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A Green Routing and Scheduling Problem in Home Health Care ⋆

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Abstract: The growing concern about the influences of anthropogenic pollutions has forced researchers and scholars to study the environmental concerns. This paper addresses a green routing and scheduling problem in home health care (HHC) with the constraints of synchronized visits and carbon emissions. In this work, the objective is to design a reasonable logistics route meanwhile reduce the effect on the environment for the HHC company. The formulated mixed-integer programming (MIP) model is solved for a set of small scale instances using Gurobi solver with a time limit of 1 hour. An efficient two-phase heuristic approach through decomposing the studied problem into a routing problem and a speed optimization problem is proposed. The heuristic approach is based on two exact methods using Gurobi solver and dynamic programming (DM) method. The proposed heuristic approach is examined by a total of 19 instances with different scales. The experimental results for the studied problem highlight the effectiveness and efficiency of the proposed heuristic approach.

Keywords: Home health care; Synchronized visits; Carbon emissions; Heuristic; Dynamic programming.

1. INTRODUCTION

Home health care (HHC) company provides the health care service for the patients at their homes in order to help them recover from illness or injury. According to a survey of the HHC companies, the HHC company conducts various logistic activities including delivering the caregivers, drugs, medical devices from the HHC company (i.e. the depot) to the patients, and biological samples (such as blood and urine) from the patients' homes to the medical laboratory for testing every day (Liu et al., 2013). The daily scheduling of the caregivers has been demonstrated to be a very difficult problem but a crucial decision activity for a HHC company (Yuan et al., 2018).

As for a HHC company, transportation cost is one of the largest operating costs in company daily activities, thus it is crucial to optimize daily traveling routes of the HHC vehicles in order to reduce the transportation cost meanwhile improving the service quality to patients. However, transportation has serious impacts on the environment, such as resource consumption, toxic effects on ecosystems and humans, noise, and the effect induced by greenhouse gas (GHG) emissions. Among these, GHG, especially carbon dioxide (CO_2) emissions, are the most concerning because CO_2 emissions have direct influences on people's health (Bektaş and Laporte, 2011). If logistics is not scheduled well, it will cause congestion and a large amount of CO_2 emissions. Therefore, it compels managers of the HHC companies to pay more attention to CO_2 emissions when designing the daily logistics activities.

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Recently, some scholars have started studying the HHC problems with the consideration of the carbon emissions. Fathollahi-Fard et al. (2018) studied the problem of the delivery the required drugs and other HHC services to patients. They firstly introduced the environmental pollution or green emissions into the HHC problems, and developed a bi-objective optimization model. Four fast heuristics are proposed to solve the problem. However, in this research, the authors didn't consider the speed of the vehicle. Xiao et al. (2018) also considered the carbon emissions in the HHC transportation problem. They used a capacity VRP (CVRP) model to describe the HHC scheduling problem and proposed an improved cuckoo search (ICS) algorithm for the problem. In this research, the authors set the speed of the vehicle as a constant.

This paper addresses a green routing and scheduling problem in home health care (HHC) with the constraints of synchronized visits and carbon emissions. In order to solve the problem, a mixed-integer programming model and a two-phase heuristic approach are developed. The rest of this paper is organized as follows. Section 2 introduces the scheduling problem and Section 3 builds the mathematical model. Section 4 develops a two-phase heuristic approach in order to solve the problem. The computational experiments are described in Section 5. Section 6 concludes the paper.

2. PROBLEM DESCRIPTION

This paper addresses a daily routing and scheduling problem of a HHC company with the constraints of synchronized visits and carbon emissions. The problem can be defined as follows. Let $G = (N, A)$ be a directed graph with a set of nodes $N = \{0, 1, ..., n, n+1\}$ and a set of arcs $A = \{(i,j) \,| i,j \in N, i \neq j\}$. Node 0 and node $n+1$ represent the depot and the medical laboratory, respectively. Nodes $P = \{1, 2, ..., n\}$ represent the patients who need care service from the HHC company.

