Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Robust backstepping control of uncertain nonlinear systems with unknown time-varying input delay

Ashish Kumar Jain * **Shubhendu Bhasin** **

* *Department of EI Engineering, M.J.P.Rohilkhand University, Bareilly, India (e-mail: akjain00@gmail.com).*
** *Department of Electrical Engineering, Indian Institute of Technology, Delhi, India (e-mail: sbhasin@ee.iitd.ac.in).*

Abstract: A robust compensator is developed for a class of strict-feedback uncertain nonlinear systems with additive disturbance and unknown time-varying input delay. The compensator is composed of a Proportional-Integral (PI) control and delay compensation term based on a finite integral of the past control values. The sufficient inequality conditions on controller gains and upper bound of input delay are derived using a Lyapunov-based stability analysis by choosing suitable L-K functionals, which guarantee a global uniformly ultimately bounded (GUUB) tracking result. Simulation results show the performance and robustness of controller for different values of time-varying input delay.

Keywords: Unknown input delay, Lyapunov-Krasovskii functionals, Strict-feedback system.

1. INTRODUCTION

Early techniques of solving the problems of linear systems with known, input delays are available in Artstein (1982), Bahill (1983), Nihtilä (1989), Krstic (2010). Motivated by the fact that, most of the applications, where exact knowledge of delay is not available, the authors in Bresch-Pietri and Krstic (2009), Bresch-Pietri et al. (2012) proposed the adaptive control based design for the problems of uncertain dynamics with unknown delay. The stabilization and control design problem of nonlinear systems with constant input delays is discussed in Krstic (2008),Krstic (2009), Mazenc and Bliman (2006), Mazenc et al. (2012), Sharma et al. (2011), Karafyllis (2011), Krstic and Smyshlyaev (2008), Mazenc et al. (2004), Mazenc et al. (2013), Pepe et al. (2008). Adaptive control law based result of nonlinear systems with constant, unknown delay is addressed in Bresch-Pietri and Krstic (2014), while the dynamics is considered as known.

The literature on nonlinear systems with time-varying input delay is available in Choi and Lim (2010), Koo et al. (2012), Merad et al. (2016), Kamalapurkar et al. (2016), Obuz et al. (2017). The authors in Choi and Lim (2010), Koo et al. (2012) proposed output feedback regulation for chain of integrator system with unknown time-varying delays. A robust control law is designed for the class of nonlinear systems with known time-varying input delay in Merad et al. (2016), Kamalapurkar et al. (2016), while, the authors in Obuz et al. (2017), presented the robust control design method for uncertain dynamics with small unknown input delay.

For nonlinear systems in strict-feedback form with arbitrarily large input delay Mazenc and Bliman (2006), used backstepping control design to prove global asymptotic stability by compensating for known constant input delay. This work is extended for the known pointwise delay in the input in Mazenc et al. (2011). The authors in Zhou et al. (2009) proposed standard backstepping design to develop an adaptive controller for a non-minimum phase system with unknown input delay and unmodeled dynamics.

The compensation of unknown input delay using predictor feedback techniques, which requires exact knowledge of delay, becomes a challenge. Still fewer results exist which solve the problem of nonlinear systems with unknown actuator delay. A recent result in Bresch-Pietri and Krstic (2014) utilized an adaptive control scheme for adaptation of the unknown constant actuator delay for unstable nonlinear systems, however, it requires exact knowledge of system dynamics. The authors in Jain and Bhasin (2020) developed a robust control law which includes a Proportional-Integral (PI) control action and delay compensator for a class of uncertain nonlinear systems with unknown constant input delay. The work in Jain and Bhasin (2020) is extended for the compensation of time-varying input delay for uncertain nonlinear system (Brunowski form) in Jain and Bhasin (2019).

This paper mainly contributes the development of a robust compensator for a class of uncertain nonlinear systems in strict-feedback form with additive disturbance and unknown time-varying input delay. This design requires knowledge of upper bound of unknown time-varing delay. The controller is composed of a PI control and delay compensator term comprising a filtered tracking error signal which includes an integral of past values of control signal where the limits of integration are based on known bound of delay. The delay terms are cancelled out in stability analysis by choosing of suitable Lyapunov-Krasovski functionals, and global uniformly ultimately bounded tracking result is obtained. Two illustrative examples are considered to demonstrate the robustness and performance of the controller.

