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Formation Control of Euler-Lagrange Systems of Leaders with Bounded Unknown Inputs

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Abstract: In this paper, we study the formation control of multiple Euler-Lagrange systems with nonautonomous leaders, which means the leaders have bounded unknown inputs. Firstly, adaptive distributed observers to the leaders' input bounds and states are designed for every follower. In addition, a discontinuous function in the adaptive distributed observer is applied to make up for the influence of the leaders' unknown inputs. Secondly, a distributed control law is constructed using the distributed observer to accomplish the formation control. Our control law achieves not only affine maneuver control but also containment control performance. All agents as a whole can rotate, shear and scale, and maneuver to destination safely.

Keywords: Formation control, Euler-Lagrange systems, multi-agent systems, distributed observer, nonautonomous leaders

1. INTRODUCTION

Multiagent control systems, including multiple unmanned ground vehicles (UGV), unmanned aerial vehicles (UAV), multiple robot manipulators and so on, have many engineering applications, such as formation control, see Gao et al. (2018), containment control, see Li et al. (2013), leaderless consensus Ren (2009) and leader-following consensus Cai and Huang (2015) for Euler-Lagrange (EL) systems and rigid bodies, among which, formation control generated considerable interest in researchers for navigation, moving target enclosing, etc.

Containment control of multiple agents, which aimes to enforce the followers to converge to the convex hull formed by the leaders, has been extensively studied in recent years. The dynamic models of systems include linear systems Li et al. (2013), EL systems Mei et al. (2012) and rigid bodies Wang et al. (2019). In their results, the formation configuration is fixed, and it extremely relies on the Laplacian matrix of the graphical topology, which limited its applications. Chen et al. (2017) solved the formation containment control of EL systems, an important class of nonlinear system that modes like underactuated surface vessels and manipulator, without using relative velocity information.

More recently, affine formation control, which aims to drive agents to maneuver as a whole such that rotation, scaling, shear and their arbitrary combinations can be conducted simultaneously, has attracted wide attention. Zhao (2018) studied the affine formation maneuver control of multiple single-integrator, double-integrator linear systems and unicycle agents. However, the leader system has no control input, so the signals the leader system can generate is limited.

It should be pointed out that for the above articles, the exosystems are autonomous systems (no control input) , no matter in containment control problem or affine formation maneuver control problem. It implies they have difficulties in implementing maneuvers to response to external environments in real time, taking obstacle avoidance as example. In practice, the leaders should have inputs to generage more general reference signals. Li et al. (2012) studied the consensus tracking problems of multiple linear systems of nonautonomous leaders. As far as we know, the formation control problem for EL systems with nonautonomous leaders is still open.

Inspired by the above facts, this paper studies the formation comtrol of EL systems of nonautonomous leaders. The main contributions of our paper are twofold. First, the leaders have bounded control inputs which are unknown to all followers and only partial followers have access to its boundaries. The problem of how to design a control law to compensate for the unknown inputs is challenging. Second, both affine control and containment control can be obtained so that formation control can be realized in cluttered environments and the whole system can reach to destination safely.

Notation: $1_N \in \mathbb{R}^N$ is a vector with all entries being 1. $I_N \in \mathbb{R}^{N\times N}$ is the identity matrix. Kronecker product is denoted by $\otimes$. $\|x\|$ denotes the Euclidean norm of a vector x. $\lambda(A)$ means the eigenvalues of matrix A.

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2. PROBLEM FORMULATION AND ASSUMPTIONS

We consider a group of N EL systems described by the following dynamic equations:

$$M_i(q_i)\ddot{q}_i(t) + C_i(q_i, \dot{q}_i)\dot{q}_i(t) + G_i(q_i) = \tau_i(t), \quad i = M+1, \cdots, M+N \tag{1}$$

where M is the number of the leader systems, for $i = M+1, \cdots, M+N$, $q_i, \dot{q}_i \in \mathbb{R}^n$ denote the vector of generalized position and velocity, respectively; $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the symmetric and positive definite inertia matrix; $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^n$ is the vector representing the Coriolis and centripetal forces; $G_i(q_i) \in \mathbb{R}^n$ is the vector of gravitational force; $\tau_i \in \mathbb{R}^N$ is the vector of control torque.

According to Slotine et al. (1991), EL systems (1) have the following properties:

Property 1: For $i = 1, \cdots, N$, $(\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i))$ is skew symmetric for $\forall\ q_i(t)$, $\dot{q}_i(t)$.

Property 2: For any $x, y \in \mathbb{R}^n$,

$$M_i(q_i)x + C_i(q_i, \dot{q}_i)y + G_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\Theta_i, \tag{2}$$

where $Y_i(q_i, \dot{q}_i, x, y) \in \mathbb{R}^{n\times p}$ is a known regression matrix and $\Theta_i \in \mathbb{R}^p$ is a constant vector consisting of the uncertain parameters of system (1).

In our formation control problem, there are M leaders whose desired generalized position vectors $q_j(t)$, for $j = 1, \cdots, M$, are assumed to be generated by the following exosystem:

$$\dot{v}_j(t) = Sv_j(t) + Rr_j(t),\ q_j(t) = Cv_j(t), \quad j = 1, \cdots, M \tag{3}$$

where $v_j \in \mathbb{R}^m$, and $S \in \mathbb{R}^{m\times m}$, $R \in \mathbb{R}^{m\times k}$, $C \in \mathbb{R}^{n\times m}$ are constant matrices.

Assumption 1: (S, R) is stabilizable.

Assumption 2: The unknown inputs of the leader system $r_j(t)$ are bounded, that is, there exists a positive constant γ_j such that $\|r_j(t)\| \le \gamma_j$.

Remark 1: In Dong et al. (2017), it is assumed that the exosystems have no control input, i.e., $\dot{v}_j(t) = S_j v_j(t)$. However, in practical systems, like robotic systems, they should react to the environment changes, avoid obstacles in cluttered environment, for example. The control input $r_j(t)$ ensures that the leader system can generate more general reference signals in real time. Furthermore, we assume that $r_j(t)$ is only known to a part of the followers to get more near practical situation.

