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Recursive Estimation of Three Phase Line Admittance in Electric Power Networks

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Abstract: Synchronized phasor measurements in power transmission and distribution networks enable real-time monitoring of voltage and currents. Such measurements can be used to monitor power flow, but also to monitor important electric parameters of the network. In this paper, it is shown how synchrophasor measurements can be used for real-time monitoring of the admittance of the connections between buses in a power network, typically the three-phase transmission or distribution lines. The objective is to formulate admittance monitoring capabilities in which changes in three-phase line admittance can be monitored in real-time and achieved by the formulation of synchrophasor-based recursive estimation techniques over short time intervals.

Keywords: Synchrophasor; Recursive Least Squares; Networks; Admittance

1. INTRODUCTION

Reliability of the modern AC power system requires adequate flexibility to address variability and uncertainty in both demand and generation resources to main voltage and frequency within acceptable ranges (US Department of Energy, 2016). An important aspect in reliability assessment is the transmission and distribution system used for power delivery (Abunima et al., 2018). Consequently, the ability to detect abrupt changes in line impedance in a power network is an important task in maintaining reliability of the power network.

The idea of monitoring three phase impedance or admittance is not new: well-known steady-state algorithms include the Détection Sélective par les Intensités Résiduels (DESIR) (Griffel et al., 1997) and Detection Differential by using the total phase-to-ground Admittance (DDA) (Welfonder et al., 2000). Both DESIR and DDA are inherently static analysis methods and carry the disadvantages of selecting threshold values that must allow for three phase asymmetry and measurement noise (Chilard et al., 1997). Dynamic extensions and improvements for DESIR and DDA have been proposed to resolve these issues, including but not limited to the work by Zamora et al. (2007) and Sagastabeitia et al. (2011) to detect high-resistance ground faults in medium-voltage (MV) power distribution networks and more recent approaches to detect a single phase to ground can be found in the work by Liu et al. (2019).

The proliferation of phasor measurement units (PMUs) to measure three phase voltage and currents have improved signal-to-noise and real-time monitoring capabilities of the power network (Phadke, 2002). Novel approaches to detect faults in a network using synchrophasor data and state estimation (Pignati et al., 2017) rely on network information in the form of a network admittance matrix, commonly seen in network modeling (Newman, 2010) and the estimation of topology of networks (Materassi and Innocenti, 2010) and physical networks (Ardakanian et al., 2019; Kivits and Van den Hof, 2019). As the entries of the network admittance matrix are the admittance/impedance connection between buses, measurement of three phase voltage and current signals can be used to characterize the significant components (Choqueuse et al., 2019) of the three phase power flow. Such a data driven approach will facilitate event detection (Zhou et al., 2019) but also detailed information on three phase impedance changes in the network.

Synchrophasor measurements are used in this paper to present a data-driven approach to recursively estimate the three phase admittance connections in a power network. Relatively less literature is available on using complex PMU data of all three phases of a line to detect phase imbalance and estimate the three phase admittance. Unlike the contribution in Ardakanian et al. (2019), the approach presented here assumes the topology of the network to be known due to the physical location of transmission or distribution lines.

The contribution of this paper is the development and testing of a recursive estimation over short time intervals to be able to detect abrupt changes in the three phase line admittance. The notion of a zero sequence voltage and residual current as proposed in Liu et al. (2019) will also be used in this work, but extended to the detection of three phase admittance changes. The singular value decomposition approach as proposed in Zhou et al. (2019) will be used to address the identifiability the three phase impedance over short time intervals. The approach will be illustrated on actual three phase synchrophasor measurements obtained from a distribution circuit. This paper states the least number of phasor measurements required to estimate the admittance parameters. This is done by formulating a Least Squares estimation problem and specifying the condition of invertability of the regressor matrix used in the estimation.

