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Leader-Tracking in a Shape-Preserving Formation With Bearing-Only Measurements

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Abstract: This paper studies the problem of shape-preserving formation control for multi-agent systems where only bearing measurements are available. Most existing methods to solve the problems assume that accurate global or local position measurements of agents are available, or use estimators to get these information without analysis of estimators' effects on the closed loop system stability. In this work, we propose a framework integrating an estimator to get relative positions using bearing-only measurements and an estimator-based controller to achieve tracking a pair of leaders in a formation with a preserved shape. The estimator is designed by exploiting orthogonality to cope with the high non-linearity of bearing measurements, based on which the controller is designed with the estimated relative positions. With rigorous theoretical analysis of the closed-loop system, we characterize the leaders which can be tracked by followers in a shape-preserving formation using bearing-only measurements, and the asymptotic stability of the closed-loop system can be guaranteed. Simulations testify the effectiveness of the proposed framework.

Keywords: Shape-preserving control, integrated estimation and control, leader-follower formation, bearing-only measurements.

1. INTRODUCTION

Due to the potential applications in many fields such as military exercises, rescue and exploration of severe environment and pollutant control, multi-robot coordination has attracted intensive attention of researchers recently (Zhu et al., 2015; Lin et al., 2013; Oh et al., 2015; Han et al., 2015; Yu and Liu, 2016; Yu et al., 2018). In multi-robot coordination problems, a group of robots are generally required to move in a specific geometric configuration to cooperatively accomplish complex tasks. One of the central problems is the formation tracking problem. In this problem, followers are often required to track the leaders and form a formation with the leaders inside, using direct or indirect measurements.

In (Ren and Sorensen, 2008), formation tracking problems for first-order multi-agent systems were studied using consensus-based approach, where the desired trajectory was provided by a virtual leader. Authors in (Brinonarranz et al., 2014) considered nonholonomic agents to track a time-varying reference signal while keeping a time-varying formation. The time-varying formation was achieved by affine transformations to extend a translation control design for circular motions while tracking a time-varying center. However, the reference signal needed to be known by all agents. In (Dong and Hu, 2017) and (Dong et al., 2017), the multiple leaders case was studied, where the states of followers form a predefined time-varying formation while tracking the convex combination of the states of multiple leaders. In (Hua et al., 2018), the authors classified the agents into tracking leaders who generate the translating trajectory, formation leaders who accomplish a time-varying formation configuration, and followers. A formation-containment tracking protocol was then proposed for the followers based on neighboring relative information. All the above works assume that accurate relative positions or positions of the agents are available. This is often not possible, especially in some severe circumstances such as GPS-denied, cluttered or disaster environment.

Compared to position or relative position information, bearing is the minimal requirement on the sensing capability of agents. The problem of bearing-only triangular formation control was considered in (Bishop, 2011), where each agent measured two inter-agent bearings locally to establish and maintain a desired angular separation relative to its neighbours. In (Zhao and Zelazo, 2016) and (Trinh et al., 2019), translational and re-scaled formations were achieved using bearing rigidity theory and the bearing-based Henneberg construction, respectively. The target formation was described by desired relative bearings, such

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that the control laws were proposed to stablize the relative bearings to the desired relative bearings. However, the methods can only guarantee fixed formations. In (Mallik et al., 2016), an approach to rigid polygonal formation which rotates around the target was investigated, but only circular motion along a common circle can be achieved based on cyclic pursuit strategy. The perfect geometry properties of circular motion were utilized to eliminate the requirement on distance measurements. In the severe circumstance however, translational or circular formation is not flexible enough to adapt to the complex environment. A flexible formation should have four degrees of freedom including rotation, rescaling, and translation, with the least requirment on its shape being preserved.

To solve the complex formation control problem using bearing-only measurement, extended Kalman filters were adopted to linearize the nonlinear bearing measurements, which was then applied to formation tracking problem (Han et al., 2015). However, the estimator's effect on the controller is not clear, such that the overall stability is still unknown for the problem. Therefore, whether the problem of time-varying formation with a preserving shape can be solved with only bearing measurements is still not clear, and is quite challenging due to the high non-linearity of bearings. Natural questions arise that what estimator can be designed to apply into the time-varying formation problem, and under what condition the estimation and formation tracking errors both converge to zero asymptotically.

In this work, to cope with the high nonlinearity of the bearing measurement in a time-varying formation problem, we propose novel framework integrating a nonlinear estimator and an estimator-based controller. A leader-follower structure is adopted for the group of agents. The estimator uses bearing-only measurements to estimate the relative positions of leader-follower and inter-followers by fully exploiting orthogonality, and a controller is designed based on estimated relative position to achieve the leader tracking in a time-varying formation while preserving its shape. By theoretically analyzing the estimator properties and their effect on the controller, we find that the bearing localizability is achieved under the persistence of excitation condition on the relative angular velocity, which is actually controlled by the estimator-based controller. The bearing localizability in this work is related to the resulting actual relative positions in the integrated system. By showing the interaction of the estimator and the controller, we characterize sufficient conditions on the leaders' trajectories, under which the estimator can be combined with the controller. Therefore the localizability is satisfied for the estimation error to converge to zero, and the formation tracking error can also asymptotically converge to zero but not a neighborhood of it, as in (Han et al., 2019). Rigorous theoretical analysis of the overall system is given, and effectiveness of the proposed framework is shown via simulations.

