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Control Lyapunov Function Design for Trajectory Tracking Problems of Wheeled Mobile Robot

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Abstract: Trajectory tracking is an important control problem that has been studied by many researchers. However, no studies have discussed the trajectory tracking problem for a wheeled mobile robot via the minimum projection method. This paper proposes a Tracking Control Lyapunov Function (TCLF) design that uses dynamic extension and the minimum projection method. The proposed method converges the two-wheeled mobile robot to a time-varying target state. Moreover, the effectiveness of the proposed method is validated through a computer simulation.

Keywords: Nonlinear Control, Control Lyapunov Function, Minimum Projection Method.

1. INTRODUCTION

Various airports are trying to roll out autonomous wheeled mobile robots such as security, cleaning, shipping, etc in recent years for reducing staff workload (SITA Inc. (2018) and SITA Inc. (2019)). Such robots motivate the need to design a tracking controller that moves the robots into the desired trajectory; the trajectory tracking is an important control problem.

Many researchers have proposed various trajectory tracking methods (Wang et al. (2015), Park et al. (2010), Pedro et al. (2007), Rosolia et al. (2017), Li et al. (2001), Ashrafiuon et al. (2017), Wu et al. (2019), Jin (2018)). For example, Pedro et al. (2007) proposed a tracking control Lyapunov function (TCLF) design from an error system by using backstepping. Li et al. (2001) proposed that trajectory tracking could be applied to passive velocity field control (PVFC) methods, which move the robot by using velocity fields. Jin (2018) proposed an iterative learning control (ILC) algorithm.

Recently, Kuga et al. (2016) proposed a static smooth control Lyapunov function (CLF) design method that uses dynamic extension and the minimum projection method. However, the applications of the proposed method for the trajectory tracking problem have not been discussed.

In this paper, we propose a control Lyapunov function design that can be applied to the trajectory tracking problem of the two-wheeled robot using dynamic extension and the minimum projection method. Moreover, we show the effectiveness of the proposed method by computer simulation.

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2. PRELIMINARIES

In this section, we introduce basic definitions of mathematical terms and fundamental properties used in the paper.

2.1 Nonlinear Control System

In this paper, we consider an input affine nonlinear control system (Kuga et al. (2016)) defined as follows:

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $x \in \mathbb{R}^n$ is a state and $u \in \mathbb{R}^m$ is an input. Mappings $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are supposed to be locally Lipschitz continuous with respect to both x and u.

2.2 Differentially Flat System

In this paper, we consider a CLF design problem for a differentially flat system. In accordance with Fliess et al. (1994), we introduce a differentially flat system defined as follows.

Definition 1. Consider (1) and the following dynamic compensator:

$$\dot{p} = a(x, p, v), \tag{2}$$

where $p \in \mathbb{R}^l$ and $v \in \mathbb{R}^m$ denote a state and an input, respectively. Moreover, we introduce the following dynamic state feedback:

$$u = b(x, p, v). \tag{3}$$

With dynamic compensation (2) and dynamic state feedback (3), we consider the following augmented system of (1) on extended state space $\mathbb{R}^{n+l}$:

$$\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} f(x) + g(x)b(x,p,v) \\ a(x,p,v) \end{bmatrix}, \tag{4}$$

where the origin is $(x,p) = (0,0)$. Moreover, we assume that there exists the following diffeomorphism $\Phi : \mathbb{R}^{n+l} \to \mathbb{R}^{n+l}$:

$$\phi = \Phi(x,p), \tag{5}$$

such that Φ transforms (4) into the following linear system:

$$\dot{\phi} = A\phi + Bv, \tag{6}$$

where the matrix (A, B) is controllable. Then, system (4) is said to be a differentially flat system.

2.3 Control Lyapunov Function Design

Definition 2. (Control Lyapunov Function). A C^1 proper function $V : \mathbb{R}^n \to \mathbb{R}_{>0}$ is said to be a control Lyapunov function (CLF) for (1) such that the following condition hold (Artstein (1983)).

