# Gap Metric Based Performance Assessment of Subcool Control in Steam Assisted Gravity Drainage Wells

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Abstract: Steam assisted gravity drainage (SAGD) is a widely adopted oil extraction technique for heavy oil reservoirs in Alberta, Canada. One of the common approaches by which the producers optimize the production from SAGD reservoirs is by controlling the emulsion level above the producer well bores, a strategy known as subcool control within the industry. In this study, we assess and compare performances of two subcool control strategies, one of which makes use of classic control strategy (PID) and the other is of advanced control strategy (model predictive controller (MPC)). As the controlled process in this case is a non-linear process, we propose a gap metric-based control performance assessment (CPA) method. By this method, the local models as well as their associated weights are determined using the gap metric. We show that the MPC-based strategy outperforms the PID loops-based strategy in subcool control application.

*Keywords:* Multi-modal control performance assessment, Gap metric, Steam assisted gravity drainage process, Subcool control, Model predictive control.

## 1. INTRODUCTION

In steam assisted gravity drainage (SAGD) production, two wells are directionally drilled one above the other and that can extend laterally within the pay zone more than a kilometer. In the top well, steam is continuously injected to lower the viscosity of the oil and allow it to mobilize through the producer well directly below it. Over time, a steam chamber is formed downhole and the mobilized bitumen and steam condensate form an emulsion inventory which is collected above the producer wellbore. The collected emulsion is moved to the surface by various artificial lift methods.

Controlling the emulsion production from each well at a rate that matches the in-flow from the reservoir as it is mobilized by the injected steam, is one of the common challenges faced in SAGD production. Variable steam injection rates and variable production rates make this a challenging production control problem with operators attempting to manage hundreds of well pairs simultaneously. Producing oil from a well at rates that exceed the inflow from the reservoir causes the well 'pump off', resulting in steam breakthrough from the injector to the producer. Steam breakthrough can cause many equipment reliability issues associated with the well resulting in damage to liners and electrical submersible pumps (ESPs) typically. On the other hand, producing at rates which are less than the inflow from the reservoir causes the emulsion level to rise in the steam chamber and eventually flood the injector if not recognized by operations. This is also not ideal, as much of the latent heat (energy) available in the steam can be lost to the emulsion layer rather than it heating the surrounding reservoir as intended under these conditions.

As the sensor technology does not yet exist to physically measure the level of emulsion above the producer wellbore, subcool is used as proxy and controlled instead. Different definitions of subcool are used in the industry depending on which pressure measurements are used for calculating the saturation conditions of the steam. Reservoir subcool is the difference between the steam saturation temperature corresponding to the Reservoir pressure and the temperature of the emulsion inventory at the producer well. Alternately, wellbore subcool is the difference between the steam saturation temperature corresponding to the producer wellbore pressure and the temperature of the emulsion at the producer well. A positive subcool measure indicates there is an available emulsion inventory to be produced. Good subcool control is desirable as it can help minimize the steam-to-oil-ratio (SOR) in a SAGD operation while protecting the downhole pump from abnormal conditions such as steam breakthrough [Edmunds et al. (1998)].

Recent studies on subcool control automation have reported the applicability of both PID and MPC [Vembadi (2015)]. Most of the reported studies make use of reservoir simulation packages to compare the performance of different control strategies without documenting results from actual field implementations. It is noteworthy that many studies in the literature discuss closed loop control of subcool based on the steam injection rates. However, the studied configuration has closed loop control based on the emulsion production rate. Subcool responds quicker to changes in production rates compared to the changes in steam injection rates which makes closing the loops on the producer wellbore a more practical option. In the studied wells, steam injection rates and pressures are still controlled through PID controllers.

One of the challenges of controlling subcool is that it responds with a large time constant to the changes in manipulated variables (production rate and injection rates). After a setpoint change to the ESP speed for example, it can be several hours before the change in emulsion inventory, and in turn, in the measured subcool. To cope with large time constant, high gain PID controllers have been used in this application which may not provide the best performance in terms of variability reduction or constraint handling [Purkayastha et al. (2015)].

MPC, on the other hand, utilizes predictive models between the controlled variable (subcool) and manipulated variable (pump speed/production rate) which captures the process dynamics. MPC can also be exploited to manage production constraints or limits, lowering the variability of the emulsion production rate, pump speed and ultimately subcool. Across the process industries, MPC has been used in multivariable constrained control applications and in applications with large time constants like the studied process.

