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Piecewise Linearization for Solving Models to Locate Urban Logistics Facilities

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Abstract: Facilities location is a strategic decision in supply chain design since it affects products and information flows through all the echelons. In urban contexts, facilities location is even more important because it shapes both the distribution activities and urban landscapes. In addition, changes in facilities location patterns have caused non-intended externalities such as congestion, emissions, noise, among others. We present a non-linear programming model to establish optimal facilities location in urban areas, modelling the city as a transportation network and considering congestion into the objective function. To solve the model, we use a piecewise linear optimization, which allows to obtain an optimal solution.

Keywords: Urban logistics, location, transportation, mathematical programming, linearization

1. INTRODUCTION

The location patterns of logistics facilities in metropolitan areas and densely populated cities have been analyzed during the last decade. These studies have established that the geography of logistics facilities location has changed since facilities have moved out to suburban and exurban areas. Some reasons for this pattern were the need to build larger and more efficient facilities to meet regional and national demand and the lack of space within cities (Aljohani and Thompson, 2016). In addition, there are other relevant aspects for location changes, such as land use restrictions for industrial activities, land cost inside the city, and proximity to important roads, ports and air terminals. This movement of logistics facilities from the inner urban area to suburban or former urban areas is known as logistics sprawl and it was defined by Dablanc and Rakotonarivo (2010) as "the movement of logistics facilities (warehouses, cross-dock facilities, intermodal terminals) towards suburban areas”

Logistics sprawl studies indicate that metropolitan areas in North America, Europe and Japan have undergone logistical dispersion by identifying changes in the location of logistic facilities, and in some cases estimating the impact of such changes (Dablanc and Ross, 2012; Woudsma, Jakubicek and Dablanc, 2016; Gupta and Garima, 2017; Aljohani and Thompson, 2018). For example, Aljohani and Thompson (2016) present a comprehensive review about logistics dispersion and how it has affected the geography of urban transport. This study shows that the distance traveled by trucks and negative environmental externalities, as higher emissions, have increased, as well as the effect on the movements of employees.

These studies also highlight that other research questions need to be answered. These questions are related to the optimization of suitable logistics facilities location in urban areas, networks distribution, and the improvement of transport system performance. This implies to determine an accessible location, modes of transport, and the assignment of clients to each facility to be opened. These decisions must explicitly consider environmental impacts and the quality of life of communities affected by the location and supply chains operation. Thus, facilities location problem, considering negative externalities caused by freight transport, is an important research topic for researchers and practitioners since it fosters sustainability in logistics operations.

In this sense, some research has included congestion into mathematical models in order to estimate its effect on total cost since it affects any business that uses road transportation for the movement of raw materials and final products along the supply chain (Jouzdani, Sadjadi and Fathian, 2013; Hwang et al., 2016; Oh, Park and Kang, 2016). In a research by Jouzdani, Sadjadi and Fathian (2013), an optimization model is proposed to solve a dynamic dairy facilities location problem considering different types of facilities (multiple products), demand uncertainty and traffic congestion. The model was tested by using a number of instances and an empirical case study; Nevertheless, authors could not obtain results for large-size instances such as Anaheim network. In addition, Hwang et al. (2016) address the high-demand facility location problem considering traffic congestion and vehicle emissions generated for both background traffic and facility demand users. Authors propose a bi-level MIP model with non-linear functions (link performance function and GHG emissions). An upper level model allows to find the optimal number and location of facilities while the lower level model addresses the traffic assignment problem of both facility demand and background traffic, using Wardrop’s user equilibrium principle. Tabu search, Memetic algorithm and Genetic algorithm meta-heuristics are implemented to solve the problem, which was tested through three different instances, including a case study of Incheon city in South Korea. Later, Oh, Park and Kang (2016) propose algorithms based on Harmony Search to solve the previous model, which proved to have a better performance in average and minimum objective function values in the large-size network. Other studies use a wide range of solution techniques, comprising Lagrange relaxation (Xie and Ouyang, 2013), Branch and Bound-based algorithms (Bai et al., 2011), and Simulated Annealing meta-heuristics (Fathian et al., 2016), among others.

