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Formation control of non-holonomic multi-agent systems under relative measurements ⋆

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Abstract: This paper addresses a formation control problem for multi-agent systems with non-holonomic constraints under relative measurements. To overcome the issue of non-holonomic constraints, we design a feedback controller deriving rotational and translational motions according to formation error. A special form of formation error is employed here, which depends only on relative positions in a local frame. Hence, the designed controller is distributed and relative, meaning that only relative measurements of neighbors are used. Because a clique-based function is used, not an edge-based one, the best performance is yielded of all distributed, relative, gradient-based controllers. Moreover, we derive a necessary and sufficient condition of graphs under which a desired formation is achieved by such controllers. The proposed method is valid regardless of the dimension of the space, and thus it is applicable to not only unmanned ground vehicles (UGVs) but also unmanned aerial vehicles (UAVs). The effectiveness of the proposed method is demonstrated by simulations.

Keywords: Multi-agent systems, distributed control, cooperative control, formation control.

1. INTRODUCTION

Multi-agent systems composed of many components that communicate and/or observe each other have been strongly focused in the field of the control engineering. For these systems, distributed control is important for reduction of computational burdens (Martínez et al., 2007), and distributed control methods for various control tasks have been developed, e.g., consensus (Olfati-Saber and Murray, 2004; Olfati-Saber et al., 2007), coverage (Cortés et al., 2004), and attitude synchronization (Igarashi et al., 2009; Ren, 2010), and so forth. Besides distributed control, relative control, relying on relative measurements of other agents, should be taken into account to utilize practical measurement data from sensing devices (Olfati-Saber et al., 2007; Lin et al., 2014).

Formation control is one of important tasks of multi-agent systems (Fax and Murray, 2004; Oh et al., 2015) for effective surveillance, inspection, and investigation. Especially, distance-based formation control is successfully achieved under distributed, relative control (Anderson et al., 2008; Krick et al., 2009; Dörfler and Francis, 2010; Lin and Jia, 2010; Sun and Anderson, 2015). This approach allows flips in the formation, which might cause an undesired configuration. This issue has been tackled in several papers by employing area constrains, matrix constraints, or so forth (Anderson et al., 2017; Sakurama et al., 2018).

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On the other hand, in many papers, motions of agents are assumed to be controllable in any directions. However, many robots including unmanned ground vehicles (UGVs), unmanned surface vehicles (USVs), unmanned underwater vehicles (UUVs), and unmanned aerial vehicles (UAVs) have non-holonomic constraints under which agents cannot slide laterally. Multi-agent systems with non-holonomic constraints have been investigated in several papers. For example, Dimarogonas and Kyriakopoulos (2008); Liu and Jiang (2013) have considered displacement-based formation control, and Montijano et al. (2016) have dealt with distance-based formation control. Zhao et al. (2018) have developed a method to drive rotational and translational motions according to gradient-based controllers.

This paper addresses a formation control problem of non-holonomic multi-agent systems under relative measurements. We focus on the method of Zhao et al. (2018) because there still remains the option for gradient-based controllers. Particularly, in the present paper, we employ a clique-based function instead of edge-based ones conventionally employed in gradient-based controllers. Note that the clique is a complete subgraph, which has a potential for enhancing control performance while maintaining distributedness of controllers.

We obtain the following contributions. First, the gradient of the clique-based function employed in the present paper depends only on relative positions in a local frame. Hence, the designed controller is distributed and relative, meaning that only relative measurements of neighbors are used. Second, we show that this clique-based function yields the best performance of all distributed, relative, gradient-based controllers under non-holonomic constraints. Third, we derive a necessary and sufficient condition of graphs under which the formation is achieved by such controllers. Forth, we can prevent the formation from flips because cliques can measure the discrepancies in formation shapes, while edges can only in distances. Finally, the proposed method does not limit the dimension of the space to 2 or 3, but is applicable to any finite dimensions.