(A1) $\bar{V}(x)$ is a proper function: for all $L > 0$, $\{x \in \mathbb{R}^n | \bar{V}(x) \le L\}$ is a compact set.
(A2) $\bar{V}(x)$ is a positive-definite function: there exists $\bar{V}(0) = 0$ and $\bar{V}(x) > 0$ with respect to all $x \in \mathbb{R}\backslash\{0\}$.
(A3) $\bar{V}(x)$ satisfies the following inequality:

$$\dot{\bar{V}} = L_f\bar{V} + L_g\bar{V} \cdot u < 0, \quad \forall x \in \mathbb{R}^n\backslash\{0\}. \tag{7}$$

This implies that $L_f\bar{V} < 0$ if $L_g\bar{V} = 0$ and $x \ne 0$, where $L_f\bar{V}$ and $L_g\bar{V}$ are Lie-derivatives defined as follows:

$$L_f\bar{V} = \frac{\partial \bar{V}}{\partial x} f(x) \quad L_g\bar{V} = \frac{\partial \bar{V}}{\partial x} g(x) \tag{8}$$

A CLF with respect to (6) can be easily designed by the following proposition.

Proposition 1. Suppose that there exists a diffeomorphism $\phi = \Phi(x)$ that transforms the system (1) into

$$\dot{\phi} = A\phi(x) + Bv(x), \tag{9}$$

where matrix A and B are controllable. Then, the function $\bar{V}(x) : \mathbb{R}^n \to \mathbb{R}$, defined by the following function, is a CLF for (9), as follows:

$$V(\phi) = \phi^T P \phi, \tag{10}$$

where P is a symmetric positive matrix.

2.4 Minimum Projection Method for CLF Design via Dynamic Extension

Yamazaki et al. (2011) proposed a static CLF design method via the minimum projection method. A CLF for augmented nonlinear control system (1) is generated from a CLF for a linear control system (6), as shown in the following theorem. In this paper, state space $X, \bar{X}$ is defined in the neighborhood of the origin.

Theorem 1. Let a continuous function $\bar{V} : \mathbb{R}^{n+l} \supset \bar{X} \to \mathbb{R}$ be a CLF for (6). Then, a function $V : \mathbb{R}^{n+l} \supset X \to \mathbb{R}$ defined by the following equation is a CLF for (1):

$$V(x) = \min_p \bar{V}(x,p). \tag{11}$$

2.5 Tracking Control Lyapunov Function

In this paper, we consider a trajectory tracking problem. A smooth time varying feedback $u = k(x,t)$ is usually considered. The tracking control Lyapunov function (TCLF), defined as follows, has been developed to design a smooth time-varying controller for trajectory tracking .

Definition 3. (Tracking Control Lyapunov Function (Nakamura (2016))) Consider system (1) and an admissible reference state $x_r(t) : [0, +\infty) \to \mathbb{R}^n$; i.e., there exists $u_r(t) \in \mathbb{R}^m$ such that $f(x_r(t)) + g(x_r(t))u_r(t) - \dot{x}_r(t) = 0$ for all $t \in \mathbb{R}_{\ge 0}$, where u_r is a reference input. A TCLF for system is a C^1 differentiable function $V(x,t) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ for stabilization of $x_r(t)$ such that the following conditions hold.