Each patient $i \in P$ has a drug and service demand q_i, and each caregiver has the same load and service capacity Q. Each patient $i \in P$ is associated with a service duration τ_i. Each patient $i \in P$ has a service time window $[a_i, b_i]$, where a_i represents the earliest time and b_i represents the latest time for visiting the patients. Each caregiver is allowed to arrive before the earliest time a_i, but the caregiver must wait until that the time is available for the patient. The caregiver is prohibited to arrive after the latest time b_i. The depot and the laboratory have the same time window, meaning the caregivers must leave from the depot and return to the laboratory between the earliest time and latest time.

Some patients may need synchronized services, which means that two or more caregivers must service these patients simultaneously. In this paper, we only consider two caregivers visit a patient simultaneously. For each patient $i \in P$ with synchronized services, a fictive patient i' who has the same locations, demand, service duration and time window with patient i is generated. We refer all fictive patients to P_f. Therefore, we define $N' \leftarrow N \cup P_f$, $P' \leftarrow P \cup P_f$, $A' = \{(i,j) \,| i,j \in N', i \neq j\}$. We adopt $(i,j) \in P^{sync}$ to represent a couple of patients $i,j \in P'$ who need synchronized services. In other words, i and j are associated to the same patient and must be serviced by two different caregivers simultaneously.

The distance between patient i and j is denoted as d_{ij}. This paper considers the constraints of the carbon emission. Speed has a great influence on carbon emission. Therefore, the speed parameter is employed in the paper. The speed of the vehicle k associated to the caregiver k is v. Based on the speed v, it is very easy to calculate the travel time between i and j. The travel time between i and j is d_{ij}/v.

The problem is developed to determine a set of routes in order to minimize the carbon emissions under the constraints of time windows, capacity and synchronized visits, and the following assumptions: (1) each caregiver has the same service capacity and is associated to a vehicle; (2) each vehicle leaves from the depot and returns to the laboratory, and visits each node at most once; (3) the unused vehicles are assumed to start from the depot and end at the laboratory, in order to prevent from adding the emission cost, we assume that the distance from depot to laboratory is 0; (4) because there are many uncertain factors in the city transportation, the speed of the vehicle is assumed to be a constant average speed; (5) for the patient with synchronized visit services requirement, a fictive patient who has the same locations, demand, service duration and time windows is generated. We assume that the patient at most needs two caregivers to service at the same time; (6) for the patient with synchronized visit services requirement, if caregiver 1 arrives earlier than caregiver 2, caregiver 1 must wait for caregiver 2 and then serving the patient together.

3. MATHEMATICAL FORMULATION

In this section, the mathematical model of the studied problem is introduced Firstly, the theory of carbon emissions is introduced; then, a mixed-integer programming (MIP) model is developed for this problem.

3.1 Carbon emissions

This paper adopts the emissions function developed by the United Kingdom Transport Research Laboratory (Hickman et al., 1999). The emissions function has been used by many researchers, such as Jabali et al. (2012), Teoh et al. (2018) and so on, which can demonstrate the effectiveness of the emission function. The emissions function $\varepsilon(v)$ is provided as follows:

$$\varepsilon(v) = L + av + bv^2 + cv^3 + dv^{-1} + ev^{-2} + fv^{-3} \quad (1)$$

where v is the speed of the vehicle in km/h, and the coefficients L, a, b, c, d, e and f will be different under the vehicles with different types and sizes.

The coefficients are adopted the settings in Hickman et al. (1999), and the values of L, a, b, c, d, e and f are 765, -7.04, 0, 0.006320, 8334, 0, 0, respectively.

The vehicle will emit $\varepsilon(v_{ij})$ g/km carbon dioxide (CO_2) when the vehicle is driven at the speed v. Therefore, the CO_2 emission of a vehicle travels from patient i to patient j can be expressed as:

$$E_{ij} = \varepsilon(v_{ij})\, d_{ij} \quad (2)$$

where the units of E_{ij} and d_{ij} are g and km, respectively.