The rest of this paper is organized as follows. In section 2, the problem is formulated with models in the complex domain. In sections 3 and 4, estimators and an estimator-based controller for agents are proposed respectively, and the theoretical analysis for the overall system is also given. Simulations testify the effectiveness of the control method in section 4. Conclusion and future work are discussed in section 5.

2. PROBLEM FORMULATION

Consider a group of n agents in the plane, whose positions and velocities are denoted by complex numbers $z_1, \dots, z_n \in \mathbb{C}$ and $v_1, \dots, v_n \in \mathbb{C}$ respectively. For complex numbers, we introduce the following notations. $\iota = \sqrt{-1}$ denotes the imaginary unit. $\mathsf{Re}(z)$ and $\mathsf{Im}(z)$ are the real and imaginary parts of a complex number $z \in \mathbb{C}$ respectively. Besides, its modulus and angle are $|z|$ and $\angle(z)$ respectively. The inner product of two complex numbers $x, y \in \mathbb{C}$ is denoted as $\langle x, y\rangle = \mathsf{Re}(xy^\dagger)$, where $y^\dagger$ is the conjugate of y. Note that the definition of the inner product is different from the standard definition.

2.1 Graph Theory

There are 2 leaders in the group, and others are followers. Suppose that coordinates of all the agents are aligned. Without loss of generality, we label leaders as $\mathcal{V}_l = \{1, 2\}$ and followers as $\mathcal{V}_f = \{3, \dots, n\}$ respectively. A digraph $\mathcal{G}$ of n nodes consists of a non-empty node set $\mathcal{V} = \{1, 2, \dots, n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to represent the sensing graph, where the edge (j, i) indicates that agent i can measure the *bearing* from agent j, namely, $\phi_{ij} = \angle(z_j - z_i)$. Denote $\mathcal{N}_i$ as the in-neighbor set of node i, i.e., $\mathcal{N}_i = \{j|(j, i) \in \mathcal{E}\}$. The complex Laplacian matrix associated to $\mathcal{G}$ is $L \in \mathbb{C}^{n\times n}$ with its (i, j)th off-diagonal entry a complex number $-w_{ij}$ if $j \in \mathcal{N}_i$ and 0 otherwise, and the (i, i)th diagonal entry $\sum_{k\in\mathcal{N}_i} w_{ik}$. It is clear that $L\mathbf{1}_n = 0$.

We assume that the digraph $\mathcal{G}$ is an acyclic graph throughout this paper. Because leaders neither measure the bearings nor receive information from any follower, the leader nodes in digraph $\mathcal{G}$ do not have incoming edges, such that the complex Laplacian of $\mathcal{G}$ takes the following form

$$L = \begin{bmatrix} 0_{2\times 2} & 0_{2\times(n-2)} \\ L_{lf} & L_{ff} \end{bmatrix}, \tag{1}$$

where L_{lf} and L_{ff} are block matrices with appropriate dimensions. They indicates the interaction between leaders and followers, and among followers, respectively.

For clarity of presentation, let the communication graph be the same with the sensing graph. The subsequent analysis can be slightly modified to fit in the case where the sensing and communication graphs are different.

2.2 Shape-preserving Formation Control Problem

Agents $i = 1, \dots, n$ are governed by the single-integrator modeled dynamics:

$$\dot{z}_i = u_i \tag{2}$$

where the control input $u_i = v_i$. Denote $z = [z_1, \dots, z_n]^\top \in \mathbb{C}^n$ as the aggregated position vector of n agents.

To define a target formation shape, let a constant complex vector $\xi = [\xi_1, \dots, \xi_n]^\top \in \mathbb{C}^n$ denote a position assignment of n agents which characterizes the formation shape that agents try to achieve. Then the shape-preserving formation control problem can be defined as follows.

Definition 1. The system of n agents is said to form a shape-preserving formation asymptotically, if there exist time-varying complex variables $c_1(t)$ and $c_2(t)$ such that

$$\lim_{t\to\infty} |z(t) - F_\xi(t)| = 0, \quad (3)$$

with $F_\xi(t) = c_1(t)\mathbf{1}_n + c_2(t)\xi$, which preserves the formation shape described by ξ.