2. PROBLEM FORMULATION & ASSUMPTIONS

Consider a nonlinear system described as

$$\begin{cases} \dot{x}_i\left(t\right)= & x_{i+1}\left(t\right)+f_i\left(x_1\left(t\right),\cdots,x_i\left(t\right),t\right), \\ & 1\leq i\leq n-1 \\ \dot{x}_n\left(t\right)= & f_n\left(x\left(t\right),t\right)+g\left(x\left(t\right),t\right)u\left(t-\tau\left(t\right)\right)+d\left(t\right) \\ y\left(t\right)= & x_1\left(t\right) \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^{n^2}$ is a vector of measurable system states defined as $x\left(t\right)\triangleq\left[\,x_1^T\left(t\right),\,x_2^T\left(t\right),\,\cdots,\,x_n^T\left(t\right)\right]^T$, $u(t)\in\mathbb{R}^n$ denotes the control input vector, $\tau\left(t\right)\in\mathbb{R}^+$ is an unknown time-varying input delay, $f_j(x_1\left(t\right),\ldots,x_j\left(t\right),t)\in\mathbb{R}^n, j=1,...,n$ and $g\left(x\left(t\right),t\right)\in\mathbb{R}^{n\times n}$ are unknown smooth functions and $d(t)\in\mathbb{R}^n$ represents disturbances. The objective is to develop a controller $u(t)$ such that the the system output trajectory $y(t)$ tracks the desired trajectory $y_d(t)$. The following assumptions are considered for subsequent development.

Assumption 1. The desired trajectory $y_d(t)$ and its' derivatives are bounded by known positive constants.

Assumption 2. The delay $\tau\left(t\right)$ is upper bounded as $\tau\left(t\right)\leq\bar{\tau}$, where $\bar{\tau}\in\mathbb{R}^+$ is a known positive constant. The derivative of $\tau\left(t\right)$ is bounded such that $\mid\dot{\tau}\left(t\right)\mid\leq\mathit{\Gamma}\leq 1$, where $\mathit{\Gamma}\in\mathbb{R}^+$ is a known constant.

Assumption 3. The unknown functions $f_j(x_1,\cdots,x_j,t)$, $1\leq j\leq n$ satisfy the following growth condition:
$\|f_j(x_1,\cdots,x_j,t)\|\leq\zeta_{j1}\parallel\bar{x}_j\parallel+\zeta_{j2}$,
where $\bar{x}_j=\left[\,x_1\left(t\right),\,x_2\left(t\right),\,\cdots,\,x_j\left(t\right)\right]^T$ and ζ_{j1},ζ_{j2} are known positive constants.

Assumption 4. The function $g(X,t)$ is lower and upper bounded as $\underline{g}\leq\parallel g\left(X,t\right)\parallel\leq\bar{g}$, where $\underline{g},\bar{g}\in\mathbb{R}^+$ are known constants.

Assumption 5. $\|d(t)\|\leq\bar{d}$, where $\bar{d}\in\mathbb{R}^+$ is a known constant.

Assumption 6. The system dynamics in (1) does not exhibit the finite escape time phenomenon for $t\in[0,\tau\left(t\right))$.

3. CONTROL DEVELOPMENT AND STABILITY ANALYSIS

This section presents the control design procedure by using the backstepping procedure:
Step 1: The position tracking error denoted by $e_1(t)\in\mathbb{R}^n$, is defined as

$$e_1(t)\triangleq x_1(t)-x_{1d}(t) \tag{2}$$

and

$$e_2\left(t\right)\triangleq x_2\left(t\right)-u_1\left(t\right), \tag{3}$$

where, $u_1\left(t\right)$ denotes the virtual control input of state $x_2\left(t\right)$. Using (1), (3) and differentiating (2), yields

$$\dot{e}_1=e_2+u_1+\triangle_1-\dot{x}_{1d}, \tag{4}$$

where the function $\triangle_1\left(f_1,t\right)\in\mathbb{R}^n$ is defined as

$$\triangle_1\left(f_1,t\right)\triangleq f_1\left(x_1,t\right). \tag{5}$$

The virtual control law can be designed as

$$u_1=\dot{x}_{1d}-k_1e_1 \tag{6}$$

Consider a Lyapunov function candidate V_1 defined as

$$V_1\triangleq\frac{1}{2}e_1^Te_1 \tag{7}$$

Using (4), (6) and differentiating (7), yields

$$\dot{V}_1=-k_1\parallel e_1\parallel^2+e_1^Te_2+e_1^T\triangle_1 \tag{8}$$

Step 2: The error signal $e_3\in\mathbb{R}^n$ for the state x_3 is defined as

$$e_3\triangleq x_3-u_2 \tag{9}$$

where u_2 is virtual control input of state x_3, defined as

$$u_2\triangleq\ddot{x}_{1d}-k_1e_2+k_1^2e_1-k_2e_2 \tag{10}$$

Differentiating (3), using (9) and (10), the derivative of e_2 is obtained as

$$\dot{e}_2=e_3-k_2e_2+\triangle_2 \tag{11}$$

where, the function $\triangle_2\left(k_1,f_1,f_2,t\right)\in\mathbb{R}^n$ is defined as

$$\triangle_2\left(k_1,f_1,f_2,t\right)\triangleq k_1f_1\left(x_1,t\right)+f_2\left(x_1,x_2,t\right) \tag{12}$$