(C1) $V(x,t)$ is a proper function: for any $L > 0$, $\{x \in \mathbb{R}^n | V(x,t) \le L\}$ is a compact set.
(C2) $V(x,t)$ is a positive-definite function: there exists $V(x_r(t),t) = 0$ and $V(x,t) > 0$ with respect to all $x - x_r(t) \in \mathbb{R}\backslash\{0\}$.
(C3) $V(x,t)$ satisfies the following inequality.

$$\dot{V} = \frac{\partial V}{\partial t} + L_fV + L_gV \cdot \tilde{u} < 0, \qquad \forall (x - x_r(t)) \in \mathbb{R}^n\backslash\{0\}. \tag{12}$$

This implies that $L_fV < 0$ if $L_gV = 0$ and $x \ne x_r(t)$, where L_fV and L_gV are Lie-derivatives defined as follows:

$$L_fV = \frac{\partial V}{\partial x}\left(f(x) - g(x)u_r(t)\right) \; L_gV = \frac{\partial V}{\partial x} g(x). \tag{13}$$

2.6 Sontag type controller (Krstić et al. (1998))

Proposition 2. Let V a TCLF for system (1). Then, the following input $\tilde{u} = u - u_r \in \mathbb{R}^m$ asymptotically tracks the origin of the system (1):

$$\tilde{u}(x,t) = \begin{cases} -\dfrac{a + \sqrt{a^2 + \|b\|^4}}{\|b\|^2} b^T & (b \ne 0) \\ 0 & (b = 0), \end{cases} \tag{14}$$

where $a = \partial V / \partial t + L_fV$ and $b = L_gV$.

3. PROBLEM STATEMENT

In this paper, we consider a two-wheeled mobile robot illustrated in Fig. 1. The robot can be modeled by the following equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \cos x_3 & 0 \\ \sin x_3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{15}$$

where $x = [x_1, x_2, x_3] \in \mathbb{R}^3$ is a state, and $u = [u_1, u_2] \in \mathbb{R}^2$ is an input. Moreover, we consider the reference state $x_r(t) \in \mathbb{R}^3$ and the reference input $u_r(t) \in \mathbb{R}^2$, as follows:

Assumption 1. The reference state $x_r(t)$ and the reference input $u_r(t)$ are known and always satisfy the following equation:

$$\begin{bmatrix} \dot{x}_{1r}(t) \\ \dot{x}_{2r}(t) \\ \dot{x}_{3r}(t) \end{bmatrix} = \begin{bmatrix} \cos x_{3r}(t) & 0 \\ \sin x_{3r}(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1r}(t) \\ u_{2r}(t) \end{bmatrix}, \tag{16}$$

where $x_r(t)$ is a class C^2.

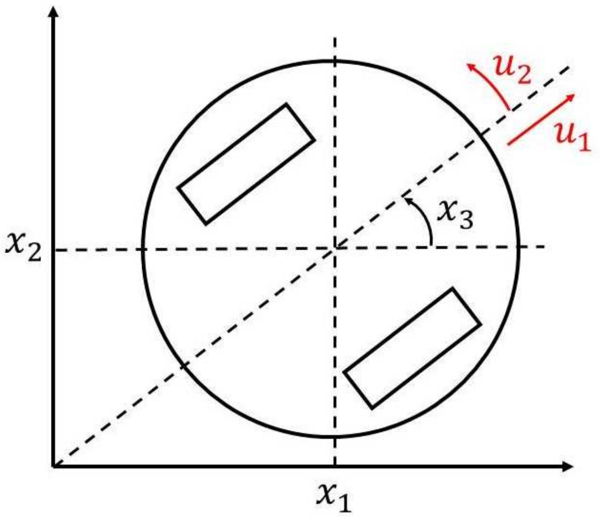

Fig. 1. Two-wheeled mobile robot system

In this paper, our objective is to design a TCLF using dynamic extension and the minimum projection method. Moreover, we design a controller with a TCLF of the robot. This TCLF makes the robot track to $x_r(t)$.

4. TCLF DESIGN VIA DYNAMIC EXTENSION AND THE MINIMUM PROJECTION METHOD

Kuga et al. (2016) proposed a time invariant CLF design method for differentially flat systems by the minimum projection method. However, the method can design only a time invariant CLF.

In this section, we design a time varying CLF by using dynamic extension and the minimum projection method.

4.1 Linearization of error system via dynamic extension

We design a tracking error system and transform the error system to a linear control system by dynamic extension.