Note that $c_1(t)$ represents the translation of the formation, and the amplitude and modulus of $c_2(t)$ represent the orientation and scaling of the formation respectively. It is worth mentioning that by properly choosing ξ, $c_1(t)$, and $c_2(t)$, we can derive general kinds of time-varying formations, e.g., desired formations in (Dong et al., 2017; Hua et al., 2018; Zhao and Zelazo, 2016; Trinh et al., 2019). Furthermore, for the followers to track a pair of leaders in order to form a shape-preserving formation, the trajectories of leaders should be feasible as $\lim_{t\to\infty} z_l(t) = c_1(t)\mathbf{1}_2 + c_2(t)\xi_l$,where $z_l = [z_1 \ \ z_2]^\top \in \mathbb{C}^2$ and $\xi_l = [\xi_1 \ \xi_2]^\top \in \mathbb{C}^2$. For such the pair of leaders, we assume that their velocities v_i, $i \in \mathcal{V}_l$ are continuously differentiable and bounded.

In this work, we aim to solve the leader-tracking problem where followers only use bearing measurements to track the leaders and achieve a shape-preserving formation. A natural question is then whether the bearing-only measurements are adequate for followers to solve a time-varying formation control problem. Furthermore, we need to answer what the leader trajectory can guarantee the successful tracking in a shape-preserving formation using bearing only measurements.

3. FORMATION CONTROL WITH BEARING-ONLY MEASUREMENTS

In this section, we propose the framework integrating a bearing-based relative estimator and an estimator-based formation controller to solve the aforementioned questions. The estimator is designed to cope with the high nonlinearity of the bearing measurements, and the controller is designed to achieve shape-preserving formation based on the estimation of relative positions. The rigorous theoretical analysis of the overall system will be given.

3.1 Relative Estimator Using Bearing-Only Measurements

Denote the relative position from vehicle i to j as $z_{ij} = z_j - z_i = |z_{ij}|e^{\iota\phi_{ij}}$, where the amplitude of z_{ij}, i.e., $|z_{ij}|$, is the relative distance from vehicle i to j. We assume that there is no collision between any pair $(j, i) \in \mathcal{E}$, i.e., $|z_{ij}(t)| > 0, \forall t > 0$ throughout this paper. This assumption can be dropped by including additional relative distance measurements into a switched control law, which could give a repulsive force between the agents when they are too close to each other. Note that the stability analysis in the rest of this paper is always valid before collision happens.

Denote the complex number with amplitude 1 and argument $\phi_{ij} + \frac{\pi}{2}$ as $\varrho_{ij} = \iota e^{\iota\phi_{ij}}$. By the orthogonal property, we obtain that $\langle z_{ij}(t), \iota e^{\iota\phi_{ij}(t)}\rangle = 0$ always holds for $t > 0$ and any $i, j \in \mathcal{V}$. For each follower i, regarding ϱ_{ij} as the measurement of the relative bearings, the relative position can be estimated only using bearing measurements. Taking the orthogonal property into account, the relative position estimator of any pair of $(j, i) \in \mathcal{E}$ can be designed in a Kalman-like form:

$$\dot{\hat{z}}_{ij} = v_{ij} - \gamma_z \varrho_{ij}\langle\varrho_{ij}, \hat{z}_{ij}\rangle, \quad (4)$$

where $v_{ij} = v_j - v_i$ is the relative velocity, and the real number $\gamma_z > 0$ is the estimator gain. By (4), it is clear that $\tilde{z}_{ij} = \hat{z}_{ij} - z_{ij}$ has its dynamics as

$$\dot{\tilde{z}}_{ij} = -\gamma_z \varrho_{ij}\langle\varrho_{ij}, \tilde{z}_{ij}\rangle. \quad (5)$$

Then we have the following result.

Lemma 1. For each pair $(j, i) \in \mathcal{E}$, denote the steady state of $\tilde{v}_{ij}(t) = \dot{\tilde{z}}_{ij}(t)$ in (5) as $\bar{\tilde{v}}_{ij}$. Then the equilibrium state $\bar{\tilde{v}}_{ij} = 0$ can be reached asymptotically.

Proof. Consider a Lyapunov functional candidate $V'(t) = \frac{1}{2}\langle\tilde{z}_{ij}(t), \tilde{z}_{ij}(t)\rangle \geq 0$. By (5) we have $\dot{V}' = -\gamma_z\langle\tilde{z}_{ij}, \varrho_{ij}\rangle^2 \leq 0$, which implies that

$$\lim_{t\to\infty}\langle\tilde{z}_{ij}(t), \varrho_{ij}\rangle = 0. \quad (6)$$

Due to $|\tilde{z}_{ij}(t)| \leq |\tilde{z}_{ij}(0)|$ for $t > 0$, $|\tilde{z}_{ij}(t)|$ must have a limit. It means $\tilde{z}_{ij}(t)$ may reach either a steady-state solution or a periodic solution. Without loss of generality, assume that $|\tilde{z}_{ij}(t)| \to \mathtt{z}_{ij}$ for some positive constant $\mathtt{z}_{ij}$ as $t \to \infty$. We now prove that $\tilde{z}_{ij}(t)$ cannot be periodic in a steady state by contradiction argument. Assume that $\tilde{z}_{ij}(t)$ is a periodic solution, such as, $\lim\limits_{t\to\infty}\dot{\tilde{z}}_{ij}(t) = \iota\omega_z\tilde{z}_{ij}(t)$ for some $\omega_z > 0$. It follows from (5) that

$$\langle\dot{\tilde{z}}_{ij}(t), \varrho_{ij}\rangle = -\gamma_z\langle\tilde{z}_{ij}(t), \varrho_{ij}\rangle\langle\varrho_{ij}, \varrho_{ij}\rangle = -\gamma_z\langle\tilde{z}_{ij}(t), \varrho_{ij}\rangle. \quad (7)$$