Notation: Let $\mathrm{SO}(d) \subset \mathbb{R}^{d\times d}$ be the special orthogonal group of dimension d, and let $\mathrm{SE}(d) = \mathrm{SO}(d) \times \mathbb{R}^d$ be the special Euclidean group. Let $\mathrm{Skew}(d) \subset \mathbb{R}^{d\times d}$ denote the set of the d-dimensional skew-symmetric matrices. The notation $\mathbf{1}_n \in \mathbb{R}^n$ denotes the vector with all components 1, and $E_n \in \mathbb{R}^{n\times n}$ does the n-dimensional identity matrix. For vectors $x_1, x_2, \ldots, x_n \in \mathbb{R}^d$ and a set $\mathcal{C} \subset \{1, 2, \ldots, n\}$ of positive integers, let $(x_j)_{j\in\mathcal{C}}$ be the collection of x_j according to $j \in \mathcal{C}$, defined as

$$(x_j)_{j\in\mathcal{C}} = (x_{j_1}, x_{j_2}, \ldots, x_{j_{|\mathcal{C}|}}),$$

where $|\mathcal{C}|$ is the number of the elements of $\mathcal{C}$, and $j_1, j_2, \ldots, j_{|\mathcal{C}|} \in \mathcal{C}$ satisfy $1 \le j_1 < j_2 < \cdots < j_{|\mathcal{C}|} \le n$. Let $\mathrm{ave}(\cdot)$ be the component-wise average of a collection of vectors, namely,

$$\mathrm{ave}((x_j)_{j\in\mathcal{C}}) = \frac{1}{|\mathcal{C}|}\sum_{j\in\mathcal{C}} x_j.$$

For $(x_j)_{j\in\mathcal{C}} \in \mathbb{R}^{d\times|\mathcal{C}|}$ and a set $\mathcal{A} \subset \mathbb{R}^{d\times|\mathcal{C}|}$, their distance is defined as

$$\mathrm{dist}((x_j)_{j\in\mathcal{C}}, \mathcal{A}) = \inf_{(y_j)_{j\in\mathcal{C}}\in\mathcal{A}} \sqrt{\sum_{j\in\mathcal{C}} \|x_j - y_j\|^2}, \quad (1)$$

where $\|\cdot\|$ denotes the Euclidean norm of vectors.

Consider a graph $G = (\mathcal{V}, \mathcal{E})$ with a vertex set $\mathcal{V} = \{1, 2, \ldots, n\}$ and an edge set $\mathcal{E}$. We say that a subgraph $G|_\mathcal{C} = (\mathcal{C}, \mathcal{E}|_\mathcal{C})$ is induced by $\mathcal{C}$ if $\{i, j\} \in \mathcal{E}, i, j \in \mathcal{C} \Leftrightarrow \{i, j\} \in \mathcal{E}|_\mathcal{C}$ holds. A *clique* is defined as a set of vertexes which induce a complete subgraph (Bolloás, 1998). A clique is said to be *maximal clique* if it is not contained by any other cliques. Let $\mathrm{clq}(G)$ be the set of all maximal cliques in G. For $i \in V$, let $\mathrm{clq}_i(G)$ denote the set of all maximal cliques in G containing vertex i. For a graph G and a collection of vectors $(x_j^*)_{j\in\mathcal{V}} \in \mathbb{R}^{d\times n}$, a pair $(G, (x_j^*)_{j\in\mathcal{V}})$ is called a *framework*. A framework $(G, (x_j^*)_{j\in\mathcal{V}}))$ is said to be *locally clique-rigid* if there exists an open set $\mathcal{O} \supset \mathcal{X}^*$ such that

$$\mathcal{X}^* = \mathcal{X}^*_{\mathrm{clq}(G)} \cap \mathcal{O} \quad (2)$$

for the sets

$$\mathcal{X}^* = \{(x_j)_{j\in\mathcal{V}} \in \mathbb{R}^{d\times n} : \exists(\Theta, \tau) \in \mathrm{SE}(d) \ \text{s. t. } x_j = \Theta x_j^* + \tau \ \forall j \in \mathcal{V}\} \quad (3)$$

$$\mathcal{X}^*_{\mathrm{clq}(G)} = \{(x_j)_{j\in\mathcal{V}} \in \mathbb{R}^{d\times n} : \forall\mathcal{C} \in \mathrm{clq}(G) \ \exists(\Theta_\mathcal{C}, \tau_\mathcal{C}) \in \mathrm{SE}(d) \ \text{ s. t. } x_j = \Theta_\mathcal{C} x_j^* + \tau_\mathcal{C} \ \forall j \in \mathcal{C}\}. \quad (4)$$