The location of industrial facilities should not be based only in construction or investment costs, but also should take into account their impacts on the infrastructure network and public users, such as traffic congestion and pavement deterioration (Hajibabai, Bai and Ouyang, 2014). Therefore, authors propose a bi-level MINLP to optimize facility locations in a two-echelons supply chain, minimizing the total costs related to the supply chain, the existing roadway users and the pavement infrastructures. Due to the model complexity, authors reformulate the model into a single MILP and use a piece-wise linear function to approximate the cost function. The results show that the joint optimization allows reducing total costs, showing the advance of considering the impacts of freight facilities into supply chain design.

We address a location problem in urban areas which is related to the last mile logistic where freight transport externalities have a greater impact on people´s quality life. A difference from previous studies is that they have been mainly focused on supply chains designs, and regional/national applications, while our model aims to determine a suitable location in urban areas. We integrate the traffic assignment problem into a facilities location problem by using a performance function to estimate the travel time considering traffic flow and the capacity of the transportation network. For this, it was necessary the use of vehicle flow instead of products flow to represent both the capacity of candidate locations and the demand to be served. We construct a nonlinear programming model to estimate the forward and return flow of vehicles that minimize the total cost of facility location and transportation. Later, similar to the linearization implemented by Luathep et al. (2011), we present a linear approximation procedure to transform the nonlinear model into a mixed integer linear programming. We use the Sioux Falls network for the numerical experiments, which has been considered for other traffic studies (Jouzdani, Sadjadi and Fathian, 2013; Liu and Wang, 2017; Zheng *et al.*, 2017).

The paper is organized as follow: first, Section 2 describes the problem to be tackled, and present the non-linear programming model for logistics facilities location in urban areas. We also present the piecewise linearization in section 2. In section 3, we detail the results of the numerical experiment designed. Finally, Section 4 offers conclusions and identifies directions for future work.

2. MODELLING FACILITIES LOCATION PROBLEM IN URBAN AREAS

2.1 Problem description

An urban freight distribution process with vehicles flows between supplier, distribution centers and demand zones is studied. Distribution centers receive cargo that must be sent to different areas in which the customers are located. Unlike other facilities location problems, the capacity of candidate nodes and demand of customer zone are expressed in number of vehicles flows instead of product flow. This approach enables consideration of the effect of congestion into the travel time, which depends of both capacity and traffic flow.

In addition, we define customer zones rather than serving each client individually. This allows the simplification of the problem since it is not necessary to carry out the routing process. We also highlight that vehicles will use the city transportation network that is already used by public transportation, passenger and cargo vehicles. As a result, it could be a congested city in which the shortest path between an O-D will not be the more efficient route.

We aim at establishing the optimal location for distribution centers and the vehicles routes, given the transportation network capacity, traffic congestion and CO2 emissions resulting of background traffic and the vehicles required to move cargo between suppliers, distribution centers and customers.

2.2 Model formulation

This section introduces the notation and formulation of the model in the context of two echelons urban freight supply chain, where distribution centers need to be located. We propose a nonlinear programming model that joins features of the fixed cost opening facilities location problem and traffic assignment problem. In this sense, the model allows determining the optimal location while assigning vehicle flows to a transportation network under congestion. The objective of our model is to minimize the total costs related to facility construction investments (fixed cost) and transportation cost expressed in terms of the value of time under congestion. For this, the fixed cost is expressed in $ by unit of time, as well as the transportation cost. The model includes not only suppliers to distribution centers (DC) and DC to customer flows, but also the return flow from the customer zones to DC and from the latter to suppliers.

In addition to the set of nodes and links, we use paths that are defined as the set of links used to connect an O-D pair, taking into account that: a) there is not necessarily a direct connection between a pair of nodes; b) a path connecting an O-D pair in a forward flow may be different from that in the reverse flow.

The notations used in this study are described as follows:

Sets

N Set of nodes (suppliers s, candidate nodes for DC j, and customer zones nodes i);

A Set of links;

K Set of possible paths between any two nodes.

