Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Direct Sliding Mode Control of a Three-Phase AC/DC Power Converter for the Velocity Regulation of a DC Motor

Yazan Alsmadi*+, Isaac Chairez, and Vadim Utkin*****

** Department of Electrical Engineering, Jordan University of Science and Technology, Irbid 22110, Jordan (e-mail:ymalsmadi@just.edu.jo).*
*** Bioprocesses Department at the UPIBI, Instituto Politecnico Nacional, Mexico City, 07340 Mexico (e-mail:jchairezo@ipn.mx, chairezoria.1@osu.edu).*
**** Electrical Engineering Department, The Ohio State University, Columbus, OH 43210 USA. (e-mail: utkin.2@osu.edu).*
+ Corresponding Author

Abstract: A DC voltage source and a DC/DC power converter can be used to control the position, speed, or torque of a DC motor. In such operational conditions, a rectifier is needed to use a DC voltage source if only a three-phase voltage source is available. The objective of this study is to replace a rectifier and DC/DC power converter by one AC/DC power converter, such that its output would be equal to the voltage needed to control a DC motor. It is assumed that the control algorithm of a DC motor is selected, which means that the desired output voltage of the AC/DC converter as a time function or function of the motor state is known. First, a sliding mode methodology is applied to control the converter's three shoulders to make the three-phase input current track the source voltages multiplied by a time-varying gain. The gain is then selected such that the converter output voltage is equal to the desired input of the DC motor. It is shown that this condition holds if the time-varying gain satisfies a first-order differential equation, which can be implemented as part of the controller. The application of Lyapunov theory confirms that the speed regulation process has a stable equilibrium point at the origin and that the time gain variation is bounded. The power efficiency is equal to one if the gain is positive. A numerical simulation demonstrates application of the developed control methodology for both constant and time-varying angular speed reference inputs.

Keywords: DC motor, Sliding mode control, AC/DC power converter, Equivalent control.

1. INTRODUCTION

DC motors are commonly used actuators for speed and position control in diverse electromechanical devices (Kuo and Golnaraghi, 1995). Robotics, rolling mills, machine tools, printing processes, excavators, and cranes are examples of control systems with DC motor applications (Witt et al, 1993). DC motors are governed by linear differential equations of the second (with respect to angle speed) or third (with respect to position) order, so the well-known methods developed in linear control theory can be applied (Hemati and Leu, 1992).

The implementation of suitable feedback controllers implies that a DC voltage source is available and the desired voltage for the input to a DC motor is obtained using pulse width modulation (PWM) in an additional DC/DC converter (Dewangan et al, 2012; Guerrero et al, 2011; Bai et al, 2008). If a three-phase source with balanced voltages is available, then an AC/DC rectifier is needed in the framework of this approach.

The design of feedback controllers for AC/DC power converters has been mainly investigated under the assumption that the DC voltage must be constant. However, direct AC/DC control of a DC motor requires the development of a new methodology to have the desired output time function. The accessibility of faster, reliable, and affordable power electronic switches justifies the possible technical solution to this problem. Replacing a rectifier and DC/DC converter with one AC/DC converter to solve the regulation of the DC motor velocity is seen as a challenging problem (Figure 1). It is assumed that the desired voltage to the input of a DC/DC motor is selected as a time function or function of the motor state. Switching elements in the three shoulders of the converter should be controlled such that its output voltage v_{dc} is equal to the preselected motor voltage. The key idea for solving the problem is selection of the phase current profiles to have the desired converter output. They can then be tracked by real currents using sliding mode control (SMC) methodology, which has been widely used for power converter control (Utkin, 2013; Sira-Ramirez et al, 2013; Hu et al, 2011; Shtessel et al, 2008).

