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# Adaptive cooperative control for high-order nonlinear multiagent time-delay systems using barrier functions

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**Abstract:** A novel adaptive backstepping controller is developed to achieve the asymptotic synchronization. The designed controller only contains the values of system states, and doesn't contain any other prior knowledge of system. Firstly, the designed controller regards the unknown system nonlinearities and the disturbance of state time-delay as "disturbance-like" terms, which are guaranteed to be bounded by using the pre-set barrier functions, such that any prior knowledge of system nonlinearities and state time-delay are released. Then, the "disturbance-like" terms are compensated adaptively by designing the novel compensator at each step, such that the synchronization errors are eliminated to zero eventually for each agent. It is proved that our developed controller guarantees the convergence on the basis of Lyapunov stability theory. Some simulations are shown to demonstrate the effectiveness and advantages of the developed method.

*Keywords:* Nonlinear multiagent state time-delay systems, Adaptive cooperative tracking control, Backstepping.

## 1. INTRODUCTION

The cooperative tracking control of multiagent systems (MASs) has become a hot topic and has been applied to many practical areas over the past decades, such as autonomous underwater vehicles, unmanned aerial vehicles and other intelligent robotic systems (Arcak (2006); Fax and Murray (2004)). The main task of the cooperative tracking control is to design a controller only using the local information, such that the states or outputs of MASs reach an agreement. Many literatures have extensively studied the such issues for linear and nonlinear MASs (Huang Y (2018); Olfatisaber and Murray (2004)). Especially, to precisely describe the practical systems, which usually possess high-order dynamics in the real world, quite a few unknown time-varying high-order nonlinear MASs cooperative tracking control problems have been studied, see Liu and Huang (2017); Jie et al. (2015).

Recently, more research results focused on the effects of time-delay. As is known to all, time-delay widely exists in physical equipments and it can lead to the instable of systems (Niculescu (2001)). Many research results for the cooperative control problems of time-delay MASs can be found in Lin and Jia (2010); Zhu and Jiang (2014); Yu et al. (2017). Unfortunately, the aforementioned works only concerned with either input time-delays or communication time-delays which were usually regarded as the linear terms. In the recent past, the cooperative control problems of nonlinear multiagent state time-delay systems have gained more attention (Chen et al. (2014); Ma et al. (2016); Chen et al. (2018)). The consensus schemes in Chen et al. (2014) and Ma et al. (2016) were only suitable to first-order MASs. Chen et al. (2018) extended the result in Chen et al. (2014)and Ma et al. (2016) to a class of nonlinear strict-feedback high-order MASs with state time-delays, where an adaptive neural controller was constructed. However, the tracking errors are only limited to a compact domain, and the prior knowledge of the state time-delay terms should be known for the designed controller.

In addition, in the aforementioned nonlinear MASs, the dynamics of agents were usually assumed to be known. However, it is difficult to establish the accurate system model for practical MASs. Therefore, the study of cooperative tracking control problems for unknown nonlinear MASs without prior knowledge of system nonlinearities is a challenging topic. The neural networks (NNs) and fuzzy logical are employed to approximate the system uncertain terms, which is usually considered as a continuous function by giving some assumptions (Das and Lewis (2010); Hua et al. (2015)). However, some training processes and external testing signals, which will require excessive computational resources, are necessary for controller design in these NNs-based and Fuzzy-based adaptive controllers. In addition, the consensus tracking errors are only limited to a compact domain, whose size depends on the design parameters and some unknown bounded terms, and the transient performance cannot be guaranteed.

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In some control systems with high accuracy requirements, prescribed performance control (PPC), which uses the prescribed performance functions (PPF) to convert the original system into a system with the required performance constraints, is developed to satisfy the specific performance requirements (Gao et al. (2018); Hashim et al. (2019)). It makes the convergence rate no less than a prescribed value and the maximum overshoot less than a sufficiently small constant, meanwhile the output or tracking error is confined within the prescribed performance bounds for all times. Inspired by PPC, prescribed performance cooperative control for MASs has attracted considerable attention. Significant strides, including input quantization, hysteresis input uncertainties, unknown control directions and switching networks (Yu et al. (2019); Cui et al. (2018)). In Gang et al. (2017), the distributed control problem of nonlinear strict-feedback MASs is addressed under directed and time-invariant communication graphs, such that each agent can achieve the prescribed performance. However, these methods can not achieve the asymptotic tracking with prescribed performance. Recently, in Wang et al. (2020), asymptotic tracking control problem for a class of strict-feedback time-varying nonlinear systems with unknown control directions is studied.

In spite of the progress, there are some problems that need to be further studied: For high-order nonlinear multiagent time-delay systems, can we design a novel controller without any prior knowledge of system nonlinearities and state time-delay to achieve the asymptotic tracking with prescribed performance?

In this paper, we develop a novel adaptive backstepping control approach to solve this problem. The designed controller regards the unknown system nonlinearities and the disturbance of state time-delay as "disturbance-like" terms, which are guaranteed to be bounded by using the pre-set constrained functions, such that any prior knowledge of system nonlinearities and state time-delay are released. Then, the "disturbance-like" terms are compensated adaptively by designing the novel compensator at each step, such that the tracking error is confined within the prescribed performance bounds and eliminated to zero eventually for each agent. In addition, it should be pointed out that the developed control scheme releases the assumption for the available of the differentiable reference trajectory $y_r$, and only one tuning law is necessary at each backstepping step.

The rest of this paper is organized as follows. The preliminaries and problem statement are presented in Section 2. Section 3 shows the developed adaptive control scheme. Simulation results are given in Section 4 to demonstrate the effectiveness and advantages of the proposed method. Section 5 concludes this paper.

## 2. PRELIMINARIES AND PROBLEM STATEMENT

### *2.1 Graph theory*

In this paper, a directed graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} \triangleq \{1, 2, \ldots, N\}$ denotes the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ describes the set of edges, is defined to narrate the communication topology among $N$ agents. Meanwhile, an ordered pair of distinct node $\xi_{ig} = (i, g)$, where $\xi_{ig} = (i, g) \in \mathcal{E}$ if and only if node $i$ can receive information from $g$, is defined to represent the directed edge. Additionally, $A = (a_{ig})_{N\times N}$ is defined as the adjacency matrix associated with $\mathcal{G}$, where $(i, g) \in \mathcal{E} \Leftrightarrow a_{ig} > 0$, and $a_{ig} = 0$ otherwise. Moreover, the graph does not contain repeated edges or self-loops. The set of neighbors of the $i$-th agent is denoted by $\mathcal{N}_i = \{g \in \mathcal{V} \| (i, g) \in \mathcal{E}, g \neq i\}$. The in-degree matrix is defined as $D = diag(d_1, \ldots, d_N) \in R^{N\times N}$, where $d_i = \sum_{g\in\mathcal{N}_i} a_{ig}$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = D - A$. A path is a sequence of ordered edges, e.g., $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$, and is called a directed path from node $i_k$ to node $i_1$. The directed graph contains a directed spanning tree if there exists at least one agent that has directed paths to all other agents.

A graph $\bar{\mathcal{G}} \triangleq (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} \triangleq \{0, 1, 2, \ldots, N\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$, is defined to depict the relationship among $N$ followers and leader(labeled by 0). $B = diag\{b_1, b_2, \ldots, b_N\}$ is defined to model the relationship between the leader and each follower, i.e., $b_i > 0$ if the agent $i$ can receive information from leader, and $b_i = 0$ otherwise. The following assumption is needed in this paper.

