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Speed-gradient Method in Adaptive Control and Identification. Historical Overview ⋆

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Abstract: The paper provides a historical overview of the Speed-gradient method and its applications to adaptive control and identification problems since mid-1970-th, when the method was originated, till the present days. It is demonstrated that it is an efficient and an useful tool for solving a wide range of engineering problems.

Keywords: speed-gradient, adaptive control, identification, nonlinear control, passification

1. INTRODUCTION

The first publications related to the speed-gradient type algorithms appeared in 1978. The general formulations were proposed simultaneously and independently by Alexander Fradkov and Yuri Neimark in January, 1978 at the 9th All-Union school on adaptive systems (Fradkov, 1979; Neimark, 1979).

Some related formulations for the identification problem were suggested by Krasovsky (1976), who considered the following plant model, known to within parameters

$$\dot{x} + f(x, a, u, \xi) = 0, \tag{1}$$

where x, u are the state and control vectors, assumed to be measured by the sensors or estimated; a denotes the vector of unknown parameters; $\xi = \xi(t)$ is the vector of disturbances, measured with a certain accuracy. The following *plant model* is taken

$$\dot{x}_m + f_m(x_m, a_m, u, \xi_m) = 0 \tag{2}$$

with state-space vector $x_m(t)$, adjustable parameters vector $a_m(t)$ and estimated disturbances vector $\xi_m(t)$. It is assumed that in the case of the arguments equivalence, $f_m(\cdot) \equiv f(\cdot)$. The problem is considered of ensuring convergence of adjustable parameters $a_m(t)$ to "true" values a. Based on (Krasovsky, 1973), the following *generalized performance criterion I*, including the integral quadratic state error $x_m(t) - x(t)$ and the norm of the parameters adjusting speed $u_a \equiv \dot{a}_m(t)$ is used in (Krasovsky, 1976)

$$I = V \exp(-t'/T_Q)\big|_{t+T} + \int_t^{t+T} Q(x_m - x) \exp(-t'/T_Q)\, \mathrm{d}t' + \frac{1}{2}\int_t^{t+T} \sum_{j=1}^{N} k_{aj}^{-2} \left(u_{aj}^2 - u_{aj\,\mathrm{opt}}^2\right) \exp(-t'/T_Q)\, \mathrm{d}t' \tag{3}$$

where $Q(\cdot) > 0$; T_Q, k_{aj} are design parameters; functions $V(\cdot)$, $u_{aj\,\mathrm{opt}}$ are defined below. Based on the approach of (Krasovsky, 1973), for (2), (3), the following "optimal" (in the sense of (3)) identification algorithm is derived

$$u_{aj} \equiv u_{aj\,\mathrm{opt}} = -k_{aj}^2 \frac{\partial V}{\partial a_{mj}}, \tag{4}$$

where V is a solution to the following partial differential equation

$$\frac{\partial V}{\partial t} - \frac{1}{T_Q} V - \sum_{i=1}^{n} f_{mi} \frac{\partial V}{\partial x_{mi}} = -Q(x_m - x), \tag{5}$$

or, defining the linear operator $L_t = \frac{\partial}{\partial t} - \sum_{i=1}^{n} f_{mi} \frac{\partial}{\partial x_{mi}}$, one can rewrite (5) as follows

$$(L_t - 1/T_Q) = -Q. \tag{6}$$

Then its solution may be represented in the form $V = T_Q(1 + T_Q L_t + T_Q^2 L_T^2 + \ldots)Q$. This leads to the following identification algorithm

$$\dot{a}_m = -T_Q k_a^2 \frac{\partial}{\partial a_m} \Big(\big(1 + T_Q L_t + T_Q^2 L_T^2 + \ldots\big) Q(x_m - x) \Big). \tag{7}$$

The simplified approximation of (7) may be written as follows, see (Krasovsky, 1976)

$$\dot{a}_m = T_Q^2 k_a^2 \frac{\partial f_m}{\partial a_m} \frac{\partial Q}{\partial x_m}, \tag{8}$$

which coincides with some adaptive identification algorithms, derived, particularly, by means of Fradkov (1980) SG-method.