2. PROBLEM SETTING

2.1 Variables in global and local frames

Consider a multi-agent system consisting of n agents, numbered from 1 to n. Let $\mathcal{V} = \{1, 2, \ldots, n\}$ be the set of the agent indexes. Let Σ be the global frame, which is common among the agents, and let $\Sigma_i(t)$ be the local frame of agent $i \in \mathcal{V}$, which is different from each other. The position coordinate of agent i in Σ is denoted as a vector $x_i(t) \in \mathbb{R}^d$, and the orientation of $\Sigma_i(t)$ from Σ is given by the matrix $R_i(t) \in \mathrm{SO}(d)$. Then, the global and local coordinates of a point are described by $p(t), p^{[i]}(t) \in \mathbb{R}^d$ satisfying

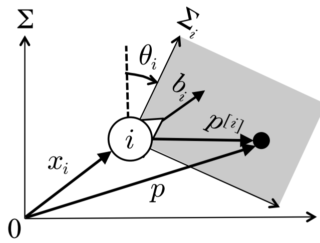

Fig. 1. Coordinates on the global and local frames ($d = 2$)

$$p(t) = R_i(t)p^{[i]}(t) + x_i(t). \quad (5)$$

The variables defined in $\Sigma_i(t)$ are denoted with the superscript $[i]$. See Fig. 1 for an illustration in the case of $d = 2$.

Let $\dot{x}_i(t) \in \mathbb{R}^d$ and $v_i^{[i]}(t) \in \mathbb{R}^d$ be the global and local velocities of agent i, defined in Σ and $\Sigma_i(t)$. From (5), these velocities have the following relation:

$$\dot{x}_i(t) = R_i(t)v_i^{[i]}(t). \quad (6)$$

2.2 Dynamics

We assume that the local speed of agent i is controllable via $u_i(t) \in \mathbb{R}$, while the direction of the local velocity is limited to $b_i \in \mathbb{R}^d$ ($\|b_i\| = 1$). Hence, the following holds:

$$v_i^{[i]}(t) = b_i u_i(t). \quad (7)$$

Moreover, the angular velocity of agent i is controllable via $S_i(t) \in \mathrm{Skew}(d)$ in the following way:

$$\dot{R}_i(t) = R_i(t)S_i(t). \quad (8)$$

Eq. (8) means that a tangent vector of $\mathrm{SO}(d)$ at $R_i(t)$ is given by $R_i(t)S_i(t)$ with a skew-symmetric matrix $S_i(t)$. This can be derived by differentiating the definition of the orthogonal matrix $R_i^\top(t)R_i(t) = E_d$ as

$$R_i^\top(t)\dot{R}_i(t) + (R_i^\top(t)\dot{R}_i(t))^\top = 0,$$

which shows that $R_i^\top(t)\dot{R}_i(t)$ is skew-symmetric and can be represented by $S_i(t)$.

Let $(R_i(t), x_i(t)) \in \mathrm{SE}(d)$ be the state of the system and let $(S_i(t), u_i(t)) \in \mathrm{Skew}(d)\times\mathbb{R}$ be the control input. Then, from (6), (7), and (8), the dynamics of the system can be described as

$$\begin{cases} \dot{R}_i(t) = R_i(t)S_i(t) \\ \dot{x}_i(t) = R_i(t)b_i u_i(t) \end{cases}. \quad (9)$$

Note that for $d = 2$, (9) is reduced to the common non-holonomic model of a rolling coin as

$$\begin{cases} \dot{\theta}_i(t) = \omega_i(t) \\ \dot{x}_i(t) = [\cos\theta(t) \ \sin\theta(t)]^\top u_i(t) \end{cases}$$

for $b_i = [1 \ 0]^\top$, $\theta_i(t) \in [0, 2\pi)$, and $\omega_i(t) \in \mathbb{R}$.

2.3 Distributed and relative controller

Agent i can obtain the information only on its neighbors $\mathcal{N}_i \subset \mathcal{V}$ through sensing. Let $\mathcal{E} = \{\{i, j\} \subset \mathcal{V} : j \in \mathcal{N}_i\}$, and graph $G = (\mathcal{V}, \mathcal{E})$ represents the network of sensing between agents. We assume that G is undirected and time-invariant.

