Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Limited Gradient Criterion for Global Source Seeking with Mobile Robots

Marcus Gronemeyer * **Mirco Alpen** * **Joachim Horn** *

* *Institute of Control Engineering, Helmut Schmidt University, Holstenhofweg 85, 22043 Hamburg, Germany (e-mail: marcus.gronemeyer@hsu-hh.de, joachim.horn@hsu-hh.de)*

Abstract: This paper presents a criterion and control scheme based on the assumption of a bound on the gradient of a field distribution which guarantees to find the global extremum of the distribution. Mobile robots move through the search space gathering information at points which are calculated as a minimization problem over part of the search space which is guaranteed to include the global extremum based on the previously gathered measurements. Position control in combination with collision avoidance drives each robot to the next position while communicating its position to the other robots. Upon arrival, the next measurement of the field distribution is performed and the next position reference is calculated by each robot until the robots narrowed the search area to a single location. Previously proposed control schemes can find single points as candidates for the global maximum but struggle to guarantee that this point is the global extremum. Simulation results with robot models show the performance in comparison to a naive approach.

Keywords: Source Seeking, Autonomous Mobile Robots, Cooperative navigation, Decentralized Control and Systems, Mission planning and decision making, Multi-vehicle systems

1. INTRODUCTION

The field of applications for robots has experienced a growing interest in research over the last years. Different contexts of applications are on the rise including disaster scenarios as elaborated in Murphy (2004) and Hoon Oh et al. (2017), logistics with cooperative vehicle like in Peter R. Wurman et al. (2008) and military applications as shown in Brian Yamauchi (2004). There are even more fields of interest like inspection of buildings and agriculture where robots can be used. For all of these fields robots perform tasks for humans which are either dangerous, tedious or repetitive. The usage of robot groups can cut the time required for the completion and even compensate the failure of robots without failing to complete the task.

One task of interest for those applications is the search of a source of a physical quantity, also called source seeking. Source seeking can be applied in any field which deals with the distribution of for example gas or radiation over an area and the goal is to find the extremum of this distribution in the area. The scope of the traced project includes the modeling and experimentation on a group of robots which can maneuver together through the area while searching for the extremum.

Works in the field tend to use a gradient based method which tries to estimate the gradient of the distribution and move along its direction to reach the extremum. This method can be paired with cooperative formation control as shown in Rosero and Werner (2014a) and Rosero and Werner (2014b), among many others, with a fixed communication topology for communication of position and scalar field value. The drawback of this method is the convergence at local extrema and therefore it can not be applied for distributions with multiple sources of different amplitude or disturbed distributions. A different control scheme has to be used for those scenarios.

This leads to the task of global source seeking, meaning the search for the highest value in presence of multiple extrema. One possible solution is the usage of kriging, a method which has its origins in geology and has been employed for groups of robots by for example Xu et al. (2011) and Kahn et al. (2015). It includes the calculation of an estimation of the distribution together with an uncertainty and exploration of points which are probable to yield high values according to the estimation. A problematic aspect with this approach is that there could exist other extrema with higher amplitude which are not found by this method.

The main contribution of this work is the elaboration of properties which are required in order to able to find the global maximum with an acceptable effort and develop a criterion for the remaining search area which is guaranteed to include the global maximum together with an according control scheme which searches this area systematically. The dynamic behavior of this approach is tested in simulation and compared to a naive approach to give an idea about the performance.

The rest of this paper is outlined as follows: The problem is defined in Section 2 and the fundamentals are introduced in Section 3. Afterwards a global source seeking scheme revolving around the choice of waypoints in Section 4 and a corresponding control scheme is presented in Section 5. This global source seeking scheme is tested and compared in simulation in Section 6 before the conclusion in Section 7.

Notation

$\mathbf{I}_q$ denotes the $q \times q$ identity matrix. $\mathbf{1}_q$ denotes a vector filled with ones in and dimension q. The set of $p \times q$ matrices is denoted by $\mathbb{R}^{p\times q}$. For the i-th row and the j-th column entry of a matrix $\boldsymbol{A}$ the notation $\boldsymbol{A}_{ij}$ is used. To describe the parallelism and communication, two Kronecker extensions are used: These are $\boldsymbol{M}_{(q)} = \boldsymbol{M} \otimes \mathbf{I}_q$ and $\hat{\boldsymbol{M}} = \mathbf{I}_N \otimes \boldsymbol{M}$.