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First (yet distinct) stability results were published in (Neimark, 1978) and in (Fradkov, 1980). Neimark (1978) considered the system where the collective of automatons changes adaptively adjustable parameters along the speed of decreasing some cost function V. The plant model is taken in the form

$$\dot{x} = f(x,u), \tag{9}$$

where x denotes the state-space vector, u stands for the vector of adaptively adjustable parameters. It is assumed that $f(0,\cdot) = 0$ and $f(\cdot,u)$ is linear on u. Cost function $V(x)$ is defined such that $V(x) \geqslant \varphi(\|x\|) \geqslant 0$, where $\varphi(\rho)$ is increasing on ρ function and $V(0) = 0$. It is also assumed that for some $u = u^*$, $\sigma > 0, M > 0$ the following inequality is fulfilled

$$\dot{V}\big(x(t)\big) \equiv \frac{\partial V}{\partial x} f(x,u^*) < -\min\{\sigma V(x), M\}. \tag{10}$$

The following Theorem is proven.

Theorem 1. (Neimark, 1978, Theorem 7.4). Let the adaptation law be taken as

$$\dot{u} = -\alpha \nabla_u \frac{\partial V}{\partial x} f(x,u), \quad \alpha > 0. \tag{11}$$

For all solutions of (9), (11) it is valid that $x(t) \to 0$, $u(t) \to \bar{u}$ as $t \to \infty$. Additionally

$$\int_0^\infty V\big(x(t)\big)\,\mathrm{d}t < \frac{1}{\sigma} V\big(x(0)\big) + \frac{1}{2\alpha}\big(u(0) - u^*\big)^2,$$

and $\bar{u}$ lies in the stability region of the closed-loop system with the exception of the case when $\big(x(0), u(0)\big) \in S_p^+$, where S_p^+ denotes the invariant sets of saddle equilibrium states for system (9), (11).

In (Neimark, 1978) the case of stochastic disturbances x (t), added to the right-hand side of plant model (9), is also discussed. For avoiding possible system instability in this case, the modification of adaptation law (11) by introducing the penalty function is suggested.

For the special case affine time-invariant controlled system $\dot{x} = f(x) + g(x)u$ and positive definite goal function $V(x)$ the control algorithm $u = -L_g V(x)$ was proposed by Jurdjevic and Quinn (1978) It is sometimes called "LgV" or "Jurdjevic–Quinn" control. Stability result in Jurdjevic and Quinn (1978) is related to the case $\dot{V} \leqslant 0$ and requires some detectability conditions (so called "Jurdjevic-Quinn" conditions), cf. (Sepulchre et al., 1997)).

Non-affine and time-varying case was first studied in (Fradkov, 1980) for differential form of SG-algorithms and in (Fradkov, 1985, 1986) for the finite form.

Various types of the speed gradient algorithms were proposed as a set of designing schemes and their applicability conditions by Alexander Fradkov in the framework of the unified *Speed-gradient method* (the SG method). This method was originated in (Fradkov, 1980) as a universal approach for solving various control problems, originally with a focus on designing the adaptation and identification algorithms, cf. (Fradkov, 1980, 1985, 1986; Seron et al., 1995; Fradkov and Pogromsky, 1998; Fradkov, 2007). The basic idea of the method is expressed by Fradkov (1980) as follows: "The paper is concerned with a scheme for design of adaptive control algorithms whereby motion is organized in the space of parameters to be adjusted along the gradient of the speed of change of an evaluative functional." During the subsequent years, the method was further developed by for elaborating the various schemes of adaptation, non-linear control, identification and synchronization. This method has found application in the works by many researchers worldwide. Jordán and Bustamante (2006) recognized it as the method, which "enables a transparent trade-off between control performance and design parameters. Furthermore the steps for controller design results are in general simple ... it has become widespread in other multiple successful applications in adaptive control mainly in Physics and Mechanics." The SG-methodology was extended to the speed-difference one which allowed to relax the matching (attainability) conditions in (Druzhinina and Fradkov, 1994).

During the last decade, the interest appeared to the SG method as an efficient tool not only for solving the engineering problems, but also for understanding the laws of nature, such as ecological systems dynamics, or fundamental laws of Physics. In this interpretation this approach is known as the *Speed-gradient principle* (Fradkov, 2007; Selivanov, 2011; Khantuleva and Shalymov, 2017, 2018; Plotnikov et al., 2016).