Let $x_j^{[i]}(t) \in \mathbb{R}^d$ be the relative position of agent $j \in \mathcal{N}_i$ from agent i, and from (5), it can be described as

$$x_j^{[i]}(t) = R_i^\top(t)(x_j(t) - x_i(t)). \tag{10}$$

Assume that information on the relative positions $x_j^{[i]}(t)$ of the neighbors $j \in \mathcal{N}_i$ is available to agent i, but its own state $(R_i(t), x_i(t))$ nor the neighbors' states $(R_j(t), x_j(t))$, $j \in \mathcal{N}_i$ are unavailable. Then, the control input $(S_i(t), u_i(t)) \in \mathrm{Skew}(d) \times \mathbb{R}$ has to be generated as

$$\begin{cases} S_i(t) = F_i((x_j^{[i]}(t))_{j\in\mathcal{N}_i}) \\ u_i(t) = f_i((x_j^{[i]}(t))_{j\in\mathcal{N}_i}) \end{cases} \tag{11}$$

with some functions $F_i : \mathbb{R}^{d\times|\mathcal{N}_i|} \to \mathrm{Skew}(d)$ and $f_i : \mathbb{R}^{d\times|\mathcal{N}_i|} \to \mathbb{R}$. We said that the controller of the form (11) is *distributed and relative.*

2.4 Control objective

Let $(x_i^*)_{i\in\mathcal{V}} \in \mathbb{R}^{d\times n}$ be a desired configuration of the agents. The control objective is that the global positions $(x_i(t))_{i\in\mathcal{V}}$ of the agents to attain a congruent shape of $(x_i^*)_{i\in\mathcal{V}}$. This can be described as

$$\begin{aligned} &\exists(\Theta(t), \tau(t)) \in \mathrm{SE}(d) \\ &\quad \text{s. t. } \lim_{t\to\infty}(x_i(t) - (\Theta(t)x_i^* + \tau(t))) = 0 \ \forall i \in \mathcal{V}, \end{aligned} \tag{12}$$

where $\Theta(t) \in \mathrm{SO}(d)$ and $\tau(t) \in \mathbb{R}^d$ represent the rotational and translational freedoms in the coordination. By using the set $\mathcal{X}^*$ in (3), (12) is represented as

$$\lim_{t\to\infty} \mathrm{dist}((x_i(t))_{i\in\mathcal{V}}, \mathcal{X}^*) = 0. \tag{13}$$

We say that $\mathcal{X}^*$ is *locally attractive* if there exists an open set $\mathcal{A} \supset \mathcal{X}^*$ such that the state $(x_i(t))_{i\in\mathcal{V}}$ from every initial state $(x_i(0))_{i\in\mathcal{V}} \in \mathcal{A}$ satisfies (13). We say that $\mathcal{X}^*$ is *globally attractive* if this holds for $\mathcal{A} = \mathbb{R}^{d\times n}$.

Now, we consider the following problem in this paper.

Problem 1. For the system (9) and graph G, design a distributed, relative controller (11) such that $\mathcal{X}^*$ is locally/globally attractive.

3. PRELIMINARY

For vectors $y_j, z_j \in \mathbb{R}^d$, $j \in \{1, 2, \ldots, m\}$, consider the optimization problem

$$\min_{(R,\tau)\in\mathrm{SE}(d)} \sum_{j\in\{1,2,\ldots,m\}} \|y_j - (Rz_j + \tau)\|^2. \tag{14}$$

Let $(\hat{R}, \hat{\tau}) \in \mathrm{SE}(d)$ be the solution of (14), which is derived as

$$\hat{R} = V\mathrm{diag}(\overbrace{1, \ldots, 1}^{d-1}, \det(UV))U^\top \in \mathbb{R}^{d\times d} \tag{15}$$

$$\hat{\tau} = \mathrm{ave}((y_j - \hat{R}z_j)_{j\in\{1,2,\ldots,m\}}), \tag{16}$$

where $U, V \in \mathbb{R}^{d\times d}$ are orthogonal matrices satisfying

$$(Z - \mathrm{ave}(Z)\mathbf{1}_m^\top)(Y - \mathrm{ave}(Y)\mathbf{1}_m^\top)^\top = USV^\top \tag{17}$$

with the diagonal matrix $S = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_d)$ of the entries $\sigma_1, \sigma_2, \ldots, \sigma_d$ ($\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_d \geq 0$), $Y = [y_1\ y_2\ \cdots\ y_m]$, and $Z = [z_1\ z_2\ \cdots\ z_m]$.