2. PROBLEM DEFINITION

The spatial distribution φ of a physical quantity is to be considered, which assigns a scalar value $\varphi(\boldsymbol{x})$ to all positions $\boldsymbol{x} \in \mathbb{R}^q$ in a compact space $\mathcal{D} \subset \mathbb{R}^q$. The distribution is assumed to be time-invariant and continuous. It is further assumed that there exists a position $\boldsymbol{x}_{max} \in \mathcal{D}$ such that $\varphi(\boldsymbol{x}_{max}) > \varphi(\boldsymbol{x})\ \forall \boldsymbol{x} \in \mathcal{D} \setminus \boldsymbol{x}_{max}$. These assumptions are extended to once continuous differentiable for the gradient criterion and n-times continuous differentiable for the higher-order criteria.

A group of N identical robots with index $i = 1 \dots N$ which are able to measure their position $\boldsymbol{y}_i \in \mathbb{R}^q$ and the scalar field value $\varphi(\boldsymbol{y}_i)$ are used to find $\boldsymbol{x}_{max}$. The linear Model $\boldsymbol{P}(s)$ describes the dynamic behavior of a robot which moves through the q-dimensional space, representing a ground ($q = 2$) or airborne ($q = 3$) robot, respectively. Measurements of the scalar field values $\varphi(\boldsymbol{y}_i)$ are taken by sensors on the robots and are communicated together with the corresponding positions to a predefined neighborhood $\mathcal{N}_i$ of the robot.

All current measurements stored at robot i can be represented as

$$\mathcal{S}_i(t_k) = \bigcup_{l=0}^{k} \{[\varphi(\boldsymbol{y}_j(t_l)), \boldsymbol{y}_j(t_l)] | j \in \mathcal{N}_i \cap \mathcal{M}(t_l)\}$$

where $t_k = k \cdot T$ with sampling period T and $\mathcal{M}(t_l)$ is the set of robots taking a measurement at time t_l.

The mission is to assure finding the global maximum of the field distribution while trying to minimize the required time and energy consumption and avoid collisions between the robots.

3. PRELIMINARIES

The presentation of the findings of this work requires some concepts already presented in the literature. These concepts are an estimation and maximization scheme called kriging as well as multi-vehicle systems and collision avoidance required for unmanned aerial systems (UAS) which are presented in the following sections.

3.1 Kriging

Kriging is a method used to estimate a field distribution with a set of sampled points first used in the context of geology Krige (1951). The distribution is considered to be modeled by a function

$$\phi : \boldsymbol{x} \in D \to \phi(\boldsymbol{x}) \in \mathbb{R}.$$

The estimation $\overline{\phi}$ of the distribution ϕ can be performed by different methods. In the recent years a Gaussian process model has been popularized as shown in Jones (2001). The Gaussian process regression yields a mean value $\overline{\phi}(\boldsymbol{x})$ and standard deviation $\sigma(\boldsymbol{x})$ for each position $\boldsymbol{x}$.

This estimation can be used to explore a spatial distribution of physical values using UAS by moving to new positions and adding them to the set of sampled points. This idea has been examined by several previous works, some of them even search for the global maximum like Kahn et al. (2015).

Their proposal is the usage of a minimization problem in order to let each UAS i calculate new sampling points $\boldsymbol{y}_{r,i}$ using its mean value $\overline{\phi}_i$ and standard deviation σ_i as

$$\begin{aligned} \boldsymbol{y}_{r,i}(t_k) &= \arg\min_{\boldsymbol{x}\in D}\{J_i(\boldsymbol{x}, \boldsymbol{y})\} \\ s.t. \quad & \overline{\phi}_i(\boldsymbol{x}) + b \cdot \sigma_i(\boldsymbol{x}) > f_{i,max}(t_k) \end{aligned}$$

where $f_{i,max}(t_k)$ is the position of the maximum value of $\overline{\phi}_i$ at time t_k, b is a tuning parameter and

$$J_i(\boldsymbol{x}, \boldsymbol{y}) = |\boldsymbol{y}_i - \boldsymbol{x}|^2 - \sum_{j\in\mathcal{N}_i} \alpha |\boldsymbol{y}_j - \boldsymbol{x}|^2$$

where α is a tuning parameter to weigh the distance to the other UAS against the distance from the own current position. This sampling point selection yields values in a region where the sum of uncertainty and the estimated value is higher than the maximum estimated value and searches for positions in this region which are close to the current position but also far away from the other UAS.