The present paper is focused on the SG-method application to adaptive control and identification problems. The rest of the paper is organized as follows. The links between the SG method and some general adaptation and identification schemes are given in Sec. 2. Some results of the SG method application to designing the adaptive and identification algorithms are outlined in Sec. 3. Concluding remarks are given in Sec. 4.

2. GENERAL ADAPTATION AND IDENTIFICATION SCHEMES IN THE SG FRAMEWORK

2.1 Model Reference Adaptive Control

Model Reference Adaptive Control (MRAC) algorithms for solving the problem are obtained in the series of fundamental works on the adaptive control theory, see e.g. (Landau, 1979), where the plant and the reference model are taken as

$$\begin{array}{ll} \dot{x} = Ax + Bu, & \text{– plant} \\ \dot{x}_M = A_M x_M + B_M r(t) & \text{– reference model.} \end{array} \tag{12}$$

The problem of asymptotical convergence of error vector $e(t) = x(t) - x_M(t)$ to zero is posed. The adaptation algorithm has the following form

$$\begin{array}{l} \nabla_{\Delta A}\,\omega(x,\theta,t) = Pe(t)x(t)^{\mathrm{T}}, \\ \nabla_{\Delta B}\,\omega(x,\theta,t) = Pe(t)r(t)^{\mathrm{T}}. \end{array} \tag{13}$$

Algorithms (13) can be inferred from the SG scheme by using the target functional $Q_t := \frac{1}{2}e(t)^{\mathrm{T}}Pe(t)$; $P = P^{\mathrm{T}} > 0$ is $n \times n$ matrix satisfying the Lyapunov equation $PA_M + A_M^{\mathrm{T}}P = -G$ for some $G = G^{\mathrm{T}} > 0$. The vector of tunable parameters in this case is a set of matrices $\Delta A(t)$, $\Delta B(t)$ elements and the differential form of SG-algorithms (Andrievskii et al., 1988; Fradkov, 1990) is used.

For preventing an unlimited growth of controller coefficients under the action of disturbances, it is recommended to use the regularized adaptation algorithm (Andrievskii et al., 1988; Fradkov, 1990) of the form

$$\begin{aligned} \frac{d}{dt}\Delta A(t) &= -\gamma\big(Pe(t)x(t)^{\mathrm{T}} + \alpha\big(\Delta A(t) - \Delta\hat{A}\big)\big), \\ \frac{d}{dt}\Delta B(t) &= -\gamma\big(Pe(t)r(t)^{\mathrm{T}} + \alpha\big(\Delta B(t) - \Delta\hat{B}\big)\big), \end{aligned} \tag{14}$$

where $\widehat{\Delta A}, \widehat{\Delta B}$ are some *a priori* estimates of tunable parameters.

2.2 Simple Adaptive Control with Implicit Reference Model

The Implicit Reference Model (IRM) approach was originated in (Fradkov, 1974), employed to adaptive tuning of PID-conroller in (Andrievsky and Fradkov, 1994) and extended to the synchronization problems in (Andrievskii and Fradkov, 2006). The IRM adaptive control laws may be derived with the help of the SG method with local objective functional $Q = \frac{1}{2}x^{\mathrm{T}}Px$, where $x \in \mathbb{R}^n$ denotes the plant state vector, $(n \times n)$ matrix P is positive-definite, $P = P^{\mathrm{T}} > 0$. The adjustable control law in the "main loop" is taken as $u = K(t)y$, where u is the control action, y is the measurable plant output, $K = K(t)$ are the controller gains, adjusted by means of the adaptation algorithm

$$\dot{K}(t) = -\gamma\delta(t)y(t), \quad \delta(t) = \sum_{i=1}^{l} g_i y_i(t), \quad u(t) = \sum_{i=1}^{l} \theta_i(t)y_i(t).$$