Authors propose selecting phase currents proportional to the phase voltages of the three-phase source with time-varying gain *K(t)*. The problem is then reduced to finding the scalar time function *K(t)*. For the proposed choice of phase current, the system behavior is like systems with the power gain equal to one (Alsmadi et al, 2018). The problem with a constant value for the output converter voltage was solved in Alsmadi et al (2018) following the above approach. In this case, the AC/DC converter can be called a controllable rectifier with the desired output DC voltage.

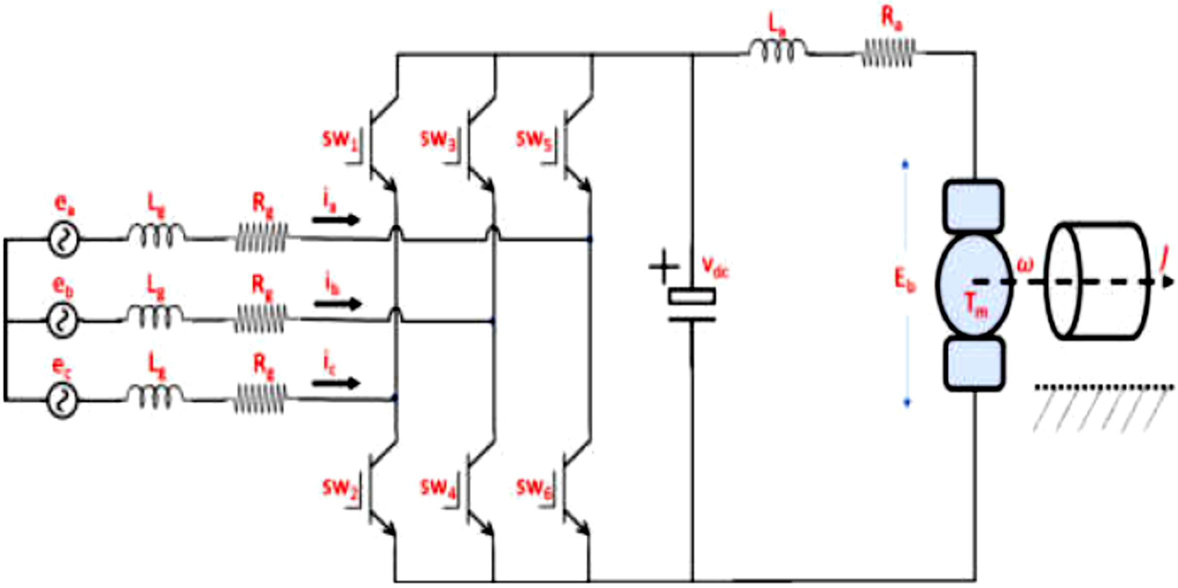

Fig. 1. Electromechanical circuit of the DC motor controlled by a three-phase converter.

This manuscript is organized as follows: Section 2 presents the mathematical models of both a DC motor and an AC/DC power converter. Section 3 presents the problem statement and the design of the feedback controller for an AC/DC converter. Section 4 details all the aspects of the numerical simulations for the velocity regulation problem of a DC motor. Section 5 concludes the study with some final remarks.

2. MATHEMATICAL MODELS

The system will be designed following the cascade principle. First, the desired input voltage of the DC motor is found such that it tracks the speed reference input. The switching logic for the converter is then selected such that its output is equal to this voltage.

2.1 DC motor

The model for a DC motor can be found in any textbook on electric machines:

$$L_a \frac{d}{dt} i_{ar} = -i_{ar} R_a - K_m \omega + v_{dc}$$
$$J \frac{d}{dt} \omega = K_i K_f i_f i_{ar} - T_L \qquad (1)$$

where i_{ar} is the armature current, ω is the angular velocity of the motor shaft, L_a is the inductance of the armature circuit, R_a is the armature resistance, J is the motor inertia, K_m defines the relationship between the angular velocity and the variation of i_{ar}, K_i and K_f are constants, i_f is the field current, and T_L is the load torque. The input, v_{dc}, is the external voltage applied to motor, which in this case corresponds to the capacitor voltage in the AC/DC power converter.