We view the system composed of (1) and (3) as a multi-agent system of (N+M) agents with (3) as the leaders and (1) as the followers. Throughout this paper, we use $\mathcal{L} = \{1, \cdots, M\}$ to denote the leaders, and $\mathcal{F} = \{M+1, \cdots, M+N\}$ to denote the followers. Suppose that the network topology of the multi-agent system is represented by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. Let $\mathcal{A} = [a_{ij}]_{i,j=1}^{M+N} \in \mathbb{R}^{(M+N)\times(M+N)}$ denote the weighted adhacency matrix of the digraph $\mathcal{G}$, where $a_{ii} = 0$, and for $i \ne j$, $a_{ij} > 0$ if and only if $(j, i) \in \mathcal{E}$, otherwise, $a_{ij} = 0$. Let $\mathcal{L} = [l_{ij}]_{i,j=1}^{M+N} \in \mathbb{R}^{(M+N)\times(M+N)}$ be the Laplacian matrix of the digraph $\mathcal{G}$ such that $l_{ii} = \sum_{j=1}^{M+N} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \ne j$. We use $\mathcal{G}^j$ to denote a subgraph of $\mathcal{G}$ by removing all edges between the leaders $1, \cdots, j-1, j+1, \cdots, M$ and all the followers. That is, in digraph $\mathcal{G}^j$, agent j is the unique leader. We use L^j to represent the Laplacian matrix of $\mathcal{G}^j$. Note that according to the partition of the leaders abnd the followers, the Laplacian matrix L^j can be written as

$$L^j = \begin{bmatrix} 0 & 0 \\ L^j_{fl} & L^j_{ff} \end{bmatrix}.$$

Problem 1: Given systems (1) and (3), and a digraph $\mathcal{G}$, find a control law such that for $j \in \mathcal{L}$ and $i \in \mathcal{F}$, for any initial conditions $v_j(0)$, $q_i(0)$ and $\dot{q}_i(0)$, $q_i(t)$ and $\dot{q}_i(t)$ exist and satisfy

$$\lim_{t\to\infty} \|q_i(t) - \sum_{j=1}^{M} m_{ij}q_j(t)\| \le \varepsilon_1,$$
$$\lim_{t\to\infty} \|\dot{q}_i(t) - \sum_{j=1}^{M} m_{ij}\dot{q}_j(t)\| \le \varepsilon_2,$$

where ε_1 and ε_2 are two small positive constants, and m_{ij} is a constant which will be analysed in the following section.

Remark 2: Unlike Dong et al. (2017), whose controller leads to $\lim_{t\to\infty}(\dot{q}_i(t) - \sum_{j=1}^{M} m_{ij}\dot{q}_j(t)) = 0$, our controller attempts to drive the formation error and its velocity to small compact sets neighboring zero, which is reasonable and acceptable in reality.

To solve Problem 1, the following assumptions are needed.

Assumption 3: For each follower, there is a directed path in the digraph $\mathcal{G}^j$, and the digraph $\mathcal{G}$ is directed.

Assumption 4: All the eigenvalues of S have non-positive real part, and $\dot{q}_j(t)$ is bounded.

Assumption 5: There exist positive constants $k_{\underline{m}}$, $k_{\overline{m}}$, k_c and k_g, such that for all $i \in \mathcal{F}$, $k_{\underline{m}}I_N \le M_i(q_i) \le k_{\overline{m}}I_N$, $\|C_i(q_i, \dot{q}_i)\| \le K_c\|\dot{q}_i\|$, $\|G_i(q_i)\| \le K_g$.

Remark 3: Assumption 3 is a standard assumption of the digraph. Assumption 5 implies the mass, the Coriolis and centripetal forces and gravitaonal force are bounded, which is in line with the reality. Under Assumption 4, the leader systems can generate a large class of signals, such as step functions, ramp functions, sinusoidal functions and their combinations.

3. ADAPTIVE DISTRIBUTED OBSERVER OF LEADERS WITH UNKNOWN INPUTS

Because the states of the leaders are only known to part of the followers, an adaptive distributed observer using information disseminated from neighbors needs to be designed. It can be shown that the distributed observer used in Dong et al. (2017) is no longer appliable to our problem due to the unknown inputs of the leader systems. If the follower systems do not know the bound of $r_j(t)$, $j \in \mathcal{L}$, i.e. γ_j, a distributed observer for γ_j is needed.

We assume that the leaders' dynamic matrices S, R, and C are known to all followers. Consider the following dynamic compensator:

$$\dot{\gamma}_i^j(t) = \mu(\sum_{k\in\mathcal{F}} a_{ik}(\gamma_i^k(t) - \gamma_i^j(t)) + a_{ij}(\gamma_j - \gamma_i^j(t))), \quad (4)$$

$$\dot{\eta}_i^j(t) = S\eta_i^j(t) - c^j RK^j \varsigma_i^j(t) + (\gamma_i^j(t) + \rho)Rf_i^j(t), \quad (5)$$
$$i \in \mathcal{F},\ j \in \mathcal{L}$$

where $\varsigma_i^j(t) = \sum_{k\in\mathcal{F}} a_{ik}(\eta_k^j(t) - \eta_i^j(t)) + a_{ij}(v_j(t) - \eta_i^j(t))$, μ and ρ are any positive constants, c^j is a constant that satisfies $c^j \geq \frac{1}{\min\{\mathrm{Re}(\lambda(\mathcal{L}_{ff}^j))\}}$,

$$f_i^j(t) = \begin{cases} \dfrac{R^{\mathrm{T}}\varsigma_i^j(t)}{\|R^{\mathrm{T}}\varsigma_i^j(t)\|}, & \|R^{\mathrm{T}}\varsigma_i^j(t)\| \neq 0 \\ 0, & otherwise \end{cases}, \quad (6)$$

$K^j = -R^{\mathrm{T}}(P^j)^{-1}$ and $P^j > 0$ is a positive-definite solution of the linear matrix inequality (LMI)

$$SP^j + P^j S^T - 2RR^T < 0. \quad (7)$$

Under Assumption 3, all eigenvalues of L_{ff}^j have positive real parts, so c^j exists. Assumption 1 is the necessary and sufficient condition on the existence of a positive-definite solution to the above LMI, see Li et al. (2013). Next, we have the following lemma.

Lemma 1: Given digraph $\mathcal{G}$, under Assumptions 1-4, for any $\mu,\ \rho > 0$, and for any initial conditions $\gamma_i^j(0)$ and $\eta_i^j(0)$, we have

$$\lim_{t\to\infty}(\gamma_i^j(t) - \gamma_j) = 0, \quad \lim_{t\to\infty}(\eta_i^j(t) - v_j(t)) = 0 \quad (8)$$

exponentially.