The main goal of this paper is to assess performances of MPC and PID which are implemented on SAGD process. Over many years, the process control community has developed several control performance assessment (CPA) metrics and techniques to assess, monitor and compare performances of the industrial controllers [Desborough and Harris (1992)],[Huang and Shah (1999)]. In industrial settings, controllers are often designed for regulatory operations, and hence decreasing the output variance is one of the most important goals in controller design. Furthermore, decreasing the variance also has implications such as improved product quality, and less wear and tear to the actuators. As a result, the Minimum Variance-based (MV) performance index has emerged as one of the most important CPA indices in the literature Huang and Shah (1999). There have been several reported applications of CPA studies on the industrial control loops.

Considering that the SAGD process is a nonlinear process [Yuan et al. (2013)], some traditional control performance benchmarks such as linear minimum variance (MV) control benchmark which is usually estimated for a single operating mode under stationary conditions are not applicable. The nonlinearity poses significant challenges in determining the minimum variance index due to the difficulties in optimizing the nonlinear minimum variance (MV) cost function. To this end, we propose to use a multi-modal CPA technique. The main idea behind the multi-modal approach is to represent the nonlinear system with a convex combination of linear models. The challenges of this approach are determining the sufficient number of local models and the operation points around which the linear models should be constructed. To overcome these challenges, a popular metric known as the gap metric is utilized in deciding the number of local linear models and the model weights [Wan and Huang (2002)].

Galán et al. (2003a) suggested using the gap metric, which measures the distance between two linear models, as a guideline for selecting local models. The advantage of the method is that a detailed nonlinear model of the system is not required. By this approach, the number of the local models can also be determined. After having all local models, the global model can be determined by combining the local models using either soft switching approaches in which the global model is formed by a weighted sum of the local models. In this paper, this method is utilized to conduct CPA. Thus, operating point selection and local controller design can be integrated in multi-model design procedure

The remainder of the paper is organized as follows: In the following section, we introduce the subcool control strategies that are being assessed in this paper. In section 3, we present the proposed gap metric based multi-modal CPA technique. In section 4, we discuss the performance assessment and comparison results . In section 5, we provide the concluding remarks.

## 2. SUBCOOL CONTROL CONFIGURATIONS

In this section, we introduce the two different subcool control configurations that are studied. The initial configuration used two PID loops working in tandem, one to control the pump speed and the second as a low override on reservoir subcool. In the second configuration, the PID controllers were replaced by an MPC for wellbore subcool control for comparison. We provide the details of both the configurations in this section.

### 2.1 Configuration 1: PID configuration.

A schematic of the control configuration is shown in Fig. 1. The strategy attempts to control electric submersible pump speed with an override on a minimum reservoir subcool.

In the application, the SAGD well is equipped with a downhole distributed temperature sensor (DTS) along the producer wellbore allowing temperatures to be measured at the multiple locations. Configured logic in the control system chooses the maximum value of the temperature measurements at a given instant as it is this maximum that corresponds to the minimum subcool along the wellbore. Reservoir pressure is inferred from the residue gas injection pressure, which in turn, is used to calculate the saturation temperature of the injected steam. The difference between the steam saturation temperature and the emulsion temperature gives the reservoir subcool measurement used by the PID controller.

In this application, the emulsion production rate is measured at the producer wellhead which is equipped with a



Fig. 1. Previously existed strategy: PID control for production control and an overriding PID control for reservoir subcool control.

wedge style emulsion flow meter. Note that it is unusual to have independent emulsion flow measurements from individual producer wells in SAGD, as typically the produced gas fraction limits their accuracy. In this application, the gas/oil ratio is low, making reliable emulsion flow measurements possible.

In the control configuration shown in Fig. 1, under normal operating conditions, the pump speed controller is active producing the oil at a rate predetermined by production engineering personnel. If the well pumps off and the subcool constraint becomes active, the subcool controller overrides the pump speed controller slowing the pump down until the subcool can recover at which time it returns control to the pump speed controller

## 2.2 Configuration 2: MPC

A schematic of the second control configuration is shown in Fig. 2 with wellbore subcool as the primary controlled variable. Wellbore pressure at the pump intake (rather than residue gas pressure) is used for calculating the saturation temperature of the steam which in turn, is used for subcool calculation. With MPC being a multivariable controller, reservoir subcool and production flow rate were incorporated as constraint variables. In this strategy, the MPC accepts a subcool setpoint and calculates the flow setpoint for the production rate controller which, in turn writes the setpoint to the pump speed controller. Should the production rate rise too high, the constraint is handled automatically by the MPC backing the production setpoint down.

In the MPC configuration, a feedforward signal is fed to the production rate controller from a production header pressure sensor. This feedforward control is used to handle the production header disturbance that occurs as the well is switched between the production header and the test separator.