Adaptive Stabilization of LTI SISO Plants. Let LTI SISO plant be modeled in the input-output form as

$$A(p)y(t) = B(p)u(t), \quad t \geqslant 0, \tag{15}$$

where u, y are scalar input and output variables, $A(p) = p^n + a_{n-1}p^{n-1} + \cdots + a_0$, $B(p) = b_m p^m + \cdots + b_0$ are polynomials in operator of differentiation on time $p \equiv d/dt$. Define k as the *relative degree* of system (15), $k = n - m > 0$. Plant (15) parameters a_i, b_j $(i = 0,\ldots,n-1,\ j = 1,\ldots,m)$ are assumed to be unknown. Desired closed-loop system performance may be expressed in the form of a certain "reference" differential equation. In the classical MRAC this equation is explicitly implemented in the adaptive controller by the Reference Model, cf. (Landau, 1979). To describe the IRM adaptive controllers, let us introduce an adaptation error signal $\sigma(t)$ as

$$\sigma(t) = G(p)y(t), \tag{16}$$

where $G(p) = p^l + g_{l-1}p^{l-1} + \cdots + g_0$ is a given Hurwitz polynomial in operator $p \equiv d/dt$. Coefficients g_i are the design parameters; they are chosen based on the desired closed-loop system dynamics. Degree l of polynomial $G(p)$ is defined below. Assuming that the adaptation law ensures tendency $\sigma(t)$ to zero let us notice that as $\sigma \equiv 0$, output $y(t)$ satisfies the following "*reference equation*"

$$G(p)y(t) = 0. \tag{17}$$

This equation describes the reference model, but this model is not implemented in the adaptive controller in the form of a certain dynamical subsystem, but introduced *implicitly* via its parameters g_i $(i = 0,1,\ldots,l-1)$. Therefore it is called Implicit Reference Model (IRM).

Let us choose the feedback control law in the following form:

$$u(t) = \sum_{i=0}^{l} k_i(t)\big(p^i y(t)\big), \tag{18}$$

where $k_i(t)$ $i = 0,\ldots,l$ are adjustable controller parameters. For the considered case the HMP property leads to the following adaptation law, see (Fradkov, 1974):

$$\dot{k}_i(t) = -\gamma\sigma(t)p^i y(t), \quad k_i(0) = k_i^0, \tag{19}$$

where $\gamma > 0$ is the *adaptation gain*, k_i^0 are given initial values of the controller gains, $i = 0,\ldots,l$. Introducing row vector $G = [g_0, g_1, \ldots, 1] \in \mathbb{R}^{l+1}$ and plant (15) transfer function $W(s)$ from input u to output vector $[y, \dot{y}, \ldots y^{(l)}]^{\mathrm{T}} \in \mathbb{R}^{l+1}$ as $W(s) = \frac{B(s)}{A(s)}\left[1, s, \ldots, s^l\right]^{\mathrm{T}}$, $s \in \mathbb{C}$, in virtue of Passification Theorem by Fradkov (1974) with respect to transfer function $GW(s)$, one may easily derive the following stability conditions of adaptive controller (18), (19):

(1) polynomial $B(s)$ is Hurwitz and $b_0 > 0$;
(2) $l = k-1$, where $k = n-m$ is a relative degree of plant model (15).

Algorithm (19) usually ensures vanishing $\sigma(t)$ essentially faster than transients in the closed-loop. As a result, changing the controller (18) gains is stopped and plant (15) output $y(t)$ obeys the IRM (17).

To avoid unlimited growth of controller (18) gains in the presence of external disturbances and measured errors, the following α-modification of (19) may be used, cf. (Ioannou and Kokotovic, 1984; Andrievskii and Fradkov, 2006)

$$\dot{k}_i(t) = -\gamma\sigma(t)p^i y(t) - \alpha\big(k_i(t) - k_i^0\big), \quad k_i^0 = k_i(0), \tag{20}$$

where the *parametric feedback gain* $\alpha \geqslant 0$ is introduced.

Adaptive Tracking Systems with IRM. Adaptive control law (18), (20) may be straightforwardly extended to the solving the tracking problem with the desired closed-loop system dynamics, see (Andrievsky and Fradkov, 1994). To this purpose let us introduce reference signal $r(t)$ and define adaptation error signal $\sigma(t)$ as

$$\sigma(t) = G(p)y(t) - D(p)r(t), \tag{21}$$

where polynomial $G(p)$ is defined above, and operator polynomial $D(p)$ has the form $D(p) = d_q p^q + \cdots + d_1 p + d_0$. Signal $\sigma(t)$ may be treated as the discrepancy in the equation

$$G(p)y(t) = D(p)r(t), \tag{22}$$

considering (22) as the IRM for the case of tracking.

