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Multi-robot energy-aware coverage control in the presence of time-varying importance regions

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Abstract: Multi-robot systems are becoming widely popular in applications where a rapid response is required or where various different robotic capabilities are required. Applications such as surveillance, or search and rescue, would require an efficient team that can be deployed and optimally dispersed over the environment. This is known as the coverage control problem. The solution to this research optimization problem is affected by several external aspects, such as characteristics of the environment as well as factors that pertain to the robotic team. This work proposes a novel solution to the complete coverage problem where the team of robots is restricted with energy limitations, and must cover an environment that has time-varying regions of importance. Our results show that in a realistic scenario, where the robots have limited energy for the task in question, the proposed solution performs significantly better than a typical coverage algorithm which disregards the energy considerations of the robotic team.

Keywords: Coverage control, multi-robot systems, guidance navigation and control, time-varying systems, control algorithms, centroidal Voronoi tessellations.

1. INTRODUCTION

For several years, practical robotic applications have been pushing the boundary of multi-robot systems research. Particularly, applications that require the coordination of a team of robots or sensors in order to perform surveillance of a known environment, are posing new challenges in this research area. Robot cooperation and coordination have been widely studied over the years. Researchers have developed several frameworks in an attempt to have a generic structure that can be applied to any heterogeneous team of robots that is employed to perform some cooperative task, as can be seen in the works by Parker (1998); Botelho and Alami (1999); Gerkey and Mataric (2002). However, there are some applications, such as monitoring and surveillance of enclosed and known areas, which may be solved optimally by mathematical tools such as Voronoi diagrams. Voronoi diagrams are computed by clustering points in the environment according to their vicinity to generator point sites. This means that a border will form along points which are equidistant between two neighbouring generating points, as explained by Cortes et al. (2002). Segmenting the environment as such is an optimal way of assigning regions in the environment to the robots or sensors in that environment. This is one of the potential solutions of the coverage control problem, where we would like to disperse a number of mobile robots or a number of static sensors optimally, to maximise coverage of some environment.

The coverage control problem is widely studied in literature. Different aspects in the environment or characteristics of the robotic team may affect the complexity and solution of this problem. For instance, some environments may have areas or regions which have higher importance and should therefore be covered better than those with lower importance. Furthermore, such higher importance areas may also move with time, and hence, the coverage problem becomes more complex since the team must disperse over the environment, and at the same time track some time-varying importance function. Additionally, each of the robots in the team often has its own constraints, such as energy and sensor limitations, which all affect the end optimal solution. In this light, most works in this research field focus on solving the coverage control problem when there is only one aspect affecting the solution. For example, in the work proposed by Lee and Egerstedt (2013), the authors only take into account the fact that an environment may have time-varying importance regions, without taking into consideration the fact that in reality the team of robots is also subject to energy limitations. Conversely, Kwok and Martínez (2007) account for the fact that robots have limited energy levels, but neglect the potential time variations in the environment.

In contrast, we propose a novel scheme that addresses these two important aspects at the same time. Together these aspects render the coverage control problem more realistic. Basically, the proposed algorithm attempts to find an optimal segmentation of the environment with a finite number of robots, each with its own energy constraints, that shall be tracking an important area which is moving with time in the environment. At the same time, the algorithm attempts to conserve as much of the energies of the robots as possible. This is expected to yield better coverage in typical realistic scenarios where robots have limited energy for the task in question.

The rest of the paper is organised as follows. In Section 2 we review existing works and the techniques that are often used to solve coverage control problems. In Section 3 we describe the novel coverage control scheme proposed in this paper. The proposed scheme is tested, evaluated and compared using realistic Monte Carlo simulations, which are presented and explained in Section 4. More specifically, we perform 100 Monte Carlo simulations to validate and compare the proposed novel algorithm against a typical non-energy aware coverage algorithm with a time-varying density function. Statistical hypothesis testing is then used to analyse the Monte Carlo results. Our results show that in a realistic scenario, the proposed energy aware coverage control algorithm performs better than the algorithms that have been proposed in literature thus far. Finally, in Section 5 we present our conclusions as well as a number of ideas for future work.

