\tau_{\theta,\rho} \tag{38}$$

$$\dot{\hat{D}} = \gamma_d\tau_{d,\rho} \tag{39}$$

$$\dot{\hat{A}} = \gamma_a\tau_{a,\rho} \tag{40}$$

Theorem 1: The closed-loop adaptive system consists of the plant (9), the control law (37), the adaptive update laws (26), (38)–(40) and filters (10)–(13). All the signals are bounded and the tracking error converges to the origin of the particular neighborhood, that is, $\lim_{t\to\infty} z_1(t) \in [-\lambda_1, \lambda_1]$, where λ_1 is a positive real design parameter.

Proof: Due to reference signal $y_r(t)$ is continues and bounded derivatives up to order ρ and the smoothness of the nonlinearities in (8), the solution of closed-loop adaptive system exists and is unique. Let its maximum interval of existence be $[0, t_f]$. Define the candidate Lyapunov function as

$$\begin{aligned}V_\rho = &\sum_{i=1}^{\rho}\frac{1}{\iota(i)}(|z_i| - \lambda_i)^{\iota(i)} f_i + \frac{1}{2}[\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} + \frac{1}{\gamma_b}|b_\rho|\tilde{\varrho}^2 \\ &+\frac{1}{\gamma_d}\tilde{D}^2 + \frac{1}{\gamma_a}\tilde{A}^2] + \sum_{i=1}^{\rho}\frac{1}{4c_{i,2}}\epsilon_a^{\mathrm{T}} P\epsilon_a\end{aligned} \tag{41}$$

Then, the time derivative of V_ρ is given by

$$\begin{aligned}\dot{V}_\rho = &\sum_{i=1}^{\rho}(|z_i| - \lambda_i)^{\iota(i)-1} f_i s_i \dot{z}_i - \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}} - \frac{1}{\gamma_b}|b_\rho|\tilde{\varrho}\dot{\hat{\varrho}} \\ &-\frac{1}{\gamma_d}\tilde{D}\dot{\hat{D}} - \frac{1}{\gamma_a}\tilde{A}\dot{\hat{A}} - \sum_{i=1}^{\rho}\frac{1}{4c_{i,2}}\epsilon_a^{\mathrm{T}}\epsilon_a\end{aligned} \tag{42}$$

substituting (24)–(25), (31)–(33), (26), (38)–(40) into (42), we have

$$\begin{aligned}\dot{V}_\rho \leq &-(c_{1,1} + c_{1,2} + \hat{b}_\rho^2)(|z_1| - \lambda_1)^{2(\iota(1)-1)} f_1 \\ &-\sum_{i=2}^{\rho}[c_{i,1} + c_{i,2}(\frac{\partial \alpha_{i-1}}{\partial y})^2 + c_{i,3} + 1] \\ &\times(|z_i| - \lambda_i)^{2(\iota(i)-1)} f_i \\ &+(|z_1| - \lambda_1)^{\iota(1)-1} f_1|\hat{b}_\rho|(|z_2| - \lambda_2^{'}) \\ &+\sum_{i=2}^{\rho}(|z_i| - \lambda_i)^{\iota(i)-1} f_i(|z_{i+1}| - \lambda_{i+1}^{'}) \\ &+\sum_{i=1}^{\rho}(|z_i| - \lambda_i)^{\iota(i)-1} f_i|\frac{\partial \alpha_{i-1}}{\partial y}||\epsilon_{a,2}| \\ &-\sum_{i=1}^{\rho}\frac{1}{4c_{i,2}}\epsilon_a^{\mathrm{T}}\epsilon_a\end{aligned} \tag{43}$$