Therefore, together with (6), $\lim\limits_{t\to\infty}\langle\tilde{z}_{ij}(t), \varrho_{ij}\rangle = 0$ and $\lim\limits_{t\to\infty}\omega_z\langle\iota\tilde{z}_{ij}(t), \varrho_{ij}\rangle = 0$ hold at the same time. It thus derives that $\lim\limits_{t\to\infty}\omega_z^2 = 0$, such that $\omega_z \to 0$ as $t \to \infty$, which contradicts with $\omega_z > 0$. Therefore, $\tilde{z}_{ij}(t)$ must converge to a steady-state solution, say $\bar{\tilde{z}}_{ij}$, which satisfies $\bar{\tilde{z}}_{ij} = \mathtt{z}_{ij} \in \mathbb{R}$. It implies that $\lim\limits_{t\to\infty}\tilde{v}_{ij} = 0$.

Besides, we can derive some boundedness result for the subsequent analysis in this paper. We have already known that $\tilde{z}_{ij}$ is bounded, thus by (5) that $\dot{\tilde{z}}_{ij}$ is also bounded. Furthermore, to obtain that $\mathtt{z}_{ij} = 0$, some extra conditions need to hold. We have the following result.

Lemma 2. The relative position estimation error $\tilde{z}_{ij}$, $(j, i) \in \mathcal{E}$ converges to zero asymptotically if the actual relative angular velocity $\omega_{ij} = \dot{\phi}_{ij}$ satisfies

(a) ω_{ij} is bounded and continuously differentiable;
(b) $\dot{\omega}_{ij}$ is bounded; and
(c) there exist $\varepsilon > 0$ and $\delta > 0$ such that

$$\int_t^{t+\delta} |\omega_{ij}(\tau)| d\tau > \varepsilon. \quad (8)$$

holds $\forall t > 0$.

Proof. We follow the proof of Lemma 1 to prove this further result. Denote $\eta_{ij} = \langle\tilde{z}_{ij}(t), \varrho_{ij}\rangle$. Under conditions (a) and (b), $\dot{\eta}_{ij} = -\gamma_z\langle\tilde{z}_{ij}(t), \varrho_{ij}\rangle + \langle\tilde{z}_{ij}(t), \iota\omega_{ij}\varrho_{ij}\rangle$ is uniformly continuous, due to the fact that $\ddot{\eta}_{ij}$ is bounded as ω_{ij}, $\dot{\omega}_{ij}$, $\tilde{z}_{ij}$, and $\dot{\tilde{z}}_{ij}$ are all bounded. Therefore, with Barbalat's Lemma (Khalil, 2002), we obtain by (6) that $\lim_{t\to\infty}\dot{\eta}_{ij} = 0$, such that $\lim_{t\to\infty}\omega_{ij}\langle\tilde{z}_{ij}(t), \iota\varrho_{ij}\rangle = 0$.

Under condition (c), there must exist a time sequence $[t_1, \ldots t_k, \ldots]$ where $t_k \to \infty$ as $k \to \infty$, for which $|\omega_{ij}(t_k)| > \varepsilon/\delta$. One can conclude with (6) that $\tilde{z}_{ij}(t_k)$ converges to be orthogonal to both the unit-modulus complex number ϱ_{ij} and its orthogonal number $\iota\varrho_{ij}$ simultaneously, i.e., $\langle \tilde{z}_{ij}(t_k), \varrho_{ij}(t_k)\rangle \to 0$, $\langle \tilde{z}_{ij}(t_k), \iota\varrho_{ij}(t_k)\rangle \to 0$ as $k \to \infty$, such that $\tilde{z}_{ij}(t_k) \to 0$ as $k \to \infty$. By the continuity of $\tilde{z}_{ij}$ and the decreasing $|\tilde{z}_{ij}(t)|$, we can derive that $\tilde{z}_{ij}(t) \to 0$ as $t \to \infty$.

The bearing localizability can be divided into two parts: the boundedness and continuity (conditions (a) and (b)); and the persistence of excitation (condition (c)). We focus on finding what conditions should be satisfied, under which the relative angular velocity ω_{ij} is still persistently excited in the presence of estimation errors $\tilde{z}_{ij}$.