The set of measurements is updated whenever one of the UAS reaches $\boldsymbol{y}_{r,i}$ and new sampling points are calculated together with a new estimation whenever the set of measurements changes.

3.2 Multi-Vehicle Systems

The position control of the group of UAS used in this work is derived from cooperative formation control. The cooperative formation control loop shown in Figure 1 as used by Pilz and Werner (2012) and Bartels and Werner (2014). It consists of 3 main blocks: the first includes the Laplacian $\boldsymbol{\mathcal{L}}$ and is not block-diagonal since it represents the communication and in contrast, the formation controller $\boldsymbol{K}(s)$ and agent dynamics $\boldsymbol{P}(s)$ which show a parallel signal flow among the vehicles and are therefore block-diagonal.

A series connection of those 3 blocks with the formation reference signal $\boldsymbol{r}$ as well as a feedback form the control loop. All signals in this setup are considered as concatenations of the signals of each agent such as $[r_1 \dots r_N]^{\mathrm{T}}$.

The Laplacian is derived from the communication topology and graph theory. It can be defined element-wise as

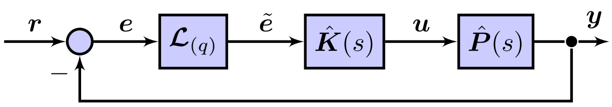

Fig. 1. Global Formation Loop from Pilz and Werner (2012)

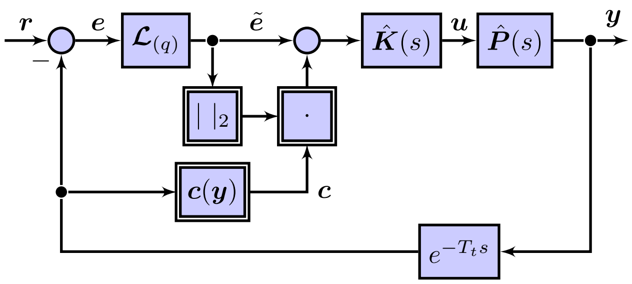

Fig. 2. Cooperative Formation Loop with Integrated Collision Avoidance Algorithm

$$\boldsymbol{\mathcal{L}}_{ij} = \begin{cases} 1 & \text{if } i = j \quad \text{and} \quad |\mathcal{N}_i| \neq 0 \\ \dfrac{1}{|\mathcal{N}_i|} & \text{if } i \neq j \quad \text{and} \quad (i,j) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

An important property of the consensus protocol is that it only controls the relative position of the agents such that for the position $\boldsymbol{y}$ holds:

$$\lim_{t\to\infty} \boldsymbol{y}(t) = \boldsymbol{r} + b \cdot \boldsymbol{1} \tag{2}$$

where b denotes an arbitrary scalar value in $\mathbb{R}$. This property will later be used for collision avoidance. If $\mathcal{L}_{(q)}$ is replaced by an identity matrix of the same size $\boldsymbol{I}_{Nq}$ this property is lost and the control loop represents position control for the UAS.

3.3 Collision Avoidance

Whenever multiple UAS move through the same space collisions among them have to be prevented. Two popular categories of collision avoidance schemes are rule-based like (Atinc et al., 2013) and prediction-based like Richards and How (2004). This work will use a rule-based approach in the form of an artificial force field which is presented in Gronemeyer and Horn (2019). The corresponding control loop is shown in Figure 2.

The collision avoidance acts upon the communicated formation error $\tilde{e}$ or in this paper where the positions are tracked on the position error by adding a component $\boldsymbol{c}\cdot|\tilde{\boldsymbol{e}}|_2$. The factor $|\tilde{\boldsymbol{e}}|_2$ weights the normalized components $\boldsymbol{c}_i$ for each UAS and $\boldsymbol{c}_i$ is defined as

$$\boldsymbol{c}_i(y) = \sum_{\mathcal{N}_j} c_{i,j}(d_{i,j}) \cdot \boldsymbol{R}(d_{i,j}) \cdot \frac{\boldsymbol{y}_j - \boldsymbol{y}_i}{|\boldsymbol{y}_j - \boldsymbol{y}_i|_2}$$

where $c_{i,j}(d_{i,j})$ gives the magnitude in dependence of the distance $d_{i,j}$ and the matrix $\boldsymbol{R}(d_{i,j})$ the rotation such that the resulting vector points away from UAS j in relation to UAS i with a rotational component for small distances in order to prevent deadlocks.