2. LITERATURE REVIEW

The idea behind coverage control is: to be able to disperse a number of robots, n, across some environment, Q, such that with their sensing capabilities, these robots are able to cover as much of the environment as possible. The position of the i^{th} robot in the environment is represented by p_i. This problem is synonymous to the facility localization problem, where user facilities must be placed optimally in a given environment. Cortés et al. (2004) describe this problem as a *spatial resource-allocation problem*. Locational optimization problems require a team of robots to be strategically placed over the environment, such that the algorithm optimizes coverage of the important areas in the environment as well as the sensing performance of the robots (Miah and Knoll (2018)). For more detail the reader is referred to Okabe and Suzuki (1997). Cortés et al. (2004) show that, given an environment Q that is a convex polytope in $\mathbb{R}^N$, Voronoi partitions are able to solve the facility localization problem. This means that when an environment is partitioned using the definition of Voronoi tessellations in (1), the cost function in (2) is being minimized,

$$V(p_i) = \{q \in \mathbb{R} | f\left(\| q - p_i \|\right) \leq f\left(\| q - p_j \|\right), \forall j \neq i\} \quad (1)$$

$$H\left(P, V\right) = \sum_{i=1}^{n} \int_{V_i} f\left(\| q - p_i \|\right) \phi(q) dq, \quad (2)$$

where P is the set of robot poses $\{p_1, p_2 ... p_i\}$, V is the set of Voronoi regions that together make up the environment Q, q is an arbitrary point in Q, $\phi(q)$ is the density function that reflects the probability of some event occuring at point q, and $f(\| q - p_i \|)$ is the performance function which provides a quantitative assessment of how well the environment is being covered. For an optimal solution where the cost H is minimized, Lloyd's Algorithm must be executed, (Lloyd (1982)). In this algorithm, the Voronoi diagram is computed according to the current robot positions, p_i, which act as the generator point sites. The centroid C_{V_i} of each Voronoi cell V_i, is calculated and the robots are then driven to these centroids. By repeatedly computing the Voronoi diagram, followed by the computation of C_{V_i} and moving the robots towards the new centroids, we can achieve a Centroidal Voronoi Tessellation (CVT) — which is an optimal segmentation of the environment — where the positions of the robots, p_i, coincide with the new centroids of the Voronoi cells.

The work carried out by Cortés et al. (2004) focuses on an environment that is represented by a convex polytope in $\mathbb{R}^N$ and which has a region of interest represented by a probability density function $\phi(q)$. In this work, Cortés et al. (2004) assume that the robots behave according to the holonomic model given by

$$\dot{p_i} = u_i, \quad (3)$$

where $\dot{p_i}$ is the speed of the robot and u_i is the control input. Furthermore, in the work by Cortés et al. (2004), it is assumed that the robots have unlimited energy. However, in a more realistic scenario the algorithm must be able to take into consideration the finite energy level of each robot when segmenting the environment. This means that high energy robots are assigned areas with a higher probability importance while low energy robots are assigned areas with a lower probability importance. Kwok and Martínez (2007) achieve a solution to the energy constrained coverage problem by using weighted Voronoi diagrams called *power diagrams*. Power diagrams are computed in a slightly different way than the usual Voronoi diagrams. The mathematical definition of a power diagram is given by

$$V_w(p_i) = \{q \in \mathbb{R} | d_{w_i}(q, p_i) \leq d_{w_j}(q, p_j), \forall j \neq i\}. \quad (4)$$

In this case, the performance function,

$$d_{w_i}(q, p_i) = \| q - p_i \|^2 - w,$$

is a power metric that includes some weight w, in the computation of the Voronoi diagram. This means that when computing a Voronoi cell, if a point q is equidistant from the generator neighbouring points p_i and p_j, then q shall be assigned to the region of the generator point that has the larger weight. For more detail on power diagrams, the reader is referred to Okabe et al. (2000); Kwok and Martínez (2007); Pavone et al. (2009).