Let $\mathcal{R}_m : \mathbb{R}^{d\times m} \times \mathbb{R}^{d\times m} \to \mathrm{pow}(\mathrm{SO}(d))$ be the set-valued function of $(y_j)_{j\in\{1,2,\ldots,m\}}$ and $(z_j)_{j\in\{1,2,\ldots,m\}}$ consisting of the matrices $\hat{R} \in \mathrm{SO}(d)$ given by (15) with any orthogonal matrices $U, V \in \mathbb{R}^{d\times d}$ satisfying (17), where $\mathrm{pow}(\cdot)$ is the power set of a set. Then, the following holds with $\hat{R} \in \mathcal{R}_m((y_j)_{j\in\{1,2,\ldots,m\}}, (z_j)_{j\in\{1,2,\ldots,m\}})$ and $\hat{\tau}$ in (16):

$$(14) = \sum_{j\in\{1,2,\ldots,m\}} \|y_j - (\hat{R}z_j + \hat{\tau})\|^2. \tag{18}$$

4. MAIN RESULT

4.1 Solution to Problem 1

For the control objective (12), we propose a distributed and relative controller (11) with the functions

$$\begin{cases} F_i((x_j^{[i]})_{j\in\mathcal{N}_i}) = (E_d - b_ib_i^\top)\phi_i((x_j^{[i]})_{j\in\mathcal{N}_i})b_i^\top \\ \qquad\qquad -((E_d - b_ib_i^\top)\phi_i((x_j^{[i]})_{j\in\mathcal{N}_i})b_i^\top)^\top \\ f_i((x_j^{[i]})_{j\in\mathcal{N}_i}) = b_i^\top\phi_i((x_j^{[i]})_{j\in\mathcal{N}_i}) \end{cases} \tag{19}$$

of $(x_j^{[i]})_{j\in\mathcal{N}_i}$ for the function

$$\phi_i((x_j^{[i]})_{j\in\mathcal{N}_i}) = \sum_{\mathcal{C}\in\mathrm{clq}_i(G)} (\mathrm{ave}((x_j^{[i]})_{j\in\mathcal{C}}) + \hat{R}_\mathcal{C}^{[i]}(x_i^* - \mathrm{ave}((x_j^*)_{j\in\mathcal{C}}))), \tag{20}$$

where $x_i^{[i]} = 0$ and $\hat{R}_\mathcal{C}^{[i]} \in \mathrm{SO}(d)$ is a matrix satisfying

$$\hat{R}_\mathcal{C}^{[i]} \in \mathcal{R}_{|\mathcal{C}|}((x_j^{[i]})_{j\in\mathcal{C}}, (x_j^*)_{j\in\mathcal{C}}). \tag{21}$$

Let us consider the meaning of the controller (19). The function $\phi_i((x_j^{[i]})_{j\in\mathcal{N}_i})$ in (20) consists of the position error of agent i from the average of the other agents belonging to each maximal clique $\mathcal{C} \in \mathrm{clq}_i(G)$. The controller (19) works to make the errors converge to zero, as illustrated as follows. From (7), (11), and (19),

$$\begin{cases} S_i(t)b_i = (E_d - b_ib_i^\top)\phi_i((x_j^{[i]}(t))_{j\in\mathcal{N}_i}) \\ v_i^{[i]}(t) = b_iu_i(t) = (b_ib_i^\top)\phi_i((x_j^{[i]}(t))_{j\in\mathcal{N}_i}) \end{cases} \tag{22}$$

is obtained, which implies the following two: (i) the local velocity $v_i^{[i]}(t)$ is given by the orthogonal projection of $\phi_i((x_j^{[i]}(t))_{j\in\mathcal{N}_i})$ to the line in the direction of b_i; (ii) the effect of the angular velocity to the local velocity, denoted by $S_i(t)b_i$, is the orthogonal complement of the projection. By the combination of the effects of $S_i(t)b_i$ and $v_i^{[i]}(t)$, $\phi_i((x_j^{[i]}(t))_{j\in\mathcal{N}_i})$ is expected to converge to zero.