*Assumption 1.* $\bar{\mathcal{G}}$ contains a directed spanning tree with the leader as the root node.

### *2.2 Problem Statement*

Consider a class of nonlinear state time-delay MASs, which consists of one leader and $N$ followers. The dynamics of the $i$-th follower is

$$\begin{cases} \dot{x}_{i,j}(t) &= x_{i,j+1}(t) + f_{i,j}(\bar{x}_{i,j}(t)) + p_{i,j}(\bar{x}_{i,j}(t - \tau_{i,j})) \\ \dot{x}_{i,n}(t) &= u_i(t) + f_{i,n}(\bar{x}_{i,n}(t)) + p_{i,n}(\bar{x}_{i,n}(t - \tau_{i,n})) \\ y_i(t) &= x_{i,1}(t), i = 1, \ldots, N; j = 1, \ldots, n-1 \end{cases} \quad (1)$$

where $n$ describes the order of the $i$-th follower; $N$ describes the number of the followers in system; $u_i \in \Re$ and $y_i \in \Re$ describe the control input and the output of the $i$-th follower; $\bar{x}_{i,j} = [x_{i,1}, \ldots, x_{i,j}]^T \in \Re^j$ and $\bar{x}_{i,n} = [x_{i,1}, \ldots, x_{i,n}]^T \in \Re^n$ are the system states; The system nonlinearities $f_{i,j}(\cdot), p_{i,j}(\cdot) : \Re^j \to \Re^n$ denote the unknown smooth functions. We assume that the system nonlinearities $f_{i,j}(\bar{x}_{i,j}(t))$ satisfy the inequality $|f_{i,j}(\cdot)| \leq \bar{f}_{i,j}(\cdot), i = 1, 2, \ldots, N; j = 1, 2, \ldots, n$, where $\bar{f}_{i,j}(\cdot)$ is continuous non-negative functions. $\tau_{i,j}$ is the unknown state time-delay, and we assume that $\tau_{i,j}$ is bounded and $\tau_{i,j} < \tau_{i,max}$ is satisfied ($\tau_{i,max}$ is a known constant). We assume that the time-delay function $p_{i,j}(\cdot)$ satisfies the inequality $\|p_{i,j}(\cdot)\| \leq \bar{p}_{i,j}(\cdot)$, where $\bar{p}_{i,j}(\cdot)$ is bounded positive smooth function.

The objective of this paper is to develop a novel adaptive controller $u_i$ for state time-delay MASs (1) without any prior knowledge of system nonlinearities, such that: (A) The state error $z_{i,j}, i = 1, \ldots, N; j = 1, \ldots, n$ of each agent satisfies $|z_{i,j}| < k_i$. $z_{i,j}$ will be defined later, $k_i(t)$ is the prescribed performance function; (B) The synchronization error asymptotically converges to zero, which means that $\lim_{t\to\infty} \|y_e\| \to 0$. $y_e = y - 1_N y_r$ is defined to denote the synchronization error, where $y_r$ is the desired trajectory, $y = [y_1, y_2, \ldots, y_N]^T$. $1_N$ is defined as $1_N = [1, 1, \ldots, 1]^T$.

The signal $y_r$ of leader is a pre-set, smooth and bounded function. $\dot{y}_r$ is bounded but may be not available. Unlike the requirements for the smooth and bounded high order derivatives of $y_r$ in existing backstepping-like approaches, the high order derivatives of $y_r$ is only needed to be bounded but not necessary to be available in our method.

*Lemma 1.* (Das and Lewis (2010)) Assumption 1 ensures $\|y_e\| \leq \|z_1\|/\amalg(L+B)$, where $\amalg(L+B)$ is the minimum singular value of $L+B$, and $z_1 = [z_{1,1}, z_{2,1}, \ldots, z_{N,1}]^T$ will be defined later as the local tracking error. The synchronization problem can be solved if the local tracking error is small enough, i.e., $\|z_1\| \to 0 \Rightarrow \|y_e\| \to 0$. Actually, $y_e$ is suitable for analysis, but it cannot be used as a feedback signal in the controller due to the influence of communication topology. Local information of agent $i$ ($z_i, i = 1, 2, \ldots, N$) can be used in the controller.

*Lemma 2.* (Li and Yang (2016); Song et al. (2019)) For any $w \in \Re$, there is a constant $\epsilon > 0$ satisfying the inequality $0 \leq |w| - \frac{w^2}{\sqrt{w^2+\epsilon^2}} \leq \epsilon$.

### 2.3 Prescribed Performance Function

In this subsection, the PPF is introduced. PPF is usually designed as a monotone exponentially decreasing time function, which starts within a predefined bigger set and reduces to a given smaller set with a systematic manner. In this paper, the smooth monotone exponentially decreasing function is defined as

$$\rho_{i,j}(t) = (\rho_{0,i,j} - \rho_{\infty,i,j})\exp(-a_{i,j}t) + \rho_{\infty,i,j} \qquad (2)$$

where $\rho_{i,j}(t) : \Re_+ \to \Re_+$; $\rho_{0,i,j}$ is the initial value of $\rho_{i,j}(t)$, and $\rho_{\infty,i,j}$ represents the ultimate value of $\rho_{i,j}(t)$; $a_{i,j}$ denotes the damped coefficient. It is clear that $\lim_{t\to\infty} \rho_{i,j}(t) \to \rho_{\infty,i,j}$. The bounds of output constrained state errors $z_{i,j}(t)$ for all $t \geq 0$ can be described as

$$-\lambda_{i,j}\rho_{i,j}(t) < z_{i,j}(t) < \rho_{i,j}(t), \quad if \quad z_{i,j}(0) \geq 0 \qquad (3)$$

$$-\rho_{i,j}(t) < z_{i,j}(t) < \lambda_{i,j}\rho_{i,j}(t), \quad if \quad z_{i,j}(0) < 0 \qquad (4)$$

where $0 \leq \lambda_{i,j} \leq 1$. A transformed error is defined as

$$\eta_{i,j} := \frac{z_{i,j}}{k_{i,j}(t)}; \quad k_{i,j}(t) = b\bar{k}_{i,j}(t) + (1-b)\underline{k}_{i,j}(t) \qquad (5)$$

where $b = 1$ if $z_{i,j}(t) \geq 0$, and $b = 0$ if $z_{i,j}(t) < 0$. $\bar{k}_{i,j}(t)$ and $\underline{k}_{i,j}(t)$ are defined as: if $z_{i,j}(0) \geq 0$, $\bar{k}_{i,j}(t) = \rho_{i,j}(t)$, $\underline{k}_{i,j}(t) = -\lambda_{i,j}\rho_{i,j}(t)$, otherwise, $\bar{k}_{i,j}(t) = \lambda_{i,j}\rho_{i,j}(t)$, $\underline{k}_{i,j}(t) = -\rho_{i,j}(t)$.

*Lemma 3.* (Han and Lee (2014)) The inequality $0 < \eta_{i,j} < 1$ (prescribed performance) can be guaranteed if and only if $\rho_{0,i,j}, \rho_{\infty,i,j}, a_{i,j}$ and $\lambda_{i,j}$ satisfy (3) and (4).