For the energy aware algorithm proposed by Kwok and Martínez (2007), the weight used to compute the weighted Voronoi cell of each robot, e_i, is the remaining energy of the i^{th} robot: $e_i = E - E_i$, where E is the maximum energy that the i^{th} robot can have, and E_i is its current energy level. The rate at which energy is consumed is modelled according to

$$\dot{E_i} = \begin{cases} -m_i u_i^2, & \text{if } E_i \geq 0, \\ 0, & \text{if } E_i = 0, \end{cases}. \quad (5)$$

where m_i represents the mass of the i^{th} robot. In this case, the Voronoi cell is computed such that points which are equidistant between two robot locations, p_i and p_j shall fall in the region of the robot that has the higher energy reserve. In this case, the cost function that is minimized in the work proposed by Kwok and Martínez (2007) is given by H_e as follows;

$$H_e = \sum_{i=1}^{n} \int_{V_i^e} \left(\| q - p_i \|^2 + (E - E_i)\right) \phi(q) dq, \quad (6)$$

where V_i^e is the Voronoi cell of the i^{th} robot when the energy levels are considered. Similarly, Pavone et al. (2009) propose a coverage control law that allows the robots to have equal workload in their Voronoi partition. Hence, the workload in any region would be equal for all the robots in the team. Similarly, Pierson et al. (2017) use the same weighted Voronoi diagrams to segment the environment according to the actuation capabilities of the different robots. A similar approach is also adopted by Marier et al. (2013) to model different sensor health.

In the algorithms discussed so far in this paper, a static environment has been considered, in that the probability density function that represents some important feature or event happening in a particular location $\phi(q)$, is not a function of time. To address this issue Cortes et al. (2002) propose the feedback plus feedforward control law Equation 7, which allows the robots to track a time-varying density function, $\phi(q,t)$. Note that (7) is the control law that governs the i^{th} robot, k is some positive constant, $\dot{C}_{V_i}$ describes how the centroids of the Voronoi cells are changing with time, M_{V_i} is the mass of each Voronoi region and $\dot{M}_{V_i}$ describes how the masses of the Voronoi cells are changing with time.

$$u_i = \dot{C}_{V_i} - \left(k + \frac{\dot{M}_{V_i}}{M_{V_i}}\right)(p_i - C_{V_i}) \tag{7}$$

One must note that the work proposed by Cortes et al. (2002) does not account for energy limitations in the robotic team. In addition, Lee and Egerstedt (2013) argue that Cortes et al. (2002) make restrictive assumptions about ϕ and hence, the control law that they propose cannot hold in general. This is because it cannot be known how $\dot{\phi}$ would behave in general, and hence Lee and Egerstedt (2013) propose their own general control law which is shown to provide a slight improvement on that proposed by Cortes et al. (2002). Lee and Egerstedt (2013) start from the notion that at any time $t > t_0$, where t_0 is the initial time, the position p_i must be equal to the centroid C_{V_i} such that the controlled variable p_i tracks the target C_{V_i}. Lee and Egerstedt (2013) assume a holonomic robot and hence they derive a control law for which the condition,

$$\| p_i(t) - C_{V_i}(p_i(t), t) \| = 0$$

holds for $\forall t \geq t_0$.

3. THE PROPOSED COVERAGE CONTROL SCHEME

In a practical application, one must consider several aspects when designing a coverage control algorithm. For instance, the environment in question might have time-varying areas of importance which the robots would need to track and cover. Moreover, real robots have limited energy, and this too must be taken into account by the coverage algorithm to enhance optimality. Additionally, a heterogeneous team would also have different sensing capabilities which the algorithm would need to exploit optimally. One should note that there are only few works in literature that address coverage control in the presence of time-varying density functions. Moreover, to the best of the authors' knowledge, there are no works that consider energy restrictions in a coverage control algorithm with time-varying density functions. For this reason, this work aims to address this open problem by considering an environment, Q, where the density function denoting the importance regions is time-varying $\phi(q,t)$, and the robots have limited energy. To achieve this aim, one should note that this is not possible simply by combining the two schemes proposed in previous works that address the two issues independently since this requires a new cost function and a control law that minimizes this cost function which is both time varying and energy aware.