By the analogy with (18) let us take the control action in the form

$$u(t) = k_r(t)\big(D(p)r(t)\big) + \sum_{i=0}^{l} k_i(t)\big(p^i y(t)\big), \tag{23}$$

where $k_r(t)$, $k_i(t)$ $(i = 0,\ldots,l)$ are tunable parameters. The adaptation law is as

$$\begin{aligned} \dot{k}_r(t) &= \gamma\sigma(t)D(p)r(t) - \alpha\big(k_r(t) - k_r^0\big), \quad k_r^0 = k_r(0), \\ \dot{k}_i(t) &= -\gamma\sigma(t)p^i y(t) - \alpha\big(k_i(t) - k_i^0\big), \quad k_i^0 = k_i(0), \end{aligned} \tag{24}$$

where $\gamma > 0$, $\alpha \geqslant 0$ are design parameters; k_r^0, k_i^0 are "guessed" initial controller gain values, $i = 0,\ldots,l$. It is worth mentioning that both degree q of polynomial $D(p)$ and its coefficients may be chosen arbitrarily by the designer.

Signal-Parametric Adaptive Controllers with IRM. Let the regulation goal $\lim_{t\to\infty} x(t) = 0$ for plant model

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \tag{25}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^m$ be posed. Following (Hsu and Costa, 1989; Andrievskii and Fradkov, 2006) introduce an auxiliary goal as ensuring the *sliding mode* motion along the predefined surface, s.t. $\sigma(t) \equiv 0$, where $\sigma(t) = Gy(y)$, G is a given $(l \times n)$-matrix. Let us use the following control law

$$u = -\gamma \operatorname{sign}\sigma, \quad \sigma = Gy, \tag{26}$$

where $\gamma > 0$ is a controller parameter. It may be proved that for system (25), (26) the posed control goal may be achieved if there exist matrix $P = P^{\mathrm{T}} > 0$ and vector K_* s.t. $PA_* + A_*^{\mathrm{T}} P < 0$, $PB = GC$, $A_* = A + BK^{\mathrm{T}} C$. As follows from Passification Theorem, the mentioned conditions are fulfilled iff: transfer function $GW(s)$ transfer function $GW(s)$ is HMP (where $W(s) = C\big(\lambda I_n - A\big)^{-1} B$); the sign of GCB is known (we assume that it is positive). Under these conditions the goal $\lim_{t\to\infty} x(t) = 0$ is achieved for sufficiently large γ (with respect to the initial conditions and actual plant parameters).

To avoid dependence of closed-loop system stability on initial conditions and plant parameters, the following "*signal-parametric*", or "*combined*" control law may be used instead of (26):

$$\begin{aligned} u &= K^{\mathrm{T}}(t)y(t) - \gamma \operatorname{sign}\sigma(y), \quad \sigma(y) = Gy(t) \\ \dot{K}(t) &= -\sigma(y)\Gamma y(t), \end{aligned} \tag{27}$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0$, $\gamma > 0$ are matrix and scalar adaptation law gains.

Remark 1. For the case of scalar control input the following control law, inspired by papers (Levant et al., 2000; Shtessel et al., 2012) may be used instead of (27):

$$\begin{aligned} u &= -k(t)\sigma(t) - \gamma_\sigma \operatorname{sign}\big(\sigma(t)\big)\sqrt{|\sigma(t)|}, \quad \sigma(t) = Gy(t) \\ \dot{k}(t) &= \gamma_k \sigma(t)^2. \end{aligned}$$

This law produces more smooth control action than the "relay" law (27).