# 3. THE PROPOSED MULTI-MODAL CPA

In this section, we introduce the proposed multi-modal performance assessment metric utilized in this paper. Gap metric is a measure of distance between two linear models. It can be utilized to select the number of local models when approximating the non-linear system with multiple



Fig. 2. Strategy 2: MPC for wellbore subcool.

local linear models [Galán et al. (2003b)]. Typically, in gap metric-based modelling, the local models are retained or merged based on the distance between them. If the gap metric is lower than the desired threshold between any pair of local models, the two will be merged. When it comes to prediction, gap metric is also used to determine the appropriate weights for the local models and provide weighted prediction. The gap metric-based approach has been utilized in robust controller design [Wan and Huang (2000), Wan and Huang (2002)]. In this paper, we propose and utilize a gap metric-based multi-modal CPA approach to assess and compare the performance of the two control configurations introduced earlier.

Gap metric: Let  $P_i$ , i=1,2 be  $p \times m$  rational transfer function matrices representing two plant models, and  $P_i = N_i M_i^{-1} = \tilde{M}_i^{-1} \tilde{N}_i$ , i = 1, 2 denote the normalized right/left coprime factorizations of  $P_1$  and  $P_2$ , respectively. In gap metric-based approach, the distance between the two models  $P_1$  and  $P_2$  in frequency domain is defined as [Tan et al. (2004)]:

$$\kappa (P_1, P_2) (\omega) = \bar{\sigma} \left( \tilde{N}_2 M_1 - \tilde{M}_2 N_1 \right) (\omega), \kappa (P_1, P_2) (\omega) \le 1 \quad \forall \omega$$
(1)

where  $\bar{\sigma}(A)(\omega)$  stands for the maximum singular value of the matrix A at frequency w. With the distance function (1), the gap metric is defined as

$$\delta(P_1, P_2) = \begin{cases} \sup_{\omega} \kappa(P_1, P_2)(\omega) & if \det(M_1^*M_2 + N_1^*N_2) \neq 0 \\ 0 & 1 & otherwise. \end{cases}$$
(2)

where det(A) stands for the determinant of the matrix A, and  $M^*(s) = M^T(-s)$  stands for the conjugate transpose of any M(s). Considering Eq. (1) and (2), the gap metric is small if the distance between the numerators and the denominators of the two systems respectively is small [Tan et al. (2004)]. The defined gap metric has some advantages over other distance measures metrics such as infinity norm owing to the following properties:

- (1)  $0 \leq \delta(P_1, P_2) \leq 1$ .
- (2) The gap metric gives a better view about distance between two linear systems. For instance, the distance between two systems  $P_1 = \frac{1}{s}$  and  $P_2 = \frac{1}{s+0.1}$  in the sense of infinity norm and gap metric is infinity and 0.1, respectively.

(3) It measures the 'distance' in the closed-loop sense. In other words, when the distance between two linear models is small in the gap metric sense it means there exists at least one feedback controller that can stabilize both linear systems [Tan et al. (2004)].

Selection of number of local models in gap metric-based modelling: In order to select the appropriate number of local models, at first the nonlinearity measure index is utilized to decompose the operation region of the nonlinear process into a set of local linear operating points as shown in [Guay et al. (1996), Helbig et al. (2000)]. In this paper, we utilize a method which measures nonlinearity in data by testing whether a nonlinear ARMAX model produces a better estimate of data than a linear ARMAX model. The estimation method can be described as

$$Y(k) = L(k) + F_n(k) + E(k)$$
 (3)

where E(k) is noise sequence, L(k) is the portion of the data to be modeled by the linear function of the above model, while  $F_n(k)$  is the portion of the data to be modeled by the nonlinear function of the above model. The nonlinearity measure index is given by the standard deviation of  $F_n(k)$ . If the nonlinear function shows that a significant portion of the data are beyond the data explained by the linear function (namely  $F_n(t)$  has higher impact than L(t), a nonlinearity is detected. This algorithm is available in a built-in MATLAB function called "isnlarx". After determining the nonlinearity measure index, a linear model is constructed around each of the local operating points. Considering that the characteristics of the process in some of these points can be similar, the identified local linear models may turn out to be redundant. Hence, the gap metric is employed to merge the linear models which are similar. Different method can be conducted for merging models, here we keep one of the model and discard others. The merging threshold depends on the application and desired total number of local models. The detailed algorithm for constructing the local models is provided in Fig. 3.



Fig. 3. Local model construction flowchart

Local model weights: To determine the weights for each local model, gap metric can be utilized. To this end,

assume there exist N local models  $(P_i, i = 1, ..., N)$  that describe the nonlinear dynamics of the process (see Figure 4). The weights are updated at discrete time interval (k).