Proof: Denote $\tilde{\gamma}_i^j(t) = \gamma_j - \gamma_i^j(t)$, then we have

$$\dot{\tilde{\gamma}}_i^j(t) = -\mu\Big(\sum_{k\in\mathcal{F}} a_{ik}(\tilde{\gamma}_i^j(t) - \tilde{\gamma}_i^k(t)) + a_{ij}\tilde{\gamma}_i^j(t)\Big). \quad (9)$$

Let $\tilde{\gamma}^j = [(\tilde{\gamma}_{M+1}^j)^{\mathrm{T}}, \cdots, (\tilde{\gamma}_{M+N}^j)^{\mathrm{T}}]^{\mathrm{T}}$, then system (9) can be put into the compact form:

$$\dot{\tilde{\gamma}}^j(t) = -\mu L_{ff}^j \tilde{\gamma}^j(t). \quad (10)$$

Then for any $\mu > 0$, $\lim_{t\to\infty}\tilde{\gamma}^j(t) = 0$, exponentially.

Next, we show that $\lim_{t\to\infty}(\eta_i^j(t) - v_j(t)) = 0$ exponentially.

To begin with, let $\tilde{\eta}_i^j(t) = \eta_i^j(t) - v_j(t)$, $\bar{r}_j(t) = 1_N \otimes r_j(t)$, $\tilde{\Pi}_i^j(t) = \tilde{\gamma}_i^j(t)Rf_i^j(t)$, $X^j = [(X_{M+1}^j)^T, \cdots, (X_{M+N}^j)^T]^T$, for $X = \tilde{\eta},\ f,\ \Pi$, and ς. Then we have

$$\begin{aligned}\dot{\tilde{\eta}}_i^j(t) =& S\tilde{\eta}_i^j(t) - c^j BK^j\varsigma_i^j(t) + (\gamma_j + \rho)Rf_i^j(t) - Rr_j(t) \\ &- \tilde{\gamma}_i^j Rf_i^j(t). \quad (11)\end{aligned}$$

Then, the compact form of (11) is as follows,

$$\begin{aligned}\dot{\tilde{\eta}}^j(t) =& (I_N \otimes S)\tilde{\eta}^j(t) - c^j(I_N \otimes RK^j)\varsigma^j(t) - (I_N \otimes R)\bar{r}_j(t) \\ &+ (\gamma_j + \rho)(I_N \otimes R)f^j(t) - \tilde{\Pi}^j(t). \quad (12)\end{aligned}$$

Since $\varsigma^j(t) = -(L_{ff}^j \otimes I_n)\tilde{\eta}^j(t)$, it holds for (12) that

$$\begin{aligned}\dot{\varsigma}^j(t) =& (I_N \otimes S + c^j L_{ff}^j \otimes RK^j)\varsigma^j(t) - (\gamma_j + \rho)(L_{ff}^j \otimes R)f^j(t) \\ &+ (L_{ff}^j \otimes R)\bar{r}_j(t) + (L_{ff}^j \otimes I_n)\tilde{\Pi}^j(t). \quad (13)\end{aligned}$$

Now, we show that $(I_N \otimes S + c^j L_{ff}^j \otimes RK^j)$ is Hurwitz.

Consider the system

$$\dot{\varpi}(t) = (I_N \otimes S + c^j L_{ff}^j \otimes RK^j)\varpi(t). \quad (14)$$

Under Assumption 3, all the eigenvalues of $\mathcal{L}_{ff}^j$ have positive real parts. Then, there exists a unitary matrix $U^j \in \mathbb{R}^{N\times N}$ that $(U^j)^{\mathrm{H}}\mathcal{L}_{ff}^j U^j = \Lambda^j$, where Λ^j is an upper-triangular matrix with $\lambda_i^j,\ i = 1, \cdots, N$, as its diagonal entries. Let $\tilde{\varpi}(t) = ((U^j)^{\mathrm{H}} \otimes I_n)\varpi(t)$. Then it follows from (14) that

$$\dot{\tilde{\varpi}}(t) = (I_N \otimes S + c^j\Lambda^j \otimes RK^j)\tilde{\varpi}(t). \quad (15)$$

It is clear that (15) is asymptotically stable if and only if the following N systems

$$\dot{\tilde{\varpi}}_i(t) = (S + c^j\lambda_i^j RK^j)\tilde{\varpi}_i(t),\ i = 1, \cdots, N. \quad (16)$$

are all asymptotically stable. Let P^j be a positive-definite solution of the LMI (7), then it follows that

$$\begin{aligned}&(S + c^j\lambda_i^j RK^j)P^j + P^j(S + c^j\lambda_i^j RK^j)^{\mathrm{T}} \\ &= SP^j + P^j S^{\mathrm{T}} + 2c^j Re(\lambda_i^j)RK^jP^j \\ &= SP^j + P^j S^{\mathrm{T}} - 2c^j Re(\lambda_i^j)RR^{\mathrm{T}} \\ &\leq SP^j + P^j S^{\mathrm{T}} - 2RR^{\mathrm{T}} < 0,\ i = 1, \cdots, N.\end{aligned}$$

That is, $(I_N \otimes S + c^j L_{ff}^j \otimes RK^j)$ is Hurwitz, then there exists a diagonal matrix $P = diag\{p_{M+1}, \cdots, p_{M+N}\}$ with $p_i > 0\ (i \in \mathcal{F})$ such that $P(I_N \otimes S + c^j L_{ff}^j \otimes RK^j) + (I_N \otimes S + c^j L_{ff}^j \otimes RK^j)^{\mathrm{T}}P = -Q$ where Q is positive definite, see Qu (2009).

Construct the following Lyapunov function candidate,

$$V(\varsigma^j) = (\varsigma^j(t))^T P\varsigma^j(t). \quad (17)$$

The derivative of $V(\varsigma^j)$ gives

$$\begin{aligned}\dot{V}(\varsigma^j) =& -(\varsigma^j(t))^{\mathrm{T}}Q\varsigma^j(t) - 2(\varsigma^j(t))^{\mathrm{T}}P(\gamma_j + \rho)(L_{ff}^j \otimes R)f^j(t) \\ &+ 2(\varsigma^j(t))^{\mathrm{T}}P(L_{ff}^j \otimes R)\bar{r}_j(t) \\ &+ 2(\varsigma^j(t))^{\mathrm{T}}P(L_{ff}^j \otimes I_n)\tilde{\Pi}^j(t). \quad (18)\end{aligned}$$