4. WAYPOINT GENERATION

The search for the global maximum and exploration of the scalar field distribution requires information to be gathered. The following paragraphs will show how $\mathcal{S}_i$ can be used to find the feasible region for the global maximum using a priori information about the maximum rate of change and curvature of the field distribution φ. Afterwards the selection method for the next position references for each UAS is presented.

4.1 Limited Gradient Criterion

An important aspect is the properties of the scalar field distribution φ. If φ is completely unknown besides being continuous and restricted to $\mathcal{D}$ and $\mathcal{S}_i(t_k)$ does not include all elements of $\mathcal{D}$ there would be no method to determine which point is the true global maximum. This is due to the fact that there could be a spike in φ at one unexplored location $\mathcal{D} \setminus \mathcal{S}_i(t_k)$ meaning a large rate of change over space.

One method to deal with this problem is finding an upper limit for the rate of change in φ. This restriction can be motivated by the fact that physical quantities like gas or heat tend to distribute over time until they reach a steady distribution. This would lead to limited directional derivatives $|\frac{\delta\varphi}{\delta x_i}| < c \;\forall i = 1 \dots q$ and in turn limit the magnitude of the gradient $\boldsymbol{g}$ to $|\boldsymbol{g}| < \sqrt{q} \cdot c$.

Using this assumption leads to the following theorem:

Theorem 1. For every set $\mathcal{S}_i(t_k)$ of measurements from a continuous field distribution $\varphi(\boldsymbol{x})$ with limited magnitude $|\boldsymbol{g}| < \sqrt{q} \cdot c$ of its gradient every position $\boldsymbol{x} \in \mathcal{D}$ for which holds

$$\begin{aligned} f_{pot,max}(t_k) &= \min_{[\varphi(\boldsymbol{y}_{\mathcal{S},i}),\boldsymbol{y}_{\mathcal{S},i}]\in\mathcal{S}_i(t_k)} \varphi(\boldsymbol{y}_{\mathcal{S},i}) + c(\boldsymbol{x} - \boldsymbol{y}_{\mathcal{S},i})^{\mathrm{T}}\boldsymbol{1}_q \\ &\geq \max_{\varphi(\boldsymbol{y}_{\mathcal{S},i})\in\mathcal{S}_i(t_k)} \varphi(\boldsymbol{y}_{\mathcal{S},i}) \end{aligned}$$

is part of a set $\mathcal{Q} \subset \mathcal{D}$ which contains all candidates for the global maximum.

Proof. Every Element $[\varphi(\boldsymbol{y}_{\mathcal{S},i}), \boldsymbol{y}_{\mathcal{S},i}]$ of $\mathcal{S}_i(t_k)$ can be used to derive a zero order Taylor series for $\boldsymbol{x}$ as

$$\varphi(\boldsymbol{x}) = \varphi(\boldsymbol{y}_{\mathcal{S},i}) + R_1(\boldsymbol{x} - \boldsymbol{y}_{\mathcal{S},i})$$

where R_1 represents the higher order terms. The upper limit for this expression is derived by substituting the Lagrange error term $|R_1(\boldsymbol{x} - \boldsymbol{y}_{\mathcal{S},i})| \leq \boldsymbol{M}(\boldsymbol{x} - \boldsymbol{y}_{\mathcal{S},i})$ to find an upper bound for R_1 and set $\boldsymbol{M} = c \cdot \boldsymbol{1}_q$ since the maximum rate of change is assumed to be known. This leads to

$$\varphi(\boldsymbol{x}) \leq \varphi(\boldsymbol{y}_{\mathcal{S},i}) + c(\boldsymbol{x} - \boldsymbol{y}_{\mathcal{S},i})^{\mathrm{T}}\boldsymbol{1}_q$$

Applying this inequality to every element in $\mathcal{S}_i(t_k)$ yields a set of inequalities $\mathcal{I}(\boldsymbol{x}, \boldsymbol{y}_{\mathcal{S},i})$. If the minimal value at any position $\boldsymbol{x}$ of $\mathcal{I}(\boldsymbol{x}, \boldsymbol{y}_{\mathcal{S},i})$ is smaller than the highest measured value $\varphi(\boldsymbol{y}_{\mathcal{S},i})$ than no possible version of φ is going to yield a global maximum at this position. All remaining $\boldsymbol{y}$ for which

$$\min(\mathcal{I}(\boldsymbol{x}, \boldsymbol{y}_{\mathcal{S},i})) \geq \max_{\varphi(\boldsymbol{y}_{\mathcal{S},i}) \in \mathcal{S}_i(t_k)} \varphi(\boldsymbol{y}_{\mathcal{S},i})$$

form $\mathcal{Q} \subset \mathcal{D}$ which contains all candidate global maximum positions.