## 3. DESIGN OF DISTRIBUTED ADAPTIVE CONTROLLER

In this section, a novel adaptive controller is developed by using barrier Lyapunov functions to achieve the asymptotic cooperative tracking performance within PPF.

The cooperative tracking control design will become more difficult owing to the unknown terms $f_{i,j}(\cdot)$, $p_{i,j}(\cdot)$, and unknown time delay $\tau_{i,j}$ are included in the MASs (1). Moreover, the state $x_{i,j}(t-\tau_{i,j})$ is uncertain because the delay term $\tau_{i,j}$ is unknown. Thus, $x_{i,j}(t-\tau_{i,j})$ can not be directly used in the controller design. In order to handle these problems, some existing papers approximate the uncertain nonlinear dynamics by using the approximation property of NNs or Fuzzy logical, and compensate the uncertainties of unknown time delay by using appropriate Lyapunov–Krasovskii functional. However, some training processes and external testing signals are necessary for controller design in these NNs-based and Fuzzy-based adaptive controllers. The tracking errors are only limited to a compact domain, and the transient performance cannot be guaranteed. In addition, the prior knowledge of $\bar{p}_{i,j}(\cdot)$ should be known for the designed controller.

In this paper, the designed controller regards the unknown system nonlinearities and the disturbance of state time-delay as “disturbance-like” terms. And a novel compensator is designed to compensate the “disturbance-like” terms adaptively. The barrier functions are used as Lyapunov function to design the controller such that the prescribed performance is guaranteed. Moreover, the designed controller doesn't require any prior knowledge of $f_{i,j}(\cdot)$, $p_{i,j}(\cdot)$, and the state time-delay term $\tau_{i,j}$.

Backstepping approach is used to construct the controller for each agent.

*step* $(i,1)$: The first change of coordinates are defined as

$$\begin{aligned} z_{i,1} = &\sum_{g\in\mathcal{N}_i} a_{ig}(x_{i,1} - x_{g,1}) + b_i(x_{i,1} - y_r) \\ &-\varrho(t)\Big[\sum_{g\in\mathcal{N}_i} a_{ig}(x^0_{i,1} - x^0_{g,1}) + b_i(x^0_{i,1} - y^0_r)\Big] \\ \eta_{i,1} := &\frac{z_{i,1}}{k_{i,1}}; \qquad \chi_{i,1} = \frac{\eta_{i,1}}{1-\eta^2_{i,1}} \qquad i = 1, 2, \ldots, N \qquad (6) \end{aligned}$$

where $x^0_{i,1}$, $x^0_{g,1}$, $y^0_r$ denote the initial values of the $x_{i,1}$, $x_{g,1}$, $y_r$, respectively; $\varrho(t)$ is a tuning function, and is used to compensate the influence of initial state errors. Thus, in our algorithm, we assume that the information of the initial condition of the each agent is known and can be transferred to other agents. In this paper, the tuning function $\varrho(t)$ is defined as $\varrho(t) = e^{-t}$. It is obvious that both $\varrho(t)$ and $\dot{\varrho}(t)$ are bounded. $k_{i,1}(t)$ is defined in (5).

The first virtual controller $\alpha_{i,1}$ of the $i$-th agent and the corresponding adaptive compensator are designed as

$$\begin{aligned} \alpha_{i,1} = &\frac{-1}{b_i + d_i}\chi_{i,1}\left(\psi_{i,1} + \frac{\iota_{i,1}\hat{M}_{i,1}}{\sqrt{\chi^2_{i,1} + \sigma^2_{i,1}(t)}}\right) \\ &+ \frac{\sum_{g=1}^N a_{ig}x_{g,2}}{b_i + d_i} \qquad (7) \end{aligned}$$

$$\dot{\hat{M}}_{i,1} = \frac{\beta_{i,1}\chi^2_{i,1}}{\sqrt{\chi^2_{i,1} + \sigma^2_{i,1}(t)}}, \hat{M}_{i,1}(0) \geq 0 \qquad (8)$$

where $\hat{M}_{i,1}$ is the estimated value of $M_{i,1}$, which will be specified later; $\hat{M}_{i,1}(0)$ is the initial condition of $\hat{M}_{i,1}$; $\psi_{i,1}, \iota_{i,1}$ are positive constants; $\sigma_{i,1}(t)$ is a positive function. There are two positive constants $\sigma_{i,1,1}$ and $\sigma_{i,1,2}$, which have $\lim_{t\to\infty}\int_0^t \sigma_{i,1}(\iota)\,d\iota \leq \sigma_{i,1,1} < +\infty$ and $|\dot{\sigma}(t)| \leq \sigma_{i,1,2} < +\infty$. $\beta_{i,1}$ is a positive constant.

*step* $(i, j(j = 2, \dots, n-1))$: The $j$-th change of coordinates are defined as

$$z_{i,j} = x_{i,j} - \alpha_{i,j-1} - \varrho(t)x^0_{i,j}$$
$$\eta_{i,j} := \frac{z_{i,j}}{k_{i,j}}; \qquad \chi_{i,j} = \frac{\eta_{i,j}}{1-\eta^2_{i,j}} \qquad i = 1, 2, \dots, N \quad (9)$$

where $x^0_{i,j}$ denotes the initial values of the $x_{i,j}$; $\alpha_{i,j-1}$ represents the virtual control law at the $(j-1)$-th step of the $i$-th agent; $k_{i,j}(t)$ is defined in (5).

The $j$-th virtual controller $\alpha_{i,j}$ of the $i$-th agent and the corresponding adaptive compensator are designed as

$$\alpha_{i,j} = -\chi_{i,j}\left(\psi_{i,j} + \frac{\iota_{i,j}\hat{M}_{i,j}}{\sqrt{\chi^2_{i,j} + \sigma^2_{i,j}(t)}}\right) \qquad (10)$$

$$\dot{\hat{M}}_{i,j} = \frac{\beta_{i,j}\chi^2_{i,j}}{\sqrt{\chi^2_{i,j} + \sigma^2_{i,j}(t)}}, \hat{M}_{i,j}(0) \geq 0 \qquad (11)$$

where $\hat{M}_{i,j}$ is the estimated value of $M_{i,j}$, which will be specified later; $\psi_{i,j}, \iota_{i,j}$ are positive constants; $\sigma_{i,j}(t)$ is a positive function. There are two positive constants $\sigma_{i,j,1}$ and $\sigma_{i,j,2}$, which have $\lim_{t\to\infty}\int_0^t \sigma_{i,j}(\iota)\,d\iota \leq \sigma_{i,j,1} < +\infty$ and $|\dot{\sigma}(t)| \leq \sigma_{i,j,2} < +\infty$. $\hat{M}_{i,j}(0)$ is the initial condition of $\hat{M}_{i,j}$; $\beta_{i,j}$ is a positive constant.

*step* $(i, n)$: The $n$-th change of coordinates are defined as

$$z_{i,n} = x_{i,n} - \alpha_{i,n-1} - \varrho(t)x^0_{i,n}$$
$$\eta_{i,n} := \frac{z_{i,n}}{k_{i,n}}; \qquad \chi_{i,n} = \frac{\eta_{i,n}}{1-\eta^2_{i,n}} \qquad i = 1, 2, \dots, N \ (12)$$

where $x^0_{i,n}$ denotes the initial values of the $x_{i,n}$, respectively; $\alpha_{i,n-1}$ represents the virtual control law at the $(n-1)$th step of the $i$-th agent; $k_{i,n}(t)$ is defined in (5).