According to the definition of $f_i^j(t)$ in (6), $(\varsigma_i^j(t))^{\mathrm{T}}Rf_i^j(t) = \|R^{\mathrm{T}}\varsigma_i^j(t)\|$, and $(\varsigma_i^j(t))^{\mathrm{T}}Rf_k^j(t) \leq \|R^{\mathrm{T}}\varsigma_i^j(t)\|\|f_k^j(t)\| \leq \|R^{\mathrm{T}}\varsigma_i^j(t)\|$, for $k \neq i$, $k \in \mathcal{F}$. Then, it follows that

$$\begin{aligned}&-2(\varsigma^j(t))^{\mathrm{T}}P(\gamma_j + \rho)(L_{ff}^j \otimes R)f^j(t) \\ &= \sum_{i=M+1}^{M+N} 2(\gamma_j + \rho)p_i(\varsigma_i^j(t))^{\mathrm{T}}R\Big(\sum_{k=1}^{M} a_{ik}(f_k^j(t) - f_i^j(t)) \\ &\quad - a_{ij}f_i^j(t)\Big) \leq -2(\gamma_j + \rho)\sum_{i=M+1}^{M+N} p_i a_{ij}\|R^{\mathrm{T}}\varsigma_i^j(t)\|. \quad (19)\end{aligned}$$

Under Assumption 3, we have

$$\begin{aligned}2(\varsigma^j(t))^{\mathrm{T}}P(L_{ff}^j \otimes R)\bar{r}_j(t) =& \sum_{i=M+1}^{M+N} 2p_i a_{ij}(\varsigma_i^j(t))^{\mathrm{T}}Rr_j(t) \\ \leq& 2\gamma_j\sum_{i=M+1}^{M+N} p_i a_{ij}\|R^{\mathrm{T}}\varsigma_i^j(t)\|. \quad (20)\end{aligned}$$

Substituting (19) and (20) into (18) gives

$$\begin{aligned}\dot{V}(\varsigma^j) \leq& -(\varsigma^j(t))^{\mathrm{T}}Q\varsigma^j(t) - 2\rho\sum_{i=M+1}^{M+N} p_i a_{ij}\|R^{\mathrm{T}}\varsigma_i^j(t)\| \\ &+ 2(\varsigma^j(t))^{\mathrm{T}}P(L_{ff}^j \otimes I_n)\tilde{\Pi}^j(t) \\ \leq& -(\varsigma^j(t))^{\mathrm{T}}Q\varsigma^j(t) + 2(\varsigma^j(t))^{\mathrm{T}}P(L_{ff}^j \otimes I_n)\tilde{\Pi}^j(t)\end{aligned}$$

$$\leq -(\lambda_{\min}(Q) - \frac{\|P(L_{ff}^j)\|^2}{\varepsilon})\|\varsigma^j(t)\|^2 + \varepsilon\|\tilde{\Pi}^j(t)\|^2. \quad (21)$$

Let $\varepsilon = \frac{2\|P(L_{ff}^j)\|^2}{\lambda_{\min}(Q)}$, then we have

$$\begin{aligned}\dot{V}(\varsigma^j) &\leq -\frac{1}{2}\lambda_{\min}(Q)\|\varsigma^j(t)\|^2 + \varepsilon\|\tilde{\Pi}^j(t)\|^2 \\ &\leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}V(\varsigma^j) + \varepsilon\|\tilde{\Pi}^j(t)\|^2 \\ &= -\lambda_1 V(\varsigma^j) + \varepsilon\|\tilde{\Pi}^j(t)\|^2, \quad t \geq 0\end{aligned} \quad (22)$$

where $\lambda_1 = \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}$. By Comparison Lemma (Lemma 3.4 of Khalil (2002)),

$$V(\varsigma^j) \leq e^{-\lambda_1 t}V(\varsigma^j(0)) + \int_0^t e^{-\lambda_1(t-\tau)}\varepsilon\|\tilde{\Pi}^j(\tau)\|^2 d\tau. \quad (23)$$

Since $\tilde{\Pi}^j(\tau) = \tilde{\gamma}_i^j(t)Rf_i^j(\tau)$, $\|f_i^j(\tau)\| \leq 1$, and $\tilde{\gamma}_i^j(t)$ converges to 0 exponentially, $\|\tilde{\Pi}^j(\tau)\|^2$ also converges to 0 exponentially. Then, there exists a positive constant λ_2 such that

$$\varepsilon\|\tilde{\Pi}^j(\tau)\|^2 \leq \varepsilon\|\tilde{\Pi}^j(0)\|^2 e^{-\lambda_2\tau}. \quad (24)$$

Without loss of generality, we assume $0 < \lambda_2 < \lambda_1$. Then, we can get

$$\begin{aligned}&\int_0^t e^{-\lambda_1(t-\tau)}\varepsilon\|\tilde{\Pi}^j(\tau)\|^2 d\tau \\ &\leq \int_0^t e^{-\lambda_1(t-\tau)}\varepsilon\|\tilde{\Pi}^j(0)\|^2 e^{-\lambda_2\tau} d\tau \\ &\leq \frac{1}{\lambda_1 - \lambda_2}\varepsilon\|\tilde{\Pi}^j(0)\|^2 e^{-\lambda_2 t}.\end{aligned} \quad (25)$$

Substituting (25) into (23), it holds that

$$\begin{aligned}V(\varsigma^j) &\leq e^{-\lambda_1 t}V(\varsigma^j(0)) + \frac{1}{\lambda_1 - \lambda_2}\varepsilon\|\tilde{\Pi}^j(0)\|^2 e^{-\lambda_2 t} \\ &\leq (V(\varsigma^j(0)) + \frac{1}{\lambda_1 - \lambda_2}\varepsilon\|\tilde{\Pi}^j(0)\|)e^{-\lambda_2 t}.\end{aligned} \quad (26)$$

We see that $\lim_{t\to\infty} V(\varsigma^j) = 0$ exponentially. Thus, we conclude that $\lim_{t\to\infty} \varsigma^j(t) = 0$ exponentially, which means $\lim_{t\to\infty} \tilde{\eta}^j(t) = 0$ exponentially because L_{ff}^j is nonsingular under Assumption 3. ■

Remark 4: Equations (4) and (5) are adaptive distributed observers for each follower to estimate input bounds and states of the jth leader, using the information of their neighbors. In our control law, the nonlinear term $(\gamma_i^j(t) + \rho)Rf_i^j(t)$ is used to make up for the influence of bounded unknown inputs of the leaders. Similar to Su and Huang (2011), in distributed observer (5), the eigenvalues of the Laplacian matrix of the graph is a priori condition for the followers.