Consider a Lyapunov function candidate V_2 defined as

$$V_2\triangleq V_1+\frac{1}{2}e_2^Te_2 \tag{13}$$

Differentiating (13), using (8) and (11), the following equation is obtained as

$$\dot{V}_2=-k_1e_1^2-k_2e_2^2+e_1^Te_2+e_2^Te_3+e_1^T\triangle_1+e_2^T\triangle_2 \tag{14}$$

Step $i\,(3\leq i\leq n-2)$**:** The error signal is defined as

$$e_i\triangleq x_i-u_{i-1} \tag{15}$$

Using (1) and differentiating (15), the dynamics of error e_i is obtained as

$$\dot{e}_i=e_{i+1}+u_i+f_i\left(x_1,x_2,\ldots,x_i\right)-\dot{u}_{i-1} \tag{16}$$

where the virtual control law u_i can be designed as

$$u_i\triangleq x_{1d}^{(i)}+\sum_{l=1}^{i}(-1)^l\left(\sum_{j=1}^{i-l+1}k_j^l\right)e_{i-l+1} + \Psi_i\left(k_1,\cdots,k_{i-1},e_2,\cdots,e_{i-1}\right) \tag{17}$$

where the function $\Psi_i\left(k_1,\cdots,k_{i-1},e_2,\cdots,e_{i-1}\right)$ contains the terms utilized to cancel out the terms in expression of (16), and $k_i\in\mathbb{R}^+$ are control gains. Substituting the expression of u_i given in (17), in (16), the closed-loop dynamics of e_i is obtained as

$$\dot{e}_i=e_{i+1}-k_ie_i+\triangle_i\left(k_1,\cdots,k_{i-1},f_1,f_2,\cdots,f_i\right). \tag{18}$$

Consider the Lyapunov function candidate

$$V_i=V_{i-1}+\frac{1}{2}e_i^Te_i. \tag{19}$$

Differentiating (19), using (18), the derivative of V_i is obtained as

$$\dot{V}_i=-\sum_{j=1}^{i}k_j\parallel e_j\parallel^2+\sum_{j=1}^{i}e_j^Te_{j+1}+\sum_{j=1}^{i}e_j^T\triangle_j\left(f_1,\cdots,f_j\right) \tag{20}$$

Step n-1: Defining following error variables

$$e_{n-1}\triangleq x_{n-1}-u_{n-2} \tag{21}$$

and

$$e_n\triangleq x_n-u_{n-1}+e_\phi+u \tag{22}$$

where e_ϕ is an auxiliary error signal, defined as

$$e_\phi\triangleq\mu\int_{t-\bar{\tau}}^{t}u(\phi)d\phi, \tag{23}$$

where $\mu\in\mathbb{R}^+$ is a known control gain. Using (1), (2) and (22), the error dynamics is given by differentiating (21) as

$$\dot{e}_{n-1}=e_n+f_{n-1}\left(x_1,x_2,\ldots,x_{n-1}\right)-\dot{u}_{n-2}+u_{n-1}-e_\phi - u. \tag{24}$$

The virtual control input u_{n-1} is defined as

$$u_{n-1} \triangleq x_{1d}^{(n-1)} + \sum_{l=1}^{n-1} (-1)^l \left(\sum_{j=1}^{n-l} k_j^l \right) e_{n-l} + \Psi_{n-1}(k_1, \cdots, k_{n-2}, e_2, \cdots, e_{n-2}) \quad (25)$$

Substituting the expression of u_{n-1} given in (25), in (24), the closed-loop dynamics of e_{n-1} is obtained as

$$\dot{e}_{n-1} = e_n - e_\phi - u - k_{n-1}e_{n-1} + \triangle_{n-1}(k_1, \cdots, k_{n-2}, f_1, f_2, \cdots, f_{n-1}) \quad (26)$$

Consider the Lyapunov function candidate

$$V_{n-1} = V_{n-2} + \frac{1}{2} e_{n-1}^T e_{n-1}. \quad (27)$$

Differentiating (27), using (26), the time derivative of V_{n-1} is obtained as

$$\begin{aligned} \dot{V}_{n-1} = &- \sum_{j=1}^{n-1} k_j \parallel e_j \parallel^2 + \sum_{j=1}^{n-1} e_j^T e_{j+1} \\ &+ \sum_{j=1}^{n-1} e_j^T \triangle_j (k_1, \cdots, k_{j-1}, f_1, \cdots, f_j) \\ &- e_{n-1}^T e_\phi - e_{n-1}^T u \end{aligned} \quad (28)$$