The actual controller $u_i$ of the $i$-th agent and the corresponding adaptive compensator are designed as

$$u_i = -\chi_{i,n}\left(\psi_{i,n} + \frac{\iota_{i,n}\hat{M}_{i,n}}{\sqrt{\chi^2_{i,n} + \sigma^2_{i,n}(t)}}\right) \qquad (13)$$

$$\dot{\hat{M}}_{i,n} = \frac{\beta_{i,n}\chi^2_{i,n}}{\sqrt{\chi^2_{i,n} + \sigma^2_{i,n}(t)}}, \hat{M}_{i,n}(0) \geq 0 \qquad (14)$$

where $\chi_{i,n} = \frac{\eta_{i,n}}{1-\eta^2_{i,n}}$; $\hat{M}_{i,n}$ is the estimated value of $M_{i,n}$, which will be specified later; $\psi_{i,n}, \iota_{i,n}$ are positive constants. There are two positive constants $\sigma_{i,n,1}$ and $\sigma_{i,n,2}$, which have $\lim_{t\to\infty}\int_0^t \sigma_{i,n}(\iota)\,d\iota \leq \sigma_{i,n,1} < +\infty$ and $|\dot{\sigma}(t)| \leq \sigma_{i,n,2} < +\infty$. $\hat{M}_{i,n}(0)$ is the initial condition of $\hat{M}_{i,n}$; $\beta_{i,n}$ is a positive constant.

The following theorem illustrates that the main control objective of MASs (1) can be achieved by using the developed control scheme.

*Theorem 1.* For high-order nonlinear multiagent time-delay systems (1), the controller (13), the virtual control laws (7), (10), and the adaptive compensators (8), (11), (14) can guarantee that: i) All signals of the resulting closed loop system are bounded; ii) The state error $z_{i,j}, i = 1, \dots, N; j = 1, \dots, n$ of each agent satisfy $|z_{i,j}| < k_{i,j}$; iii) The output synchronization error $y_e$ converges to zero asymptotically, i.e., $z_1 \to 0 \Rightarrow y_e \to 0$ for $i = 1, 2, \dots, N$. $z_1 = [z_{1,1}, z_{2,1}, \dots, z_{N,1}]$.

*Proof:* It is clear that the virtual control laws $\alpha_{i,1}, \dots, \alpha_{i,n}$ and the actual control law $u_i$ of the $i$-th agent can be described as function of the vector $\mathcal{S}_i = [\hat{M}_{i,1}, \dots, \hat{M}_{i,n}, \eta_{i,1}, \dots, \eta_{i,n}]$ as follows

$$\alpha_{i,1} := \alpha^*_{i,1}(t, \hat{M}_{i,1}, \eta_{i,1})$$
$$\alpha_{i,j} := \alpha^*_{i,j}(t, \hat{M}_{i,j}, \eta_{i,j})$$
$$u_i := u^*_i(t, \hat{M}_{i,n}, \eta_{i,n})$$

In addition, the states $x_{i,1}, \dots, x_{i,n}$ of the $i$-th agent can be described as

$$\begin{aligned} x_{i,1} &= \frac{1}{b_i + d_i}\Big\{k_{i,1}\eta_{i,1} + \sum_{g=1}^{N} a_{ig}x_{g,1} + b_i y_r \\ &\quad + \varrho(t)\Big[\sum_{g\in\mathcal{N}_i} a_{ig}(x^0_{i,1} - x^0_{g,1}) + b_i(x^0_{i,1} - y^0_r)\Big]\Big\} \\ &:= x^*_{i,1}(t, \eta_{i,1}) \\ x_{i,j} &= k_{i,j}\eta_{i,j} + \alpha_{i,j-1} + \varrho(t)x^0_{i,j} \\ &:= x^*_{i,j}(t, \hat{M}_{i,j-1}, \eta_{i,j-1}, \eta_{i,j}), \quad j = 2, \dots, n. \end{aligned}$$

Thus, we have

$$\begin{aligned} \dot{z}_{i,1} &= (b_i + d_i)\big\{x^*_{i,2}(t) + f_{i,1}(x^*_{i,1}(t)) \\ &\quad + p_{i,1}(x^*_{i,1}(t - \tau_{i,1}))\big\} - \sum_{g=1}^{N} a_{ig}\dot{x}_{g,1} + b_i\dot{y}_r \\ &\quad - \dot{\varrho}(t)\Big[\sum_{g\in\mathcal{N}_i} a_{ig}(x^0_{i,1} - x^0_{g,1}) + b_i(x^0_{i,1} - y^0_r)\Big] \\ \dot{z}_{i,j} &= x^*_{i,j+1}(t) + f_{i,j}(\bar{x}^*_{i,j}(t)) \\ &\quad + p_{i,j}(\bar{x}^*_{i,j}(t - \tau_{i,j})) - \frac{\partial\alpha^*_{i,j-1}}{\partial t} \\ &\quad - \frac{\partial\alpha^*_{i,j-1}}{\partial\hat{M}_{i,j-1}}\dot{\hat{M}}_{i,j-1} - \dot{\varrho}(t)x^0_{i,j}, \quad j = 2, \dots, n-1 \\ \dot{z}_{i,n} &= u^*_i + f_{i,n}(\bar{x}^*_{i,n}(t) + p_{i,n}(\bar{x}^*_{i,n}(t - \tau_{i,n})) \\ &\quad - \frac{\partial\alpha^*_{i,n-1}}{\partial t} - \frac{\partial\alpha^*_{i,n-1}}{\partial\hat{M}_{i,n-1}}\dot{\hat{M}}_{i,n-1} - \dot{\varrho}(t)x^0_{i,n} \end{aligned}$$

where $\bar{x}^*_{i,j} = [x^*_{i,1}, \dots, x^*_{i,j}], j = 1, \dots, n$.

Consequently, the closed loop system of the vector $\mathcal{S}_i$ is written as

$$\begin{aligned} \dot{\hat{M}}_{i,j} &= \frac{\beta_{i,j}\chi^2_{i,j}}{\sqrt{\chi^2_{i,j} + \sigma^2_{i,j}(t)}} \\ &:= h_{i,j}(t, \eta_{i,j}) \quad j = 1, \dots, n. \\ \dot{\eta}_{i,1} &= \frac{1}{k_{i,1}}\Big\{\dot{z}_{i,1} - \dot{k}_{i,1}\eta_{i,1}\Big\} \end{aligned}$$

$$
\begin{aligned}
&:= h_{i,n+1}(t, \hat{M}_{i,1}, \eta_{i,1}, \eta_{i,2}) \\
\dot{\eta}_{i,j} &= \frac{1}{k_{i,j}}\left\{\dot{z}_{i,j} - \dot{k}_{i,j}\eta_{i,j}\right\} \\
&:= h_{i,n+j}(t, \hat{M}_{i,1}, \ldots, \hat{M}_{i,j}, \eta_{i,1}, \ldots, \eta_{i,j+1}) \\
&\quad j = 2, \ldots, n-1 \\
\dot{\eta}_{i,n} &= \frac{n}{k_{i,n}}\left\{\dot{z}_{i,n} - \dot{k}_{i,n}\eta_{i,n}\right\} \\
&:= h_{i,2n}(t, \hat{M}_{i,1}, \ldots, \hat{M}_{i,n}, \eta_{i,1}, \ldots, \eta_{i,n})
\end{aligned}
$$