As is shown in Eq. (1), it is very clear that the CO_2 emissions rate $\varepsilon(v)$ will vary with different speed. Therefore, an optimal speed can be found in order to reduce the CO_2 emissions. However, it is very difficult to control the speed particularly during the peak hours in real life. Thus in this paper, the speed is assumed to be an average speed in every arc. Define two different speed as v_1 and v_2, and define the corresponding CO_2 emissions at an arc between patient i and j as E^1_{ij} and E^2_{ij}. A proposition can be obtained as follows.

Proposition 1. As for an arc between patient i and j, if $\varepsilon(v_1) \leq \varepsilon(v_2)$, then $E^1_{ij} \leq E^2_{ij}$.

Proof. It is obvious that the distance is the same as for the same arc between patient i and j. Therefore,

$$\begin{aligned} & E^1_{ij} - E^2_{ij} \\ & = d_{ij} * \varepsilon(v_1) - d_{ij} * \varepsilon(v_2) \\ & = d_{ij} * (\varepsilon(v_1) - \varepsilon(v_2)) \\ & \leq 0, \end{aligned}$$

namely $E^1_{ij} \leq E^2_{ij}$.

3.2 Mixed-integer programming model

In this section, we will describe the mixed-integer programming (MIP) model of the problem. Firstly, the model notations of the parameters for the problem are summarized as follows:

V: set of all vehicles.
N: set of all nodes, including the patients, the depot and

the laboratory.
N': set of all nodes, including the patients, the fictive patients, the depot and the laboratory.
A': set of arcs, $A' = \{(i,j)\,|i,j \in N', i \neq j\}$.
P: set of all patients.
P': set of all patients, including the fictive patients .
Q: capacity of each caregiver.
P^{sync}: set of synchronized visits.
d_{ij}: the distance from node i to node j.
u_{ij}: the demand of patients up to node i, and transported in arc (i,j).
q_i: the demand of patient i.
τ_i: the service duration for node i.
$[a_i, b_i]$: the availability time window of patient i.
$v_{ij} \in [v_{lb}, v_{ub}]$: the speed of vehicle in arc (i,j), v_{lb} and v_{ub} are the lower and upper bound of speed.
$\varepsilon(v_{ij})$: the carbon emissions function.
M: a large positive value.

Then, we will introduce the variables of the studied problem. The first binary decision variable is presented as follows:

$$x_{ijk} = \begin{cases} 1, & \text{if caregiver } k \text{ travels from } i \text{ to } j, \text{ in which } i \neq j; \\ 0, & \text{otherwise.} \end{cases}$$

The secondary decision variable is denoted as follows:
y_i: the start working time of node i.

In this paper, v_{ij} is a variable, so it is obvious that the studied model is nonlinear. In order to linearize the mathematical model, we use a set of speed levels $R = \{1, 2, ..., r, ...\}$ to discretize the speed. Each speed level $r \in R$ is corresponding to a speed v_r. And we introduce a new binary decision variable z_{ijkr} which is denoted as follows:

$$z_{ijkr} = \begin{cases} 1, & \text{if } k \text{ travels from } i \text{ to } j \text{ with speed level } r \text{ ;} \\ 0, & \text{otherwise.} \end{cases}$$

The relationship between decision variables z_{ijkr} and x_{ijk} is presented as follows:

$$\sum_{r \in R} z_{ijkr} = x_{ijk}, \ \forall i \in N', j \in P', k \in V, i \neq j \tag{3}$$

where R is the discrete speed levels $R = \{1, 2, ..., r, ...\}$. Thus, the MIP model is presented as follows:

$$Minimize \sum_{(i,j) \in A} \sum_{k \in V} \varepsilon(v_r)\, d_{ij} z_{ijkr} \tag{4}$$

subject to,

$$\sum_{k \in V} \sum_{j \in N'} x_{ijk} = 1, \ \forall i \in P' \tag{5}$$

$$\sum_{j \in N'} x_{jik} - \sum_{j \in N'} x_{ijk} = 0, \ \forall i \in P', k \in V \tag{6}$$