Therefore, we propose a new cost function $H_{e,t}$, as follows

$$H_{e,t} = \sum_{i=1}^{n} \int_{V_i^e} \left(\| q - p_i \|^2 + (E - E_i)\right) \phi(q,t) dq \tag{8}$$

Inspired by the work of Kwok and Martínez (2007), to obtain the optimal segmentation of an environment that has areas of higher importance than others (denoted through $\phi(q,t)$), power diagrams (introduced and previously described briefly in Section 2) are used with weights related to the energy levels of the robots E_i. This introduces the energy weights in our cost function as $e_i = E_i - E$. In addition, to manoeuvre the robots to the Voronoi centroids, after each power diagram computation, we propose a novel energy dependent tracking control law which in form resembles that in (5), originally proposed by Cortes et al., but with the difference that the design parameter k is now adjusted in real time according to the energy of the respective robot, as follows

$$u_i = \dot{C}_{V_i^e} - \left(\frac{E_i}{E} + \frac{\dot{M}_{V_i^e}}{M_{V_i^e}}\right)(p_i - C_{V_i^e}), \tag{9}$$

where V_i^e is the i^{th} robot energy constrained Voronoi cell. This shall make our coverage control scheme energy conscious on two counts. Firstly, energy awareness is included in the segmentation of the environment by using power diagrams, and secondly, the speed of the robots $\dot{p}_i$ is restricted by the current energy levels of the robots themselves. Therefore, if the robots are low on energy, they are assigned smaller areas and they conserve energy by driving slower towards the target centroids.

The concept behind this work is that in an environment where the important features are varying their position with time, we need to exploit the energy capabilities of the robots, but at the same time conserve it as much as possible in order to complete the task successfully and efficiently. This means that high energy robots should be covering high importance areas, as indicated by the probability in $\phi(q,t)$. As time passes, the energy of these robots starts to dissipate, to a point where some robots might end up with no energy. More specifically, when the energy of a robot reaches a preset minimum threshold, the particular robot is withdrawn from the team (considered completely inactive) and the environment needs to be covered by the rest of the team from that point in time onwards.

4. MONTE CARLO SIMULATIONS AND RESULTS

In this section we are presenting the algorithm of our scheme. In our simulation study, we evaluate the proposed novel energy aware coverage control scheme and compare it to the published algorithm by Cortes et al. (2002). In this simulation, we assume that our robots behave according to the holonomic model given in Equation 3 and we consider the energy dynamics shown in Equation 5. The time-varying probability density function used in this study is defined in Equation 10, where q_x and q_y represent the (x, y) coordinate of an arbitrary point q in space, t is the time variable and τ is the time constant of the sinusoid. This probability density function is a Gaussian function that moves across the environment in a cyclic manner. This is also reflected in the cost values as they vary with time, where the profile of the cost over time also has a cyclic nature.

$$\phi(q,t) = e^{-\left(q_x - 2\sin\frac{t}{\tau}\right)^2 - q_y^2} \qquad (10)$$

The proposed energy aware algorithm with a time-varying density function is outlined and simulated in Algorithm 1. To render the simulation study more powerful and realistic, we run it several times, each time varying a number of system parameters randomly. More specifically, we set the initial energy of the robots with random values. For each trial run, the masses of the robots themselves are also varied in order to observe how different teams of robots would fare in the same environment. Furthermore, the movement of the robots to the target locations, $C_{V_i^e}$, as well as the energy dissipation of the robots are simulated by solving the differential equations in Equations 3 and 5 using an ODE solver. If after this movement, the robots still have enough energy above the minimum threshold, they will form part of the team in the next time step. Otherwise, they would no longer be considered part of the team, and are left out of the optimization problem from that point in time onwards. This algorithm is repeated for a preset amount of time.