Parameters

CF_j Fixed cost of candidate node $j \in N$;

C_j Capacity of candidate node $j \in N$ expressed in terms of number of vehicles;

h_i Demand of customer zone $i \in N$ expressed in terms of number of vehicles;

Q_a Capacity of link $a \in A$;

b_a Background traffic of link $a \in A$;

t_a^0 Free travel time of link $a \in A$;

d_a Length (distance) of link $a \in A$;

α Value of time;

Δ_{ji}^{ak} 1 if candidate node $j \in N$ is connected to customer zone $i \in N$ using link $a \in A$ on a feasible path $k \in K$; 0 otherwise;

Δ_{ij}^{ak} 1 if customer zone $i \in N$ is connected to candidate node $j \in N$ using the link $a \in A$ on a feasible path $k \in K$; 0 otherwise;

Δ_{sj}^{ak} 1 if supplier $s \in N$ is connected to candidate node $j \in N$ using the link $a \in A$ on a feasible path $k \in K$; 0 otherwise;

Δ_{js}^{ak} 1 if candidate node $j \in N$ is connected to supplier $s \in N$ using the link $a \in A$ on a feasible path $k \in K$; 0 otherwise.

Decision variables

Y_j 1 if the distribution center is built at node j; 0 otherwise;

f_{ji}^k Number of vehicles originating from candidate node $j \in N$ to customer zone $i \in N$ using the path $k \in K$;

fr_{ij}^k Number of vehicles returning to candidate node $j \in N$ from customer zone $i \in N$ using the path $k \in K$;

f_{sj}^k Number of vehicles originating from supplier $s \in N$ to candidate node $j \in N$ using the path $k \in K$;

fr_{js}^k Number of vehicles returning from candidate node $j \in N$ to supplier $s \in N$ using the path $k \in K$.

Auxiliary variables

x_a Total traffic flow on the link $a \in A$

The model can be formulated as follows:

$$Minimize\ Z = \sum_{j \in E} CF_j Y_j + \sum_{a \in A} \alpha x_a t_a(x_a) \quad (1)$$

where

$$t_a(x_a) = t_a^0 \left(1 + 0.15 \left(\frac{x_a}{Q_a}\right)^4\right) \quad \forall a \in A \quad (2)$$

s.t.

$$x_a = b_a + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} f_{ji}^k \Delta_{ji}^{ak} + \sum_{s \in N} \sum_{j \in N} \sum_{k \in K} f_{sj}^k \Delta_{sj}^{ak} + \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} fr_{ij}^k \Delta_{ij}^{ak} + \sum_{j \in N} \sum_{s \in N} \sum_{k \in K} fr_{js}^k \Delta_{js}^{ak} \quad \forall a \in A, i \neq j \quad (3)$$

$$\sum_{j \in N} \sum_{k \in K} f_{ji}^k = h_i \quad \forall i \in N \quad (4)$$

$$\sum_{j \in N} C_j Y_j \geq \sum_{i \in N} h_i \quad (5)$$

$$\sum_{i \in N} \sum_{k \in K} f_{ji}^k \leq C_j Y_j \quad \forall j \in N \quad (6)$$

$$\sum_{s \in N} \sum_{k \in K} f_{sj}^k = \sum_{i \in N} \sum_{k \in K} f_{ji}^k \quad \forall j \in N \quad (7)$$

$$\sum_{k \in K} f_{ji}^k = \sum_{k \in K} fr_{ij}^k \quad \forall i \in N, j \in N \quad (8)$$

$$\sum_{k \in K} f_{sj}^k = \sum_{k \in K} fr_{js}^k \quad \forall s \in N, j \in N \quad (9)$$