Hence, the $\dot{\mathcal{S}}_i$ can be written as

$$
\dot{\mathcal{S}}_i = h_i(t, \dot{\mathcal{S}}_i) = \begin{bmatrix} h_{i,1}(t, \eta_{i,1}) \\ \cdots \\ h_{i,n}(t, \eta_{i,n}) \\ h_{i,n+1}(t, \hat{M}_{i,1}, \eta_{i,1}, \eta_{i,2}) \\ \cdots \\ h_{i,2n}(t, \hat{M}_{i,1}, \ldots, \hat{M}_{i,n}, \eta_{i,1}, \ldots, \eta_{i,n}) \end{bmatrix} \tag{15}
$$

Furthermore, define the open set:

$$
\Omega_i = R^n \times \underbrace{(-1,1) \times \ldots \times (-1,1)}_{n-times} \tag{16}
$$

It is clear that $\mathcal{S}_i(0) = [\hat{M}^0_{i,1}, \ldots, \hat{M}^0_{i,n}, 0, \ldots, 0] \in \Omega_i$. The set $\Omega_i$ is nonempty and open. The constrains function $k_{i,j}, j = 1, \ldots, n$ have been selected to satisfy $k_{i,j}(0) > z_{i,j}(0), j = 1, \ldots, n$. Thus, $|\eta_{i,j}(0)| < 1, j = 1, \ldots, n$, which results in $\eta_{i,j}(0) \in \Omega_i$. Meanwhile, $h_i(t, \dot{\mathcal{S}}_i) : R_+ \times \Omega_i \to R_{2n}$ is piecewise continuous in $t$ and locally Lipschitz in $\Omega_i$. Owing to the fact that the desired trajectory $y_r$, the constrained functions $k_{i,j}, j = 1, \ldots, n$, and the tuning function $\varrho(t)$ are bounded and differentiable, the nonlinearities $f_{i,j}, q_{i,j}$ are continuously differentiable functions, the actual control laws $\alpha_{i,j}, j = 1, \ldots, n-1$ and the actual control law $u_i$ are smooth over $\Omega_i$. According to Theorem in Sontag (1990), the conditions on $h_i$ guarantees the existence and uniqueness of a maximal solution $\mathcal{S}_i$ on the time interval $[0, t_{max})$.

Next, seeking a contradiction. Consider a barrier Lyapunov function candidate $V_{i,j}$ as follows.

$$
V_{i,j} = \frac{1}{2}\log\frac{1}{1-\eta^2_{i,j}} + \frac{\iota_{i,j}\rho_{i,j}}{2\beta_{i,j}}\tilde{M}^2_{i,j} \tag{17}
$$

where $\tilde{M}_{i,j} = M_{i,j} - \hat{M}_{i,j}$; $\rho_{i,j}$ is an unknown constant and will be specified later. It is clear that $V_{i,j}, i = 1, \ldots, N; j = 1, \ldots, n$ are positive definite and continuously differentiable in the set $\Omega_i$.

*step* $(i, 1)$: The time derivative of $V_{i,1}$ leads to

$$
\begin{aligned}
\dot{V}_{i,1} = \frac{\chi_{i,1}}{k_{i,1}}\Bigg( & (b_i + d_i)\big(z_{i,2} + \alpha_{i,1} + \varrho(t)x^0_{i,2}\big) \\
&+ (b_i + d_i)\big(f_{i,1}(\bar{x}_{i,1}(t)) + p_{i,1}(\bar{x}_{i,1}(t - \tau_{i,1}))\big) \\
&- \sum_{g=1}^{N} a_{ig}x_{g,2} - \sum_{g=1}^{N} a_{ig}f_{g,1}(\bar{x}_{g,1}(t)) \\
&- \sum_{g=1}^{N} a_{ig}p_{g,1}(\bar{x}_{g,1}(t - \tau_{g,1})) - b_i\dot{y}_r \\
&- \dot{\varrho}(t)Q^0_{i,1} - K_{i,1}z_{i,1}\Bigg) - \frac{\iota_{i,1}\rho_{i,1}}{\beta_{i,1}}\tilde{M}_{i,1}\dot{\hat{M}}_{i,1}
\end{aligned} \tag{18}
$$

where $Q^0_{i,1} = \sum_{g\in\mathcal{N}_i} a_{ig}(x^0_{i,1} - x^0_{g,1}) + b_i(x^0_{i,1} - y^0_r)$ is a constant; $K_{i,1} = \frac{\dot{k}_{i,1}}{k_{i,1}}$.

Substituting the virtual control law (7) into (18), it yields

$$
\begin{aligned}
\dot{V}_{i,1} \le & -\psi_{i,1}\mathcal{H}_{i,1}\chi^2_{i,1} - \frac{\iota_{i,1}\mathcal{H}_{i,1}\chi^2_{i,1}\hat{M}_{i,1}}{\sqrt{\chi^2_{i,1} + \sigma^2_{i,1}(t)}} + \mathcal{F}_{i,1}|\chi_{i,1}| \\
& - \frac{\iota_{i,1}\rho_{i,1}}{\beta_{i,1}}\tilde{M}_{i,1}\dot{\hat{M}}_{i,1}
\end{aligned} \tag{19}
$$

where $\mathcal{H}_{i,1} = \frac{1}{k_{i,1}}$, and $\mathcal{F}_{i,1} = \frac{1}{k_{i,1}}\Big(\big|(b_i + d_i)\bar{f}_{i,1}(\bar{x}_{i,1}(t))\big| + \big|z_{i,2}\big| + \big|\sum_{g=1}^{N} a_{ig}\bar{f}_{g,1}(\bar{x}_{g,1}(t))\big| + \big|\sum_{g=1}^{N} a_{ig}\bar{p}_{g,1}(\bar{x}_{g,1}))\big| + \big|(b_i + d_i)\varrho(t)x^0_{i,2}\big| + \big|K_{i,1}z_{i,1}\big| + \big|\bar{p}_{i,1}(\bar{x}_{i,1})\big| + \big|b_i\dot{y}_r\big| + \big|\dot{\varrho}(t)Q^0_{i,1}\big|$.

It is known that $|\eta_{i,j}| < 1$, $k_{i,1}, \dot{k}_{i,1}, y_r, \dot{y}_r$, $\varrho, \dot{\varrho}, x_{i,1}$ are bounded. We can obtain that $0 \le \mathcal{H}_{i,1,L} \le \mathcal{H}_{i,1} \le \mathcal{H}_{i,1,U}$ and $0 \le \mathcal{F}_{i,1,L} \le \mathcal{F}_{i,1} \le \mathcal{F}_{i,1,U}$, where $\mathcal{H}_{i,1,L}$, $\mathcal{H}_{i,1,U}, \mathcal{F}_{i,1,L}, \mathcal{F}_{i,1,U}$ are unknown constants. Thus, substituting the corresponding adaptive compensator (8) into (19) and using the Lemma2, we have

$$
\begin{aligned}
\dot{V}_{i,1} \le & -\psi_{i,1}\mathcal{H}_{i,1,L}\chi^2_{i,1} - \frac{\iota_{i,1}\rho_{i,1}\chi^2_{i,1}M_{i,1}}{\sqrt{\chi^2_{i,1} + \sigma^2_{i,1}(t)}} \\
& + \iota_{i,1}\rho_{i,1}M_{i,1}|\chi_{i,1}| \\
\le & -\psi_{i,1}\mathcal{H}_{i,1,L}\chi^2_{i,1} + \mathcal{F}_{i,1,U}\sigma_{i,1}(t)
\end{aligned} \tag{20}
$$

where $M_{i,1} := \frac{\mathcal{F}_{i,1,U}}{\mathcal{H}_{i,1,L}\iota_{i,1}}, \rho_{i,1} := \mathcal{H}_{i,1,L}$.