We consider the following error system:

$$\dot{e} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} u_1 \cos x_3 - \dot{x}_{1r}(t) \\ u_1 \sin x_3 - \dot{x}_{2r}(t) \\ u_2 - \dot{x}_{3r}(t) \end{bmatrix}. \tag{17}$$

where $e = x - x_r(t)$ is a state error. Then, we consider the virtual state ϕ and input v as follows:

$$\begin{aligned}\phi(x,t,u_1) = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} &= \begin{bmatrix} x_1 - x_{1r}(t) \\ u_1 \cos x_3 - \dot{x}_{1r}(t) \\ x_2 - x_{2r}(t) \\ u_1 \sin x_3 - \dot{x}_{2r}(t) \end{bmatrix} \\ &= \alpha(x)u_1 + \beta(x,t),\end{aligned} \tag{18}$$

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos x_3 & -u_1 \sin x_3 \\ \sin x_3 & u_1 \cos x_3 \end{bmatrix} \begin{bmatrix} \dot{u_1} \\ u_2 \end{bmatrix} + \begin{bmatrix} -\ddot{x}_{1r} \\ -\ddot{x}_{2r} \end{bmatrix} \tag{19}$$

where, $\phi = [e_1, \dot{\phi}_1, e_2, \dot{\phi}_3]^T$ and $v = [\dot{\phi}_2, \dot{\phi}_4]$. System (17) can transform into the following linear control system:

$$\dot{\Phi} = \begin{bmatrix} 0&1&0&0 \\ 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \end{bmatrix} \Phi + \begin{bmatrix} 0&0 \\ 1&0 \\ 0&0 \\ 0&1 \end{bmatrix} v. \tag{20}$$

4.2 Dynamic TCLF design

We design a dynamic TCLF from (20).

We consider the following symmetric positive matrix P:

$$P = \begin{bmatrix} A & O \\ O & A \end{bmatrix}, \tag{21}$$

where A and O denote the following matrix:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix}, \quad O = \begin{bmatrix} 0&0 \\ 0&0 \end{bmatrix}, \tag{22}$$

where A_{11},A_{12} and A_{22} satisfy the following condition:

$$A_{11}A_{22} - A_{12}^2 > 0 \quad A_{11}, A_{22} > 0. \tag{23}$$

Thus, we can design the dynamic CLF $\bar{V}(\phi)$ as follows:

$$\begin{aligned}\bar{V}(\phi) &= \phi^T P \phi \\ &= A_{11}\phi_1^2 + A_{22}\phi_2^2 + A_{11}\phi_3^2 + A_{22}\phi_4^2 \\ &\quad +2A_{12}\phi_1\phi_2 + 2A_{12}\phi_3\phi_4,\end{aligned} \tag{24}$$

where (24) satisfies the following proposition.

Proposition 3. The function $\bar{V}(\phi)$ satisfying the following conditions is a TCLF.

$$A_{11}A_{22} - A_{12}^2 > 0 \quad A_{11}, A_{12}, A_{22} > 0. \tag{25}$$

Proof. Since P is a positive definite symmetric matrix, A_{11},A_{12} and A_{22} satisfy the following condition:

$$A_{11}A_{22} - A_{12}^2 > 0 \quad A_{11}, A_{22} > 0.$$

Then, we consider dynamic CLF (24) whether it satisfies the Definition 2. Firstly, $\bar{V} \to \infty$ with respect to $||\phi|| \to \infty$; this implies $\bar{V}$ satisfies condition (A1). Secondly, ϕ is a quadratic function; hence $\bar{V}$ satisfies (A2). Finally, $\bar{V}$ satisfies (A3) on condition that $A_{12} > 0$ because $L_f\bar{V}$ and $L_g\bar{V}$ have to satisfy the following equation:

If $L_g\bar{V} = 0$, then

$$\begin{aligned}L_g\bar{V} &= [A_{12}\phi_1 + A_{22}\phi_2 \;\; A_{12}\phi_3 + A_{22}\phi_4] \\ &= 0, \\ L_f\bar{V} &= 2(A_{11}\phi_1\phi_2 + A_{12}\phi_2^2 + A_{11}\phi_3\phi_4 + A_{12}\phi_4^2) \\ &= -2\left[\left(\frac{A_{11}A_{22} - A_{12}^2}{A_{12}}\right)\phi_2^2 + \left(\frac{A_{11}A_{22} - A_{12}^2}{A_{12}}\right)\phi_4^2\right] \\ &< 0.\end{aligned}$$

Therefore, the function $\bar{V}(\phi)$ is a TCLF. □

Moreover, the dynamic TCLF can be designed as follows from (18):

$$\begin{aligned}\bar{V}(\phi) &= \bar{V}(x,t,u_1) \\ &= A_{11}\left(x_1 - x_{1r}(t)\right)^2 + A_{22}\left(u_1 \cos x_3 - \dot{x}_{1r}(t)\right)^2 \\ &\quad + A_{11}\left(x_2 - x_{2r}(t)\right)^2 + A_{22}\left(u_1 \sin x_3 - \dot{x}_{2r}(t)\right)^2 \\ &\quad + 2A_{12}\left(x_1 - x_{1r}(t)\right)\left(u_1 \cos x_3 - \dot{x}_{1r}(t)\right) \\ &\quad + 2A_{12}\left(x_2 - x_{2r}(t)\right)\left(u_1 \sin x_3 - \dot{x}_{2r}(t)\right).\end{aligned} \tag{26}$$

4.3 TCLF design via minimum projection method

We apply the minimum projection method to the dynamic TCLF (26) and design a TCLF for system (15).

Theorem 2. Consider system (15), reference state x_r and reference input u_r satisfying (16). Suppose $\bar{V}(\phi)$ is a TCLF of (20). Then, function $V(x,t)$ defined by the following equation is a dynamic TCLF for system (15):

$$\begin{aligned} V(x,t) &= \min_{u_1} \bar{V}(\phi) \\ &= \min_{u_1} \bar{V}(x,t,u_1) \quad (27) \\ &= \bar{V}(x,t,z(x,t)), \quad (28) \end{aligned}$$

where $z(x,t)$ is an argument such that $\bar{V}(x,t,z(x,t)) = \min \bar{V}(x,t,u_1)$. Moreover, the following relation holds.

$$\frac{\partial \bar{V}}{\partial u_1} = 0, \quad \forall u_1 \in \{z(x,t) | x \in \mathbb{R}^3, t \in \mathbb{R}\}. \tag{29}$$

For the proof of Theorem 2, we prepare the following six lemmas.

Lemma 1. (29) is equivalent to the following equation:

$$G(x)u_1 = H(x,t), \tag{30}$$

where $G(x) = 2\alpha(x)^T P\alpha(x)$ and $H(x,t) = -2\alpha(x)^T P\beta(x,t)$.

Proof. (29) can be calculated from (18) as follows:

$$\begin{aligned} \frac{\partial \bar{V}}{\partial u_1}(x,t,u_1) &= 2\Phi^T P \frac{\partial \bar{\Phi}}{\partial u_1} \\ &= 2\Phi^T P\alpha(x) \\ &= 2\alpha^T(x)P(\alpha(x)u_1 + \beta(x,t)) \\ &= 2\alpha^T(x)P\alpha(x)u_1 + 2\alpha(x)^T P\beta(x,t). \end{aligned}$$

Then, the following implication holds.

$$\frac{\partial \bar{V}}{\partial u_1}(x,t,u_1) = 0 \Longleftrightarrow G(x)u_1 = H(x,t). \tag{31}$$

□

Lemma 2. $G(x)$ is regular.