4.2 Extension to Higher Order

The criterion presented in the last section can be extended to higher orders of the Taylor series

$$T_n(\boldsymbol{x}, \boldsymbol{y}_{\mathcal{S},i}) = \sum_{\alpha \leq n} \frac{1}{\alpha!} \delta^\alpha \varphi(\boldsymbol{x})(\boldsymbol{x} - \boldsymbol{y}_{\mathcal{S},i})^\alpha.$$

This would require measurements or calculations of all derivatives up to the n-th order $\varphi(\boldsymbol{y})^{(n)}$ and a limit for the $n+1$-th order derivative $\delta^{n+1}\varphi(\boldsymbol{x})$. This leads to a criterion

$$\min_{[\varphi(\boldsymbol{y}_{\mathcal{S},i}), \boldsymbol{y}_{\mathcal{S},i}] \in \mathcal{S}_i(t_k)} T_n(\boldsymbol{x}, \boldsymbol{y}_{\mathcal{S},i}) + \sum_{\alpha=n+1} \frac{1}{\alpha!} \boldsymbol{M}(\boldsymbol{y}_{\mathcal{S},i})(\boldsymbol{x} - \boldsymbol{y}_{\mathcal{S},i})^\alpha$$
$$\geq \max_{\varphi(\boldsymbol{y}) \in \mathcal{S}_i(t_k)} \varphi(\boldsymbol{y})$$

with a proof similar to the proof of theorem 1. This criterion would give a smaller set $\mathcal{Q}$ for the same number of sampling points but requires more information regarding measurements and the limit in the higher derivatives which might not be realistic for a scenario of UAS with limited computational capacity and limited load.

4.3 Waypoint Selection

The selection of waypoints $\boldsymbol{y}_{r,i}$ is a key aspect of exploration. Selection should regard the information gained but also distribute the exploration load on all UAS of the group and consider the dynamics of each UAS as well as their spacing.

Section 3.1 presents a selection scheme from (Kahn et al., 2015) which fulfills most requirements, but it is not shown that it guarantees finding the global maximum. This can be remedied by using one of the criteria presented in the previous section instead of $\overline{\phi}_i(\boldsymbol{x}) + b \cdot \sigma_i(\boldsymbol{x}) > f_{i,max}(t_k)$. Additionally the spacing needs to be considered which can be represented by the term

$$\Delta_{i,j} = |\boldsymbol{y}_{r,j} - \boldsymbol{y}_{r,i}|_2 \quad \text{for} \quad i, j \in 1 \dots N$$

leading to the minimization problem

$$\boldsymbol{y}_{r,i}(t_k) = \arg\min_{\boldsymbol{x} \in D} \{J_{i,aug}(\boldsymbol{x}, \boldsymbol{y})\}$$
$$s.t. \quad \boldsymbol{x} \in \mathcal{Q} \quad \wedge \quad \Delta_{i,j} < d_2 \quad for \quad i \neq j$$

which minimizes

$$J_{i,aug}(\boldsymbol{x}, \boldsymbol{y}) = |\boldsymbol{y}_i - \boldsymbol{x}|^2 - \sum_{j \in \mathcal{N}_i} \alpha |\boldsymbol{y}_j - \boldsymbol{x}|^2 - \beta \cdot f_{pot,max}(t_k)$$

with d_2 as a threshold for the collision avoidance as described in Gronemeyer and Horn (2019) and β as a tuning parameter for the weighting of positions with potentially high values against the other two terms. This choice of boundary conditions for the minimization ensures that the waypoints are not too close together in order to prevent deadlocks caused by the cancellation of the position control and collision avoidance. The term $f_{pot,max}(t_k)$ gives the opportunity to prioritize points with potentially high values in $\varphi(\boldsymbol{x})$.