2.2 Power converter

The dynamics of the AC side of the power converter satisfies

$$L_g \frac{d}{dt} i_a = e_a - R_g i_a - v_{an}$$
$$L_g \frac{d}{dt} i_b = e_b - R_g i_b - v_{bn} \qquad (2)$$
$$L_g \frac{d}{dt} i_c = e_c - R_g i_c - v_{cn}$$

Here, i_a, i_b, and i_c represent the three-phase AC input currents and R_g and L_g correspond to the grid-side resistance and inductance, respectively. The terms e_a, e_b, and e_c are the balanced three-phase AC voltages representing the infinite bus, and v_{an}, v_{bn}, and v_{cn} are the voltages on the AC side corresponding to the neutral power point n. The balanced three-phase AC voltages satisfy

$$e_a = E_0 \, sin(\omega_s t)$$
$$e_b = E_0 \, sin(\omega_s t - \frac{2}{3}\pi) \qquad (3)$$
$$e_c = E_0 \, sin(\omega_s t + \frac{2}{3}\pi)$$

where E_0 is the amplitude of the phase voltage and ω_s is the AC power source frequency. If vectors $E = [e_a \quad e_b \quad e_c]^T$, $E_n = [v_{an} \quad v_{bn} \quad v_{cn}]^T$, and $I = [i_a \quad i_b \quad i_c]^T$ are defined, then the set of ordinary differential equations (2) can be represented as

$$L_g \frac{d}{dt} I = E - R_g I - E_n \qquad (4)$$

Switching elements in the electrical power bridge are assumed to be ideal, with the switching logic

$$SW_j = \begin{cases} 1 & if \quad SW_j \text{ is not activated} \\ -1 & if \quad SW_j \text{ is activated} \end{cases} \tag{5}$$

where $j \in \{a,\ b,\ c\}$. Based on the switching description, the voltage E_n can be described as

$$E_n = v_c \Omega S, \quad \Omega = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \tag{6}$$

with $S = \begin{bmatrix} S_a & S_b & S_c \end{bmatrix}^T$. Then, (4) can be represented as

$$L_g \frac{d}{dt} I = E - R_g I - v_c \Omega S \tag{7}$$

On the DC side of the converter, the output voltage is governed by

$$C \frac{d}{dt} v_{dc} = -i_{ar} + I^T S \tag{8}$$

where vector S functions as the control.

3. DESIGN OF DC MOTOR INPUT VOLTAGE

The input voltage v_{dc} of the DC motor should be selected such that the angle speed tracks the time-varying smooth-enough reference input $\omega^*(t)$. From the DC motor equations, $\omega = \omega^*(t)$ is the solution if $v_{dc} = u^*$, which can be found from

$$\begin{gathered} L_a \frac{d}{dt} i_{ar}^* + i_{ar}^* R_a + K_m \omega^* = u^*(t) \\ \frac{1}{K_i K_f i_f} \left(J \frac{d}{dt} \omega^* + T_L \right) = i_{ar}^* \end{gathered} \tag{9}$$

If $v_{dc} = u^* + \Delta u$, $\omega = \omega^* + \Delta\omega$, and $i_{ar} = i_{ar}^* + \Delta i_{ar}$, then

$$\begin{gathered} L_a \frac{d}{dt} \Delta i_{ar} = -\Delta i_{ar} R_a - K_m \Delta\omega + \Delta u \\ \frac{1}{K_i K_f i_f} J \frac{d}{dt} \Delta\omega = \Delta i_{ar} \end{gathered} \tag{10}$$

Let $\Delta u = -\alpha \cdot \Delta\omega$ (where α is constant) to improve the transient process. $\Delta\omega$, Δi_{ar}, and Δu then tend to zero and ω tends to ω^*. The control problem is solved if the output voltage of the power converter v_{dc} is equal to

$$v_{dc}^* = u^* - \alpha \cdot \Delta\omega \tag{11}$$

It is assumed that the range of reference inputs ω^* is such that v_{dc}^* is positive. According to the proposed plan, phase current I should track reference input I^*, which is selected such that the output voltage is equal to the desired function v_{dc}^*.