Proof. $G(x)$ is a quadratic function with respect to $\alpha(x)$. Moreover, P is symmetric and positive definite. Thus, $G(x)$ is a regular matrix. □

Lemma 3. $z(x,t)$ for any $x \in \mathbb{R}$ defined by (29) is determined uniquely. Moreover, $z(x,t)$ is an argument such that $\bar{V}(x,t,z(x,t)) = \min \bar{V}(x,t,u_1)$.

Proof. By Lemma 2, $G(x)$ is regular for an arbitrary x. Thus, (30) can transform the following equation:

$$z(x,t) = u_1 = G^{-1}(x)H(x,t). \tag{32}$$

Hence, $z(x,t)$ is an extreme value satisfying (29). Moreover, $\bar{V}(x,t,u_1)$ is a proper and bounded below function. Then, $z(x,t)$ minimizes $\bar{V}(x,t,u_1)$ with respect to u_1 and is determined uniquely. □

Lemma 4. $V(x,t)$ is a smooth function.

Proof. Consider the following equation F using (30):

$$F(x,t,u_1) = G(x)u_1 - H(x,t). \tag{33}$$

The partial derivation of F with respect to u_1 can be calculated as follows:

$$\frac{\partial F}{\partial u_1} = G(x) \tag{34}$$

Note that $\partial F/\partial u_1$ is regular by Lemma 2. Therefore, by the implicit function theorem, there exists a smooth mapping $u_1(x,t)$ such that the following equation holds:

$$F(x,t,u_1(x,t)) = 0 \tag{35}$$

Therefore, $V(x,t)$ is a smooth function. □

Lemma 5. The following inequality holds with respect to $\dot{V}$:

$$\dot{V}(x,t) \le \dot{\bar{V}}(x,t,u_1). \tag{36}$$

Proof. The following equation holds by definition of the function V:

$$V(x,t) = \min_{u_1} \bar{V}(x,t,u_1),$$
$$\forall z(x,t) \in \operatorname{argmin} \bar{V}(x,t,u_1).$$

Therefore, the following relationship holds:

$$\begin{aligned} V(x(t),t) &= \min_{u_1} \bar{V}(x(t),t,u_1(t)) \\ &= \bar{V}(x(t),t,z(x(t),t)). \quad (37) \end{aligned}$$

Let $z(x(t_0),t_0) = u_1(t_0)$ in any fixed time $t_0 \in \mathbb{R}$. As $\bar{V}$ is the TCLF, the following inequality holds:

$$\begin{aligned} V(x(t_1),t_1) &= \bar{V}(x(t_1),t_1,z(x(t_1),t_1)) \\ &\le \bar{V}(x(t_1),t_1,u_1(t_1)), \quad (38) \end{aligned}$$

where $t_1 = t_0 + \Delta t$. Thus, the following inequality holds with respect to $\dot{\bar{V}}$:

$$\begin{aligned} &\dot{V}(x(t),t) \\ &= \lim_{\Delta t \to 0} \frac{V(x(t_1),t_1) - V(x(t_0),t_0)}{\Delta t} \\ &\le \lim_{\Delta t \to 0} \frac{\bar{V}(x(t_1),t_1,u_1(t_1)) - \bar{V}(x(t_0),t_0,u_1(t_0))}{\Delta t} \\ &= \dot{\bar{V}}(x(t_0),t_0,u_1(t_0)). \end{aligned}$$

Therefore, the following inequality holds:

$$\dot{V}(x,t) \le \dot{\bar{V}}(x,t,u_1). \tag{39}$$

□

Remark 1. Even if $z(x(t_0),t_0) = u_1(t_0)$, $z(x(t_1),t_1) = u_1(t_1)$ does not hold in general. Thus, inequality of (38) is required.