$$Y_j = \{0,1\} \quad \forall j \in N \quad (10)$$

$$f_{ji}^k \geq 0 \quad \forall j \in N, i \in N, k \in K \quad (11)$$

$$f_{sj}^k \geq 0 \quad \forall j \in N, s \in N, k \in K \quad (12)$$

$$fr_{ij}^k \geq 0 \quad \forall j \in N, i \in N, k \in K \quad (13)$$

$$fr_{js}^k \geq 0 \quad \forall j \in N, s \in N, k \in K \quad (14)$$

The objective function (1) minimizes the cost for opening/building facilities, and the transportation cost between suppliers, distribution centers and customer zones. The transportation cost is calculated as the cost of the total travel time under congestion. For this, the second term of the objective function (2) is a performance function of each link in relation to the travel time, where t_a^0 and Q_a are the free travel time and link capacity, respectively (Daskin, 1985). Parameter α converts travel time to travel cost. Constraint (3) states that traffic flow on a link is equal to the sum of the background traffic and the equivalent to passenger vehicles of freight vehicles used to move the cargo within the supply chain, including Supplier-CD-Supplier flows and CD-Customer zones-CD flows. Since there is not vehicle flow between nodes i and j when i is equal to j, these variables are excluded from this constraint set. Constraint (4) guarantees that all customer zones must be served. Constraints (5) and (6) ensure that the total demand in zones i does not exceed the total capacity of facilities j, and that outflows from each distribution center do not exceed their capacity. Constraint (7) enforces the flow conservation at distribution centers; i.e., the inbound vehicle flow must be equal to the outbound vehicle flow. Constraints (8)-(9) ensure that vehicles return to their origin. Constraints (10)-(14) define the binary and nonnegative variables.

2.3 Piecewise linearization for objective function

In this section we present a linear approximation procedure to deal with the nonlinear term in objective function (1). The linearization method is used to approximate the functions of link travel time $t_a(x_a)$ and total link travel time $\overline{t_a}$ where $\overline{t_a} = x_a t_a(x_a)$. This approximation follows the procedure developed by Luathep et al (2011). The nonlinear objective function (1) includes a link performance function that allows to calculate travel time on a link (15):

$$t_a(x_a) = t_a^0 \left(1 + 0.15 \left(\frac{x_a}{Q_a}\right)^4\right) \quad \forall a \in A \quad (15)$$

Then, the functions of link travel time $t_a(x_a)$ and total link travel time $\overline{t_a}$ can also be expressed as follows:

$$t_a(x_a) = t_a^0 + b_a\left(\frac{x_a}{Q_a}\right)^{n_a} \tag{16}$$

Since $\bar{t_a} = x_a t_a(x_a)$, then:

$$\bar{t_a}(x_a) = t_a^0 x_a + b_a \frac{(x_a)^{n_a+1}}{(Q_a)^{n_a}} \tag{17}$$

Notice that in equation (16) the term b_a must be equal to 0.15*$t_a^0$and $n_a$will be 4. This ensures consistency regarding equation (15) and reduce the complexity of the linearization process. It is important to mention that t_a and $\bar{t_a}$ are convex and monotonically increasing nonlinear functions in x_a, which is essential for the linearization process applied by Luathep et al. (2011) and described below:

Given a bounded interval for total vehicle flow x_a on a link $a \in A$ $[x_a^0, x_a^M]$, where x_a^0 takes the value of zero and x_a^M the maximum value per link, which depends on the available data. The interval is divided into M segments $[x_a^{m-1}, x_a^m]$ with m=1, 2, …, M. Thus, t_a and $\bar{t_a}$ are approximated by linear interpolations over the M segments. M should be large enough so the intervals could be small, and the approximated values of t_a and $\bar{t_a}$ can be as close as possible to the real values. Likewise, x_a^M should be large enough to include all possible valued of vehicle flow on the link.

Let $t_a^m = t_a(x_a^m)$ y $\bar{t}_a^m = \bar{t}_a(x_a^m)$ the values of t_a and $\bar{t_a}$, respectively. These values are estimated for each x_a^m, and taken as parameters of the model. Furthermore, the linearization method requires the introduction of two decision variables: a binary variable k_a^m and a continuous variable λ_a^m for m = 1, 2, ..., M. The binary variable indicates the segment in which x_a falls by doing a comparison between x_a and x_a^{m-1}. On the other hand, the continuous variable allows to evaluate the distance between x_a and x_a^{m-1}, so that $\lambda_a^m = x_a^m - x_a^{m-1}$ if $x_a \geq x_a^{m-1}$; $\lambda_a^m = x_a - x_a^{m-1}$ otherwise.