Then, using Youngs inequality, specific as follows

$$-\hat{b}_\rho^2(|z_1| - \lambda_1)^{2(\iota(1)-1)} f_1 + (|z_1| - \lambda_1)^{\iota(1)-1} f_1|\hat{b}_\rho|(|z_2| - \lambda_2^{'})$$

$$\leq \frac{1}{4}(|z_2| - \lambda_2^{'})^2 \quad (44)$$

$$-(|z_i| - \lambda_i)^{2(\iota(i)-1)} f_i + (|z_i| - \lambda_i)^{\iota(i)-1} f_i (|z_{i+1}| - \lambda_{i+1}^{'})$$

$$\leq \frac{1}{4}(|z_{i+1}| - \lambda_{i+1}^{'})^2, i = 2, \ldots, \rho - 1 \quad (45)$$

$$(|z_i| - \lambda_i)^{\iota(i)-1} f_i |\frac{\partial \alpha_{i-1}}{\partial y}||\epsilon_{a,2}|$$

$$\leq c_{i,2}(\frac{\partial \alpha_{i-1}}{\partial y})^2 (|z_i| - \lambda_i)^{2(\iota(i)-1)} f_i + \frac{1}{4c_{i,2}}|\epsilon_{a,2}|^2, i = 1, \ldots, \rho \quad (46)$$

Let the selection of $c_{i,3}(|z_i| - \lambda_i)^{2(\iota(i)-1)} f_i$ satisfies that

$$c_{i,3}(|z_i| - \lambda_i)^{2(\iota(i)-1)} f_i > \frac{1}{4}(|z_i| - \lambda_i^{'})^2, \forall |z_i| \geq \lambda_i^{'} \quad (47)$$

substituting (44)–(47) into (43) , it makes V_ρ negative semidefinite, that is

$$\dot{V}_\rho \leq -\sum_{k=1}^{\rho} c_{k,1}(|z_k| - \lambda_k)^{2(\iota(k)-1)} f_k \leq 0 \quad (48)$$

that implies $z_i, i = 1, \ldots, \rho$, $\tilde{\varrho}$, $\tilde{\theta}$, $\tilde{A}$, $\tilde{D}$, ϵ_a are bounded. Further, we get $\hat{\varrho}$, $\hat{\theta} \in L_\infty$. y is also bounded because of the boundedness of z_1 and y_r. Then, the the boundedness of ζ can be ensured from (10), and Ξ is bounded from (13). The boundedness of ϵ is ensured from *Lemma 3.1.* From (11), when $i = 1, \ldots, n$ we can obtain

$$\xi_i = \frac{s^{i-1} + k_1 s^{i-2} + \cdots + k_{i-1}}{K(s)}[\sigma(y)u] \quad (49)$$

where $K(s) = \det(sI - A_0) = s^n + k_1 s^{n-1} + \cdots + k_n$. In addition, from the system (8), one can show that

$$\frac{d^n y}{dt^n} = \sum_{i=1}^{n} \frac{d^{n-i}}{dt^{n-i}}[f_i(y) + \Psi_{(i)}(y)d] + \sum_{j=\rho}^{n} b_j \frac{d^{n-j}}{dt^{n-j}}[\sigma(y)u] \quad (50)$$

Noticing the second term on the right side of the equation is $B(s)\sigma(y)u$, substituting it into (49), we get

$$\xi_i = \frac{s^{i-1} + k_1 s^{i-2} + \cdots + k_{i-1}}{K(s)B(s)} \times \{\frac{d^n y}{dt^n} - \sum_{i=1}^{n} \frac{d^{n-i}}{dt^{n-i}}[f_i(y) + \Psi_{(i)}(y)d]\} \quad (51)$$

The boundedness of y, the smoothness of $f_i(y)$ and $\Psi(y)$, Assumption 2 and (51), imply that $\xi_1, \ldots, \xi_n$ are bounded, that means $v_{i,j}, i = \rho, \ldots, n, j = 1, \ldots, n$ is bounded. Then from (14),we have the boundedness of $\hat{x}$, and the boundedness of x further. In the end, because $\sigma(y)u$ is bounded, from (37), u is bounded. Thus, we have shown that all the signals of the closed-loop system are bounded on $[0, t_f]$. It can be further concluded that $(|z_i| - \lambda_i)^{\iota(i)-1} f_i \in L^2$. Furthermore, from $\frac{d}{dt}(|z_i| - \lambda_i)^{\iota(i)-1} f_i = \frac{\partial}{\partial z_i}[(|z_i| - \lambda_i)^{\iota(i)-1} f_i]\dot{z}_i$, we have $\frac{d}{dt}(|z_i| - \lambda_i)^{\iota(i)-1} f_i \in L_\infty$. According to Barbalats lemma, we conclude that,

$$\lim_{t \to \infty}(|z_i| - \lambda_i)^{\iota(i)-1} f_i = 0 \quad (52)$$

In particular, tracking error z_1 converges to $[-\lambda_1, \lambda_1]$.