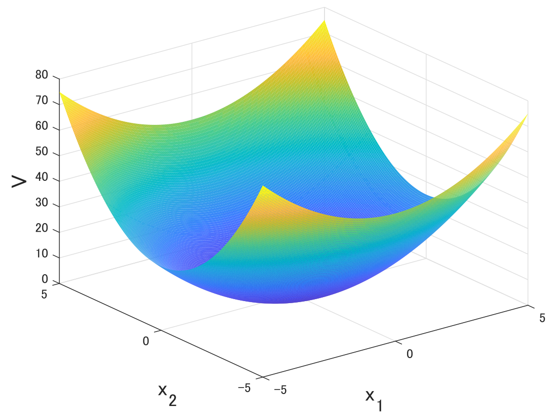

Fig. 2. Tracking Control Lyapunov Function: $x_1 - x_2$

Lemma 6. The time derivative of a TCLF V satisfies the following inequality:

$$\dot{V} < 0. \tag{40}$$

Proof. The function $\bar{V}$ is a dynamic TCLF. Therefore, the following equation holds:

$$\dot{\bar{V}} < 0. \tag{41}$$

Thus, $\dot{V}$ satisfies the following inequality by Lemma 5:

$$\dot{V} \leq \dot{\bar{V}} < 0. \tag{42}$$

□

Therefore, Theorem 2 is proven by using the above lemmas.

Proof. A function V defined by (27) can be determined uniquely by (29). Moreover, V is a TCLF for (15). □

Now, we can design a TCLF $V(x,t)$ for the system (15) using Theorem 2 as follows:

$$\begin{aligned} V(x,t) = &\left(A_{11} - \frac{A_{12}^2}{A_{22}}\right)(x_1 - x_{1r}(t))^2 \\ &+\left(A_{11} - \frac{A_{12}^2}{A_{22}}\right)(x_2 - x_{2r}(t))^2 \\ &+\frac{1}{A_{22}}\{[A_{22}\dot{x}_{1r}(t) - A_{12}(x_1 - x_{1r}(t))]\sin x_3 \\ &-[A_{22}\dot{x}_{2r}(t) - A_{12}(x_2 - x_{2r}(t))]\cos x_3\}^2, \end{aligned} \tag{43}$$

where $z(x,t)$ satisfies the following equation:

$$\begin{aligned} z(x,t) = &\frac{1}{A_{22}}[(A_{22}\dot{x}_{1r}(t) - A_{12}(x_1 - x_{1r}(t)))\cos x_3 \\ &+(A_{22}\dot{x}_{2r}(t) - A_{12}(x_2 - x_{2r}(t)))\sin x_3]. \end{aligned} \tag{44}$$

We illustrate a TCLF as follows.

Figure 2 shows the TCLF (43) with respect to x_1 and x_2, where $x_3 = 0$, $x_r = 0$, $A_{11} = 2$, $A_{12} = 1$ and $A_{22} = 1$. Figure 3 shows the TCLF (43) with respect to x_1 and x_3, where $x_2 = 0$, $x_r = 0$, $A_{11} = 2$, $A_{12} = 1$ and $A_{22} = 1$.

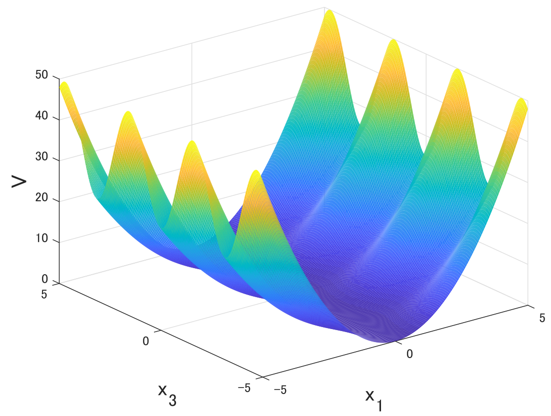

Fig. 3. Tracking Control Lyapunov Function: $x_1 - x_3$

5. CONTROLLER DESIGN

In this section, we design a trajectory tracking controller using the obtained TCLF.