Step n: Differentiating (22), using (1), (23), the following equation is obtained as

$$\begin{aligned} \dot{e}_n = &f_n(x(t), t) + g(x(t), t) u(t - \tau(t)) + d(t) \\ &+ \mu u - \mu u(t - \bar{\tau}) + \dot{u} - \dot{u}_{n-1} \end{aligned} \quad (29)$$

where, $\dot{u} \in \mathbb{R}^n$ is defined as

$$\dot{u} \triangleq -\mu u - k e_n \quad (30)$$

where $k \in \mathbb{R}^+$ is a known control gain. Adding and subtracting the term $g(x(t), t) u(t - \bar{\tau})$ in (29), using (25) and (30), cancelling the common terms, the closed-loop equation of error signal $e_n(t)$ is obtained as

$$\begin{aligned} \dot{e}_n = &- (\mu - g(x(t), t)) u(t - \bar{\tau}) \\ &+ g(x(t), t) \{u(t - \tau(t)) - u(t - \bar{\tau})\} \\ &+ d(t) - \left(k - \sum_{j=1}^{n-1} k_j \right) e_n - \sum_{j=1}^{n-1} k_j e_\phi \\ &+ \sum_{j=1}^{n-1} k_j u + \sigma_1 e_1 + \sigma_2 e_2 + \cdots + \sigma_{n-1} e_{n-1} \\ &+ \triangle_n (k_1, \cdots, k_{n-1}, f_1, \cdots, f_n) \end{aligned} \quad (31)$$

where, $\sigma_1, \sigma_2, \cdots, \sigma_{n-1}$ are constant functions of virtual control gains $k_1, k_2, \cdots, k_{n-1}$. The corollary of the Mean Value Theorem (Theorem 5.19 in Rudin et al. (1964)), is utilized on $u(t - \bar{\tau}) - u(t - \tau)$ given in (31), yielding

$$\parallel u(t - \bar{\tau}) - u(t - \tau(t)) \parallel \leq | (\tau(t) - \bar{\tau}) | \parallel \dot{u}(t - \hat{\tau}(t)) \parallel \quad (32)$$

where $\hat{\tau}(t) \in (\tau(t), \bar{\tau})$ is an auxiliary delay, defined as

$$\hat{\tau}(t) = \theta \tau(t) + (1 - \theta) \bar{\tau} \quad (33)$$

where $\theta \in (0, 1)$ is a constant. Consider a Lyapunov-Krasovskii (LK) functional V_n defined as

$$V_n = V_{n-1} + \frac{1}{2} e_n^T e_n + \frac{1}{2} u^T u + P + Q + R + S \quad (34)$$

where, P, Q, R and S are LK functionals defined as

$$P(t) \triangleq \xi \int_{t - \hat{\tau}(t)}^{t} \|\dot{u}(\phi)\|^2 d\phi \quad (35)$$

$$Q(t) \triangleq \frac{\omega_1}{2\bar{\tau} k^2 \mu^2} \int_{t - \hat{\tau}(t)}^{t} \left(\int_s^t \|\dot{u}(\phi)\|^2 d\phi \right) ds \quad (36)$$

$$R(t) \triangleq \frac{n+6}{2k} (\mu - \underline{g})^2 \int_{t - \bar{\tau}}^{t} \|u(\phi)\|^2 d\phi \quad (37)$$

$$S(t) \triangleq 3 \left(\sum_{j=1}^{n-1} k_j \right)^2 \left(\frac{n+6}{2k} \right) \bar{\tau} \mu^2 \int_{t - \bar{\tau}}^{t} \int_s^t \|u(\phi)\|^2 d\phi ds. \quad (38)$$

Differentiating (34), using (28), (30) and (31), the derivative of V_n is obtained as

$$\begin{aligned} \dot{V}_n = &- \sum_{j=1}^{n-1} k_j \parallel e_j \parallel^2 + \sum_{j=1}^{n-1} e_j^T e_{j+1} \\ &+ \sum_{j=1}^{n} e_j^T \triangle_j (k_1, \cdots, k_{j-1}, f_1, \cdots, f_j) \\ &- e_{n-1}^T e_\phi + e_{n-1}^T u + u^T(-\mu u - k e_n) \\ &- \left(k - \sum_{j=1}^{n-1} k_j \right) \parallel e_n \parallel^2 - \sum_{j=1}^{n-1} k_j e_n^T e_\phi \\ &+ \sum_{j=1}^{n-1} k_j e_n^T u - e_n^T (\mu - g(x(t), t)) u(t - \bar{\tau}) \\ &+ e_n^T g(x(t), t) \{u(t - \tau(t)) - u(t - \bar{\tau})\} \\ &+ \sigma_1 e_n^T e_1 + \sigma_2 e_n^T e_2 + \cdots + \sigma_{n-1} e_n^T e_{n-1} \\ &+ e_n^T d(t) + \dot{P} + \dot{Q} + \dot{R} + \dot{S}. \end{aligned} \quad (39)$$