$$\sum_{j \in N'} x_{0jk} \leq 1, \ \forall k \in V \tag{7}$$

$$\sum_{i \in N'} x_{i(n+1)k} \leq 1, \ \forall k \in V \tag{8}$$

$$\sum_{i \in N'} u_{ji} - \sum_{i \in N'} u_{ij} = q_j, \ \forall j \in P' \tag{9}$$

$$u_{ij} \leq Q \sum_{k \in V} x_{ijk}, \ \forall (i,j) \in A' \tag{10}$$

$$\begin{gathered} y_i - y_j + \tau_i + d_{ij} z_{ijkr}/v_r \leq M(1 - z_{ijkr}), \\ \forall i \in N', j \in P', k \in V, r \in R, i \neq j \end{gathered} \tag{11}$$

$$a_i \leq y_i \leq b_i, \ \forall i \in N' \tag{12}$$

$$y_i = y_j, \ \forall (i,j) \in P^{sync} \tag{13}$$

$$x_{ijk} \in \{0,1\}, \ \forall (i,j) \in A', k \in V \tag{14}$$

$$z_{ijkr} \in \{0,1\} \ \forall (i,j) \in A', k \in V, r \in R \tag{15}$$

$$\sum_{r \in R} z_{ijkr} = x_{ijk}, \ \forall i \in N', j \in P', k \in V, i \neq j \tag{16}$$

$$u_{ij} \geq 0, \ \forall (i,j) \in A' \tag{17}$$

$$y_i \geq 0, \ \forall i \in P' \tag{18}$$

The objective function (4) is the total carbon emission cost based on the speed of the vehicle, the planed distance and the carbon emissions function. Constraint (5) guarantees that each patient is visited only once. Constraint (6) ensures the flow balance of the vehicles, i.e., the caregiver visits the patient and then will leave the patient. Constraints (7) and (8) ensure that the vehicles start at the depot and end at the medical laboratory. Constraint (9) is the flow equation for the demand of patients, and constraint (10) is the capacity constraints. Constraint (11) denotes that the caregiver k can't arrive at j before $y_i + \tau_i + d_{ij}/v_r$, the reason is that the caregiver k needs the service duration τ_i and travel time from i to j. Here, M is a large positive value. Constraint (12) ensures the time window of the patient i. Constraint (13) guarantees the synchronized services. Constraints (14) and (15) are two binary variables. Constraint (16) is the relationship between these two binary variables. Constraints (17) and (18) ensure the non-negative.

The VRPTW has been proven that it is a non-deterministic polynomial hard (NP-hard) problem. The studied problem is a combination of routing problem and speed optimization problem, which is more difficult than VRPTW. Therefore, the studied problem is also a NP-hard problem.

3.3 Speed optimization model

In this paper, we design a two-phase heuristic algorithm to solve the studied MIP model. We decompose the studied problem as a route optimization problem and a speed optimization problem. In this section, we will introduce the mathematical model of the speed optimization problem.

Define a set of routes $S = \{1, 2, ..., s, ...\}$, each route s has m nodes including the depot, the lab and all the patients. And each node is corresponding to the original number $id \in N'$. The speed optimization problem can be formulated as follows:

$$Minimize \sum_{s=1}^{S} \sum_{i=0}^{m-1} \varepsilon\left(v^s_{i,i+1}\right) d^s_{i,i+1} \tag{19}$$

subject to,

$$y^s_{i+1} - y^s_i - d^s_{i,i+1}/v^s_{i,i+1} \geq 0, \ \forall s \in S, i = 1, 2, ..., m-1 \tag{20}$$

$$a_{id} \leq y_{id} \leq b_{id}, \ \forall id \in N' \tag{21}$$

$$y_{id1} = y_{id2}, \ \forall (id1, id2) \in P^{sync} \tag{22}$$

$$v_{lb} \leq v^s_{i,i+1} \leq v_{ub}, \ \forall s \in S, i = 1, 2, ..., m-1 \tag{23}$$

The objective function (19) is the total carbon emission cost in the fixed routes. Constraints (20) and (21) ensure the time window of the node id. Constraint (22) guarantees the synchronized services. Constraint (23) is the speed selection scope.