Let $\tilde{u} = [\tilde{u}_1, \tilde{u}_2]^T = [u_1 - u_{1r}(t), u_2 - u_{2r}(t)]^T$; thus, we transform the system (15) into the following system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} u_{1r}(t)\cos x_3 \\ u_{1r}(t)\sin x_3 \\ u_{2r}(t) \end{bmatrix} + \begin{bmatrix} \cos x_3 & 0 \\ \sin x_3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix}$$

Therefore, by Proposition 2, we can design a controller for the two-wheeled mobile robot.

6. COMPUTER SIMULATION

In this section, we perform a computer simulation for TCLF (27) for system (15) to demonstrate the effectiveness of the proposed method. The simulation period is 20 s. The reference state is defined as follows:

$$\begin{bmatrix} x_{1r}(t) \\ x_{2r}(t) \end{bmatrix} = \begin{bmatrix} r_1 \sin 2\omega t \\ r_2 \cos \omega t \end{bmatrix}, \tag{45}$$

where $r_1 = 5$, $r_2 = 5$ and $\omega = \pi/6$. Note that the reference state marks like "8". Moreover, we consider the following positive symmetrical matrix P:

$$P = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \tag{46}$$

We show computer simulation results in Fig.s 4 and 5 with initial condition $x_0 = (-2, 6, 0)$. Fig. 4 shows the trajectory by the proposed method. Fig. 5 shows the input history to the proposed method. The figure above shows that we can confirm the trajectory draws "8" and the proposed method converges to the reference state.

7. CONCLUSION

In this paper, we proposed a TCLF design via dynamic extension and the minimum projection method for a two-wheeled mobile robot. Moreover, the effectiveness of the proposed method was confirmed by computer simulation.

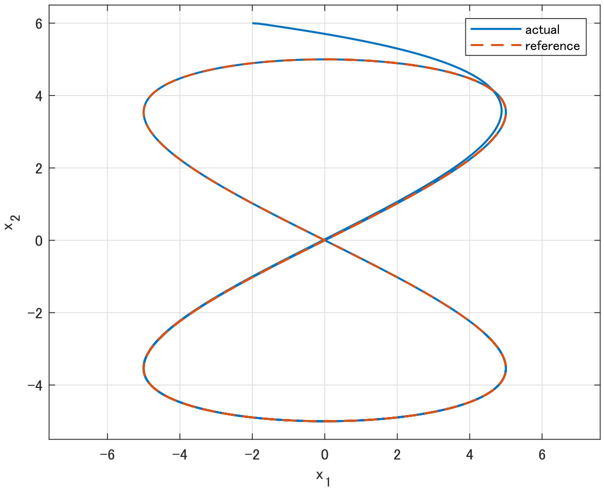

Fig. 4. Trajectory by the proposed method

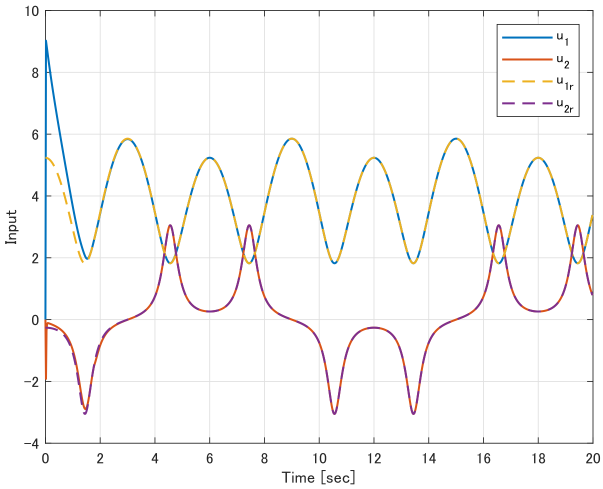

Fig. 5. Input response by the proposed method

However, as can be seen from Fig. 3, the proposed method is the local TCLF with respect to the angle. Solving the problem remains future work.

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