Parameter Identification Let the plant with uncertain parameters be modeled as

$$\dot{x}(t) = A_* x(t) + B_* u(t),. \tag{28}$$

with unknown constant matrices A_*, B_*, a certain Hurwitz matrix G, and measurable state and input vectors $x(t) \in \mathbb{R}^n$, $u \in \mathbb{R}^m$. For identification, introduce the following *Adjustable Model* (Narendra and Kudva, 1974)

$$\dot{x}_M(t) = Gx_M(t) + \big(A(t) - G\big)x(t) + B(t)u(t), \tag{29}$$

with state-space vector $x_M(t) \in \mathbb{R}^n$ and matrices $A(t)$, $B(t)$ which serve as estimates of A_*, B_*. In this case vector θ is defined as $\theta(t) = \operatorname{col}(A, B)$. Introduce the error signal $e(t) = x_M(t) - x(t)$, and the goal function

$$Q_t = 1/2 e(t)^{\mathrm{T}} Pe(t), \quad \text{where} P = P^{\mathrm{T}} > 0. \tag{30}$$

Following the SG design procedure, one obtains

$$\omega = \dot{Q}_t = 1/2 e(t)^{\mathrm{T}} P\big(Ge(t) + (A - A_*)x(t) + (B - B_*)u(t)\big) \tag{31}$$

$$\nabla_A \omega = Pe(t)x(t)^{\mathrm{T}}, \quad \nabla_B \omega = Pe(t)u(t)^{\mathrm{T}}, \tag{32}$$

$$\begin{cases} \dot{A}(t) = -\gamma Pex^{\mathrm{T}}, \quad \gamma > 0, \\ \dot{B}(t) = -\gamma Peu^{\mathrm{T}}. \end{cases} \tag{33}$$

3. LAST YEARS APPLICATION RESULTS FOR ADAPTIVE CONTROL

3.1 Adaptive Wing Rock Suppression

The *wing rock* phenomenon is a self-excitation of a swinging motion in a strong attack angle. When a winge rock appears, the roll angle undergoes oscillations of increasing amplitude, asymptotically converging to a stable limit cycle (Ng et al., 1991; Katz, 1999). The wing rock dynamics are represented The wing rock dynamics are represented by a substantially nonlinear model whose parameters vary over a wide range depending on the flight conditions (height, Mach number, payload mass, etc.) and attack angle.

Lee et al. (2016) proposed the SG adaptation algorithm in the final form for the roll angle control, which simultaneously suppresses the wing rock motion. The following model of the roll angle dynamics is used:

$$\ddot{\varphi} + a_0\varphi + a_1\dot{\varphi} + a_2|\dot{\varphi}|\dot{\varphi} + a_3\varphi^3 + a_4\varphi^2\dot{\varphi} = bu, \tag{34}$$

where φ denotes the roll angle, u is the control action (the aileron deflection), $a_i = a_i(\alpha)$, $b = b(\alpha) > 0$ are unknown aircraft model parameters, depending on the angle of attack α. The problem of roll angle φ tracking the reference variable φ^* is considered, which means bringing the system state to the target manifold $\psi(\varphi, t) \equiv \dot{e} + \lambda e = 0$, where $e = \varphi - \varphi^*(t)$.

For employing the SG design method, the objective functional is taken as

$$Q = \frac{1}{2}\int_0^t \big(\dot{\psi}(\varphi, s, \theta) + \gamma(\psi(\varphi, s, \theta))\big)^2 \mathrm{d}s \tag{35}$$

where $\gamma(\psi) = k_1\psi + k_2\psi^3$, $k_1, k_2 > 0$ are design parameters. The adjustable control law in the main loop is chosen in the form

$$u = -\theta(t)^{\mathrm{T}}\chi(\varphi, \dot{\varphi}, t). \tag{36}$$

with function χ on $[\varphi, \dot{\varphi}, |\dot{\varphi}|\dot{\varphi}, \varphi^3, \varphi^2\dot{\varphi}]^{\mathrm{T}}$, subjected to determination, where $\theta(t) \in \mathbb{R}^5$. Implementation of the consequent SG-procedure steps leads to the following expressions: $\omega(\cdot) \equiv \dot{Q}_t = \frac{1}{2}(\psi + \gamma)^2$, $\frac{\partial\omega(\cdot)}{\partial\theta} = -b\big(\psi + \gamma(\psi)\big)\chi(x, t)$. Then the following *finite-differential form* of the SG-algorithm is obtained:

$$\theta = \Gamma\big(\theta_p + \theta_I\big), \quad \text{where} \tag{37}$$

$$\theta_p(x, t) = \psi(x, t)\chi(x, t) - \Psi(x, t), \tag{38}$$

$$\dot{\theta}_I(x, t) = -\psi\left(\frac{\partial\chi}{\partial t} + \frac{\partial\chi}{\partial\varphi}\dot{\varphi}\right) + \frac{\partial\Psi}{\partial t} + \frac{\partial\Psi}{\partial\varphi}\dot{\varphi} + \zeta(\psi)\chi, \tag{39}$$