4. PROPOSED APPROACH

As mentioned before, the proposed model is a NP-hard problem, which is very difficult to solve by using an exact method for the large scale problems. In order to simplify this problem, we design a two-phase heuristic approach through decomposing the studied problem into a routing problem and a speed optimization problem. The proposed method is based on the following two-phase approach:

- *Route construction phase*: a set of routes is built in the first phase. In this phase, we degenerate the MIP model, and use exact method Gurobi solver to solve the degenerated problem with the constant speed.
- *Speed optimization phase*: a set of routes have been calculated in the first phase. In this phase, we design a dynamic programming method for the speed optimization in the fixed routes.

In this section, the proposed dynamic programming (DM) method for the speed optimization problem is detailed.

4.1 Dynamic programming

In this section, a dynamic programming (DP) method is used to solve the carbon emission optimization problem in multiple routes with the constraints of time windows and synchronized visits. Qian and Eglese (2014) have used a DP method to optimize the cost in terms of fuel emissions in a time-varying network. However, rather than in only a single fixed route, the problem with synchronized visits is more complicated.

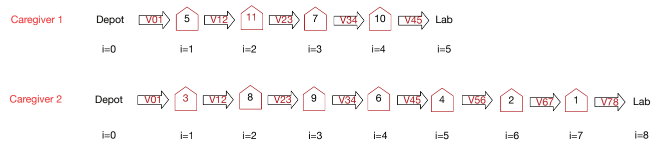

Fig. 1. Example of a solution to an instance with 10 patients.

In order to visualize the model, we take an example of a solution to an instance with 10 patients, which is shown in Fig. 1. It should be noted that the patient 11 is fictitious, and is actually patient 3 who has the demand of synchronized visits. However, due to the constraint of synchronized visits, we can't optimize the speed in a single fixed route, but in multiple routes which have the patiens with the demand of synchronized visits. The caregiver 1 and caregive 2 must serve the patient 3 at the same time, which increase the difficulty of speed optimization.

The speed optimization problem is solved in two steps, and each step involves a recurrence. First, the optimal carbon emissions for p_{i-1} to p_i, where $p_{i-1}, p_i \in P_{sub}$, with different start times, finishing times, and start working time are computed. Second, the optimal carbon emissions for the fixed route through all the patients are calculated.

Dynamic programming recurrence for the adjacent patients Define $\mathbb{C}\left(p_i, t_{start}, t_{finish}, y_i\right)$ as the optimal carbon emissions of traveling from patient p_{i-1} at time t_{start}, arriving at patient p_i at t_{finish}, and start working at y_i. The start working time can be calculated as follows:

$$y_i = \begin{cases} a_i, & if\ t_{finish} \leq a_i \\ t_{finish}, & if\ a_i < t_{finish} \leq b_i \\ \infty, & otherwise \end{cases} \tag{24}$$

where a_i and b_i are the lower bound and upper bound of the time windows at node i, respectively. It should be noticed that for the patients $(i, j) \in P^{sync}$ who needs synchronized services, the start working time is $max\{y_i, y_j\}$. For each pair of adjacent patients $p_{i-1}, p_i \in P_{sub}$, the carbon emissions with all possible starting, finishing, and start working times should be calculated.

Define $f\left(i, t_i, y_i\right)$ as the minimum carbon emissions from the start node to patient i with the associated arrival time t_i and start working time y_i. Define $g\left(arc_{ij}, t_i, t_j\right)$ as the carbon emissions along arc arc_{ij} when the caregiver travels from node i at t_i and arrives node j at t_j. Therefore, it is easy to calculate the speed along arc_{ij} as follows:

$$v_{ij} = d_{ij} / \left(t_j - t_i\right) \tag{25}$$

where d_{ij} is the distance of arc_{ij}. Based on the time windows constraint, it is clear that the smallest speed v_{ij}^{lb} along arc_{ij} can be calculated as follows:

$$v_{ij}^{lb} = d_{ij} / \left(b_j - y_i - \tau_i\right) \tag{26}$$

where τ_i is the service time at node i. Then, the carbon emissions along arc_{ij} can be calculated as follows:

$$g\left(arc_{ij}, t_i, t_j\right) = \begin{cases} \varepsilon\left(v_{ij}\right) d_{ij}, & if\ v_{ij} \geq v_{ij}^{lb} \\ \infty, & otherwise \end{cases} \tag{27}$$