As the expectation, $\phi_i((x_j^{[i]}(t))_{j\in\mathcal{N}_i})$ always converges to zero, and the control objective (12) is achieved under some conditions on graph G.

Theorem 1. For a connected graph G, consider the system (9) and the distributed and relative controller (11) with (19) for the function $\phi_i(\cdot)$ in (20). Then, (i) Φ is globally attractive, where

$$\Phi = \{(x_i)_{i\in\mathcal{V}} \in \mathbb{R}^{d\times n} : \forall i \in \mathcal{V},\ \exists \bar{R}_i \in \mathrm{SO}(d) \\ \text{s. t.}\ \ \phi_i((\bar{R}_i^\top (x_j - x_i))_{j\in\mathcal{N}_i}) = 0\}, \quad (23)$$

(ii) $\mathcal{X}^*$ is locally attractive if and only if the framework $(G, (x_j^*)_{j\in\mathcal{V}})$ is locally clique-rigid, and (iii) $\mathcal{X}^*$ is globally attractive if G is complete. □

Due to the limit of the space, only a sketch of the proof is given for Theorem 1. (i) According to Sakurama et al. (2018), the function $\phi_i(\cdot)$ in (20) satisfies

$$\phi_i((R_i^\top (x_j - x_i))_{j\in\mathcal{N}_i}) = -R_i^\top \frac{\partial v}{\partial x_i}((x_j)_{j\in\mathcal{V}}) \quad (24)$$

with the function $v : \mathbb{R}^{d\times n} \to \mathbb{R}$ given by

$$v((x_j)_{j\in\mathcal{V}}) = \sum_{\mathcal{C}\in \mathrm{clq}(G)} \frac{1}{2}(\mathrm{dist}((x_j)_{j\in\mathcal{C}}, \mathcal{D}_\mathcal{C}))^2 \quad (25)$$

for the set

$$\mathcal{D}_\mathcal{C} = \{(x_j)_{j\in\mathcal{C}} \in \mathbb{R}^{d\times|\mathcal{C}|} : \exists (R_\mathcal{C}, \tau_\mathcal{C}) \in \mathrm{SE}(d) \\ \text{s. t.}\ \ x_j = R_\mathcal{C} x_j^* + \tau_\mathcal{C}\ \forall j \in \mathcal{C}\}. \quad (26)$$

By using (24), (19) is represented as

$$\begin{cases} F_i((x_j^{[i]})_{j\in\mathcal{N}_i}) = -(E_d - b_i b_i^\top)R_i^\top \dfrac{\partial v}{\partial x_i}((x_j)_{j\in\mathcal{V}})b_i^\top \\ \qquad\qquad + ((E_d - b_i b_i^\top)R_i^\top \dfrac{\partial v}{\partial x_i}(((x_j)_{j\in\mathcal{V}})b_i^\top)^\top \\ f_i((x_j^{[i]})_{j\in\mathcal{N}_i}) = -b_i^\top R_i^\top \dfrac{\partial v}{\partial x_i}((x_j)_{j\in\mathcal{V}}) \end{cases}. \quad (27)$$

In this way, the proposed controller (19) is based on the gradient-based controller with the objective function $v((x_j)_{j\in\mathcal{V}})$ in (25). Hence, its gradient converges to zero, which is mentioned in Theorem 1(i) from (24).

(ii) Locally, $v((x_j)_{j\in\mathcal{V}}) = 0$ holds if and only if its gradient is zero, and thus $v((x_j(t))_{j\in\mathcal{V}})$ locally converges to zero. Then, from (25), for any $\mathcal{C} \in \mathrm{clq}(G)$, $(x_j(t))_{j\in\mathcal{V}}$ converges to $\mathcal{D}_\mathcal{C}$. Note that $\cap_{\mathcal{C}\in\mathrm{clq}(G)}\mathcal{D}_\mathcal{C} = \mathcal{X}^*_{\mathrm{clq}(G)}$ from (4) and (26). From the assumption that G is locally clique-rigid, (2) holds, which implies that locally $\mathcal{X}^*_{\mathrm{clq}(G)} = \mathcal{X}^*$ holds. Hence, $(x_j(t))_{j\in\mathcal{V}}$ locally converges to $\mathcal{X}^*$. From (3), (13) is locally achieved, and Theorem 1(ii) is obtained.