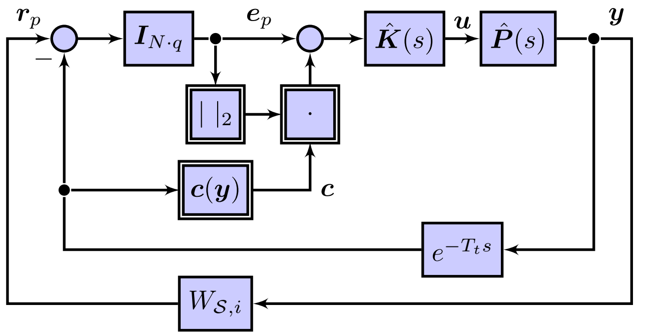

Fig. 3. Cooperative Formation Loop with Integrated Collision Avoidance Algorithm and $W_{\mathcal{S},i}$

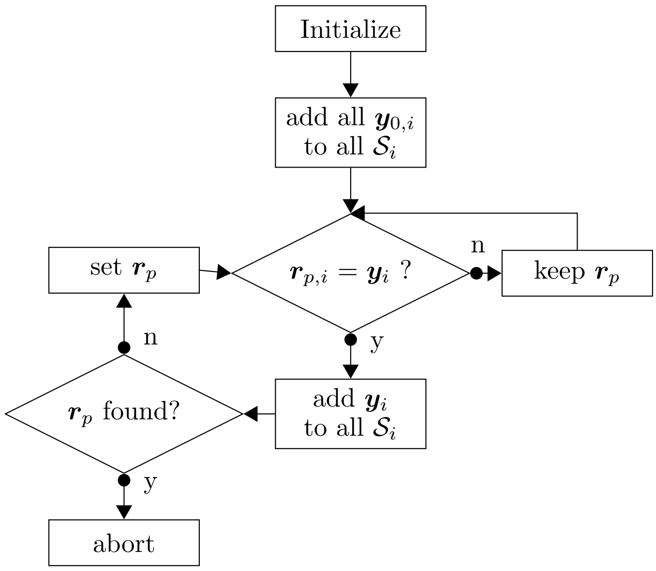

Fig. 4. Flow Chart for Waypoint Generation $W_{\mathcal{S},i}$

In the case that $\boldsymbol{x} \in \mathcal{Q} \quad \wedge \quad \Delta_{i,j} < d_2 \quad$ for $\quad i \neq j$ is empty the UAS should move sufficiently far away from $\mathcal{Q}$ and stop the exploration since it can't explore without possibly disturbing another UAS in its search.

5. CONTROL SCHEME

The general control scheme is presented in Figure 3. A position control loop is created by the substitution of $\boldsymbol{I}_{N\cdot q}$ for $\boldsymbol{\mathcal{L}}_q$. The position control loop is combined with collision avoidance and a feedback for the position reference $\boldsymbol{r}_p$ over the waypoint generation block $W_{\mathcal{S},i}$ creating a second feedback loop. $W_{\mathcal{S},i}$ generates event based position references in dependence of $\boldsymbol{y}_i$ and $\mathcal{S}_i$ following the flowchart in Figure 4.

In the initialization phase, the starting positions and the corresponding measurements are added to $\mathcal{S}_i$ and the first

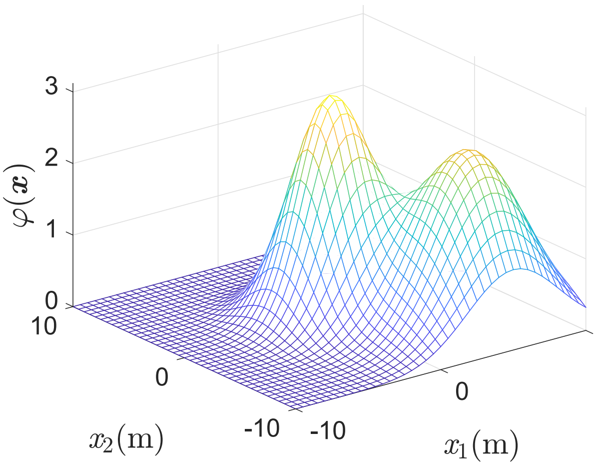

Fig. 5. Spatial distribution $\varphi(\mathbf{x})$

$\boldsymbol{r}_p$ is calculated by solving the minimization problem from section 4.3. Afterwards, the position control continues until a UAS reaches its position reference. In this case all $S_i(t_k)$ as well as $\boldsymbol{r}_p$ is updated and every UAS with updated $\mathcal{S}_i$ solves the minimization problem.

If the update of $\boldsymbol{r}_p$ does not yield a new position reference for a UAS this specific UAS stops exploring and the others continue with their new position reference. This happens when $\mathcal{Q}$ is empty or the area spanned by $\Delta_{i,j} < d_2$ fully covers $\mathcal{Q}$. It is important to ensure that the UAS moves sufficiently far away from $\mathcal{Q}$ after finishing their exploration to not interfere with the UAS which are still exploring.