We assign the weights for each local model using following formula

$$\omega_i(k) = \frac{1 - \delta_i}{\sum_{i=1}^N (1 - \delta_i)} \tag{4}$$

where  $\delta_i$  is the gap metric between the local model *i* and the process model at time instance *k*. Figure 4 illustrates the scheme for single-input-single-output nonlinear model. In figure 4, the associated weight for each local linear model is assigned using its gap-metric-based distance from process model at sample *k*.



Fig. 4. Illustration of the proposed weighting method for the nominal linearized models  $P_1, \ldots, P_N$ .

Multi-modal CPA: Now suppose a nonlinear dynamic model is linearized around N equilibrium points using a set of ARMAX models as:

$$y_{i}(k+d) = A_{i}(z^{-1}) y(k) + B_{i}(z^{-1}) u(k) + C_{i}(z^{-1}) e(k+d) \quad \text{for} \quad i = 1, \dots, N$$
(5)

where y(k) and u(k) denote the process output and input at time k, respectively, e(k) denotes process noise with zero mean and variance  $\sigma_e^2$ , and d denotes system delay time for the local model i.  $A_i(z^{-1})$ ,  $B_i(z^{-1})$  and  $C_i(z^{-1})$ are the transfer functions expressed as polynomials in backshift operator  $z^{-1}$ .

Now, the weights determined by the gap metric can be utilized to combine the local models (5) as the following,

$$y(k+d) = \frac{\sum_{i=1}^{N} (1-\delta_i) y_i(k+d)}{\sum_{i=1}^{N} (1-\delta_i)}$$
  
= 
$$\frac{\sum_{i=1}^{N} (1-\delta_i) \times (A_i(z^{-1}) y_i(k) + B_i(z^{-1}) u_i(k)))}{\sum_{i=1}^{N} (1-\delta_i)} + \frac{\sum_{i=1}^{N} (1-\delta_i) \times (C_i(z^{-1}) e(k+d))}{\sum_{i=1}^{N} (1-\delta_i)}$$
(6)

Further,  $C_i(z^{-1})$  can be split into unpredictable  $F_i$  and predictable  $R_i$  components using the Diophantine identity [Huang and Shah (1999)] for each operation region *i*:

$$C_i(z^{-1}) = F_i(z^{-1}) + z^{-d}R_i(z^{-1})$$
(7)

where  $F_i$  is polynomial in  $z^{-1}$ ,  $deg(F_i) = d - 1$ , i.e.

$$F_i(z^{-1}) = \sum_{j=0}^{d-1} f_{ij} z^{-j}$$
(8)

where deg stands for degree of the polynomial and  $f_{ij}$  are feedback invariant terms.

Eq. (6) can be rewritten as

$$y(k+d) = \frac{\sum_{i=1}^{N} (1-\delta_i) \left(A_i(z^{-1}) y(k) + B_i(z^{-1}) u(k)\right)}{\sum_{i=1}^{N} (1-\delta_i)} + \frac{\sum_{i=1}^{N} (1-\delta_i) \left(R_i(z^{-1}) e(k)\right)}{\sum_{i=1}^{N} (1-\delta_i)} + \frac{\sum_{i=1}^{N} (1-\delta_i) F_i(z^{-1}) e(k+d)}{\sum_{i=1}^{N} (1-\delta_i)}$$
(9)

Above equation can be utilized to calculate the benchmark value. For doing this, the minimum variance cost function is defined as [Huang and Shah (1999)]

$$J_{MV} = \min_{u(k)} E\left\{y^2 \left(k+d\right)\right\}$$
(10)

where  ${\cal E}$  denotes the expectation operator.

By substituting Eq. (12) in above equation, and considering that  $E\{u(k) e(k+d)\} = 0$  and  $E\{y(k) e(k+d)\} =$ 0, and the term  $\frac{\sum_{i=1}^{N} (1-\delta_i)F_i(z^{-1})e(k+d)}{\sum_{i=1}^{N} (1-\delta_i)}$  is invariant with respect to u(t), the optimal value of the cost function (10) is  $J_{MV-Optimal} =$ 

$$E \left\{ \left[ \frac{\sum_{i=1}^{N} (1-\delta_i) F_i(z^{-1}) e(k+d)}{\sum_{i=1}^{N} (1-\delta_i)} \right]^2 \right\}$$
(11)

and corresponding optimal control law u(k) can be achieved by solving the following equation.