Integrating (20) over $[0, t]$, it yields

$$
\begin{aligned}
V_{i,1}(t) + \int_0^t \psi_{i,1}\mathcal{H}_{i,1,L}\chi^2_{i,1}(s)ds \le \\
V_{i,1}(0) + \mathcal{F}_{i,1,U}\int_0^t \sigma_{i,1}(s)ds, \forall t \in [0, +\infty).
\end{aligned} \tag{21}
$$

which indicates that

$$
\begin{aligned}
&\frac{1}{2}\log\frac{1}{1-\eta^2_{i,1}} \le V_{i,1} \le V_{i,1}(0) + \mathcal{F}_{i,1,U}\sigma_{i,1,1} = \mathcal{D}_{i,1} \\
&\frac{\iota_{i,1}\rho_{i,1}}{2\beta_{i,1}}\tilde{M}^2_{i,1} \le V_{i,1} \le \mathcal{D}_{i,1}.
\end{aligned}
$$

Thus, we can obtain

$$
\begin{aligned}
|\eta_{i,1}| &\le \sqrt{1 - e^{-2\mathcal{D}_{i,1}}} \le 1 \\
|\hat{M}_{i,1}| &\le \sqrt{\frac{2\beta_{i,1}\mathcal{D}_{i,1}}{\iota_{i,1}\rho_{i,1}}} + M_{i,1}.
\end{aligned}
$$

Then, it deduces that $\alpha_{i,1}$ and $x_{i,2}$ are bounded.

*step* $(i, j(j = 2, \ldots, n-1))$: Taking the time derivative of $V_{i,j}$, it yields

$$\dot{V}_{i,j} = \frac{\chi_{i,j}}{k_{i,j}}\Big((z_{i,j+1} + \alpha_{i,j} + \varrho(t)x^0_{i,j+1}) + f_{i,j}(\bar{x}_{i,j}(t)) + p_{i,j}(\bar{x}_{i,j}(t-\tau_{i,j})) - \dot{\alpha}_{i,j-1} - \dot{\varrho}(t)x^0_{i,j} - K_{i,j}z_{i,j}\Big) - \frac{\iota_{i,j}\rho_{i,j}}{\beta_{i,j}}\tilde{M}_{i,j}\dot{\hat{M}}_{i,j} \quad (22)$$

where $K_{i,j} = \frac{\dot{k}_{i,j}}{k_{i,j}}$.

Employing the virtual control law (10), we have

$$\dot{V}_{i,j} \le -\psi_{i,j}\mathcal{H}_{i,j}\chi^2_{i,j} - \frac{\iota_{i,j}\mathcal{H}_{i,j}\chi^2_{i,j}\hat{M}_{i,j}}{\sqrt{\chi^2_{i,j} + \sigma^2_{i,j}(t)}} + \mathcal{F}_{i,j}|\chi_{i,j}| - \frac{\iota_{i,j}\rho_{i,j}}{\beta_{i,j}}\tilde{M}_{i,j}\dot{\hat{M}}_{i,j} \quad (23)$$

where $\mathcal{H}_{i,j} = \frac{1}{k_{i,j}}$, $\mathcal{F}_{i,j} = \frac{1}{k_{i,j}}\big(|z_{i,j+1}| + |\varrho(t)x^0_{i,j+1}| + |\bar{f}_{i,j}(\bar{x}_{i,j})| + |\bar{p}_{i,j}(\bar{x}_{i,j})| + |\dot{\alpha}_{i,j-1}| + |\dot{\varrho}(t)x^0_{i,j}| + |K_{i,j}z_{i,j}|\big)$.

It is known that $|\eta_{i,j}| < 1, k_{i,j}, \dot{k}_{i,j}, y_r, \dot{y}_r, \varrho, \dot{\varrho}, x_{i,1}, \ldots, x_{i,j}, \dot{\alpha}_{j-1}$ are bounded. We can obtain that $0 \le \mathcal{H}_{i,j,L} \le \mathcal{H}_{i,j} \le \mathcal{H}_{i,j,U}$ and $0 \le \mathcal{F}_{i,j,L} \le \mathcal{F}_{i,j} \le \mathcal{F}_{i,j,U}$, where $\mathcal{H}_{i,j,L}, \mathcal{H}_{i,j,U}, \mathcal{F}_{i,j,L}, \mathcal{F}_{i,j,U}$ are unknown constants.

Next, employing (11) and Lemma2, (23) becomes

$$\begin{aligned}\dot{V}_{i,j} &\le -\psi_{i,j}\mathcal{H}_{i,j,L}\chi^2_{i,j} - \frac{\iota_{i,j}\rho_{i,j}\chi^2_{i,j}M_{i,j}}{\sqrt{\chi^2_{i,j} + \sigma^2_{i,j}(t)}} + \iota_{i,j}\rho_{i,j}M_{i,j}|\chi_{i,j}| \\ &\le -\psi_{i,j}\mathcal{H}_{i,j,L}\chi^2_{i,j} + \mathcal{F}_{i,j,U}\sigma_{i,j}(t) \end{aligned} \quad (24)$$

where $M_{i,j} := \frac{\mathcal{F}_{i,j,U}}{\mathcal{H}_{i,j,L}\iota_{i,j}}, \rho_{i,j} := \mathcal{H}_{i,j,L}$.

Integrating (24) over $[0, t]$, it yields

$$V_{i,j}(t) + \int_0^t \psi_{i,j}\mathcal{H}_{i,j,L}\chi^2_{i,j}(s)ds \le V_{i,j}(0) + \mathcal{F}_{i,j,U}\int_0^t \sigma_{i,j}(s)ds, \forall t \in [0, +\infty)$$
$$j = 2, \ldots, n. \quad (25)$$

which indicates that

$$\frac{1}{2}\log\frac{1}{1-\eta^2_{i,j}} \le V_{i,j} \le V_{i,j}(0) + \mathcal{F}_{i,j,U}\sigma_{i,j,1} = \mathcal{D}_{i,j}$$
$$\frac{\iota_{i,j}\rho_{i,j}}{2\beta_{i,j}}\tilde{M}^2_{i,j} \le V_{i,j} \le \mathcal{D}_{i,j}, j = 2, \ldots, n.$$

Thus, we can obtain

$$|\eta_{i,j}| \le \sqrt{1 - e^{-2\mathcal{D}_{i,j}}} \le 1$$
$$|\hat{M}_{i,j}| \le \sqrt{\frac{2\beta_{i,j}\mathcal{D}_{i,j}}{\iota_{i,j}\rho_{i,j}}} + M_{i,j}, w = 2, \ldots, n.$$