In (5), the discontinuous of the nonlinear term $f_i^j(t)$ will cause chattering to the control input. Next, to avoid this situation, we replace $f_i^j(t)$ by $\bar{f}_i^j(t)$, a continuous function,

$$\bar{f}_i^j(t) = \frac{R^{\mathrm{T}}\varsigma_i^j(t)}{\|R^{\mathrm{T}}\varsigma_i^j(t)\| + \kappa_i^j} \quad (27)$$

where κ_i^j is a small positive constant.

Lemma 2: Given digraph $\mathcal{G}$, under Assumptions 1-4, using the continuous function $\bar{f}_i^j(t)$ in (27) for the distributed observer $\eta_i^j(t)$ in (5), then for any μ, $\rho > 0$, and for any initial conditions $\gamma_i^j(0)$ and $\eta_i^j(0)$, the estimation error $\tilde{\eta}_i^j(t)$ is uniformly ultimately bounded.

Proof: The Lyapunov function candidate is the same as (17). Similar to the proof of Lemma 1, $(\varsigma_i^j(t))^{\mathrm{T}}R\bar{f}_i^j(t) = \frac{\|R^{\mathrm{T}}\varsigma_i^j(t)\|^2}{\|R^{\mathrm{T}}\varsigma_i^j(t)\|+\kappa_i^j}$, and $(\varsigma_i^j(t))^{\mathrm{T}}R\bar{f}_k^j(t) \leq \|R^{\mathrm{T}}\varsigma_i^j(t)\|$. It can be verified that

$$\begin{aligned}&-2(\varsigma^j(t))^{\mathrm{T}}P(\gamma_j+\rho)(L_{ff}^j\otimes R)\bar{f}^j(t)+2(\varsigma^j(t))^{\mathrm{T}}P(L_{ff}^j\otimes R)\bar{r}_j \\ &\leq 2(\gamma_j + \rho)\sum_{i=M+1}^{M+N} p_i\kappa_i^j\left(\sum_{k=M+1}^{M+N} a_{ik} + a_{ij}\right).\end{aligned} \quad (28)$$

Denoting $\iota = 2(\gamma_j + \rho)\sum_{i=M+1}^{M+N} p_i\kappa_i^j(\sum_{k=M+1}^{M+N} a_{ik} + a_{ij})$, then the derivative of $V(\varsigma^j)$ along (13) satisfies

$$\dot{V}(\varsigma^j) \leq -(\varsigma^j(t))^{\mathrm{T}}Q\varsigma^j(t) + 2(\varsigma^j(t))^{\mathrm{T}}P(L_{ff}^j \otimes I_n)\tilde{\Pi}^j(t) + \iota. \quad (29)$$

Based on the Comparison Lemma and (26), we have $V(\varsigma^j) \leq (V(\varsigma^j(0)) + \frac{1}{\lambda_1-\lambda_2}\varepsilon\|\tilde{\Pi}^j(0)\|)e^{-\lambda_2 t} + \iota/\lambda_1$. Thus, $V(\varsigma^j)$ is ultimately bounded with the upper bound ι/λ_1, and it means that $\varsigma_i^j(t)$ and $\eta_i^j(t)$ are ultimately bounded.

In the definition of ι, since γ_j, ρ, p_i, a_{ij} are constants, ι is a linear combination of κ_i^j. If κ_i^j is small enough, ι is also small enough. Because $V(\varsigma^j) = (\varsigma^j(t))^{\mathrm{T}}P\varsigma^j(t)$ is ultimately bounded with the upper bound ι/λ_1, $\varsigma^j(t)$ is ultimately bounded with a small enough constant, so is $\varsigma_i^j(t)$ and $\tilde{\eta}_i^j(t)$. Therefore, the estimation error $\tilde{\eta}_i^j(t)$ can be as small as desired by setting appropriate value for κ_i^j. ■

In the following section, we use the continuous term $\bar{f}_i^j(t)$ instead of $f_i^j(t)$ for the distributed observer $\eta_i^j(t)$. In addition, $\bar{f}_i^j(t)$ guarantees the existence and continuous of $\ddot{\eta}_i^j(t)$, which plays a key role for the design of the followers' control law.

4. FORMATION CONTROL OF MULTIPLE EL SYSTEMS

In this section, the adaptive distributed observer (4), (5), and (27) are applied to synthesize a distributed control law for multiple EL systems to achieve the formation control.

To begin with, we define a new variable,

$$\dot{q}_{ri}(t) = \sum_{j=1}^{M} m_{ij}C\dot{\eta}_i^j(t) - \alpha(q_i(t) - \sum_{j=1}^{M} m_{ij}C\eta_i^j(t)) \quad (30)$$

where α is a positive constant. It follows that

$$\ddot{q}_{ri}(t) = \sum_{j=1}^{M} m_{ij}C\ddot{\eta}_i^j(t) - \alpha(\dot{q}_i(t) - \sum_{j=1}^{M} m_{ij}C\dot{\eta}_i^j(t)). \quad (31)$$

According to (5), we have

$$\ddot{\eta}_i^j(t) = S\dot{\eta}_i^j(t)+RK^j\dot{\varsigma}_i^j(t)+\dot{\gamma}_i^j(t)R\bar{f}_i^j(t)+(\gamma_i^j(t)+\rho)R\dot{\bar{f}}_i^j(t) \quad (32)$$

where $\dot{\gamma}_i^j(t)$ is defined in (4), $\dot{\varsigma}_i^j(t) = \sum_{k\in\mathcal{F}} a_{ik}(\dot{\eta}_k^j(t) - \dot{\eta}_i^j(t)) + a_{ij}(\dot{v}_j(t) - \dot{\eta}_i^j(t))$, and

$$\dot{\bar{f}}_i^j(t) = \frac{R^{\mathrm{T}}\dot{\varsigma}_i^j(t)(\|R^{\mathrm{T}}\varsigma_i^j(t)\| + \kappa_i^j) - g_i^j(t)}{(\|R^{\mathrm{T}}\varsigma_i^j(t)\| + \kappa_i^j)^2} \quad (33)$$

where

$$g_i^j(t) = \frac{R^{\mathrm{T}}\varsigma_i^j(t)(\varsigma_i^j(t))^{\mathrm{T}}RR^{\mathrm{T}}\dot{\varsigma}_i^j(t)}{\|R^{\mathrm{T}}\varsigma_i^j(t)\|}.$$