In our comparative Monte Carlo study, we compare the cost in Equation 8 for the proposed algorithm, which has a time-varying density function, $\phi(q, t)$ and which is energy aware, with a time-varying algorithm that does not account for the limited energy of the robots, namely the algorithm proposed by Cortes et al. (2002). The algorithm proposed by Cortes et al. (2002) represents a class of coverage algorithms which have a time-varying density function but are not aware of the energy limitations of the robots. To do this, we consider different, random initial energy levels of the robots and different, random robot masses in each trial. This allows us to analyse different scenarios and to show that the proposed energy aware coverage algorithm with a time-varying density function repeatedly yields a lower cost than an algorithm which is not energy aware. In each trial, the initial conditions and the robot masses are the same for the two algorithms, such that a fair comparison is made.

The first part of our results section shall consider a typical single trial. For this trial, we plot the instantaneous cost against time for the two algorithms under test, as shown in Figure 1. It is expected that the cost, $H_{e,t}$, would be higher for the algorithm that is not energy aware, than that of the algorithm that is energy aware. This is because in the cost function in Equation 8, we aim to minimize the energy cost together with the cost of coverage. Since we expect the non-energy aware algorithm to use up more energy of the robots, then the energy cost across all the team is expected to be higher. In fact, Figure 1 shows that the combined instantaneous cost over time, is generally higher for the non-energy aware algorithm than for the

Algorithm 1 Single trial simulation of the proposed energy aware coverage control scheme.

procedure (p_i, Q)
  $n = 5$ % Number of robots
  $E_{max} = 2$
  $E_{min} = 0.1$
  $E_i = (E_{max} - E_{min}) \cdot rand(N, 1) + E_{min}$
  $weights = E_i - E_{max}$
  $mass + max = 5$
  $mass_{min} = 1$
  $m_i = (mass_{max} - mass_{min}) rand(N, 1) + mass_{min}$
  $t_{final} = 125$
  $k_{prop} = 0.3$

  for $t = 0 : 0.5 : t_{final}$ **do**
    Compute values for ϕ for time t
    $[v, r] =$ `PowerDiagram`(p_i, Q) % Computes energy constrained power diagram
    for $i = 1 : N$ **do**
      $C_{V_i^e} =$ `PolyCentroid`$(v(r_i), Q, \phi(q))$
% Simulate movement of robots and update the system states
      $\dot{p}_i = \dot{C}_{V_i^e} - \left(\frac{E_i}{E} + \frac{\dot{M}_{V_i^e}}{M_{V_i^e}}\right)(p_i - C_{V_i^e})$
      $\dot{E}_i = -\left(\dot{C}_{V_i^e} - \left(\frac{E_i}{E} + \frac{\dot{M}_{V_i^e}}{M_{V_i^e}}\right)(p_i - C_{V_i^e})\right)^2$

      $p_{i_{new}} =$ `ODEsolver` $(\dot{p}_i, p_i)$
      $E_{i_{new}} =$ `ODEsolver` $(\dot{E}_i, E_i)$
      if $E_i \geq E_{min}$ **then**
        update $p_i = p_{i_{new}}$
        update $E_i = E_{i_{new}}$
        update $weight_i = E_i - E_{max}$
      else
% Stop movement of the robot
        $\dot{p}_i = 0$
        $\dot{E}_i = 0$
      end if
    end for
  end for
  return p_i, E_i
end procedure

energy aware algorithm. The RMS value of the combined cost of the non-energy aware algorithm over the whole trial duration is 12.55 while that of the energy aware algorithm is 8.9, which further shows that an energy aware algorithm performs better than the non-energy aware algorithm, in a realistic scenario where energy conservation is important. In an algorithm that is not energy aware, we expect the energy cost (Equation 11) to be greater than that of an algorithm which is energy aware.

$$H_{energy} = \sum_{i=1}^{n} \int_{V_i^e} (E - E_i)\, \phi(q,t) dq \qquad (11)$$