The constraints added to the model are the following:

$$t_a = t_a^0 + \sum_{m=1}^{M} \frac{t_a^m - t_a^{m-1}}{x_a^m - x_a^{m-1}} \lambda_a^m \tag{18}$$

$$\bar{t}_a = \bar{t}_a^0 + \sum_{m=1}^{M} \frac{\bar{t}_a^m - \bar{t}_a^{m-1}}{x_a^m - x_a^{m-1}} \lambda_a^m \tag{19}$$

$$x_a = x_a^0 + \sum_{m=1}^{M} \lambda_a^m \tag{20}$$

$$\lambda_a^m \geq (x_a^m - x_a^{m-1}) k_a^m \tag{21}$$

$$\lambda_a^{m+1} \leq (x_a^{m+1} - x_a^m) k_a^m \tag{22}$$

$$\lambda_a^1 \leq x_a^1 - x_a^0 \tag{23}$$

$$\lambda_a^M \geq 0 \tag{24}$$

$$\lambda_a^m \geq 0 \ \forall a \in A, m \in M \mid m > 1 \tag{25}$$

$$k_a^m \in \{0,1\} \ \forall a \in A, m \in M \tag{26}$$

By adding the above constraints, the nonlinear programming model can be transformed into a mixed integer linear programming (MILP). As a result, the solution can be obtained by using usual techniques, such as branch and bound algorithms. Then, the MILP is presented below:

$$Minimize\ Z = \sum_{j \in N} CF_j Y_j + \sum_{a \in A} \alpha \bar{t_a} \tag{27}$$

Subject to constraints (3)-(14) and (18)-(26)

3. NUMERICAL EXAMPLE

In this section, the proposed MILP are applied to the Sioux Falls network, which consist of 24 nodes and 76 links (Fig. 1). The free flow speed, link distance, link capacity, as well as parameters of the performance function are equal to the network provided in the website of Transportation Networks, a repository for transportation research available at https://github.com/bstabler/TransportationNetworks. In this example, supplier is in node 1, nodes 2 to 24 are candidates for distribution facility. The hypothetical building cost is tabulated in Table 1 and α is equal to \$17/hour. In addition, we establish the following assumptions:

- The paths are explicitly enumerated. We identified at least two paths connecting O-D nodes, unless they are directly connected.
- Supplier nodes cannot be candidate nodes or demand nodes. This assumption takes place since suppliers are usually located outside the cities and we aim to locate a DC within an urban area.
- A customer zone can be served by several facilities.
- The background traffic for most of the links is equal or greater than link capacity, in order to simulate a congested urban area.

The presolved model contains 2301 variables and 995 constraints. The problem is solved using Xpress® on a personal computer equipped with 3.00 GHz CPU and 16 GB of memory and it took 14 seconds to achieve an optimal solution.

In the optimal solution, nodes 3 and 10 are selected to locate logistics facilities, as shown in Fig. 2. The facility in node 3 meets the demand of zones 2, 4, 5, 6, 8, 11, 12, 13, 14, 21, 23, 24, while facility in node 10 serve zones 7, 9, 15, 16, 17, 18, 19, 20, and 22. The total cost is \$1,212,080,000.00. Note that even when a single facility can serve all the demand, the model aims to open as many facilities as possible in order to reduce the transportation cost, which is dependent on the total traffic flow per links, since as the traffic increases, so does the total travel time.