6. SIMULATION

An examination of the proposed source seeking scheme is performed in this section. It includes the simulation setup which is used with a naive method and the proposed method and the results are compared afterwards to give an idea of how much improvement can be achieved in comparison to a naive method. The simulations are performed with MATLAB®/Simulink®.

6.1 Simulation Setup

The simulation is performed on a squared area of $400m^2$. The area is discretized to a grid with spacing of $0.5m$. The field distribution is assumed to be a superposition of 2 multivariate normal distributions at $[0\ 0]^{\mathrm{T}}$ and $[5\ -6]^{\mathrm{T}}$ with a diagonal variance matrix with values 10 and 20. The amplitudes are 3 and 2.5 and the full distribution shown in Figure 5.

The exploration is performed by $N = 3$ robots model youBot by KUKA as an example for UAS. They are modeled as second-order systems $\boldsymbol{T}_{you}(s)$ with an integrator since they are velocity controlled. Their dynamics are decoupled since the youBots can move omnidirectionally. This yields a linear System $\boldsymbol{P}(s) = \boldsymbol{T}_{you}(s) \cdot \frac{1}{s}$. It is important to note that the velocity is limited and therefore the system input is limited to $\pm 0.6\frac{m}{s}$ as well. Further details can be taken from (Gronemeyer and Horn, 2019). Their starting positions are

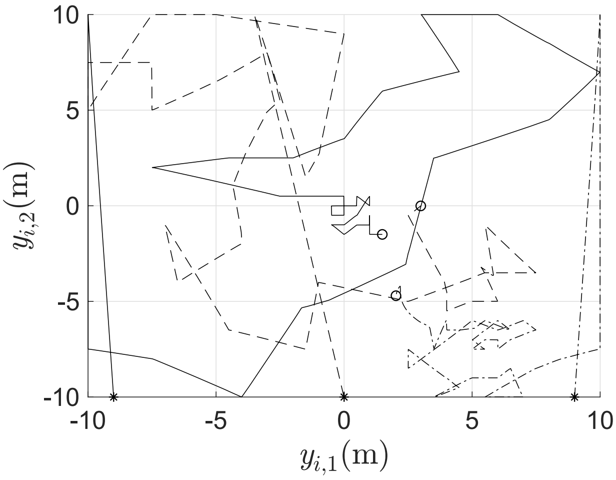

Fig. 6. Simulation Results of Robot Trajectories

$$\boldsymbol{y}_0 = [-9\ -10\ 0\ -10\ 9\ -10]^{\mathrm{T}}$$

in order to display a scenario where the robots are deployed at the edge of an unknown and dangerous area. The communication of positions for the collision avoidance and waypoint generation is full, meaning all positions are available for all robots. The tuning parameters for the minimization problem are set as $\alpha = 1$ and $\beta = 4$.

The performance of the proposed method is compared against a naive global source seeking scheme. This naive approach sets up predefined sampling points as a grid over the search space. The spacing of the grid points is chosen for the simulation. The search area is split up such that each robot explores the same number of gridpoints which are closest to its starting position. The robots are supposed to move to the nearest unexplored grid point until all grid points are explored. It is ensured that the current $\boldsymbol{r}_p$ is spaced in a manner which avoids deadlocks caused by collision avoidance.

6.2 Results

The simulation results for the setup from the previous section and use of the proposed algorithm utilizing the limited gradient criterion are shown in Figure 6. The starting positions are marked with stars and the final positions marked with circles. The trajectory of robot 1 is shown as a solid line, for robot 2 as a dashed line and for robot 3 as dash-dotted.

It can be observed that all of the robots move to the opposite side of the area at first since the potential maximum value is very large there initially due to the limited gradient criterion. Afterwards the robots explore different regions of the search space. One robot ends up exploring the region around the origin, another one the region around the local maximum while the other searches in the rest of the search space. The first robot finishes its movement after 378 seconds while the last robot finishes after about 450 seconds.

The influence of all factors of the minimization problem can be observed in the trajectories. The influence of the potential maximum term dominates at the beginning favoring exploration over the other terms. The factor of the distance between the robots causes them to divide the search space among them and when they arrive at a region with a small change in scalar value the factor of distance to the current position dominates.

When examining the difference in finish time it is evident that not all robots are used until the end. The unused ones could perform other tasks or a second phase could be triggered switching to a control scheme that can utilize all robots in a small space in order to find the exact position of the global maximum.