3.2 Estimator-based Controller

Based on estimated relative positions $\hat{z}_{ij}$ for any pair $(j, i) \in \mathcal{E}$, the controller is proposed for followers to form a preserving-shape formation. The integrated controller for $i \in \mathcal{V}_f$ is proposed as

$$\dot{z}_i = \sum_{j \in \mathcal{N}_i} w_{ij}(v_j + \gamma_v \hat{z}_{ij})/w_{ii}, \tag{9}$$

where $\gamma_v > 0$ is the controller gain, and w_{ij} is the associated weight in the complex Laplacian matrix. The controller, as shown in (9), does not require extra information but only needs neighbor's velocity.

Before presenting the main result, we establish the following preliminary result.

Lemma 3. Assume that $\xi \in \mathbb{C}^n$ satisfies $\xi_i \neq \xi_j$ for $i \neq j$. Then the equilibrium state of $\dot{x} = -Lx$ where $x \in \mathbb{C}^n$ is $\bar{x} = c_1 \mathbf{1}_n + c_2 \xi$ with

$$\begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} 1 & \xi_1 \\ 1 & \xi_2 \end{bmatrix}^{-1} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}, \tag{10}$$

where $\bar{x}_1 = \lim_{t\to\infty} x_1(t)$ and $\bar{x}_2 = \lim_{t\to\infty} x_2(t)$, if and only if $L\xi = 0$ and $\det(L_{ff}) \neq 0$.

A similar result could be found in (Lin et al., 2013). The difference lies in the property that $\bar{x}_i$ for $i \in \mathcal{V}_l$ are no longer constant, such that the configuration c_1 and c_2 are time-varying. The idea of the proof is almost the same and thus omitted due to page limitation.

Notice now that, the controller (9) is proposed based on the estimator (4). We say that the estimator can be integrated with the controller and the asymptotic stability of the overall system can be guaranteed, if the localizability can be satisfied in the leader-tracking formation problem. To answer the question of under what conditions the bearing localizability of the overall system can be satisfied by the controller *in existence of the estimation error*, we must analyze the closed-loop interaction of the estimator and the controller, which is the main challenge of this work. Out of the observation that the pairwise relative position z_{ij} inside a formation is actually controlled by the estimator-based controller (9) and steered by the pair of leaders, we propose the following condition on the trajectories of the leaders.

Condition 1. Let $v_{ll} = v_2 - v_1$ be the relative velocity of the pair of leaders, where $|v_{ll}(t)| > 0$, $\forall t > 0$. There exist $\varepsilon_v > 0$ and $\delta_v > 0$ such that the persistence of excitation condition on the leaders relative velocity holds $\forall t > 0$,

$$\int_t^{t+\delta_v} |\langle \dot{v}_{ll}(\tau), \varrho_{ll}(\tau)\rangle| d\tau > \varepsilon_v, \tag{11}$$

where $\varrho_{ll} = \iota e^{\iota\theta_{ll}}$ with $\theta_{ll} = \angle(v_{ll})$.

Then we present our main result in this work.

Theorem 1. Assume that $L\xi = 0$, $\det(L_{ff}) \neq 0$, and $\xi \in \mathbb{C}^n$ satisfies $\xi_i \neq \xi_j$ for $i \neq j$. Then for a pair of leaders and a group of followers (2) with the estimator (4) and the controller (9), the leaders can be tracked while forming a shape-preserving formation asymptotically, if Condition 1 on the trajectories of leaders holds.

Proof. We firstly give the overall system which consists of the estimator and the controller. By rewriting (9), we can obtain that for $i \in \mathcal{V}_f$,

$$w_{ii}v_i - \sum_{j\in\mathcal{N}_i} w_{ij}v_j = \gamma_v \sum_{j\in\mathcal{N}_i} w_{ij}z_j - \gamma_v w_{ii} z_i + \gamma_v \sum_{j\in\mathcal{N}_i} w_{ij}\tilde{z}_{ij}.$$

We now can write in the aggregated form that $Lv = -\gamma_v Lz + \tilde{z}$, where $v = [v_1, \ldots, v_n] \in \mathbb{C}^n$ is the aggregated velocity vector, and $\tilde{z} \in \mathbb{C}^n$ with its ith element being $\tilde{z}(i) = \gamma_v \sum_{j\in\mathcal{N}_i} w_{ij}\tilde{z}_{ij}$ for $i \in \mathcal{V}_f$ and 0 for $i \in \mathcal{V}_l$. Taking the time derivative of its both sides of we have

$$L\dot{v} = -\gamma_v Lv + \tilde{v}, \tag{12}$$

where $\tilde{v} \in \mathbb{C}^n$ with its ith element being

$$\tilde{v}(i) = \gamma_v \sum_{j\in\mathcal{N}_i} w_{ij}\tilde{v}_{ij} = -\gamma_v\gamma_z \sum_{j\in\mathcal{N}_i} w_{ij}\langle \tilde{z}_{ij}, \varrho_{ij}\rangle \varrho_{ij} \tag{13}$$