3.1 Tracking system

Since the sum of the phase currents is equal to zero, only two of the three components of vector I can be made equal to the desired values. Equations with respect to phase currents i_a and i_b can be derived from (4):

$$\begin{gathered} L_g \frac{d}{dt} I_{a,\,b} = E_{a,\,b} - R_g I_{a,\,b} - v_{dc} \Omega_{a,\,b} S \\ C \frac{d}{dt} v_{dc} = -i_L + I^T S \\ I_{a,\,b} = C_{a,\,b} I, \quad E_{a,\,b} = C_{a,\,b} E \quad \Omega_{a,\,b} = C_{a,\,b} \Omega \\ C_{a,\,b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \Omega_{a,\,b} = \frac{-1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \end{bmatrix} \end{gathered} \tag{12}$$

The objective of the tracking system is to reduce the two-dimensional vector σ to zero:

$$\sigma = L_g \left(I_{a,b} - I_{a,b,ref} \right) \tag{13}$$

where $I_{a,b,ref}$ is the reference input to be selected. As follows from (13), the dynamics of σ satisfy

$$\frac{d}{dt}\sigma = E_{a,\,b} - R_g I_{a,\,b} - v_{dc} \Omega_{a,\,b} S - L_g \frac{d}{dt} I_{a,b,ref} \tag{14}$$

A Lyapunov candidate function is selected as

$$V = 0.5 \|\sigma\|^2 . \tag{15}$$

The discontinuous control S should then enforce the sliding mode in the manifold $\sigma = 0$. This means that

$$\frac{d}{dt} V = 0.5 \sigma^T \left(F - v_{dc} \Omega_{a,b} S \right) \tag{16}$$

should be negative, with $F = E_{a,b} - R_g I_{a,b} - L_g \frac{d}{dt} I_{a,b,ref}$. Taking $\sigma^* = \Omega_{a,\,b} \sigma$, and $\sigma^* = \begin{bmatrix} \sigma_1^* & \sigma_2^* & \sigma_3^* \end{bmatrix}^T$, the time derivative of the Lyapunov candidate function satisfies

$$\frac{d}{dt} V = \left(\sigma^*\right)^T \Omega_{a,\,b}^T \left[\Omega_{a,\,b} \Omega_{a,\,b}^T \right]^{-1} F - v_{dc} \left(\sigma^*\right)^T S \tag{17}$$

The time derivative of the Lyapunov function is negative for the switching logic

$$SW_j = \begin{cases} 1 & if \quad \sigma_j^* > 0 \\ -1 & if \quad \sigma_j^* < 0 \end{cases}, \quad j = a, b, c \tag{18}$$

for high enough positive values of v_{dc}. This means that a sliding mode occurs on the manifold $\sigma = 0$ and

$$I_{a,b} = I_{a,b,ref} \tag{19}$$

3.2 Control of output voltage

To derive the sliding mode equation, the equivalent control $\left(\Omega_{a,\,b}S\right)_{eq}$ should be found from the equation $\frac{d}{dt}\sigma = 0$ and substituted into the equation for v_{dc} in (6) (Utkin, 1992):

$$\left(\Omega_{a,\,b}S\right)_{eq} = \frac{1}{v_{dc}}\left(E_{a,\,b} - R_g I_{a,\,b} - L_g \frac{d}{dt} I_{a,\,b,ref}\right) \qquad (20)$$

The last term in the equation for v_{dc} satisfies

$$C\frac{d}{dt}v_{dc} = -i_L + I^T S \qquad (21)$$

which can be written in the form

$$I^T S = \left(i_a, i_b, -i_a - i_b\right)S = \left(i_a - i_c, i_b - i_c\right)\Omega_{a,b}S \qquad (22)$$