(iii) For the complete graph G, globally $\mathcal{X}^*_{\mathrm{clq}(G)} = \mathcal{X}^*$ holds, and $v((x_j)_{j\in\mathcal{V}}) = 0$ holds if and only if its gradient is zero. Hence, the discussion for (ii) globally holds, and Theorem 1(iii) is obtained.

4.2 Discussion on local clique-rigidity

The local clique-rigidity is essential for the attraction of $\mathcal{X}^*$. Actually, if a framework $(G, (x_j^*)_{j\in\mathcal{V}}))$ is not locally clique-rigid, we cannot achieve the attraction of $\mathcal{X}^*$ with for the gradient-based controller (27) not only with (25), but also with any objective functions. The following theorem describes this fact.

Theorem 2. For a graph G, consider system (9) with the distributed and relative controller (11) which is given by the gradient-based controller (27) with some $v((x_j)_{j\in\mathcal{V}})$ satisfying $v((x_j)_{j\in\mathcal{V}}) = 0$ for $(x_j)_{j\in\mathcal{V}} \in \mathcal{X}^*$. Then, there exists a function $v((x_j)_{j\in\mathcal{V}})$ such that $\mathcal{X}^*$ is locally attractive if and only if the framework $(G, (x_j^*)_{j\in\mathcal{V}}))$ is locally clique-rigid.

Proof. The sufficiency follows Theorem 1 (ii).

To prove the necessity, assume that $\mathcal{X}^*$ is an equilibrium set and is locally attractive with a function $v((x_j)_{j\in\mathcal{V}})$ satisfying $v((x_j)_{j\in\mathcal{V}}) = 0$ for $(x_j)_{j\in\mathcal{V}} \in \mathcal{X}^*$ whose gradient-based controller is distributed and relative. Because $\mathcal{X}^*$ is locally attractive,

$$v^{-1}(0) \cap \mathcal{A} = \mathcal{X}^* \quad (28)$$

holds for any open set $\mathcal{A} \supset \mathcal{X}^*$. Otherwise, there exists $(\psi_j)_{j\in\mathcal{V}} \in v^{-1}(0) \cap \mathcal{A}$ such that $(\psi_j)_{j\in\mathcal{V}} \notin \mathcal{X}^*$. Then, the solution $(x_j(t))_{j\in\mathcal{V}}$ from $(x_j(0))_{j\in\mathcal{V}} = (\psi_j)_{j\in\mathcal{V}}$ satisfies $(x_j(t))_{j\in\mathcal{V}} = (\psi_j)_{j\in\mathcal{V}} \notin \mathcal{X}^*$for all $t \geq 0$ because $v^{-1}(0)$ is a set of critical points. Hence, $(x_j(t))_{j\in\mathcal{V}}$ does not converge to $\mathcal{X}^*$

From Sakurama et al. (2015, 2018), the following holds:

$$\mathcal{X}^* \subset \hat{v}^{-1}(0) \subset v^{-1}(0)\ \forall v((x_j)_{j\in\mathcal{V}}) \in \mathcal{F}, \quad (29)$$

where $\hat{v}((x_j)_{j\in\mathcal{V}})$ is the function defined in (25), $\mathcal{F}$ is the set of the functions $v((x_j)_{j\in\mathcal{V}})$ such that $\mathcal{X}^* \subset v^{-1}(0)$ and its gradient is distributed and relative, namely there exists a function $\phi_i(\cdot)$ such that (24) holds. From (4), (25), and (26),

$$\hat{v}^{-1}(0) = \mathcal{X}^*_{\mathrm{clq}(G)} \quad (30)$$

holds. Because of $\hat{v}((x_j)_{j\in\mathcal{V}}) \in \mathcal{F}$ and $\mathcal{A} \supset \mathcal{X}^*$, from (28) and (29), we obtain

$$\mathcal{X}^* = \mathcal{X}^* \cap \mathcal{A} \subset \hat{v}^{-1}(0) \cap \mathcal{A} \subset v^{-1}(0) \cap \mathcal{A} = \mathcal{X}^*,$$

which leads to $\mathcal{X}^* = \hat{v}^{-1}(0) \cap \mathcal{A}$. From this equation and (30), (2) is achieved for $\mathcal{O} = \mathcal{A}$. Hence, $(G, (x_j^*)_{j\in\mathcal{V}}))$ is locally clique-rigid. □

There are two comments on Theorem 2. First, the objective function is not limited to (25) in this theorem. Hence, the local clique-rigidity is an essential property of the network topology, not depending on objective functions. Second, we do not have to consider any gradient-based controllers other than the proposed one (19) because if the framework is locally clique-rigid, the attraction of $\mathcal{X}^*$ is achieved by this controller; otherwise, the attraction cannot be achieved by any other gradient-based controllers. In this sense, the proposed controller (19) is the best of all such controllers.