Then, we define

$$s_i(t) = \dot{q}_i(t) - \dot{q}_{ri}(t), \quad i \in \mathcal{F}. \tag{34}$$

By Property 2, there exists a known matrix $Y_i(q_i, \dot{q}_i, \ddot{q}_{ri}, \dot{q}_{ri})$ and an unknown constant vector Θ_i such that

$$M_i(q_i)\ddot{q}_{ri}(t)+C_i(q_i,\dot{q}_i)\dot{q}_{ri}(t)+G_i(q_i) = Y_i(q_i,\dot{q}_i,\ddot{q}_{ri},\dot{q}_{ri})\Theta_i. \tag{35}$$

The distributed control law we proposed is as follows,

$$\tau_i(t) = -K_i s_i(t) + Y_i(t)\hat{\Theta}_i(t), \quad i \in \mathcal{F},\ j \in \mathcal{L}, \tag{36a}$$

$$\dot{\hat{\Theta}}_i(t) = -\Lambda_i^{-1}Y_i^{\mathrm{T}}(t)s_i(t), \tag{36b}$$

$$\dot{\gamma}_i^j(t) = \mu\Big(\sum_{k\in\mathcal{F}} a_{ik}(\gamma_i^k(t) - \gamma_i^j(t)) + a_{ij}(\gamma_j - \gamma_i^j(t))\Big), \tag{36c}$$

$$\dot{\eta}_i^j(t) = S\eta_i^j(t) - c^j RK^j\varsigma_i^j(t) + (\gamma_i^j(t)+\rho)R\bar{f}_i^j(t), \tag{36d}$$

where $K_i \in \mathbb{R}^{n\times n}$ and $\Lambda_i \in \mathbb{R}^{p\times p}$ are two symmetric and positive definite matrices, $\hat{\Theta}_i$ denotes the state of the dynamic compensator (36b).

Theorem 1. Given the followers in (1), the leaders in (3), and a digraph $\mathcal{G}$, under Assumptions 1-5, Problem 1 is solvable by the control law composed of (36a)-(36d).

Proof: Substituting (30) into (34) gives

$$(\dot{q}_i(t) - \sum_{j=1}^{M} m_{ij}C\dot{\eta}_i^j(t)) + \alpha(q_i(t) - \sum_{j=1}^{M} m_{ij}C\eta_i^j(t)) = s_i(t). \tag{37}$$

Since $\alpha > 0$, (37) is a stable first order linear system in $(q_i(t) - \sum_{j=1}^M m_{ij}C\eta_i^j(t))$ with input $s_i(t)$. If $\lim_{t\to\infty} s_i(t) = 0$, then both $(q_i(t) - \sum_{j=1}^M m_{ij}C\eta_i^j(t))$ and $(\dot{q}_i(t) - \sum_{j=1}^M m_{ij}C\dot{\eta}_i^j(t))$ converges to 0 as t tends to infinity. Next, we show that $\lim_{t\to\infty} s_i(t) = 0$.

To this end, substituting (36a) into (1) gives

$$M_i(q_i)\ddot{q}_i(t)+C_i(q_i,\dot{q}_i)\dot{q}_i(t)+G_i(q_i) = -K_i s_i(t)+Y_i(t)\hat{\Theta}_i(t). \tag{38}$$

Let $\tilde{\Theta}_i(t) = \hat{\Theta}_i(t) - \Theta_i$. Combing (34), (35) and (38) gives

$$M_i(q_i)\dot{s}_i(t)+C_i(q_i,\dot{q}_i)s_i(t) = -K_i s_i(t)+Y_i(t)\tilde{\Theta}_i(t), \tag{39}$$

$$\dot{\tilde{\Theta}}_i(t) = -\Lambda_i^{-1}Y_i^{\mathrm{T}}(t)s_i(t). \tag{40}$$

Define the following Lyapunov candidate,

$$V(t) = \sum_{i=M+1}^{M+N} \frac{1}{2}(s_i^{\mathrm{T}}(t)M_i(q_i)s_i(t) + \tilde{\Theta}_i^{\mathrm{T}}(t)\Lambda_i\tilde{\Theta}_i(t)). \tag{41}$$

By (31) and (34), since $\dot{s}_i(t)$ exists, so is $V(t)$. Taking the derivative of $V(t)$, by Property 1 of the EL systems, we have

$$\begin{aligned}\dot{V} &= \sum_{i=M+1}^{M+N} (s_i^{\mathrm{T}}(t)M_i(q_i)\dot{s}_i(t) + \frac{1}{2}s_i^{\mathrm{T}}(t)\dot{M}_i(q_i)s_i(t) + \tilde{\Theta}_i^{\mathrm{T}}(t)\Lambda_i\dot{\tilde{\Theta}}_i(t))\\ &= \sum_{i=M+1}^{M+N}\Big(s_i^{\mathrm{T}}(t)(-C_i(q_i,\dot{q}_i)s_i(t) - K_i s_i(t) + Y_i(t)\hat{\Theta}_i(t)) + \frac{1}{2}s_i^{\mathrm{T}}(t)\dot{M}_i(q_i)s_i(t) + \tilde{\Theta}_i^{\mathrm{T}}(t)\Lambda_i\dot{\tilde{\Theta}}_i(t)\Big)\\ &= -\sum_{i=M+1}^{M+N} s_i^{\mathrm{T}}(t)K_i s_i(t) \le 0.\end{aligned} \tag{42}$$

Note that (42) implies that both $s_i(t)$ and $\tilde{\Theta}_i(t)$ are bounded. To show $\dot{V}(t)$ is uniformly continuous for all $t \ge 0$, we need to further show that $\dot{s}_i(t)$ is bounded. To this end, from (39), because $M_i(q_i)$ is positive definite, we need to show $C_i(q_i,\dot{q}_i)$ and $Y_i(t)\tilde{\Theta}_i(t)$ are bounded.

By (37), both $(\dot{q}_i(t) - \sum_{j=1}^M m_{ij}C\dot{\eta}_i^j(t))$ and $(q_i(t) - \sum_{j=1}^M m_{ij}C\eta_i^j(t))$ are bounded because of the boundedness of s_i. According to Lemma 2, where we modify the discontinuous $f_i^j(t)$ in Lemma 1 to a continuous term $\bar{f}_i^j(t)$ in (27), $\eta_i^j(t)$ is bounded under Assumption 4, then by (5), $\dot{\eta}_i^j(t)$ is also bounded. Thus $\dot{q}_i(t)$ is bounded, which implies $C_i(q_i,\dot{q}_i)$ is bounded.