5. SIMULATION EXAMPLE

In this section, in order to illustrate the effectiveness of the proposed adaptive backstepping NN control approach, we give one mathematical simulation example.

The system model is described by the following second-order differential equation

$$\begin{cases} \dot{x}_1 = x_2 + ye^{-y^2} + y\sin(0.1t) \\ \dot{x}_2 = y^2 + 0.2y^3 + bu + 0.1\cos(0.1t) \\ y = x_1 \end{cases} \quad (53)$$

where the high-frequency gain $b = 2$ is assumed to be unknown, $f_1(y) = ye^{-y^2}$, $f_2(y) = y^2 + 0.2y^3$ are the unknown functions; $\Psi(y) = [y, 1]^{\mathrm{T}}$; $d_1 = \sin(0.1t)$ and $d_2 = 0.1\cos(0.1t)$ are the disturbances; the reference output is taken as $y_r(t) = 2 + \sin(2t)$.

The real control law is designed as

$$\begin{aligned} u = &-[(c_{2,1} + c_{2,2}(\frac{\partial \alpha_1}{\partial y})^2 + c_{2,3})(|z_2| - \lambda_2)^{\iota(2)-1} \\ &\times f_2 + \hat{D}S(|\frac{\partial \alpha_1}{\partial y}|)(g_1 + S(\|\Psi_{(1)}(y)\|)) \\ &+ \hat{A}S(|\frac{\partial \alpha_1}{\partial y}|)(g_2 + 1)]s_2 + \beta_2 + \frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma\tau_{\theta,2} \\ &+ \frac{\partial \alpha_1}{\partial \hat{D}}\gamma_d\tau_{d,2} + \frac{\partial \alpha_1}{\partial \hat{A}}\gamma_a\tau_{a,2} + \hat{\varrho}\ddot{y}_r \end{aligned} \quad (54)$$

And the adaptive laws are given as follows

$$\begin{aligned} \dot{\hat{\theta}} = &\Gamma[(|z_1| - \lambda_1)^{\iota(1)-1} f_1 s_1(\omega - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1)e_1) \\ &- \frac{\partial \alpha_1}{\partial y}\omega(|z_2| - \lambda_2)^{\iota(2)-1} f_2 s_2] \end{aligned} \quad (55)$$

$$\begin{aligned} \dot{\hat{D}} = &\gamma_d[(|z_1| - \lambda_1)^{\iota(1)-1} f_1(g_1 + S(\|\Psi_{(1)}(y)\|)) \\ &+ (|z_2| - \lambda_2)^{\iota(2)-1} f_2 S(|\frac{\partial \alpha_1}{\partial y}|) \\ &\times (g_1 + S(\|\Psi_{(1)}(y)\|)) \end{aligned} \quad (56)$$

$$\begin{aligned} \dot{\hat{A}} = &\gamma_a[(|z_1| - \lambda_1)^{\iota(1)-1} f_1(g_2 + 1) \\ &+ (|z_2| - \lambda_2)^{\iota(2)-1} f_2 S(|\frac{\partial \alpha_1}{\partial y}|)(g_2 + 1)] \end{aligned} \quad (57)$$