2. NOTATION AND NETWORK TOPOLOGY

For illustration of the methodology presented in this paper, a small power network of five buses with three phase line connections between the buses as given in Fig.1 is considered. From a topology point of view, the distribution network has a know topology (Ardakanian et al., 2019) but with unknown 3×3 admittance values between the buses. From a physical network point of view (Kivits and Van den Hof, 2019), the buses are considered the nodes of the network. The nodal three phase voltages are considered as the node signals whereas the three phase branch currents are the external input signals used for estimation of the 3×3 admittance.

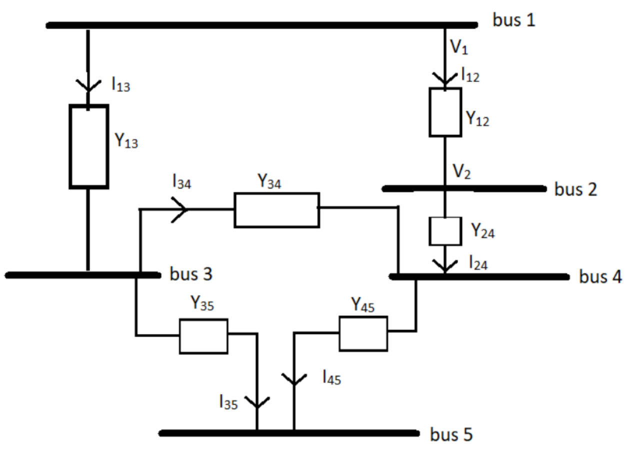


Fig. 1. A sample network with 3 phase admittance connections Y_{ij} between the buses.

Since the current and voltage at each node are linearly related and there is no intrinsic relation between current and voltage at each node, the equation given in Kivits and Van den Hof (2019), can be written as

$$\sum_{k \in N_j} A_{jk}[V_j(t) - V_k(t)] = I_j(t) \quad V_j(t), V_k(t), I_j(t) \in \mathbb{C}^{3\times1} \tag{1}$$

where N_j denotes the node number present in the network. Here A_{jk} is the element of Laplacian matrix A. It should be noted that each entry A_{jk} of the Laplacian matrix A has size 3×3, reflecting the three phase admittance. Furthermore, we assume the power network only provides synchrophasor information of voltages and currents at each time instance t. In polar coordinates, the synchrophasor information consists of the sine voltage/current magnitude and phase shift but essentially reflect complex numbers $V_j(t), V_k(t), I_j(t) \in \mathbb{C}^{3\times1}$ at each time instance t of the (linear) dynamics of A_{jk} at the main AC frequency of the grid. As a result, the admittance to be estimated is simply a complex matrix $A_{jk} \in \mathbb{C}^{3\times3}$, making A a block Laplacian matrix.

With the known topology of a power network but connections (i.e. admittance between two buses) to be unknown and 3 dimensional, this paper formulates the recursive estimation of the individual connections between two nodes and parametrized as 3×3 admittance matrix entries A_{jk}. The recursive estimation must be done over short time intervals, preferably using only three distinct time intervals for $V_j(t), V_k(t), I_j(t) \in \mathbb{C}^{3\times1}$, to be able to detect abrupt changes in the three phase line admittance. In the network given in Fig.1 the currents and voltages between any two buses can be related using Ohm's law

$$I_{12} = -I_{21} = Y_{12}[V_1 - V_2] \tag{2}$$

where subscript ij denotes that the parameter belongs to the connection between ith bus (node) and jth bus (node). Hence I_{ij} is the current and Y_{ij} is the admittance of the connection between ith node and jth node whereas V_i is the voltage at the ith node. The input signal current in (1) can be written as the sum of its components as

$$\sum_{k \in N_j} I_{jk}(t) = I_j(t) \tag{3}$$

where

$$A_{jk}[V_j(t) - V_k(t)] = I_{jk}(t) \tag{4}$$

Combining (2) and (4) leads to a physical network estimation problem where $A_{jk} = Y_{jk}$ in (1). More importantly, the complex matrix $A_{jk} \in \mathbb{C}^{3\times3}$ has a symmetric structure that can be exploited during the estimation of the three phase admittance.