for $i \in \mathcal{V}_f$ and 0 for $i \in \mathcal{V}_l$. Then (12) can be written equivalently in the form as $\dot{x} = -\gamma_v x + \tilde{v}$, by denoting $x = Lv$. We firstly prove some boundedness for checking the bearing localizability of the overall system. Its solution

$$x(t) = e^{-\gamma_v t}x(0) + \int_0^t e^{-\gamma_v(t-\tau)}\tilde{v}(\tau)d\tau$$

is bounded if $\tilde{v}(t)$ is always bounded. By (13) we have $\tilde{v}$ being bounded due to the bounded $\tilde{z}_{ij}$ for any pair of $(j, i) \in \mathcal{E}$. Then we can obtain that $x = Lv$ is always bounded. By $v_l = [v_1\ v_2]^\top$ and $v_f = [v_3, \ldots, v_n]^\top$, we can derive equivalently that $L_{lf}v_l + L_{ff}v_f$ is bounded. Due to bounded v_l, $L_{fl}v_l$ is bounded and by the triangular inequality $L_{ff}v_f$ is bounded, such that v_f is bounded. We now can conclude that $v = [v_l^\top\ v_f^\top]^\top$ is bounded, such that for any pair of $i, j \in \mathcal{V}$, v_{ij} is bounded. With bounded $\tilde{v}$ and v, by (12) we can similarly derive that $\dot{v}$ and thus $\dot{v}_{ij}$ are also bounded. Let $d_{ij} = |z_{ij}|$ then $v_{ij} = \dot{z}_{ij} = \dot{d}_{ij}e^{\iota\phi_{ij}} + \iota\omega_{ij}d_{ij}e^{\iota\phi_{ij}}$, yielding ω_{ij} being bounded. Furthermore, taking the time derivative of v_{ij} yields $\dot{v}_{ij} = (\ddot{d}_{ij} - \omega_{ij}^2 d_{ij})e^{\iota\phi_{ij}} + (2\omega_{ij}\dot{d}_{ij} + \dot{\omega}_{ij}d_{ij})\iota e^{\iota\phi_{ij}}$. It implies that $\dot{\omega}_{ij}$ is bounded, such that ω_{ij} is continuously differentiable and uniformly bounded. It can be concluded that the first two conditions in Lemma 2 are satisfied for any pair $(j, i) \in \mathcal{E}$.

We now start to check the persistence of excitation condition of the localizability for integrated system. As shown in Lemma 1, for any pair $(j, i) \in \mathcal{E}$, $\tilde{v}_{ij} \to 0$ as $t \to \infty$. Then each element of $\tilde{v}$ converges to zero as time goes to infinity. We can obtain by (Ryan, 2005) that for $\gamma_v > 0$ there is $x \to 0$, that is, $Lv \to 0$ as $t \to \infty$. By the result in Lemma 3, we derive that

$$\lim_{t\to\infty} v = c_1'(t)\mathbf{1}_n + c_2'(t)\xi, \tag{14}$$

where $c_2'(t) = (v_2(t) - v_1(t))/(\xi_2 - \xi_1) = v_{ll}(t)/(\xi_2 - \xi_1)$. Denote $\xi_{ij} = \xi_j - \xi_i$, then for any $i, j \in \mathcal{V}$ there is

$$\lim_{t\to\infty} v_{ij} = v_{ll}(t)\xi_{ij}/\xi_{12}. \tag{15}$$

In the following it will be proved that the actual relative position satisfies the persistence of excitation condition (8) if condition (11) holds. We prove this using the contradiction argument. Suppose that (8) fails, then it can be proved by continuity that for a uniformly bounded ω_{ij}, $\lim_{t\to\infty} \omega_{ij} = 0$, such that $\lim_{t\to\infty}\langle v_{ij}, \varrho_{ij}\rangle/d_{ij} = 0$. By $|d_{ij}| > 0$, we further obtain that

$$\lim_{t\to\infty} \langle v_{ij}, \varrho_{ij}\rangle = 0. \tag{16}$$

Denote $\zeta_{ij} = \langle v_{ij}, \varrho_{ij}\rangle$, and its first and second order derivatives are

$$\dot{\zeta}_{ij} = \langle \dot{v}_{ij}, \varrho_{ij}\rangle + \omega_{ij}\langle v_{ij}, \iota\varrho_{ij}\rangle \tag{17}$$

$$\ddot{\zeta}_{ij} = \langle \ddot{v}_{ij}, \varrho_{ij}\rangle + \langle 2\dot{v}_{ij}, \iota\omega_{ij}\varrho_{ij}\rangle - \langle v_{ij}, \omega_{ij}^2\varrho_{ij}\rangle \tag{18}$$

respectively. We now prove that $\ddot{\zeta}_{ij}$ is bounded such that ζ_{ij} is uniformly bounded. We have proven that v_{ij}, $\dot{v}_{ij}$ and ω_{ij} are bounded. Taking the time derivative of $\tilde{v}_{ij}$ and by (5) yields