Using Assumption 3, $\triangle_j (k_1, \cdots, k_{j-1}, f_1, \cdots, f_j)$ in (39) can be upper bounded by the following inequality

$$\parallel \triangle_j (k_1, \cdots, k_{j-1}, f_1, \cdots, f_j) \parallel \leq \gamma_{j1} \parallel z \parallel + \gamma_{j2} \quad (40)$$

where, $z \in \mathbb{R}^{(n+2)n}$ is a vector, defined as

$$z \triangleq \left[e_1^T \; e_2^T \; \cdots \; e_n^T \; e_\phi^T \; u^T \right]^T \quad (41)$$

and γ_{j1}, γ_{j2} are known positive constants. Using the inequality given in (32) and (40), Assumption 2,4,5 in (39), the following inequality is obtained as

$$\begin{aligned} \dot{V}_n \leq &- k_1 \parallel e_1 \parallel^2 - k_2 \parallel e_2 \parallel^2 - \cdots - k_{n-1} \parallel e_{n-1} \parallel^2 \\ &- \left(k - \sum_{j=1}^{n-1} k_j \right) \parallel e_n \parallel^2 - \mu \parallel u \parallel^2 \\ &+ k \parallel u \parallel \parallel e_n \parallel + \parallel e_1 \parallel \parallel e_2 \parallel \\ &+ \parallel e_2 \parallel \parallel e_3 \parallel + \cdots + \parallel e_{n-2} \parallel \parallel e_{n-1} \parallel \\ &+ (\sigma_{n-1} + 1) \parallel e_{n-1} \parallel \parallel e_n \parallel + \parallel e_\phi \parallel \parallel e_{n-1} \parallel \\ &+ \parallel u \parallel \parallel e_{n-1} \parallel + \sum_{j=1}^{n-1} k_j \parallel e_\phi \parallel \parallel e_n \parallel \\ &+ \sum_{j=1}^{n-1} k_j \parallel u \parallel \parallel e_n \parallel + (\mu - \underline{g}) \parallel u(t - \bar{\tau}) \parallel \parallel e_n \parallel \\ &+ \bar{g} \bar{\tau} \parallel \dot{u}(t - \hat{\tau}(t)) \parallel \parallel e_n \parallel + \sigma_1 \parallel e_n \parallel \parallel e_1 \parallel \\ &+ \sigma_2 \parallel e_n \parallel \parallel e_2 \parallel + \cdots + \sigma_{n-2} \parallel e_n \parallel \parallel e_{n-2} \parallel \\ &+ \parallel e_1 \parallel (\gamma_{11} \parallel z \parallel + \gamma_{12}) + \parallel e_2 \parallel (\gamma_{21} \parallel z \parallel + \gamma_{22}) \\ &+ \cdots + \parallel e_n \parallel (\gamma_{n1} \parallel z \parallel + \gamma_{n2}) + \bar{d} \parallel e_n \parallel \\ &+ \dot{P} + \dot{Q} + \dot{R} + \dot{S}. \end{aligned} \quad (42)$$

Based on (30) and Young's inequality i.e. $\parallel a \parallel \parallel b \parallel \leq \frac{1}{2} \left(\parallel a \parallel^2 + \parallel b \parallel^2 \right)$, the following inequality is obtained as

$$\|\dot{u}(t)\|^2 \leq 2\left(\mu^2\|u\left(t\right)\|^2 + k^2\|e_n\left(t\right)\|^2\right). \tag{43}$$

Using (35)-(38), (43), applying Young's inequality, the inequality $\int_{t-\bar{\tau}}^{t}\left(\int_{s}^{t}\|u(\phi)\|^2 d\phi\right)ds \leq \bar{\tau}\int_{t-\bar{\tau}}^{t}\|u(\phi)\|^2 d\phi$, the Cauchy-Schwarz inequality $\| e_\phi\left(t\right)\|^2 \leq \mu^2\bar{\tau}\int_{t-\bar{\tau}}^{t}\|u(\phi)\|^2 d\phi$ and square completion in certain terms of (42), yields