Then, it deduces that $\alpha_{i,j}$ and $x_{i,j+1}$ are bounded. Differentiating $\alpha_{i,j}$, we have

$$\begin{aligned}\dot{\alpha}_{i,j} = -\Bigg\{&\dot{\chi}_{i,j}\Bigg(\psi_{i,j} + \frac{\iota_{i,j}\hat{M}_{i,j}}{\sqrt{\chi^2_{i,j} + \sigma^2_{i,j}(t)}} - \frac{\iota_{i,j}\hat{M}_{i,j}\chi^2_{i,j}}{(\chi^2_{i,j} + \sigma^2_{i,j}(t))^{\frac{3}{2}}}\Bigg) + \chi_{i,j}\Bigg(\frac{\iota_{i,j}\beta_{i,j}\chi^2_{i,j}}{\chi^2_{i,j} + \sigma^2_{i,j}(t)} - \\ &\frac{\iota_{i,j}\hat{M}_{i,j}\sigma_{i,j}(t)\dot{\sigma}_{i,j}(t)}{(\chi^2_{i,j} + \sigma^2_{i,j}(t))^{\frac{3}{2}}}\Bigg)\Bigg\}, \forall t \in [0, t_{max}] \\ &j = 2, \ldots, n-1 \end{aligned} \quad (26)$$

The time derivative of $\chi_{i,j}$ is

$$|\dot{\chi}_{i,j}| \le \frac{1 + \eta^2_{i,j}}{k_{i,j}(1 - \eta^2_{i,j})^2}\Big(|z_{i,j+1}| + |\bar{f}_{i,j}(\bar{x}_{i,j})| + |\alpha_{i,j}| + |\bar{p}_{i,j}(\bar{x}_{i,j})| + |\dot{\varrho}(t)x^0_{i,j}| + |K_{i,j}z_{i,j}|\Big) \quad (27)$$

which is straightforward to deduce the boundedness of $\dot{\chi}_{i,j}$. Immediately, $\dot{\alpha}_{i,j}$ is also bounded.

*step* $(i, n)$: The time derivative of $V_{i,n}$ is

$$\dot{V}_{i,n} = \frac{\chi_{i,n}}{k_{i,n}}\Big(u_i + f_{i,n}(\bar{x}_{i,n}(t)) - \dot{\alpha}_{i,n-1} + p_{i,n}(\bar{x}_{i,n}(t-\tau_{i,n})) - \dot{\varrho}(t)x^0_{i,n} - K_{i,n}z_{i,n}\Big) - \frac{\iota_{i,n}\rho_{i,n}}{\beta_{i,n}}\tilde{M}_{i,n}\dot{\hat{M}}_{i,n} \quad (28)$$

where $K_{i,n} = \frac{\dot{k}_{i,n}}{k_{i,n}}$.

Substituting the actual control law (13) into (28), it follows that

$$\dot{V}_{i,n} \le -\psi_{i,n}\mathcal{H}_{i,n}\chi^2_{i,n} - \frac{\iota_{i,n}\mathcal{H}_{i,n}\chi^2_{i,n}\hat{M}_{i,n}}{\sqrt{\chi^2_{i,n} + \sigma^2_{i,n}(t)}} + \mathcal{F}_{i,n}|\chi_{i,n}| - \frac{\iota_{i,n}\rho_{i,n}}{\beta_{i,n}}\tilde{M}_{i,n}\dot{\hat{M}}_{i,n} \quad (29)$$

where $\mathcal{H}_{i,n} = \frac{1}{k_{i,n}}$, and $\mathcal{F}_{i,n} = \frac{1}{k_{i,n}}\big(|\bar{f}_{i,n}(\bar{x}_{i,n}(t))| + |\bar{p}_{i,n}(\bar{x}_{i,n})| + |\dot{\alpha}_{i,n-1}| + |\dot{\varrho}(t)x^0_{i,n}| + |K_{i,n}z_{i,n}|\big)$.

Based on the fact that $|\eta_{i,n}| < 1, k_{i,n}, \dot{k}_{i,n}, y_r, \dot{y}_r, \varrho, \dot{\varrho}, x_{i,1}, \ldots, x_{i,n}, \dot{\alpha}_{n-1}$ are bounded, we can know that $0 \le \mathcal{H}_{i,n,L} \le \mathcal{H}_{i,n} \le \mathcal{H}_{i,n,U}$ and $0 \le \mathcal{F}_{i,n,L} \le \mathcal{F}_{i,n} \le \mathcal{F}_{i,n,U}$, where $\mathcal{H}_{i,n,L}, \mathcal{H}_{i,n,U}, \mathcal{F}_{i,n,L}, \mathcal{F}_{i,n,U}$ are unknown constants.

Furthermore, employing (14), (29) becomes

$$\begin{aligned}\dot{V}_{i,n} &\le -\psi_{i,n}\mathcal{H}_{i,n,L}\chi^2_{i,n} - \frac{\iota_{i,n}\rho_{i,n}\chi^2_{i,n}M_{i,n}}{\sqrt{\chi^2_{i,n} + \sigma^2_{i,n}(t)}} + \iota_{i,n}\rho_{i,n}M_{i,n}|\chi_{i,n}| \\ &\le -\psi_{i,n}\mathcal{H}_{i,n,L}\chi^2_{i,n} + \mathcal{F}_{i,n,U}\sigma_{i,n}(t) \end{aligned} \quad (30)$$

where $M_{i,n} := \frac{\mathcal{F}_{i,n,U}}{\mathcal{H}_{i,n,L}\iota_{i,n}}, \rho_{i,n} := \mathcal{H}_{i,n,L}$.

Integrating (20), (24) and (30) over $[0, t]$, it yields

$$V_{i,n}(t) + \int_0^t \psi_{i,n}\mathcal{H}_{i,n,L}\chi^2_{i,n}(s)ds \le$$

$$V_{i,n}(0) + \mathcal{F}_{i,n,U} \int_0^t \sigma_{i,n}(s)ds, \forall t \in [0, +\infty) \quad (31)$$

which indicates that

$$\frac{1}{2}\log\frac{1}{1-\eta_{i,n}^2} \leq V_{i,n} \leq V_{i,n}(0) + \mathcal{F}_{i,n,U}\sigma_{i,n,1} = \mathcal{D}_{i,n}$$
$$\frac{\iota_{i,n}\rho_{i,n}}{2\beta_{i,n}}\tilde{M}_{i,n}^2 \leq V_{i,n} \leq \mathcal{D}_{i,n}.$$

Thus, we can obtain

$$|\eta_{i,n}| \leq \sqrt{1-e^{-2\mathcal{D}_{i,n}}} \leq 1$$
$$|\hat{M}_{i,n}| \leq \sqrt{\frac{2\beta_{i,n}\mathcal{D}_{i,n}}{\iota_{i,n}\rho_{i,n}}} + M_{i,n}.$$

Thus, there exists a compact subset $\Omega_i^s = [-\hat{M}_{i,1}, \hat{M}_{i,1}] \times \ldots \times [-\hat{M}_{i,n}, \hat{M}_{i,n}] \times [-\eta_{i,1}, \eta_{i,1}] \times \ldots \times [-\eta_{i,n}, \eta_{i,n}] \subset \Omega_i$ such that the maximal solution of (15) satisfies $\mathcal{S}_i \in \Omega_i^s$ for all $t \in 0, t_{max})$. As stated by the result of Bechlioulis and Rovithakis (2016), it follows that $|\eta_{i,j}| \leq 1, j = 1, 2, \ldots, n, \forall t \in [0, +\infty)$. Consequently, we can show that $\alpha_{i,1}, \ldots, \alpha_{i,n-1}, u_i, x_{i,1}, \ldots, x_{i,n}, \hat{M}_{i,1}, \ldots, \hat{M}_{i,n}$ are bounded.