In (37), $\Gamma = \Gamma^{\mathrm{T}} > 0$ stands for the adaptation gain matrix; $\frac{\partial\Psi}{\partial\dot{\varphi}} = \psi\frac{\partial\chi}{\partial\varphi}$. In (Lee et al., 2016) is proved that $\varphi(t)$, $\dot{\varphi}(t)$, $\tilde{\theta}^{\mathrm{T}}\chi(x, t)$ tend to zero as $t \to \infty$.

In (Lee et al., 2016) the comparative simulation results are presented for the SG control law and the law obtained by the I&I approach, see (Astolfi et al., 2008; Lee and Singh, 2014). Both the adaptation laws include an integral update rule and an algebraic state-dependent vector function. Simulation results showed that both the adaptive systems are capable of suppressing the wing rock motion, despite uncertainties in the model parameters at various angles of attack.

The similar control problem is considered by Andrievsky et al. (2019). The aircraft roll motion is modeled by (34). For adaptive wing rock suppression, the simple adaptive control with IRM, described in Sec. 2.2 is employed, which leads to the following adaptive controller for the roll motion:

$$e(t) = \varphi(t) - \varphi^*(t), \quad \dot{\xi}(t) = e(t), \ \ \xi(0) = 0 \tag{40}$$

$$\sigma(t) = \tau\varphi(t) + e(t) \quad \text{– adaptation error,} \tag{41}$$

$$u(t) = -\big(k_i\xi(t) + k_p(t)e(t) + k_d(t)\varphi(t)\big), \tag{42}$$

$$\dot{k}_p(t) = \gamma\sigma(t)e(t) - \lambda\left(k_p(t) - k_p^0\right), \ \ k_p(0) = k_p^0, \tag{43}$$

$$\dot{k}_d(t) = \gamma\sigma(t)\varphi(t) - \lambda\left(k_d(t) - k_d^0\right), \ \ k_d(0) = k_d^0, \tag{44}$$

where $\varphi(t)$ is the roll reference signal, $\tau > 0$, $\gamma > 0$, $\lambda > 0$ are the design parameters; k_p^0, k_d^0 denote initial values of the tunable proportional and derivative gains, found based on the available *a priori* information on plant model parameters (the design parameters). If the *a priori* data gives no opportunity for meaningful choice of k_p^0, k_d^0 it is reasonable to pick up $k_p^0 = 0$, $k_d^0 = 0$. It should be noted that integral gain k_i in (42) is not subjected to tuning.

3.2 Robust Control of Aircraft Lateral Movement

Robust output feedback continuous control design for time-continuous linear plants under parametric uncertainties and external bounded disturbance is considered in (Furtat et al., 2014). The proposed algorithm tracks the output of the plant to the reference output with the required accuracy. Application of the algorithm to the control of lateral movement of an aircraft under parametric and external disturbances is presented and comparison of the proposed algorithm with H_∞ and the SG control is given. The simulation results illustrate the efficiency and robustness of the suggested control system.

3.3 Airfoil Flutter Suppression by IRM Adaptive Controller

In (Andrievsky et al., 2018), the Simple Adaptive Control scheme based on the IRM and the SG method is designed for active suppressing the airfoil flutter. Two-dimensional wing, oscillating in pitch and plunge is considered. The SG method is applied for active suppressing the airfoil flutter. Pitch angle α is a wing inclination with respect to the axis of elasticity. The matrix-vector equation form for deflection and torsion of the wing is taken as (Chen et al., 2012; Abdelkefi et al., 2013)