The DP recurrence for updating the value of $f\left(j, t_j, y_j\right)$ is described as follows:

$$f\left(j, t_j, y_j\right) = \min_{t_j \in \{t_i + d_{ij}/v_{ij}\}} \left[f\left(i, t_i, y_i\right) + g\left(arc_{ij}, t_i, t_j\right)\right] \tag{28}$$

The value of $f\left(j, t_j, y_j\right)$ is calculated based on the greedy rules from the start node to node j with the arrival time being t_j and start working time y_j. It is obvious that the value of $f\left(j, t_j, y_j\right)$ may not be the minimum carbon emissions from the start node to node j with the arrival time being t_j and start working time y_j. The value of $f\left(j, t_j, y_j\right)$ can only be considered as the upper bound of the minimum carbon emissions during this trip. Of course, if the start node and j are two adjacent points, $f\left(j, t_j, y_j\right)$ is the optimal carbon emissions. The iterations will be stopped when the value of $f\left(j, t_j, y_j\right)$ cannot be reduced anymore.

Therefore, if the start node is p_{i-1} and the start time is t_{start}, the value of optimal carbon emissions $\mathbb{C}\left(p_i, t_{start}, t_{finish}, y_i\right)$ can be calculated as follows:

$$\mathbb{C}\left(p_i, t_{start}, t_{finish}, y_i\right) = f\left(i, t_{finish}, y_i\right) \tag{29}$$

Dynamic programming recurrence for the fixed route Define $\mathbb{F}\left(p_i, t_{finish}, y_i\right)$ as the optimal carbon emissions from the depot to patient p_i with the arrival time at p_i being t_{finish} and start working time y_i. The following DP recurrence will be utilized to calculate the optimal carbon emissions for the complete route.

$$\mathbb{F}\left(p_i, t_{finish}, y_i\right) = \min_{a_i \leq y_{i-1} + \tau_{i-1} \leq b_i} \{\mathbb{F}\left(p_{i-1}, t_{i-1}, y_{i-1}\right) + \mathbb{C}\left(p_i, y_{i-1} + \tau_{i-1}, t_{finish}, y_i\right)\} \tag{30}$$

Algorithm 1 Dynamic Programming

```
Input: $S_{Input}$;
Output: Optimal carbon emissions $F$, speed $v$;
$N \leftarrow length(S_{Input})$, set $t^1_{arrive} \leftarrow 0$, $t^1_{start} \leftarrow 0$, $F^1 \leftarrow 0$.
for k ← 2,..., N do
    Calculate all the $\{t^k_{arrive}\}$, $\{t^k_{start}\}$, $\{v^{k-1,k}\}$ and $\{F^k\}$ based on $\{t^{k-1}_{start}\}$, speed bound, time windows and synchronized visits constraint;
    if $\{t^k_{start}\}' \subseteq \{t^k_{start}\}$, and all the values of $\{t^k_{start}\}'$ are same then
        $F^k_{min} \leftarrow \min\left(\{F^k\}\right), \ \forall t^k_{start} \in \{t^k_{start}\}'$.
        Update $\{t^k_{arrive}\}$, $\{t^k_{start}\}$, $\{v^{k-1,k}\}$ and $\{F^k\}$.
    end if
end for
```

In the process of recurrence, if there are some decisions with the same start working time y_i at patient p_i, we can compare these decisions and find the optimal carbon emissions at patient p_i associated the same start working time y_i. Because the same start working time will not have an influence on the latter patients, the patient p_i can be considered as a new "depot" for the latter patients. The dynamic programming method is presented in Algorithm 3, $\{t^k_{arrive}\}$, $\{t^k_{start}\}$, $\{v^{k-1,k}\}$ and $\{F^k\}$ are the sets of arrival time, start working time, speed and carbon emissions at node k, respectively. If the synchronized visits constraint doesn't be considered in the process of DP method, the value of $\mathbb{F}(p_i, t_{finish}, y_i)$ can only be considered as the lower bound of optimal carbon emissions of the complete route.