$$\begin{aligned}
\dot{V}_n \leq &-\frac{1}{2}\left(k_1 - 1 - (n+6)\frac{\sigma_1^2}{2k}\right)\| e_1 \|^2 \\
&-\frac{1}{2}\sum_{i=2}^{n-2}\left(k_i - 2 - (n+6)\frac{\sigma_i^2}{2k}\right)\| e_i \|^2 \\
&-\frac{1}{2}\left(k_{n-1} - 3 - (n+6)\frac{(\sigma_{n-1}+1)^2}{2k}\right)\| e_{n-1} \|^2 \\
&-\left\{\frac{k}{2(n+6)} - \sum_{j=1}^{n-1}k_j - \frac{1}{2}\bar{g}^2\bar{\tau}^2k^2\mu^2\right. \\
&\left.-2\left(\xi + \frac{\omega_1}{2k^2\mu^2}\right)k^2\right\}\| e_n \|^2 \\
&-\left\{\mu - \frac{k}{2} - \frac{1}{2} - \frac{n+6}{2k}\left(\sum_{j=1}^{n-1}k_j\right)^2 - \frac{n+6}{2k}\left(\mu - \underline{g}\right)^2\right. \\
&-3\left(\sum_{j=1}^{n-1}k_j\right)^2\left(\frac{n+6}{2k}\right)\bar{\tau}^2\mu^2 \\
&\left.-2\left(\xi + \frac{\omega_1}{2k^2\mu^2}\right)\mu^2\right\}\| u \|^2 \\
&+\left(\sum_{i=1}^{n-1}\frac{\| \gamma_{i1} \|^2}{k_i}\right)\| z \|^2 + \frac{n+6}{2k}\| \gamma_{n1} \|^2\| z \|^2 \\
&+\sum_{i=1}^{n-1}\frac{\| \gamma_{i2} \|^2}{k_i} + \left(\frac{n+6}{2k}\right)\left(\| \gamma_{n2} \|^2 + \bar{d}^2\right) \\
&-\left(\xi\left(1-\mathit{\Gamma}\right) - \frac{1}{2k^2\mu^2}\right)\| \dot{u}\left(t - \hat{\tau}\left(t\right)\right) \|^2 \\
&-\frac{\omega_1\left(1-\mathit{\Gamma}\right)}{4\xi k^2\mu^2}P\left(t\right) - \frac{\left(1-\mathit{\Gamma}\right)}{2\bar{\tau}}Q\left(t\right) \\
&-\bar{\tau}\left(\sum_{j=1}^{n-1}k_j\right)^2\left(\frac{\mu}{\mu - \underline{g}}\right)^2 R\left(t\right) - \frac{1}{3\bar{\tau}}S\left(t\right).
\end{aligned} \tag{44}$$

Provided following sufficient gain conditions

$$k_1 > 1 + (n+6)\frac{\sigma_1^2}{2k} \tag{45}$$

$$k_i > 2 + (n+6)\frac{\sigma_i^2}{2k} \quad 2 \leq i \leq n-2 \tag{46}$$

$$k_{n-1} > 3 + (n+6)\frac{(\sigma_{n-1}+1)^2}{2k} \tag{47}$$

$$\xi > \frac{1}{2\left(1-\mathit{\Gamma}\right)k^2\mu^2} \tag{48}$$

$$\begin{aligned}
\mu >& \frac{k}{2} + \frac{1}{2} + \frac{n+6}{2k}\left(\sum_{j=1}^{n-1}k_j\right)^2 + \frac{n+6}{2k}\left(\mu - \underline{g}\right)^2 \\
&+3\left(\sum_{j=1}^{n-1}k_j\right)^2\left(\frac{n+6}{2k}\right)\bar{\tau}^2\mu^2 + 2\left(\xi + \frac{\omega_1}{2k^2\mu^2}\right)\mu^2
\end{aligned} \tag{49}$$

$$\bar{\tau} < \frac{1}{\bar{g}k\mu}\sqrt{\frac{k}{(n+6)} - 2\sum_{j=1}^{n-1}k_j - 4\left(\xi + \frac{\omega_1}{2k^2\mu^2}\right)k^2} \tag{50}$$

the inequality given in (44) is rewritten as

$$\begin{aligned}
\dot{V}_n \leq &-\left(\beta - \sum_{i=1}^{n-1}\frac{\| \gamma_{i1} \|^2}{k_i} - \frac{n+6}{2k}\| \gamma_{n1} \|^2\right)\| z \|^2 \\
&-\frac{\omega_1\left(1-\mathit{\Gamma}\right)}{4\xi k^2\mu^2}P\left(t\right) - \frac{\left(1-\mathit{\Gamma}\right)}{2\bar{\tau}}Q\left(t\right) \\
&-\bar{\tau}\left(\sum_{j=1}^{n-1}k_j\right)^2\left(\frac{\mu}{\mu - \underline{g}}\right)^2 R\left(t\right) - \frac{1}{3\bar{\tau}}S\left(t\right) \\
&+\sum_{i=1}^{n-1}\frac{\| \gamma_{i2} \|^2}{k_i} + \left(\frac{n+6}{2k}\right)\left(\| \gamma_{n2} \|^2 + \bar{d}^2\right)
\end{aligned} \tag{51}$$