In simulation, for the k-filters, set $k_1 = 3, k_2 = 2$, For the first RBF vector $h_1(y)$ contains 10 nodes with the center $\mu_j, (j = 1, \ldots, 10)$ evenly placed on [-5,5] and the width $\eta = 0.56$, and the second RBF vector $h_2(y)$ contains 20 nodes with the center $\mu_j, (j = 1, \ldots, 20)$ evenly placed on [-5,5] and the width $\eta = 1.05$. The design parameters are taken as $\Gamma = \mathrm{diag}10$, $c_{1,1} = c_{1,2} = 1$, $c_{2,1} = c_{2,2} = 0.35$, $c_{2,3} = 0.25$, $\lambda_2^{'} = 2$, $\gamma_d = 0.5$, $\gamma_a = 0.15$. We used $S(|y|) = \sqrt{y^2 + 1})$. All initial conditions are set to be zero. The tracking accuracy is required to be $\lambda_1 = \lambda_2 = 0.05$, i.e. $|y - y_r| \leq 0.05$, when $t \to \infty$.

The simulation results are shown in Fig. 1–2. From Fig.1, it can be seen that the tracking error can converge to

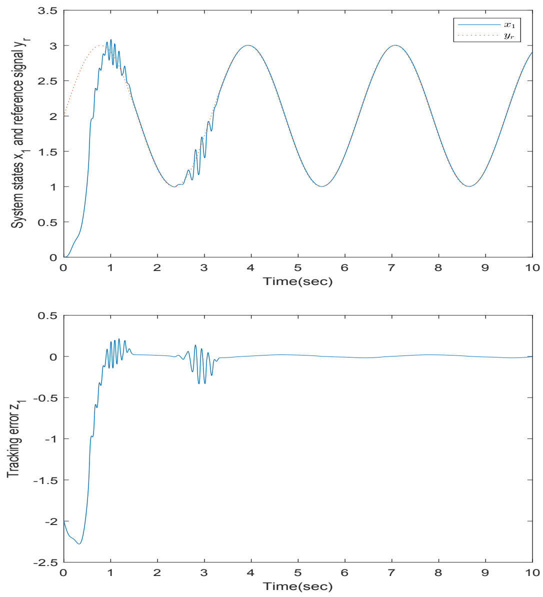

Fig. 1. Trajectories of x_1, y_r and tracking error z_1.

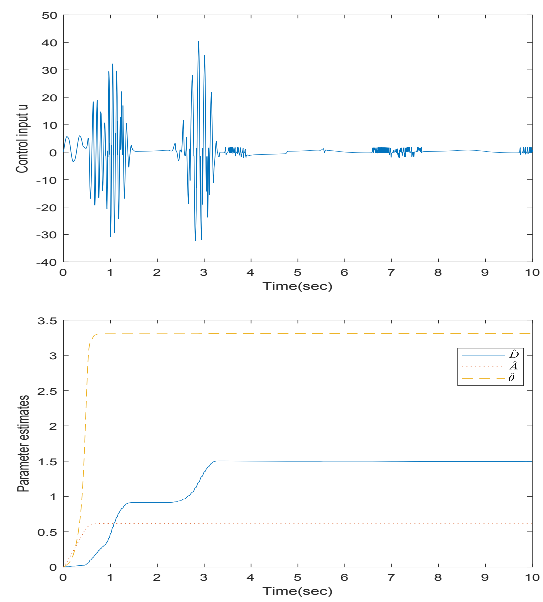

Fig. 2. The controller u and trajectories of $\hat{\theta}$, $\hat{D}$, $\hat{A}$.

a small neighborhood around the origin after 10s, but a large tracking error and chattering also exist in the initial phase. However, better tracking performance is gradually acquired along with the learning of neural network, and the chattering also disappears after 10s. Fig.2 shows the trajectories of u, $\hat{\theta}$, $\hat{A}$, $\hat{D}$, from which, we can see the boundednesses of them.

6. CONCLUSION

In this paper ,the requirements of the bound of uncertain parameters and the bound of the disturbances have been removed via control design. The design and analysis is based on adaptive backstepping with a flat zone introduced in the Lyapunov function, where the flat zone enables the bound estimation to be incorporated in the backstepping design. The main advantage of the obtained ANNC scheme is that the boundedness of all the variables are pledged , as well as tracking error converges to an arbitrarily small neighbourhood of the origin.

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