$$\dot{\tilde{v}}_{ij} = -\gamma_z\iota\omega_{ij}\varrho_{ij}\langle\varrho_{ij}, \tilde{z}_{ij}\rangle - \gamma_z\omega_{ij}\varrho_{ij}\langle\iota\varrho_{ij}, \tilde{z}_{ij}\rangle + \gamma_z^2\varrho_{ij}\langle\varrho_{ij}, \tilde{z}_{ij}\rangle,$$

such that $\dot{\tilde{v}}_{ij}$ for any pair $(j, i) \in \mathcal{E}$ is bounded. Therefore $\dot{\tilde{v}}$ is bounded. By taking time derivative of both sides of (12), we obtain that $L\ddot{v} = -\gamma_v L\dot{v} + \dot{\tilde{v}}$. Following the same idea of proving the boundedness of $\dot{v}_{ij}$ as aforementioned, we can similarly derive that $\ddot{v}_{ij}$ is also bounded. Now we can conclude that $\ddot{\zeta}_{ij}$ is bounded, such that $\dot{\zeta}_{ij}$ is uniformly bounded. Therefore with (16), by Barbalat's Lemma again, $\lim_{t\to\infty} \dot{\zeta}_{ij} = 0$. Since $\lim_{t\to\infty}\omega_{ij} = 0$, it follows from (17) that

$$\lim_{t\to\infty} \langle \dot{v}_{ij}, \varrho_{ij}\rangle = 0. \tag{19}$$

Recall (12) and (14), we can derive that $\lim_{t\to\infty} L\dot{v} = 0$ as $\lim_{t\to\infty}\tilde{v} = 0$, such that $\lim_{t\to\infty}\dot{v} = c_1''\mathbf{1}_n + c_2''\xi$, with $c_2'' = \dot{v}_{ll}/\xi_{12}$. It yields that

$$\lim_{t\to\infty} \dot{v}_{ij} = \dot{v}_{ll}(t)\xi_{ij}/\xi_{12}. \tag{20}$$

We now can consider (15), (16), (19), and (20) to derive the following equations which are satisfied at the same time:

$$\lim_{t\to\infty} \langle v_{ll}(t), \varrho_{ij}\rangle = 0, \ \lim_{t\to\infty} \langle \dot{v}_{ll}(t), \varrho_{ij}\rangle = 0, \tag{21}$$

for any pair $(j, i) \in \mathcal{E}$, where ξ_{ij}/ξ_{12} is constant thus we can neglect this term in above equations. Under Condition 1, we now obtain the orthogonal decomposition of $\dot{v}_{ll}$ onto the orthogonal basis $\{v_{ll}, \iota v_{ll}\}$ as $\dot{v}_{ll} = \langle\dot{v}_{ll}, v_l\rangle v_{ll}/|v_{ll}| + \langle\dot{v}_{ll}, \iota v_{ll}\rangle\iota v_{ll}/|v_{ll}|$. Substituting it into the second equation in (21) and combine it with the first equation, we can easily derive $\lim_{t\to\infty}\langle\dot{v}_{ll}, \iota v_{ll}\rangle\langle\iota v_{ll}, \varrho_{ij}\rangle/|v_{ll}| = 0$. Consider this result together with (21) again, one has

$$\lim_{t\to\infty} \langle\dot{v}_{ll}, \iota v_{ll}\rangle/|v_{ll}| = \lim_{t\to\infty}\langle\dot{v}_{ll}, \varrho_{ll}\rangle = 0, \tag{22}$$

which contradicts with the sufficient condition (11). Then we can draw the conclusion that under the condition (11), the uniformly bounded ω_{ij} for any pair $(j, i) \in \mathcal{E}$ must satisfy condition (8) in Lemma 2.

Up to this point, it is proved that all the three conditions in Lemma 2 are satisfied. We can derive that for any pair $(j, i) \in \mathcal{E}$, $\lim_{t\to\infty}\tilde{z}_{ij} = 0$, which means $\tilde{z} \to 0$ as $t \to \infty$. Then $Lz \to 0$, such that $\lim_{t\to\infty} z = c_1(t)\mathbf{1}_n + c_2(t)\xi$ and a time-varying formation with a preserving-shape is achieved by the group of agents.

Remark 1. Note that Condition 1 is rather mild. Leaders do not need to keep $|\langle\dot{v}_{ll}(t), \varrho_{ll}(t)\rangle|$ always greater than zero or as a constant. It provides flexibility for the leaders in obstacle-cluttered environments.