5. COMPUTATIONAL EXPERIMENTS

To the best of our knowledge, there is no existing benchmark instance for our HHC scheduling problem. Therefore, in order to obtain effective benchmark instances, we generate the test instances based on the classical VRPTW benchamark instances designed by Solomon (1987). We use the proposed heuristic approach to solve the studied problem. At the same time, the Gurobi solver is also applied to solve the MIP model.

5.1 Test instances and experiment settings

There are no similar problem in the existing researches, so we generate the test instances based on the classical Solomon VRPTW benchmark instances. In the Solomon VRPTW benchmark instances, the information includes the location of the customers and depot, demand, time windows(ready time, due time), and the service time.

In the Solomon VRPTW benchmark instances, the speed is standardized to 1. It is very necessary to adjust the proportion of the data in the Solomon VRPTW benchmark instances to suit the proposed problem. According to the survey, the normal speed limit is 50 km/h in the city of France. However, the drivers often need to slow down and accelerate during driving when driving to the intersection, so it is difficult to keep an average speed at 50 km/h. In this paper, the HHC scheduling activities happens at a city or a town. Therefore, an average speed 10m/s (namely 36 km/h) is very suitable in the test instances of the proposed problems. In the basis of the Solomon VRPTW benchmark instances, the rules of generating the test instances of the proposed problems are as follows: we set the coordinate of the medical laboratory as (30,40); the distance is 100 times the original, the time window and service time are 10 times the original; other parameters will not be changed. The unit of the Xcoord and Ycoord is meter(m), and the unit of the time windows and service time is second(s).

As for the speed level settings, we set two speed level in the paper. The first is 30 km/h, and the second is 40 km/h. As for the synchronized visits constraint, we set the third patient in every ten patients as the synchronized-service patient.

In this paper, we use two methods to solve the studied problem. The first method is mixed-integer programming (MIP) solved by Gurobi solver, and the second method is a two-phase heuristic approach. The proposed heuristic approach solves the probelm through decomposing the studied problem into a routing problem and a speed optimization problem. The routing probelem is a degenerated probelm of the studied probelm with constant speed 40 km/h, and solved by Gurobi solver. All the experiments are conducted on Intel Core i7-3770, 8 Duo 3.4 GHZ in order to solve the proposed problem.

5.2 Experimental results

In this part, the proposed MIP model is solved. Based on the Proposition 1, for a route between patient i and j, the carbon emission with the speed of 30 km/h is smaller than carbon emission with the speed of 40 km/h. Therefore, for degenerated probelm, if the speed of some route between the patients can be optimized under all the constraints, then the carbon emissions can be reduced again by optimizing the speed. In other words, it is a speed optimization problem in the fixed route under the constraints of time windows and synchronized visits (Wang and Meng, 2012; Qian and Eglese, 2014).

However, due to the constraint of synchronized visits, we cannot optimize the speed in a single fixed route, but in multiple routes with the constraint of synchronized visits. Therefore, we design a dynamic programming method for the speed optimization problem based on the best results solved by Gurobi solver for the degenerated model.

The experimental results for the studied problem are presented in Table. 1. It is obvious that the Gurobi solver can only solve 12 instances (63.16%) for the MIP model, and give a best lower bound and upper bound for other instances in 1 hour, which proves that the studied problem is very complicated and Gurobi solver is very difficult to solve all the instances in 1 hour. Therefore, we design a two-phase heuristic approach for the studied problem. The proposed heuristic approach can solve all the studied instances, and the calculating time of the heuristic approach is much smaller than Gurobi solver, which can demonstrate efficiency of the proposed heuristic approach. The gap between the proposed heuristic approach and Gurobi solver is smaller than 0.16%, which can prove the effectiveness of the proposed heuristic approach.