The current I_{ij} is 3 phase vector and can be written in matrix form by reformulating (4) into

$$I_{12} = \begin{bmatrix} I_{12}^A \\ I_{12}^B \\ I_{12}^C \end{bmatrix} = \begin{bmatrix} Y_{12}^A & Y_{12}^{AB} & Y_{12}^{AC} \\ Y_{12}^{AB} & Y_{12}^B & Y_{12}^{BC} \\ Y_{12}^{AC} & Y_{12}^{BC} & Y_{12}^C \end{bmatrix} \left(\begin{bmatrix} V_1^A \\ V_1^B \\ V_1^C \end{bmatrix} - \begin{bmatrix} V_2^A \\ V_2^B \\ V_2^C \end{bmatrix} \right) \tag{5}$$

where Y_{ij}^x is the admittance of phase x with $x \in \{A, B, C\}$ and I_{ij}^x, V_{ij}^x are the phase currents and voltages. To simplify notations, we drop the node i and j dependency and simplify the notation of (5) to

$$I = YV, \text{ with}$$

$$Y = \begin{bmatrix} Y^A & Y^{AB} & Y^{AC} \\ Y^{AB} & Y^B & Y^{BC} \\ Y^{AC} & Y^{BC} & Y^C \end{bmatrix}, \; I = \begin{bmatrix} I^A \\ I^B \\ I^C \end{bmatrix}, \; V = \begin{bmatrix} V^A \\ V^B \\ V^C \end{bmatrix} \tag{6}$$

for the problem of estimating admittance between two nodes. For the sign convention, I in (6) is considered to be the current flowing *into* the node, whereas V is the voltage at the node, and Y is the equivalent Thevenin admittance at the node or between nodes.

3. ESTIMATION OF NETWORK ADMITTANCE

3.1 Three Phase Admittance Estimation

The admittance matrix Y with its symmetric structure given in (6) can be estimated by rewriting I, Y and V in a linear regression equation and minimizing the error using a standard Least Squares (LS) problem. Exploiting the symmetry of Y in (6), the parameters of the admittance matrix Y are given by the vector

$$\theta_Y = \begin{bmatrix} Y^A & Y^{AB} & Y^{AC} & Y^B & Y^{BC} & Y^C \end{bmatrix}^T \tag{7}$$

and (6) evaluated at a single time stamp can be written in terms of θ_Y as

$$I(t) = \Psi_V(t)\theta_Y \tag{8}$$

where

$$\Psi_V(t) = \begin{bmatrix} V^A(t) & V^B(t) & V^C(t) & 0 & 0 & 0 \\ 0 & V^A(t) & 0 & V^B(t) & V^C(t) & 0 \\ 0 & 0 & V^A(t) & 0 & V^B(t) & V^C(t) \end{bmatrix} \quad (9)$$

and $I(t) = \left[I^A(t)\ I^B(t)\ I^C(t)\right]^T$. With the vectorization of the 3×3 symmetric admittance matrix in (7), a linear regression equation for the estimation of θ_Y is of the form

$$\boldsymbol{I} = \boldsymbol{\Psi_V}\theta_Y + e(t, \theta_Y) \quad (10)$$

where

$$\boldsymbol{\Psi_V} = \begin{bmatrix} \Psi_V(t_1) \\ \Psi_V(t_2) \\ . \\ \Psi_V(t_N) \end{bmatrix}, \ \boldsymbol{I} = \begin{bmatrix} I(t_1) \\ I(t_2) \\ . \\ I(t_N) \end{bmatrix} \quad (11)$$

To estimate a value for the the complex vector θ_Y, the complex equation error $e(t, \theta_Y)$ can be minimized using a LS problem of the form

$$\hat{\theta}_Y = \arg\min_{\theta_Y} |e(t, \theta_Y)|_2^2 = \arg\min_{\theta_Y} e(t, \theta_Y)e^*(t, \theta_Y) \quad (12)$$

where $e^*(t, \theta_Y)$ denotes the complex conjugate transpose of $e(t, \theta_Y)$. The analytic solution to the LS problem (12) can be written in terms of the normal equations

$$\left[\frac{1}{N}\boldsymbol{\Psi_V^*\Psi_V}\right]\hat{\theta}_Y = \left[\frac{1}{N}\boldsymbol{\Psi_V^* I}\right] \quad (13)$$

and can be solved uniquely if $\boldsymbol{\Psi_V}$ has full column rank. Non-singularity of the product $\boldsymbol{\Psi_V^*\Psi_V}$ highly depends on the information content of the voltage signals included in the regressor defined in (9).