$$\begin{bmatrix} I_\alpha & m_w x_\alpha b \\ m_w x_\alpha b & m_t \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{h} \end{bmatrix} + \begin{bmatrix} c_\alpha & 0 \\ 0 & c_h \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{h} \end{bmatrix} + \begin{bmatrix} k_\alpha(\alpha) & 0 \\ 0 & k_h(h) \end{bmatrix} \begin{bmatrix} \alpha \\ h \end{bmatrix} = \begin{bmatrix} M \\ -L \end{bmatrix}, \qquad (45)$$

where m_t is the total mass of the main wing and the support structure, m_w is the mass of the main wing, x_α is the dimensionless distance between the center of mass and the bending axis; I_α is the moment of inertia; b is the average chord of the wing; c_α, c_h are the coefficients of the damping by plunge displacement and the pitch angle, respectively; $k_h(h)$ and $k_\alpha(\alpha)$ are the spring stiffness coefficients for the displacement and pitch angle, respectively, so that $\alpha k_\alpha(\alpha)$ is a nonlinear term $\alpha k_\alpha(\alpha) = k_1\alpha + k_2\alpha^2$.

The following proportional-derivative IRM adaptive control law is proposed for active flutter suppression system:

$$u(t) = k_p(t)\alpha(t) + k_d(t)\dot{\alpha}(t), \qquad (46)$$

$$\sigma(t) = \dot{\alpha}(t) + g_0\alpha(t), \qquad (47)$$

$$\dot{k}_p(t) = \gamma\sigma(t)\alpha(t) - \lambda\left(k_p(t) - k_p^0\right), \quad k_p(0) = k_p^0, \qquad (48)$$

$$\dot{k}_d(t) = \gamma\sigma(t)\dot{\alpha}(t) - \lambda\left(k_d(t) - k_d^0\right), \quad k_d(0) = k_d^0, \qquad (49)$$

3.4 Angular Velocity Stabilization for a Spinning Satellite

Andrievsky and Guzenko (2014) considered the problem of angular velocity stabilization for a spinning satellite, supplied by and small resistojets and the damper, centered on the body fixed X-axis and has a point mass m. That mass moves along an axis perpendicular to X-axis at the some distance of the principal axis Z. The following model by Meehan and Asokanthan (2006) is used:

$$\begin{cases} (I+m(1-\mu)y^2)\dot{\omega}+2m(1-\mu)y\dot{y}\omega-mb\ddot{y}=M(t), \\ m(1-\mu)\ddot{y}+c\dot{y}+\left(k-(1-\mu)\omega^2\right)y-b\dot{\omega}=0, \end{cases} \qquad (50)$$

where ω, y are satellite angular velocity and damper mass displacement; I, m, k c are the satellite moment of inertia about Z-axis, damper mass, spring constant and viscous resistance gain (respectively); $\mu = m/m_T$, where m_T denotes a total mass of the system. The external torque $M(t)$ is a sum of excitation torque $M_E(t)$ and control torque, i.e. $M_C(t)$ s.t. $|M_C(t)| \leqslant \bar{M}$. Application of the energy-based GS design method gives the following “proportional” and “relay” control laws:

$$M_C = \gamma\big(H_{\mathrm{ref}} - H(y,\dot{y},\omega)\big)\cdot\big(\omega + \dot{\tilde{y}}(\tilde{I}+\tilde{y}^2-1)^{-1}\big),$$

$$M_C = \gamma\,\mathrm{sign}\left(H_{\mathrm{ref}} - H(y,\dot{y},\omega)\right)\cdot\mathrm{sign}\left(\omega + \dot{\tilde{y}}(\tilde{I}+\tilde{y}^2-1)^{-1}\right),$$

where $\tilde{y} = (1-\mu)b^{-1}y$, $\tilde{I} = (1-\mu)m^{-1}b^{-2}I$ are introduced. These control laws can be directly implemented by means of the on-off operating resistojets.

4. CONCLUSIONS

The present paper presents a historical overview of the SG-method with the focus of its application to adaptive control and identification problems. It is demonstrated that it is a useful and an efficient tool for solving a wide range of engineering problems, confirming that it “enables a transparent trade-off between control performance and design parameters” (Jordán and Bustamante, 2006). Today, after 40 years of development the SG-method is used by many authors who employ it to tackle various application problems of identification and control.

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