Consequently, the sliding mode motion (9) becomes

$$C\frac{d}{dt}v_{dc} = -i_L + \frac{1}{v_{dc}}\left(i_{a,ref} - i_{c,ref}, i_{b,ref} - i_{c,ref}\right)\cdot \left(E_{a,\,b} - R_g I_{a,\,b} - L_g \frac{d}{dt} I_{a,\,b,ref}\right) \qquad (23)$$

Phase current reference inputs are then selected to be proportional to the source voltages, with a time-varying coefficient:

$$I_{a,\,b,ref} = K(t) E_{a,\,b}, \qquad (24)$$

$$\left(i_{a,ref} - i_{c,ref}, i_{b,ref} - i_{c,ref}\right) = K(t)\left(E_a - E_c, E_b - E_c\right), \qquad (25)$$

and

$$\frac{d}{dt} I_{a,\,b,ref} = \left(\frac{d}{dt}K(t)\right) E_{a,\,b} + K(t)\left(\frac{d}{dt}E_{a,\,b}\right), \qquad (26)$$

The terms on the right-hand side are then preliminarily calculated after substitution for the balanced source voltages:

$$K(t)(E_a - E_c, E_b - E_c)K(t)\begin{pmatrix}\frac{d}{dt}E_{a,} \\ \frac{d}{dt}E_b\end{pmatrix} = K^2(t)\left(E_a\frac{d}{dt}E_{a,}, E_b\frac{d}{dt}E_b, E_c\frac{d}{dt}E_c\right) = 0 \qquad (27)$$

Considering the second term in (26), one then has

$$K(t)\left(E_a - E_c, E_b - E_c\right)\frac{d}{dt}K(t)\begin{pmatrix}E_{a,} \\ E_b\end{pmatrix} = K(t)\frac{d}{dt}K(t)\left(E_a^2 + E_b^2 + E_c^2\right) = \frac{3}{2}K(t)\frac{d}{dt}K(t)E_0^2 \qquad (28)$$

Here, the following identity was used:

$$K(t)\left(E_a - E_c, E_b - E_c\right)\begin{pmatrix}E_{a,} \\ E_b\end{pmatrix} = \frac{3}{2}K(t)E_0^2 \qquad (29)$$

$$K(t)\left(E_a - E_c, E_b - E_c\right)K(t)\begin{pmatrix}E_{a,} \\ E_b\end{pmatrix} = \frac{3}{2}K^2(t)E_0^2 \qquad (30)$$

The result of the substitution of (30) into (26) has the following form:

$$C\frac{d}{dt}v_{dc} = -i_L + \frac{1}{v_{dc}}\left[-\frac{3}{2}K(t)\left(\frac{d}{dt}K(t)\right)E_0^2 L_g + \frac{3}{2}K(t)E_0^2 - \frac{3}{2}R_g K^2(t)E_0^2\right] \qquad (31)$$

or, for $v_{dc} > 0$, $i_L > 0$, and $y = v_{dc}^2$,

$$\left(\frac{1}{2}C\frac{d}{dt}y + i_L\sqrt{y}\right)\frac{2}{3K(t)E_0^2} = -L_g\left(\frac{d}{dt}K(t)\right) + 1 - R_g K(t) \qquad (32)$$

Equation (12) can then be implemented in the controller with $y^* = \left(v_{dc}^*\right)^2$,

$$\left(\frac{1}{2}C\frac{d}{dt}y^* + i_L\sqrt{y^*}\right)\frac{2}{3K(t)E_0^2} = -L_g\left(\frac{d}{dt}K(t)\right) + 1 - R_g K(t) \qquad (33)$$