This is because the non-energy aware algorithm is not concerned with how the energy of the team is spent, but rather, the aim is focused on dispersing the robots as much as possible over the environment. Although it might be obvious that an energy-aware algorithm will always outperform a non-energy aware one, this can actually depend on the chosen scenario. If the maximum energy of each robot, E_{max}, is very high for the given environment, the non-energy aware algorithm can have an advantage since it can be more greedy in its energy consumption, achieve better coverage than the energy-aware algorithm and still leave the robots with ample energy to continue their task. For this reason, in our simulations we set E_{max} to a fair and realistic value. In our single trial experiment, we can see this from the RMS value of the energy cost presented in Table 1. The RMS value of the energy cost of the energy aware algorithm is lower than that of the non-energy aware algorithm. It then follows that since the non-energy aware algorithm attempts to maximise coverage at the expense of energy, then we expect the coverage cost (Equation 12) of such an algorithm to be somewhat lower than that of an energy-aware algorithm.

$$H_c = \sum_{i=1}^{n} \int_{V_i^e} \left(\| q - p_i \|^2 \right) \phi(q,t) dq \qquad (12)$$

This is because an energy aware algorithm would be more reluctant to allow robots to spend their energies freely. Rather, it would attempt to strike a balance between energy expenditure and maximising coverage of the environment. This can be seen from the RMS values in Table 1, where the RMS value of the coverage cost of the energy aware algorithm is larger than that of the non-energy aware algorithm. It should also be emphasized that in a realistic scenario, where the robots' energies are limited, the proposed scheme might even lead to lower coverage cost than its competition in a longer trial period. This is because its energy conserving nature allows the robots to operate for a longer period of time and hence, continue to contribute to the team to lower the coverage cost. In contrast, in the non-energy aware algorithm, the robots will be depleted of their energy and stopping earlier, thus sacrificing coverage from that point onwards. This argument stems from the fact that perfect coverage (zero cost) may be obtained with an infinite number of robots. Hence, decreasing the size of the team (due to robots stopping without energy), increases the overall cost. Moreover, there is a higher likelihood that the size of the team diminishes quickly with non-energy aware algorithms than with the proposed energy aware algorithm.

To further show that the hypothesis that an energy aware algorithm performs better than a non-energy aware algorithm in a typical scenario where the robots' energies are limited, 100 Monte-Carlo simulations were conducted where, for each trial the RMS value of the cost in Equation 8, $H_{e,t}$, is recorded for both algorithms. For this purpose, the initial energy level as well as the mass of each robot were randomly generated for each trial. The mass of each robot affects the rate of energy consumption so this also makes our simulation more realistic. For each trial, the initial energy levels and the robot masses were the same for both algorithms. All other parameters were left constant across the different trials and across the two algorithms. Furthermore, the density function $\phi(q,t)$ could have an affect on the rate of energy consumption depending on the value of τ. This is because the smaller the value of τ, the faster the movement of the environment and hence the higher the speed of the robots. Since speed directly effects the rate of energy consumption, then one would expect this rate to be higher for smaller values of τ. In the current study, τ was not varied.

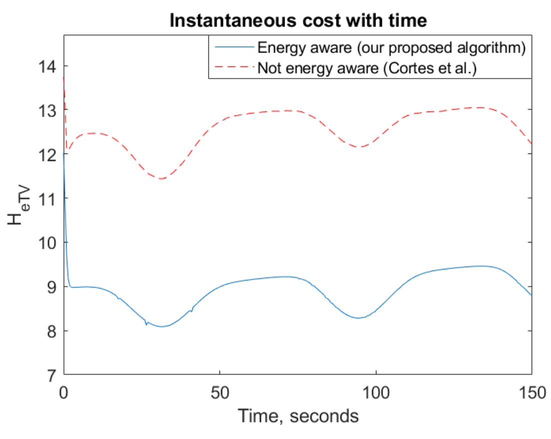

Fig. 1. Plot of the instantaneous cost $H_{e,t}$.

The average RMS value of the cost $H_{e,t}$ for the energy aware algorithm is 5.03, while that of the non-energy aware algorithm is 5.46. The mean and variance of the two data sets are tabulated in Table 2.