where, $\beta \in \mathbb{R}^+$ is defined as

$$\begin{aligned}
\beta = \min\Bigg[&\frac{1}{2}\left(k_1 - 1 - (n+6)\frac{\sigma_1^2}{2k}\right), \cdots, \\
&\frac{1}{2}\sum_{i=2}^{n-2}\left(k_i - 2 - (n+6)\frac{\sigma_i^2}{2k}\right), \\
&\frac{1}{2}\left(k_{n-1} - 3 - (n+6)\frac{(\sigma_{n-1}+1)^2}{2k}\right), \\
&\left\{\frac{k}{2(n+6)} - \sum_{j=1}^{n-1}k_j - \frac{1}{2}\bar{g}^2\bar{\tau}^2k^2\mu^2\right. \\
&\left.-2\left(\xi + \frac{\omega_1}{2k^2\mu^2}\right)k^2\right\}, \\
&\left\{\mu - \frac{k}{2} - \frac{1}{2} - \frac{n+6}{2k}\left(\sum_{j=1}^{n-1}k_j\right)^2 - \frac{n+6}{2k}\left(\mu - \underline{g}\right)^2\right. \\
&\left.-3\left(\sum_{j=1}^{n-1}k_j\right)^2\left(\frac{n+6}{2k}\right)\bar{\tau}^2\mu^2 - 2\left(\xi + \frac{\omega_1}{2k^2\mu^2}\right)\mu^2\right\}\Bigg]
\end{aligned} \tag{52}$$

Using (34), it can be shown that

$$V_n \leq \lambda \| y\left(t\right) \|^2 \tag{53}$$

where, $\lambda \in \mathbb{R}^+$ is a known constant and

$$y\left(t\right) \triangleq \left[\, z^T\left(t\right) \;\sqrt{P\left(t\right)} \;\sqrt{Q\left(t\right)} \;\sqrt{R\left(t\right)} \;\sqrt{S\left(t\right)}\,\right]^T \tag{54}$$

Further, the inequality in (51), yields

$$\dot{V}_n \leq -\frac{\bar{\beta}}{\lambda}V_n + \epsilon \tag{55}$$

where,

$$\epsilon = \sum_{i=1}^{n-1}\frac{\| \gamma_{i2} \|^2}{k_i} + \left(\frac{n+6}{2k}\right)\left(\| \gamma_{n2} \|^2 + \bar{d}^2\right) \tag{56}$$

and

$$\bar{\beta} = \min \left[\left(\beta - \sum_{i=1}^{n-1} \frac{\| \gamma_{i1} \|^2}{k_i} - \frac{n+6}{2k} \| \gamma_{n1} \|^2 \right), \frac{\omega_1 (1-\Gamma)}{4\xi k^2 \mu^2}, \frac{(1-\Gamma)}{2\bar{\tau}}, \bar{\tau} \left(\sum_{j=1}^{n-1} k_j \right)^2 \left(\frac{\mu}{\mu - \underline{g}} \right)^2, \frac{1}{3\bar{\tau}} \right] \tag{57}$$

provided

$$\beta > \sum_{i=1}^{n-1} \frac{\| \gamma_{i1} \|^2}{k_i} + \frac{n+6}{2k} \| \gamma_{n1} \|^2 . \tag{58}$$

The solution of differential inequality in (55) can be obtained as

$$V_n(t) \leq \left(V_n(0) - \frac{\lambda \epsilon}{\bar{\beta}} \right) \exp\left(-\frac{\bar{\beta}}{\lambda} t \right) + \frac{\lambda \epsilon}{\bar{\beta}}. \tag{59}$$

Using (34) & (59), the error $e(t)$ can be expressed as

$$\| e_1(t) \| \leq \sqrt{2 \left(V(0) - \frac{\lambda \epsilon}{\bar{\beta}} \right) \exp\left(-\frac{\bar{\beta}}{\lambda} t \right) + 2 \frac{\lambda \epsilon}{\bar{\beta}}}. \tag{60}$$

The expression in (60) can be used to prove GUUB result. Based on (54), it is concluded that $e_1(t), \cdots, e_n(t), e_\phi(t), u(t), P(t), Q(t), R(t), S(t) \in \mathcal{L}_\infty$. Since $e_1(t), \cdots, e_n(t), e_\phi(t), u(t), x_d(t), x_d^{(i)}(t) \in \mathcal{L}_\infty$, (2) indicates that $x_1(t), \cdots, x_i(t) \in \mathcal{L}_\infty$.

4. SIMULATION

4.1 Example 1

The dynamics of an input-delayed second-order nonlinear system is considered as

$$\dot{x}_1 = \theta_1 x_1 \sin(x_1) + x_2$$
$$\dot{x}_2 = \theta_2 x_1 x_2 + \theta_3 x_1 + 10 \left(1 + 0.5 \cos^2(x_1)\right) u(t - \tau(t)) + d \tag{61}$$

where $\theta_1 = -1$, $\theta_2 = 0.1$ and $\theta_3 = -0.2, \tau(t) \in \mathbb{R}^+$ is unknown input delay, $u(t)$ is the control input and $d(t)$ denotes the external disturbance given by $d(t) = 0.1 \sin(t)$. The desired trajectory is considered as $x_{1d}(t) = 5 \sin(t/4)$.

Table 1. Tracking errors for different values of delay.