6. CONCLUSIONS

Transportation cost is one of the largest operating costs in HHC company daily activities, thus it is crucial to

Table 1. The experimental results for the studied problem.

Instance			Gurobi solver		Heuristic-(Gurobi+DP)			
Name	NP	NSync	Cost(kg)	CpuT(s)	Cost(kg)	Gap(%)	CpuT1(s)	CpuT2(s)
HHC_C105	10	1	11.43	0.71	11.43	0.00	0.63	0.002
HHC_C203	10	1	[16.30,20.39]	3,600.00	20.39	0.00	236.05	0.002
HHC_C204	10	1	[15.73,19.29]	3,600.00	19.29	0.00	680.41	0.004
HHC_C205	10	1	22.11	22.37	22.11	0.00	5.99	0.002
HHC_R103	10	1	26.59	159.77	26.82	0.86	48.23	0.012
HHC_R104	10	1	24.25	206.68	24.25	0.00	258.99	0.003
HHC_R105	10	1	30.11	0.34	30.11	0.00	0.50	0.002
HHC_R203	10	1	23.34	191.42	23.34	0.00	123.20	0.002
HHC_R204	10	1	21.94	246.91	21.94	0.00	41.22	0.003
HHC_R205	10	1	23.48	11.15	23.48	0.00	1.39	0.002
HHC_RC103	10	1	22.47	2,945.68	22.74	1.20	1,291.29	0.015
HHC_RC105	10	1	24.47	678.81	24.47	0.00	423.20	0.002
HHC_RC203	10	1	[17.74,21.04]	3,600.00	21.04	0.00	1,647.13	0.002
HHC_RC204	10	1	[17.66,20.52]	3,600.00	20.52	0.00	2,249.72	0.003
HHC_RC205	10	1	[21.91,22.30]	3,600.00	22.30	0.00	593.23	0.005
HHC_C105	25	3	29.73	1,762.08	29.73	0.00	23.14	0.016
HHC_C205	25	3	[31.37,35.81]	3,600.00	34.45	<0.00	1,817.45	0.013
HHC_R105	25	3	64.13	424.90	64.80	1.04	37.08	0.018
HHC_R205	25	3	[44.15,48.53]	3,600.00	47.63	<0.00	214.58	0.009
AVG				1,676.36		<0.16	510.18	0.006

[1] The set [a,b] represents that Gurobi solver doesn't give an exact solution in 3600s, a and b are the best lower bound and upper bound, respectively.
[2] CupT1 is the calculating time in the first phase, and CupT2 is the the calculating time in the second phase of the the proposed two-phase heuristic algorithm.
[3] $Gap = \frac{Heuristic.Cost - Gurobi.Cost}{Gurobi.Cost} \times 100\%$.

optimize the routes of the HHC vehicles in order to reduce the transportation cost meanwhile improving the service quality to patients. However, transportation has serious impacts on the environment. Therefore, it compels the managers to pay more attention to carbon emissions when designing the daily logistics activities. This study addresses a green routing and scheduling problem in home health care with the constraints of synchronized visits and carbon emissions. We formulated the problem as a mixed-integer programming (MIP) model. The MIP model is solved by the optimization solver Gurobi. A two-phase heuristic approach through decomposing the studied problem into a routing problem and a speed optimization problem is proposed. The approach is based on two exact methods using Gurobi solver and dynamic programming (DM) method. The proposed heuristic approach has the advantage to perform joint route and speed optimization within Gurobi solver and DP method, and thus performs much better on difficult instances than single approach.

There are several interesting research directions to this work. On the one hand, it could be very interesting to extend the problem with considering traffic congestion issues in order to reduce even further carbon emissions. On the other hand, we are currently studying the opportunity to develop column generation based exact methods to solve the studied problem.

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