One can already see that there is a considerable difference among the mean cost of these two algorithms, however, a statistical test can show whether this difference is statistically significant or not. The RMS values data set of the two algorithms, were tested for normality. This revealed that the collected data in both data sets is normally distributed. This can further be seen in a boxplot of the two data sets, as shown in Figure 2. To confirm that there is a statistical difference between the two sample means, a two-sample t-test is used to test the *null hypothesis* that the difference between the mean cost of the two algorithms is due to chance and not due to the intrinsic differences between the algorithms themselves. The *alternative hypothesis* would be that the difference between the means of the two algorithms is statistically significant and not due to chance. The result of the statistical t-test strongly rejected the null hypothesis, with a level of significance of 0.05 and hence, shows that the difference between the performance of the two algorithms is statistically significant, meaning that the proposed scheme truly leads to lower overall costs.

5. CONCLUSIONS AND FURTHER WORK

Although the works presented in literature propose several solutions for coverage control using Voronoi diagrams, to date these works have been mostly limited to those that account only for one constraining element at a time,

Table 1. Table of RMS Values for Total cost ($H_{e,t}$), Energy Cost(H_e), Coverage Cost (H_c) and

Algorithm	$RMS(H_{e,t})$	$RMS(H_{energy})$	$RMS(H_c)$
Not Energy aware	12.55	12.57	1.06
Energy aware	8.93	6.92	2.02

Table 2. Table of Mean and Variance of the two cost data sets

Algorithm	Mean Cost	Variance of Cost
Not Energy aware	5.46	0.65
Energy aware	5.03	0.72

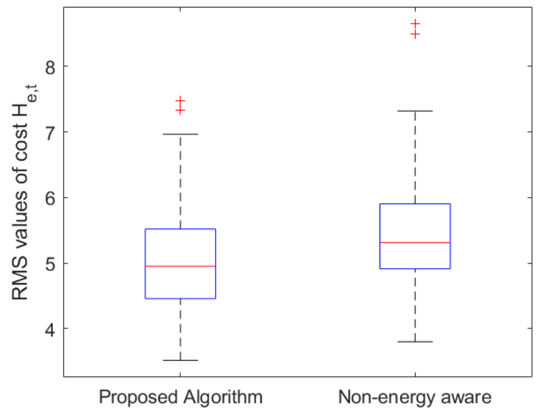

Fig. 2. Boxplot of the datasets of our proposed algorithm and the non-energy aware algorithm

such as robot energy constraints, sensor capabilities, time variations in the environment and so on. Our work proposes a solution that considers multiple elements in the optimization problem, particularly having a time-varying probability density function in the environment and also taking into account robot energy constraints. In our novel coverage control scheme, energy is conserved by segmenting the environment in a way such that low energy robots are given smaller coverage areas, as well as not allowing the speed of low energy robots to increase significantly, through the time-varying parameter $\frac{E_i}{E}$ in the control law. To show that an energy aware algorithm performs better than an algorithm which is not energy aware, even in an environment that has a time-varying density function, we simulate the two algorithms under the same conditions and compare their energy aware costs with time. For a single trial, it could already be seen that the energy aware algorithm performs better than the non-energy aware algorithm. However, to further show that this is not the case for some singular particular conditions, we perform 100 Monte Carlo simulations, each time-varying the initial energy levels of the robots and the masses of the robots themselves. Across these 100 trials, we show that there is a statistical difference between the mean RMS values of the costs of the two algorithms. This shows that we can confidently say that the energy aware algorithm, generally performs better than that which is not energy aware, when the environment has a time-varying density function and the robots have limited energies, as is the case in typical practical applications.

In the future we aim to implement and demonstrate this algorithm on a real life system including different robots and once again analyse the results. Furthermore, we believe that this work may be expanded to include other criteria in the cost function, such as criteria related to sensors. This problem has been studied in isolation as shown in the work by Li and Liu (2017); Papatheodorou et al. (2016). However, it would be beneficial in a practical application if these sensor limitations are also included in the coverage algorithm. In our future work, we intend on investigating the inclusion of such elements, among other issues, in our existing novel algorithm.

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