Moreover, from (31), we have

$$\int_0^t \psi_{i,1}\mathcal{H}_{i,1,L}\chi_{i,1}^2(s)ds \leq V_{i,1}(0) + \mathcal{F}_{i,1,U}\sigma_{i,1,1} \quad (32)$$

By using the Barbalat lemma, we can know that $\lim_{t\to\infty}\chi_{i,1} \to 0$. Then, we can know that $\lim_{t\to\infty} z_{i,1} \to 0$ for all $i = 1, 2, \ldots, N$. Furthermore, using Lemma1, we can conclude that the synchronization error $\|y_e\|$ converges to zero, which means that the output asymptotic synchronization is achieved.

## 4. SIMULATION RESULTS AND DISCUSSION

### *4.1 Simulation results*

In this section, the developed control scheme will be verified by employing it to the example in Chen et al. (2018). Each follower is modeled as follow.

$$\begin{cases} \dot{x}_{i,1} &= x_{i,2} + x_{i,1}^2 + 0.1\sin(t)x_{i,1} + \cos(x_{i,1}(t-\tau_{i,1})) \\ \dot{x}_{i,2} &= x_{i,3} + x_{i,1}x_{i,1}^2 + 0.1\cos(t)x_{i,2}^2 + \\ & \quad x_{i,1}(t-\tau_{i,2})x_{i,2}(t-\tau_{i,2}) \\ \dot{x}_{i,3} &= u_i + x_{i,1}^2 x_{i,3} + 0.05\cos(\pi t)x_{i,1}x_{i,1}x_{i,3} + \\ & \quad x_{i,1}(t-\tau_{i,3})x_{i,3}(t-\tau_{i,3}) \\ y_i(t) &= x_{i,1}(t), \ i = 1, \ldots, N. \end{cases}$$

The considered MASs consists of a leader and five followers, the communication topology depicting their relationship is given in Fig.1.

The trajectory of the leader is $y_r(t) = 0.5(\sin(t) + \sin(0.5t))$. The control objective is to achieve that the outputs $y_i(t) = x_{i,1}(t)$ of all the followers can track the desired trajectory $y_r(t)$, asymptotically. In this simulation,

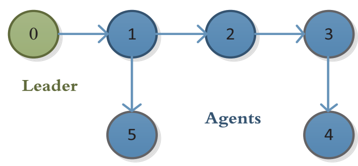
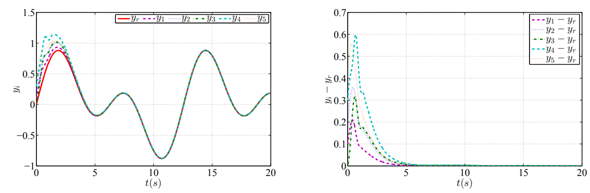


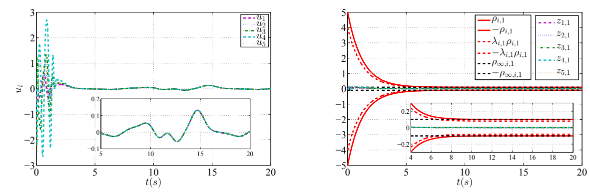

Fig. 1. Communication topology for the MASs

the state initials are set as: $\tau_{i,1} = 0.1, \tau_{i,2} = 0.15, \tau_{i,3} = 0.2$. The state initials are set as: $x_{1,1}^0 = x_{1,2}^0 = x_{1,3}^0 = 0.1, x_{2,1}^0 = x_{2,2}^0 = x_{2,3}^0 = 0.2, x_{3,1}^0 = x_{3,2}^0 = x_{3,3}^0 = 0, x_{4,1}^0 = x_{4,2}^0 = x_{4,3}^0 = 0.3, x_{5,1}^0 = x_{5,2}^0 = x_{5,3}^0 = 0.4$. The initial values of other coefficients are set as: $\hat{M}_{i,1}(0) = 0.5, \hat{M}_{i,2}(0) = 1.5, \hat{M}_{i,3}(0) = 12$ for $1 \leq i \leq 5$. The other control coefficients are set as: $\iota_{i,1} = 3, \iota_{i,2} = 2, \iota_{i,3} = 2, \psi_{i,1} = 40, \psi_{i,2} = 40, \psi_{i,3} = 80, \beta_{i,1} = \beta_{i,2} = \beta_{i,3} = 5, \sigma_{i,1}(t) = \sigma_{i,2}(t) = \sigma_{i,3}(t) = \frac{1}{t^2+0.1}$ for $1 \leq i \leq 5$. Constrained functions are set as: $\rho_{0,i,1} = 5, \rho_{0,i,2} = 5, \rho_{0,i,3} = 20, \rho_{\infty,i,1} = 0.1, \rho_{\infty,i,2} = 0.5, \rho_{\infty,i,3} = 15, \lambda_{i,1} = \lambda_{i,2} = \lambda_{i,3} = 0.8, a_{i,1} = a_{i,2} = a_{i,3} = 0.8; i = 1, 2, 3, 4, 5$; $A = (a_{ig})_{5\times5}$, $B = diag\{b_1, b_2, \ldots, b_N\}$ are set as:

$$A = \begin{bmatrix} 0&0&0&0&0 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 1&0&0&0&0 \end{bmatrix}, B = \begin{bmatrix} 1&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$$



(a) Tracking performance of outputs $y_i(t)$ (b) Output tracking error $y_i(t) - y_r(t)$



(c) Control input $u_i$ (d) State errors $z_{i,1}$

Fig. 2. The simulation results using the control scheme developed in this paper:(a) Tracking performance of outputs $y_i(t)$; (b) Output tracking error $y_i(t) - y_r(t)$; (c) Control input $u_i$; (d) State errors $z_{i,1}$

The simulation results are shown in Fig.2. Fig.2(a) shows the tracking performance curves of the $y_i(t)$. The output of the each agent tracks the desired trajectory asymptotically. Fig.2(b) shows the output tracking error of each agent, and the output tracking error converges to zero eventually. Fig.2(c) shows the control input of each agent. Fig.2(d) shows the state error of each agent, it is clear that the state error $z_{i,1}$ of each agent is limited in the defined PPF, which means $|z_{i,1}| \leq k_{i,1}$. All shown signals in Fig.2

are guaranteed to be bounded. Obviously, the simulation performance confirms our theoretical result.

## 5. CONCLUSION

This paper developed a novel adaptive controller for a class of high-order nonlinear multiagent state time-delay systems. The unknown system nonlinearities and state time-delay terms were regarded as "disturbance-like" terms, which were guaranteed to be bounded by using the barrier functions. A novel compensator was designed to eliminate the "disturbance-like" terms, which leads to the asymptotic synchronization. The synchronization errors are limited in the prescribed tracking functions. Some simulations were considered to illustrate the effectiveness of the developed controller.

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