The solution to the equation with respect to a mismatch satisfies

$$\left(\frac{1}{2}C\frac{d}{dt}\Delta y + i_L\frac{\Delta y}{\sqrt{y^*} + \sqrt{y}}\right) = 0 \qquad (34)$$

with $\Delta y = y^* - y$. The solution to equation (34) is asymptotically stable, meaning that the output voltage v_{dc} tends to the desired value v_{dc}^* (5). Note that $\frac{d}{dt}y^*$ depends on $\frac{d}{dt}\omega$, which can be found from the DC motor equation. Let M be the upper bound

$$\left|\frac{1}{2}C\frac{d}{dt}y^* + i_L\sqrt{y^*}\right| < M \qquad (35)$$

There then exists $K_0 > 0$ such that, for values of $\left|K(t)\right| \geq K_0$ such that K and $\frac{d}{dt}K(t)$ have opposite signs, gain K is bounded. On the other hand, if $R_g K(t) < 1$ and $E_o > E_{o,m}$, there exists $E_{o,m} > 0$ such that $\frac{d}{dt}K(t) > 0$. This means that K is a positive bounded function for initial condition $\varepsilon \leq R_g K(0) \leq K_0$ and corresponds to a power factor equal to one, with $i_{a,b,c} = K(t)E_{a,b,c}$ for a high enough amplitude of the voltage source.

4. SIMULATIONS

This section illustrates the application of the controllers proposed in this study. For this numerical example, the components of the power converter and the DC motor circuits are $L_a = 0.012 \ \ Henry$, $R_a = 3.9 \ \ Ohm$, $J = 0.9 \ \ kg \cdot m^2$, $K_m = 0.05 volts \cdot s / rad$, $K_i = 5.0$, $K_f = 3.0$, $i_f = 2.0$ $Amperes$, $T_L = 5 \ \ N \cdot m]$, $R_g = 15 * 10^{-3} Ohm$, and $L_g = 2 * 10^{-3} \ \ Henry$ (Yao et al, 2013). The simulated DC rate was selected as 0.5 hp. Figure (2) shows the tracking of the reference $\omega^* = 10$ by the actual angular velocity ω. Notice that the tracking error reduces asymptotically to zero due to the application of the controller proposed for this part of the DC motor circuit.

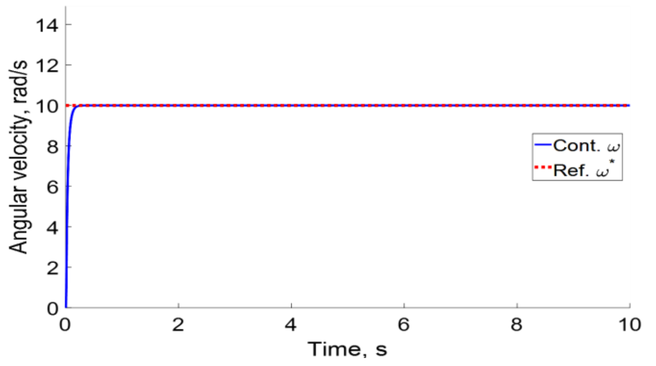

Fig. 2. Comparison of the reference angular velocity ω^* and the controlled DC motor ω.

The reference voltage v_{dc}^* is calculated according to equation (11). The AC/DC power converter can then be controlled based on the first-order SMC, which forces tracking of the reference voltage within the first 0.2 s (Figure 3), which in turn ensures the tracking of the reference angular velocity ω^*. The SMC introduces a transient period (0.1 s) of adjustment of the input currents followed by the regular sinusoidal tracked forms (Figure 4). Here, it is noticeable that the input currents have a transient evolution with an exponential decay to the steady sinusoidal regime, which corresponds to the expected restricted relationship between the input currents.