4. NUMERICAL EXAMPLES

In this section we testify the effectiveness of our method with numerical examples. We consider 9 agents in the simulations. The desired formation shape is described by the target configuration ξ which is illustrated by Fig.1(a). The topology of the communication and sensing network

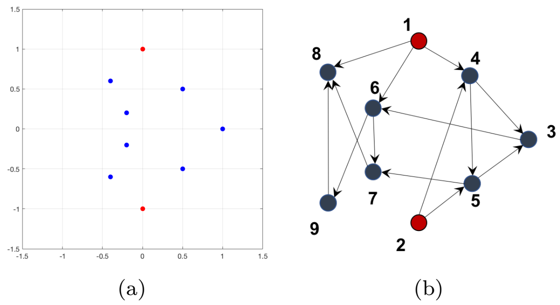


Fig. 1. Formation shape ξ and digraph $\mathcal{G}$.

is shown by Fig.1(b). A feasible way to choose the complex weights w_{ij} and its influence on the convergence speed can be found in (Lin et al., 2015). Select the initial parameters and positions of followers randomly.

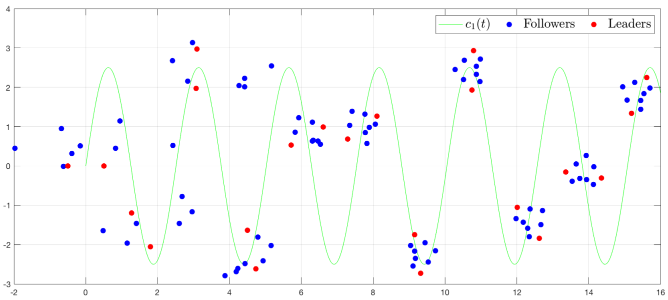
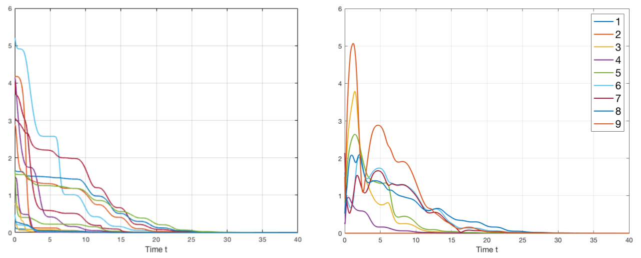


Fig. 2. Snapshots of agent positions.

To enable the followers to track the leaders in a shape-preserving formation, the leaders' trajectories are pairwisely designed to control the translation and rotation of the formation. In this simulation we let the desired scale of the formation, which is controlled by leaders, be invariant. It is desired that the formation could asymptotically translate at a sin wave, while the orientation of the formation is varying with the curve. Therefore, we let $c_1(t) = 0.5t + \iota\sin t$, and the orientation be $\angle(c_2(t)) = \frac{\pi}{4}(1 + \sin t)$. Then the trajectories of the leaders are $z_i(t) = c_1(t) + 0.5e^{\iota\frac{\pi}{4}(1+\sin t)}\xi_i$ for $i = 1, 2$. The velocities $v_i(t)$, i.e., the control input of the leaders are derived by taking time derivative of $z_i(t)$ for $i = 1, 2$. Snapshots of the leaders positions at time $t = 0, 3.85, 7.7, 11.55, 15.4, 19.25, 23.1, 26.94, 30.8, 34.65$, and 38.5 are shown by red in Fig.2. Set the gains of



(a) (b)

Fig. 3. Evolution of $|\tilde{z}_{ij}(t)|$ for $(j, i) \in \mathcal{E}$ and $|z_i(t) - (c_1(t)\mathbf{1}_n + c_2(t)\xi_i)|$ for $i \in \mathcal{V}$.

the estimator and controller as $\gamma_z = 4$ and $\gamma_v = 5$ respectively. Then the snapshots of the follower positions are shown by blue in Fig.2. It can be observed that the followers could asymptotically track the leaders in a shape-preserving formation where the desired shape is described by Fig.1(a). Fig.3(a) shows the estimation error $\tilde{z}_{ij}$ for each pair $(j, i) \in \mathcal{E}$. The formation tracking error could converge to zero with estimator error, as shown in Fig.3(b). To see that Condition 1 is satisfied, we show it in Fig.4.

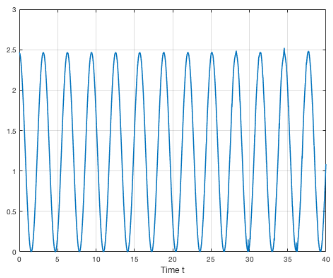


Fig. 4. Evolution of $|\langle \dot{v}_{ll}, \varrho_{ll} \rangle|$ in Eq. (11).

Since the leaders are modeled with single-integrator, $\dot{v}_{ll}$ are approximated by differentiating the control inputs of the leaders. It is observed that the persistence of excitation condition is always satisfied.

5. CONCLUSION

In this paper, we studied the problem of tracking a pair of leaders in a shape-preserving formation with bearing-only measurements. To cope with the nonlinearity of the bearing measurements, we develop an estimator to derive the relative position and a controller with the estimated relative position to form a time-varying formation. To enable the stability of the overall system, the leaders' relative velocity must satisfy the persistence of excitation condition. Then the equilibrium of the overall system can be reached asymptotically. In the future, more general topology of the leader-follower network and more general models of the agents will be considered.

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