6.3 Comparison

The comparison against the naive approach will be performed by comparison of performance measures shown in Table 1 where t_{end} is the time where the last robot stops. $\sum |\boldsymbol{u}_i|_2$ gives the sum of the squared control signal values over the complete time and summed over all robots which gives an indication about the required energy since $\boldsymbol{u}_i$ is the velocity reference. $|\mathcal{S}_i|$ is the total number of sampling points which is identical for all i.

Table 1. Performance measures

	t_{end}	$\sum \lvert\boldsymbol{u}_i\rvert_2$	$\lvert\mathcal{S}_i\rvert$
Naive	4004	54	1681
Limited Gradient	452	40	105

The numbers show that the time, as well as number of sampling points, is about a factor 10 lower if the limited gradient criterion is used in this simulation. The consumed energy is not that much lower since the average distance between the sampling points is greater. This factor can be influenced by the weighting parameters for J_i and will lead to a trade-off between required time and energy used. All in all the approach utilizing the limited gradient criterion dominates the performance measures of the naive approach.

7. CONCLUSION AND OUTLOOK

This work presented a global source seeking scheme which can guarantee that the exploration yields the global maximum using UAS exploring a field distribution with a limited gradient. This was achieved by the establishment of a criterion that yields a region of candidates for the global maximum and exploring this area systematically until the global maximum is found. Simulations show that this approach achieves better performance than a naive approach.

Further work will include the implementation on the mentioned youBot platform and the handling of uncertainties from measurements of the position and scalar field value. Another interesting idea is the development of a second phase of the exploration which can increase the exploration speed in the phase where not all of the robots contribute to the search anymore.

REFERENCES

Atinc, G.M., Stipanovic, D.M., Voulgaris, P.G., and Karkoub, M. (2013). Collision-free trajectory tracking while preserving connectivity in unicycle multi-agent systems. *American Control Conference (ACC).*

Bartels, M. and Werner, H. (2014). Cooperative and consensus-based approaches to formation control of autonomous vehicles. *19th IFAC World Congress.*

Brian Yamauchi (2004). Packbot: A versatile platform for military robotics. *Defense and Security.*

Gronemeyer, M. and Horn, J. (2019). Collision avoidance for cooperative formation control of a robot group. *IFAC-PapersOnLine*, 52(8), 434–439. doi: 10.1016/j.ifacol.2019.08.096.

Hoon Oh, B., Kim, K., Choi, H.L., and Hwang, I. (2017). Integer linear program approach for evacuation planning for disaster victims with different urgency levels. *Preprints of the 20th World Congress.*

Jones, D.R. (2001). A taxonomy of global optimization methods based on response surfaces.

Kahn, A., Marzat, J., Piet-Lahanier, H., and Kieffer, M. (2015). Global extremum seeking by kriging with a multi-agent system. *IFAC-PapersOnLine*, 48(28), 526–531. doi:10.1016/j.ifacol.2015.12.182.

Krige, D.G. (1951). A statistical approach to some basic mine valuation problems on teh witwaterstrand. *Journal of the Chemical Metallurgurical & Mining Society of South Africa.*

Murphy, R.R. (2004). Human–robot interaction in rescue robotics. *IEEE Transactions on Systems, Man and Cybernetics, Part C (Applications and Reviews)*, 34(2), 138–153. doi:10.1109/TSMCC.2004.826267.

Peter R. Wurman, Raffaello D'Andrea, and Mick Mountz (2008). Coordinating hundreds of cooperative, autonomous vehicles in warehouses. *AI Magazine, vol. 29, no. 1.*

Pilz, U. and Werner, H. (2012). An H_∞/l_1 control approach to cooperative control of multi-agent systems. *51st Conference on Decision and Control.*

Richards, A. and How, J. (2004). Decentralized model predictive control of cooperating uavs. In *43rd IEEE Conference on Decision and Control (CDC) (IEEE Cat. No.04CH37601)*, 4286–4291 Vol.4. doi: 10.1109/CDC.2004.1429425.

Rosero, E. and Werner, H. (2014a). Cooperative source seeking via gradient estimation and formation control (part 1). *2014 UKACC International Conference on Control.*

Rosero, E. and Werner, H. (2014b). Cooperative source seeking via gradient estimation and formation control (part 2). *2014 UKACC International Conference on Control.*

Xu, Y., Choi, J., and Oh, S. (2011). Mobile sensor network navigation using gaussian processes with truncated observations. *IEEE Transactions on Robotics*, 27(6), 1118–1131. doi:10.1109/TRO.2011.2162766.