$$\frac{\sum_{i=1}^{R} (1-\delta_i) (k) \left(A_i \left(z^{-1}\right) y (k) + B_i \left(z^{-1}\right) u (k)\right)}{\sum_{i=1}^{R} (1-\delta_i) (k)} + \frac{\sum_{i=1}^{R} (1-\delta_i) (k) \left(R_i \left(z^{-1}\right) e (k)\right)}{\sum_{i=1}^{R} (1-\delta_i) (k)} = 0$$
(12)

Using Eq. (4), (8) and (11), the MV benchmark for multimodal (5) can be written as follows:

$$J_{MV-Optimal}(k) = \sum_{i=1}^{N} w_i(k) \sum_{j=1}^{d} f_{ij}^2 \sigma_e^2$$
(13)

MV benchmark is theoretically achievable minimum value for the variance of the process output. Actual variance in the process variables can be compared against the theoretically achievable minimum value as the following,

$$\eta(k) = \frac{J_{MV-Optimal}(k)}{\sigma_y^2} = \left[\sum_{i=1}^N w_i(k) \sum_{j=1}^d f_{ij}^2\right] \frac{\sigma_e^2}{\sigma_y^2}$$
(14)

where  $\eta$  is the MV index which is obtained as the ratio between the MV benchmark ( $J_{MV-Optimal}$ ) and the actual 5

process output variance  $(\sigma^2_y)$ . The MV index is a scalar whose range is given by [0, 1]. The index values close to 0 indicate poor performance, and values close to 1 mean good control in comparison with minimum variance control.

#### 4. RESULTS

In this section, performances of the PID and MPC controllers implemented on SAGD process are assessed and compared by the proposed multi-modal performance assessment method.

PID controller configuration was replaced by the MPC configuration on the  $11^{th}$  of March 2017. In this study, we make use of data from the  $11^{th}$  of January 2017 to the  $10^{th}$  of March 2017 for assessing the performance of configuration 1 and data from the  $11^{th}$  of March to the  $15^{th}$  of April 2017 for assessing the performance of the MPC based configuration. Two adjacent time periods were chosen for the comparison to ensure that the effect of potential long-term process changes would not affect the performance metrics. Over the evaluation period process changes were considered negligible. From both periods, data were sampled at every 30 seconds archived in data historian. Fig. 5 shows the trends of the key variables under control of PID (classic control strategy) and MPC (advanced control strategy), respectively. For proprietary reasons, the values of the process variables are masked by removing the y-axis ticks of the plots. However, similar yaxis scales (minimum and maximum limits) are retained for the plots that compare trends between existing and new control configurations.



Fig. 5. Trends of emulsion flow rate, reservoir subcool and wellbore subcool from the selected data for analysis.

To set up comparison between the performances of MPC and PID controllers, the proposed multi-modal MV benchmark is applied on SAGD data by following the steps introduced in Algorithm 1. Figure 6 shows the MV benchmark for each local linear model. Now, for each time interval (1000 samples in this case), the multi-modal MV index can be calculated using Eq. (13). The weights  $(w_i(k))$  are determined by calculating gap metric between the linear local models and the model obtained by operation data at samples  $k, \ldots, k + 1000$ . As the nonlinear SAGD model is not known, the routine operating data of the process is utilized to construct the ARMAX models with input, output and noise polynomial order of all 3. Considering the frequent change of set-point, sufficient excitation is expected. The selected operating points for constructing local models are shown in Fig. 6. The flow rate and subcool are input and outputs of the local models, respectively. Figure 7 shows the obtained results for multi-modal MV benchmark based on the proposed approach. It shows that the overall MV benchmark varies between 0.05 and 0.2. Now the equation (14) can be utilized to compare the performance of the MPC and PID. It is determined that average performance index for PID and MPC is 0.12 and 0.32, respectively. Based on the proposed performance assessment result, the implemented controller configuration with MPC has led to the reduction in the overall variability of the key process variables, emulsion flow and subcool.



Fig. 6. The MV benchmark values of each local linear models.



Fig. 7. The MV multi-modal benchmark Curve for SAGD system which is function of time.

## 5. CONCLUSIONS

This paper proposed a method to assess the subcool control performances implemented on SAGD production well that has a strong nonlinearity. The performance assessment was conducted primarily based upon the variability in production flow and subcool measurements. Considering the nonlinearity in subcool control loop, the gap metric-based multi-modal control performance assessment technique is proposed. Using the gap metric, the required number of local models is determined and the associated model combination weights are calculated. From the obtained multi-model, control performance benchmark was calculated for the subcool control. It is concluded that the MPC configuration has better performance in decreasing the variability of the key variables of interest.

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