By (35), $Y_i(t)\tilde{\Theta}_i(t)$ is bounded if and only if $\dot{q}_{ri}(t)$ and $\ddot{q}_{ri}(t)$ are bounded. $\dot{q}_{ri}(t)$ is bounded from (30), because we have shown that both $\eta_i^j(t)$ and $\dot{\eta}_i^j(t)$ are bounded. Considering the right hand of (32), $\varsigma_i^j(t)$ and $\dot{\varsigma}_i^j(t)$ are bounded based on Assumption 4 and the boundedness of $\eta_i^j(t)$ and $\dot{\eta}_i^j(t)$. So from (27), $\dot{\bar{f}}_i(t)$ is bounded. Therefore, $\ddot{q}_{ri}$ is bounded.

Up to now, we have shown that $\lim_{t\to\infty} s_i(t) = 0$, which means $\lim_{t\to\infty}(q_i(t) - \sum_{j=1}^M m_{ij}C\eta_i^j(t)) = 0$ and $\lim_{t\to\infty}(\dot{q}_i(t) - \sum_{j=1}^M m_{ij}C\dot{\eta}_i^j(t)) = 0$. On the one hand, by Lemma 2, since $\tilde{\eta}_i^j(t) = \eta_i^j(t) - v_j(t)$ converges to a small bounded compact set neighboring zero. Then, we have

$$\begin{aligned}&\lim_{t\to\infty} \|q_i(t) - \sum_{j=1}^{M} m_{ij}q_j(t)\|\\ &= \lim_{t\to\infty} \|q_i(t) - \sum_{j=1}^{M} m_{ij}Cv_j(t)\|\\ &= \lim_{t\to\infty} \|q_i(t) - \sum_{j=1}^{M} m_{ij}(\eta_i^j(t) - \tilde{\eta}_i^j(t))\|\\ &\le \lim_{t\to\infty} \|q_i(t) - \sum_{j=1}^{M} m_{ij}C\eta_i^j(t)\| + \lim_{t\to\infty} \|\sum_{j=1}^{M}\tilde{\eta}_i^j(t)\|\\ &= \lim_{t\to\infty} \|\sum_{j=1}^{M}\tilde{\eta}_i^j(t)\|.\end{aligned} \tag{43}$$

Therefore, there exists a small constant ε_1 such that $\lim_{t\to\infty}\|q_i(t) - \sum_{j=1}^M m_{ij}q_j\| \le \varepsilon_1$. ε_1 can be as small as desired by turning κ_i^j small enough in (27).

On the other hand,

$$\begin{aligned}&\dot{\eta}_i^j(t) - \dot{v}_j(t)\\ &= S\eta_i^j(t)+RK^j\varsigma_i^j(t)+(\gamma_i^j(t)+\rho)R\bar{f}_i^j(t)-(Sv_j(t)+Rr_j(t))\\ &= S\tilde{\eta}_i^j(t)+RK^j\varsigma_i^j(t)+(\gamma_j+\rho)R\bar{f}_i^j(t)-Rr_j(t)+\tilde{\gamma}_i^j(t)R\bar{f}_i^j(t),\end{aligned}$$

since $\tilde{\eta}_i^j(t)$ and $\varsigma_i^j(t)$ converge to a small bounded compact set neighboring zero, $\|\bar{f}_i^j(t)\|$ and $\|r_j(t)\|$ are bounded, and $\tilde{\gamma}_i^j(t)$ decays to zero exponentially, $(\dot{\eta}_i^j(t) - \dot{v}_j(t))$ also converges to a small bounded compact neighboring zero. Similarly,

$$\begin{aligned}
&\lim_{t\to\infty} \|\dot{q}_i(t) - \sum_{j=1}^{M} m_{ij}\dot{q}_j(t)\| \\
&= \lim_{t\to\infty} \|\dot{q}_i(t) - \sum_{j=1}^{M} m_{ij}C(\dot{\eta}_i^j(t) - (\dot{\eta}_i^j(t) - \dot{v}_j(t)))\| \\
&\leq \lim_{t\to\infty} \|\dot{q}_i(t) - \sum_{j=1}^{M} m_{ij}C\dot{\eta}_i^j(t)\| \\
&\quad + \lim_{t\to\infty} \|\sum_{j=1}^{M} m_{ij}C(\dot{\eta}_i^j(t) - \dot{v}_j(t))\| \\
&= \lim_{t\to\infty} \|\sum_{j=1}^{M} m_{ij}C(\dot{\eta}_i^j(t) - \dot{v}_j(t))\|. \qquad (44)
\end{aligned}$$

Therefore, there exists a small constant ε_2 such that $\lim_{t\to\infty} \|\dot{q}_i(t) - \sum_{j=1}^{M} m_{ij}C\dot{v}_j(t)\| \leq \varepsilon_2$. ■

Remark 5: Theorem 1 shows that both the formation error (43) and the velocity error (44) depend on $\lim_{t\to\infty} \eta_i^j(t)$, which depends on the combination of κ_i^j, as Lemma 2 shows. Therefore, the formation error and the velocity error finally converge to small bounded compact sets neighboring zero by setting κ_i^j small enough. And it can be shown that κ_i^j makes $\bar{f}_i^j(t)$ continuous, which guarantees the continuity and boundedness of $\dot{\eta}_i^j(t)$ and $\ddot{\eta}_i^j(t)$.

Remark 6: In our control law (36a) and (36b), agents have the freedom to set values of m_{ij}. In affine formation maneuver control and containment control, the tracking error $\lim_{t\to\infty} \delta_f(t) = \lim_{t\to\infty}(p_f(t)+(L_{ff}^{-1}L_{fl}\otimes I_n)p_l(t)) = 0$, where $p_f(t)$ and $p_l(t)$ are column vectors composed of states of followers and leaders, respectively. Then, by setting $m_{ij} = -(L_{ff}^{-1}L_{fl})_{ij}$, our control law implements the same results as affine formation maneuver control and containment control.

5. AN ILLUSTRATIVE EXAMPLE

In this section, we consider a group of two EL systems, each of which describes a fully actuated mobile robot Cheah et al. (2009), whose motion equation is :

$$M_i\ddot{q}_i(t) + C_i\dot{q}_i(t) = \tau_i(t),\ i = 4, 5$$

where $q_i = [x_i, y_i]^{\mathrm{T}}$ represents its position in the 2D horizontal plane, M_i denotes the mass, C_i is the damping constant, and $\tau_i(t)$ stands for the force applied to it. In our example, M_i and C_i are unknown to every agent, so $\Theta_i = [M_i, C_i]^{\mathrm{T}}$. Their actual values are 1kg and 0.5, respectively. Thus the actual value of Θ_i is $\Theta_{4,5} = [1, 0.5]^{\mathrm{T}}$. By Property 2, since Y_i satisfies (43), we have $Y_i = [\ddot{q}_{ri}, \dot{q}_{ri}]$.