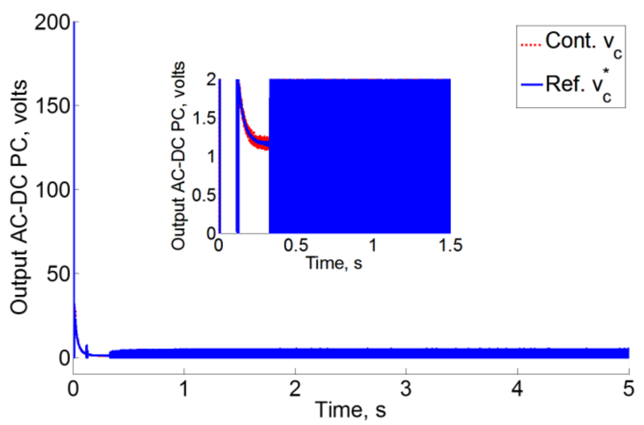

Fig. 3. Comparison of the reference v_{dc}^* and the controlled DC motor v_{dc}.

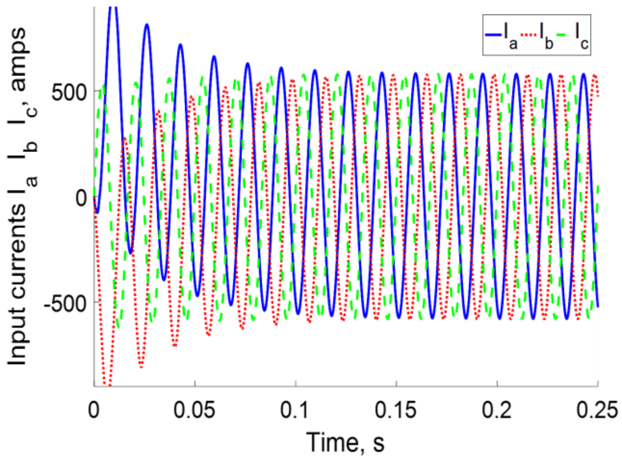

Fig. 4. Time variation of the input currents for the three-phase AC/DC power converter.

In this case, the dynamics of the gain keep their form in correspondence with ordinary differential equation (33). The gain as a time function appears in Figure 5. During the process, the gain $K(t)$ remains positive.

The proposed controller is simulated by considering a time-dependent angular velocity $\omega^*(t) = 5\sin(0.5t) + 8$. The gain remains positive, justifying the claim regarding the high-power efficiency (Figure 6).

5. CONCLUSION

A design method is developed to control a DC motor without a DC voltage source. A three-phase power converter is directly connected to a DC motor. The proposed sliding mode control approach is intended to solve the problem of tracking the angle speed reference input. Formally, control is three-dimensional (three shoulders of the converter), but we deal with a singular case, and only two state variables can be controlled. Two converter input currents are selected to be controlled in the sliding mode such that they are proportional to the source voltage with time-varying positive gain. The operation mode is similar to that of systems with sinusoidal signals and power factors equal to one. The next step is selecting the gain such that the output voltage is precisely equal to the desired input of the DC motor. The gain should satisfy a first-order differential equation. The positiveness of the gain seems to be an innovative option for solving the velocity regulation of a DC motor based on the application of the sliding mode control and a reduced version of a three-phase AC/DC power converter. This result appears to be an alternative that may reduce the complexity of controller design. The dynamic system governed by this equation is implemented in the controller. Theoretical results are confirmed by simulations with both constant and time-dependent angular velocity references.

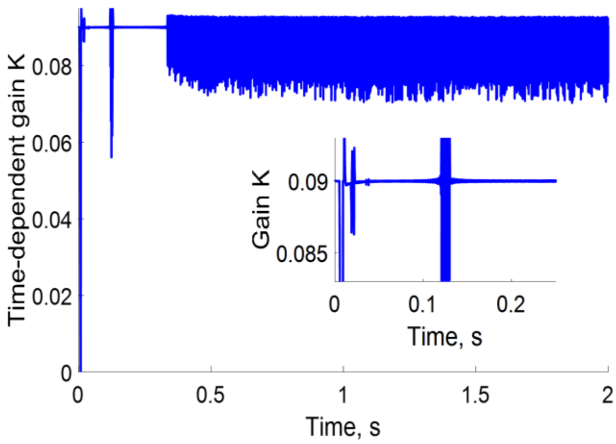

Fig. 5. Time dependence of the gain $K(t)$.