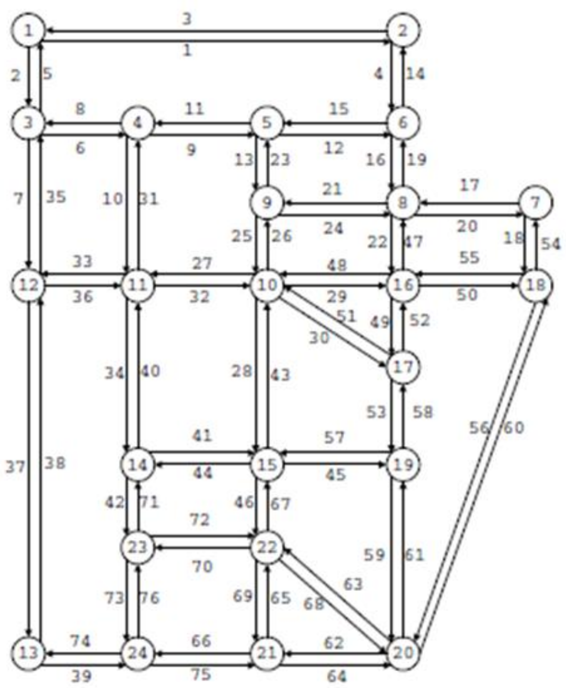

Fig. 1. Sioux Falls Network

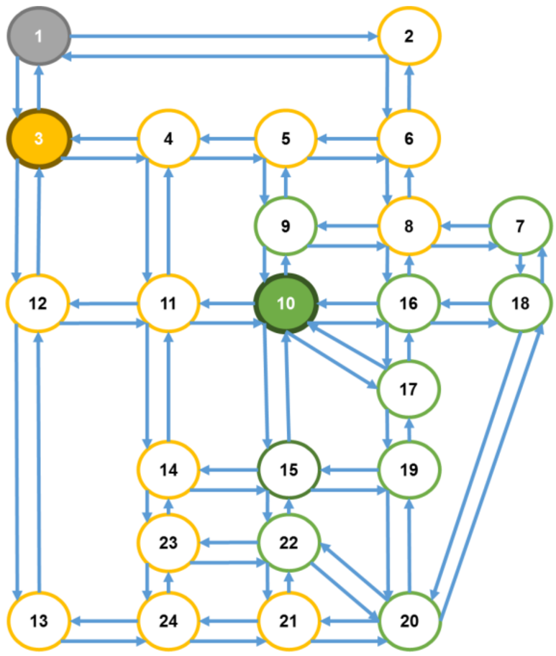

Fig. 2. Sioux Falls Network with optimal location for logistics facilities

Table 1. Opening/Building cost of distribution center facilities

Node	Opening/Building Cost ($/h)	Node	Opening/Building Cost ($/h)
2	790	14	650
3	700	15	500
4	560	16	630
5	620	17	540
6	700	18	560
7	400	19	630
8	400	20	600
9	500	21	480
10	780	22	860
11	670	23	520
12	860	24	560
13	490		

4. CONCLUSIONS

According to Hwang et al. (2016), the location of new facilities increases the traffic on the roads around them, affecting transport conditions and the quality of life of the nearby communities. Because of that, it is necessary to consider the effects of traffic congestion when location decisions take place in urban areas, where the transport infrastructure could be used at its maximum capacity. This situation is already experienced by cities in both developed and developing countries, which makes necessary doing research in this matter. As we mentioned above, one of the research topics are related to the location of logistics facilities in urban areas, supporting changes and trends in supply chain operations, such as e-commerce, and sustainability in urban freight. Therefore, an MINLP optimization to address the location of logistics facilities in urban areas is proposed. The model comprises the facilities location issue and the assignment of the traffic generated by the new facility, minimizing the total cost of the logistics facility. This approach can be used to establish urban consolidation centers and city hubs to improve the distribution process in terms of transportation and environmental costs. Also, solutions obtained with this model can be used as an input to integrate urban freight in city planning.

We use a piece-wise linearization to approximate the non-linear link travel time function, formulating it as a mixed integer linear programming model that can be solved using algorithms available in any optimization software. To illustrate the application of the linearization, we use hypothetical data for a network with 24 nodes and 76 links. The MILP has proven to be computationally efficient. In this study, using FICO® Xpress®, it took about 14 seconds to reach optimality for the Sioux Falls network. However, the computational time may substantially increase as the problem size increases.

Consequently, there is a need to apply efficient algorithm and optimization techniques to solve medium and large-scale networks.

Several simplifying assumptions were made due to the complexity of this problem. In future work, these assumptions should be relaxed to make a more extensive analysis. In addition, a more sophisticated model can be formulated, taking notice of dynamic background traffic, traffic restrictions, different types of vehicles, and dynamic planning horizon.

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