The three leaders are single-integrator agents:

$$\dot{p}_j(t) = v_j(t),\ j = 1, 2, 3$$

with p_j and v_j being the position and velocity, respectively. By Assumption 2, v_j is bounded. In our example, we assume $\|v_j(t)\| \leq 2$. In our example, the leader systems can generate proper trajectories automatically using carried sensors, but it is beyond the scope of our research.

The graph topology of the system can be found in Fig. 1 (a). Parameters in our example are: $\mu = 2$, $\rho = 2$, $c^j = 2$, $\kappa_i^j = 0.001$, $\alpha = 2$, $\Lambda_i = 0.02I_5$, $K_i = 30I_2$, $K_j = \begin{bmatrix} 1 & 0 & 1.7321 & 0 \\ 0 & 1 & 0 & 1.7321 \end{bmatrix}$ for $j = 1, 2, 3$ and $i = 4, 5$. It can be verified that Assumptions 1-5 are satisfied.

In our example, the set of m_{ij} is designed as follows:

Table 1. Values of m_{ij} in different time periods

$t\backslash m_{ij}$	m_{41}	m_{42}	m_{43}	m_{51}	m_{52}	m_{53}
$0 \leq t \leq 77.5s$	-1	1.5	0.5	-1	0.5	1.5
$77.5s < t \leq 107.5s$	1/3	1/3	1/3	2/3	1/6	1/6
$t > 107.5s$	0.5	0.5	0.5	0.5	0	0.5

The change of formation shape at $t = 77.5s$ and $t = 107.5s$ can be seen in Fig. 1 (a).
As illustrated in Fig. 1 (d)-(e), the accuracy for formation and velocity are $10^{-3}m$ and $10^{-3}m/s$, respectively, which has the same order of magnitude as κ_i^j, and it implies the formation error and velocity error converge to small compact sets neighboring zero. Note that the huge change of the formation error in Fig. 1 (b) and the velocity error in Fig. 1 (c) at $t = 77.5s$ and $t = 107.5s$ is because of change of formation shape, and the existence of abrupt velocity error in Fig. 1 (c) at $t = 17.5s$ is due to the sudden change of the leaders' velocity for avoiding obstacles.

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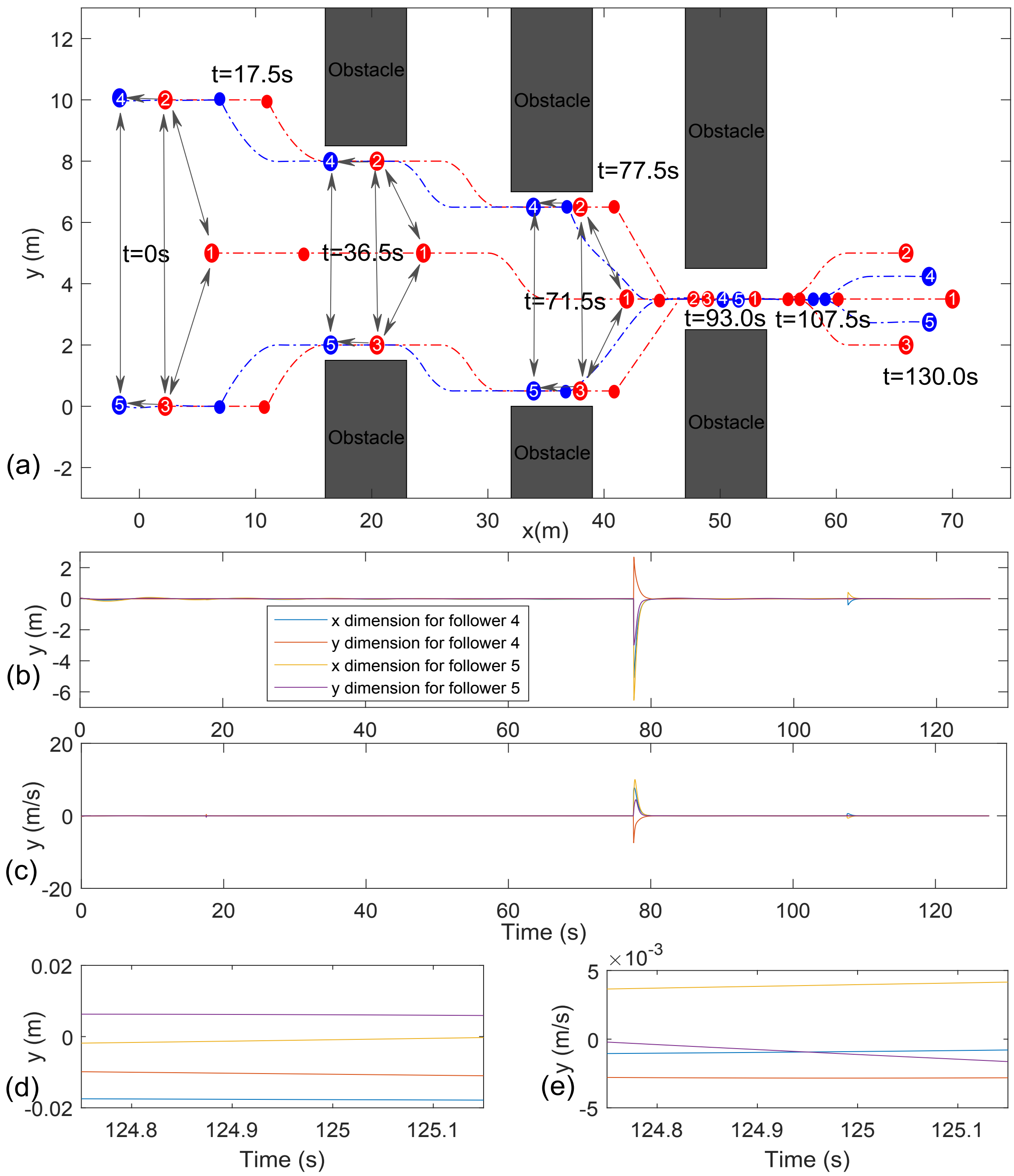

Fig. 1. Trajectories (a), formation error (b), and velocity error (c) of agents. (d) and (e) are formation error and velocity error between $124.75s \leq t \leq 125.15s$. The simulation animation can be found at https://youtu.be/03BygQ8lN10.

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