Case	Input delay τ (m.sec.)	k	k_1	μ	RMS value of error $(x_1 - x_{1d})$
1	20sin(2t)+22	200	120	30	0.0584
2	40sin(2t)+44	30	6	0.1	0.4398

The simulation shows the performance of controller given in (30) for the different values of sinusoidal delay i.e. Case 1 and 2 given in Table 1. The delay bound of 100 msec is fixed to obtain the errors and control response shown in Fig.1 for the dynamics given in (61). It is observed that the tracking performance depends upon the value of input delay. As the delay increases, the errors increases, however, remains bounded. The control gains k, k_1, μ and RMS value of errors are mentioned in the Table 1. The initial values of states are chosen in simulation as $[\,0.3 \;\; 0.1\,]$.

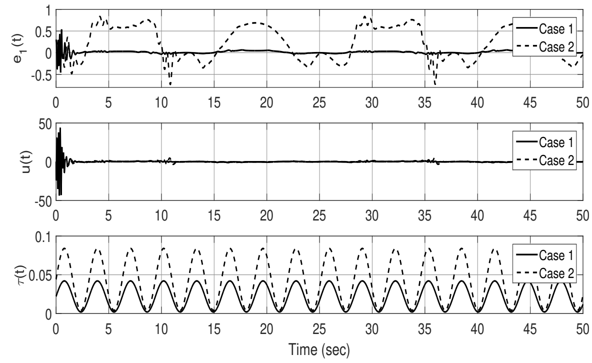

Fig. 1. Tracking error and control input response for the delay bound of 100 msec.

4.2 Example 2

The mathematical model of the motion control of a one link manipulator with time-varying input delay is given by Gao et al. (2016)

$$\begin{cases} J\ddot{q} + b\dot{q} + N \sin(q) & = T + T_d \\ L\dot{T} + RT & = u(t - \tau(t)) - k_m \dot{q} \end{cases} \tag{62}$$

where, $q, \dot{q}$ and $\ddot{q}$ denote the link position, velocity and acceleration, respectively, $J = 1Kg.m^2$ is the mechanical inertia, $b = 1Nm.sec/rad$ is the viscous friction coefficient, $N = 10$ is a positive constant, T is the torque, T_d represents the additive disturbance, $L = .1H$ and $R = 1$ are the armature inductance and resistance of the motor, $K_m = 0.2Nm/A$ is the back electromotive force coefficient, $u(t - \tau(t))$ is the delayed control input which denotes the electromechanical torque, and $\tau(t)$ is the time- varying input delay. The desired trajectory $q_d(t) = \sin(t) + \sin(0.5t)\, rad$. The time-varying input delay and the disturbance term are chosen as $\tau(t) = 10 \sin(0.5t) + 30ms$ and $T_d = \sin(t)\, Nm$, respectively.

The dynamics given in (62) can be rewritten in following form as

$$\begin{cases} \dot{x}_1 & = & x_2 \\ J\dot{x}_2 = & & x_3 - bx_2 - N \sin(x_1) + T_d \\ L\dot{x}_3 = & & u(t - \tau(t)) - k_m x_2 - Rx_3 \end{cases} \tag{63}$$

where, state variables are defined as $[\, x_1 \; x_2 \; x_3 \,]^T \triangleq [\, q \; \dot{q} \; T \,]^T$. The Simulation is performed on (63), choosing initial states as $[\, x_1(0) \; x_2(0) \; x_3(0) \,] = [\, 0.3 \;\; 0.1 \;\; 0.5 \,]$ and control gains $k = 50,\ k_1 = 20,\ k_2 = 6$ and $\mu = .1$ in (30), the tracking error and control input responses are obtained as shown in Fig. 2.

5. CONCLUSION

This paper extends the results of Jain and Bhasin (2019) for a class of uncertain nonlinear systems in strict-feedback form with unknown time-varying input delay. The upper bound on unknown time-varying input delay and lower bounds on control gains are determined in stability analysis by choosing suitable L-K functionals to prove global uniformly ultimately bounded tracking. Simulation results with different values of the input delay demonstrate the effectiveness of the proposed controller.

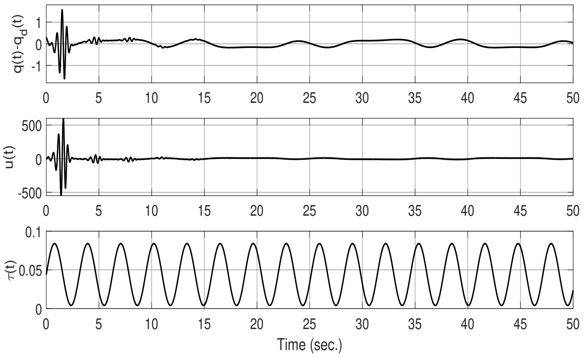

Fig. 2. Tracking error and control input response for the delay bound of 100 msec.

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