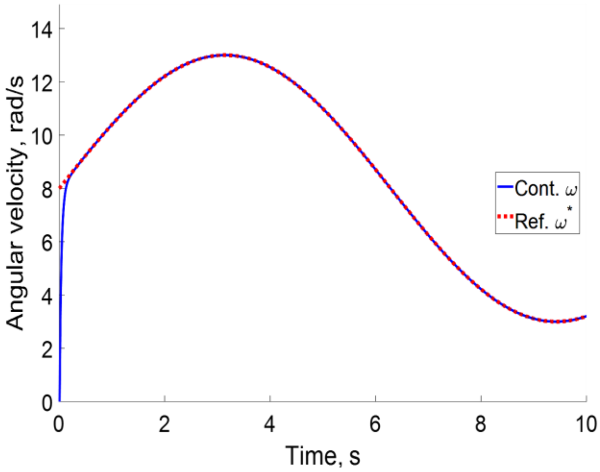

Fig. 6. Tracking of a time-dependent angular velocity reference.

REFERENCES

Alsmadi, Y. M., Utkin, V., Haj-Ahmed, M., Xu, L., & Abdelaziz, A. Y. (2018). Sliding-mode control of power converters: AC/DC converters & DC/AC inverters. International Journal of Control, 91(11), 2573-2587.

Bai, H., & Mi, C. (2008). Eliminate reactive power and increase system efficiency of isolated bidirectional dual-active-bridge DC-DC converters using novel dual-phase-shift control. IEEE Transactions on Power Electronics, 23(6), 2905.

De Wit, C. C., Ortega, R., & Seleme, S. I. (1993, May). Robot motion control using induction motor drives. In [1993] Proceedings IEEE International Conference on Robotics and Automation (pp. 533-538). IEEE.

Dewangan, A. K., Chakraborty, N., Shukla, S., & Yadu, V. (2012). PWM based automatic closed loop speed control of DC motor. International journal of engineering trends and technology, 3(2), 110-112.

Guerrero, J. M., Vasquez, J. C., Matas, J., De Vicuña, L. G., & Castilla, M. (2010). Hierarchical control of droop-controlled AC and DC microgrids—A general approach toward standardization. IEEE Transactions on industrial electronics, 58(1), 158-172.

Hemati, N., & Leu, M. C. (1992). A complete model characterization of brushless DC motors. IEEE Transactions on Industry Applications, 28(1), 172-180.

Hu, J., Shang, L., He, Y., & Zhu, Z. Q. (2010). Direct active and reactive power regulation of grid-connected DC/AC converters using sliding mode control approach. IEEE transactions on power electronics, 26(1), 210-222.

Kuo, B. C., & Golnaraghi, F. (1995). Automatic control systems (Vol. 9). Englewood Cliffs, NJ: Prentice-Hall.

Liu, J., Laghrouche, S., & Wack, M. (2014). Observer-based higher order sliding mode control of power factor in three-phase AC/DC converter for hybrid electric vehicle applications. International Journal of Control, 87(6), 1117-1130.

Shtessel, Y., Baev, S., & Biglari, H. (2008). Unity power factor control in three-phase AC/DC boost converter using sliding modes. IEEE Transactions on Industrial Electronics, 55(11), 3874-3882.

Sira-Ramírez, H., Luviano-Juárez, A., & Cortés-Romero, J. (2013). Robust input–output sliding mode control of the buck converter. Control Engineering Practice, 21(5), 671-678.

Utkin, V. (2013). Sliding mode control of DC/DC converters. Journal of the Franklin Institute, 350(8), 2146-2165.

Utkin, V. I. (1992). Sliding modes in control and optimization. London, Springer Verlag.

Yao, J., Jiao, Z., & Ma, D. (2013). Adaptive robust control of DC motors with extended state observer. IEEE Transactions on Industrial Electronics, 61(7), 3630-3637.