5. NUMERICAL EXAMPLES

The effectiveness of the proposed method is demonstrated through numerical examples for $n = 6$ agents in the $d = 2$-dimensional space. The control objective is given as (12) with the desired configuration $(x_i^*)_{i\in\mathcal{V}} \in \mathbb{R}^{2\times 6}$ in Fig. 2. The edges of graph G are given as Fig. Fig. 2.

Simulations are carried out for the model (9) with the distributed, relative controller (11), (19). Figs. 3 and 4 show the simulation results from different initial states. The trajectories $(x_i(t))_{i\in\mathcal{V}}$ of agent positions from $t = 0$ to 20 are drawn by the dotted lines, and the final positions are described by the squares. Figs. 3 and 4 show that from any initial states the agents attain the desired configuration in Fig. 2 with some rotations. The rotations are determined by the initial positions. Anyway, in any cases, the control objective (13) is achieved.

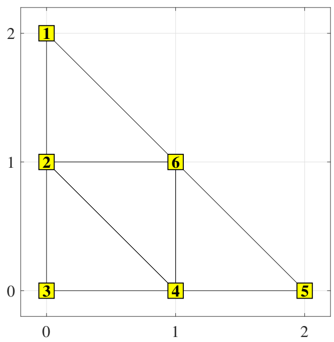

Fig. 2. Desired configuration $(x_i^*)_{i\in\mathcal{V}} \in \mathbb{R}^{2\times d}$ and the desired velocity $v^* = [-1\ 1]^\top$

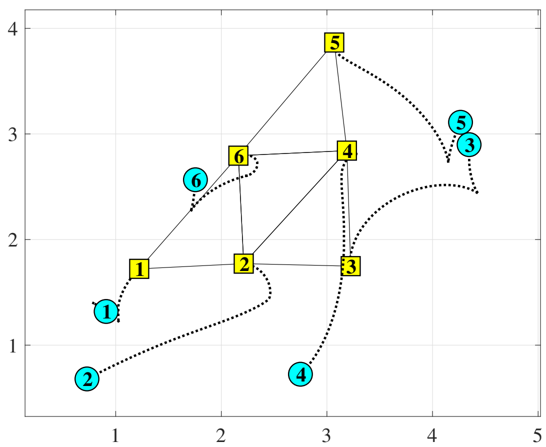

Fig. 3. Simulation result: case 1

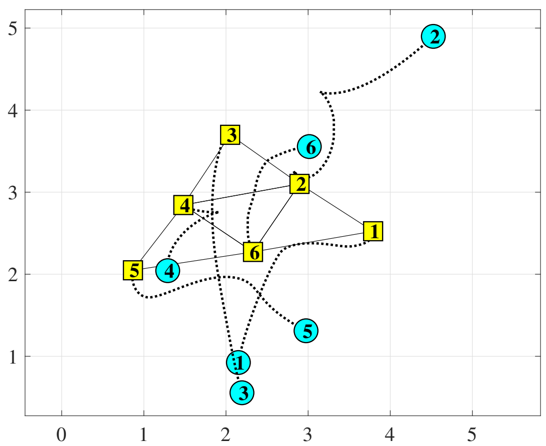

Fig. 4. Simulation result: case 2

6. CONCLUSION

This paper addressed a formation control problem for multi-agent systems with non-holonomic constraints under relative measurements. We designed a feedback controller deriving rotational and translational motions according to a particular type of formation error. Actually, this formation error depends only on relative positions in a local frame, and thus the designed controller is distributed and relative. Moreover, the formation error is based on a clique-based function, which yields the best performance of all distributed, relative, gradient-based controllers. Next, a necessary and sufficient condition of graphs was derived with which a desired formation is achievable. Finally, the effectiveness of the proposed method was demonstrated by simulations.

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