3.2 Requirements on Excitation

Clearly, a unique solution $\hat{\theta}_Y$ to (13) exists if and only if $\text{rank}(\boldsymbol{\Psi_V}) = 6$. For the specific situation of synchrophasor data of a possibly balanced three phase AC power network, conditions on the rank of $\boldsymbol{\Psi_V}$ and the identifiability of $\hat{\theta}_Y$ can be formulated and are summarized in the following.

Proposition 1. with $\Psi_V(t)$ be given in (9), $\text{rank}(\Psi_V(t)) = 3$ if and only if $V^A(t) \neq 0$ or $V^B(t) \neq 0$ or $V^C(t) \neq 0$.

Proof. Assume, there exists an arbitrary vector $\lambda = [\lambda_1\ \lambda_2\ \lambda_3]$ such that

$$\lambda\Psi_V(t) = [0\ 0\ 0\ 0\ 0\ 0], \quad \lambda \neq [0\ 0\ 0] \quad (14)$$

Suppose $V^A \neq 0$, $V^B = 0$ and $V^C = 0$, then

$$\Psi_V(t) = \begin{bmatrix} V^A(t) & 0 & 0 & 0\ 0\ 0 \\ 0 & V^A(t) & 0 & 0\ 0\ 0 \\ 0 & 0 & V^A(t) & 0\ 0\ 0 \end{bmatrix} \quad (15)$$

The only λ satisfying (14) is $\lambda = [0\ 0\ 0]$. The same can be said when only $V^B(t) \neq 0$ and $V^C(t) \neq 0$. Hence $\text{rank}(\Psi_V(t)) = 3$ when only one of the phase voltages is non zero. The same conclusion can be drawn when only one of the phase voltages is zero and the rest two are non-zero. Conversely, for $V^A(t) \neq 0, V^B(t) \neq 0$, and $V^C(t) \neq 0$, the equation for the first, fourth and sixth column of (14) implies

$$\lambda_1 V^A(t) = 0, \lambda_2 V^B(t) = 0, \lambda_3 V^C(t) = 0 \\ \implies \lambda = [0\ 0\ 0]$$

Therefore, there is no vector which satisfies (14), hence $\text{rank}(\Psi_V(t)) = 3$. Finally, given $\text{rank}(\Psi_V(t)) = 3$, it is easy to conclude that at least $V^A(t) \neq 0$, since if $V^A(t) = V^B(t) = V^C(t) = 0$ then $\text{rank}(\Psi_V(t)) = 0$. □

It is worth noting that in the situation of a balanced three phase circuit and defined by

$$V^A(t) + V^B(t) + V^C(t) = 0 \quad (16)$$

the rank of $\Psi_V(t)$ remains 3, as the voltages of the three phases are linearly dependent as given in (16).

Conjecture 2. Let $V^A(t) + V^B(t) + V^C(t) = 0$ with $V^A(t) \neq 0 \implies \text{rank}(\Psi_V(t)) = 3$

To facilitate fast detection of admittance changes, it may be tempting to conclude that at least 2 independent measurements of a complex 3 phase voltage phasor should suffice to estimate the 6 dimensional complex admittance parameter θ_Y. However, such measurements still do not provide enough excitation as summarized in the following result.

Proposition 3. Consider $\Psi_V(t_1)$,$\Psi_V(t_2)$ given in (9) with $\text{rank}(\Psi_V(t_1)) = 3$ and $\text{rank}(\Psi_V(t_2)) = 3$ where

$$\begin{bmatrix} V^A(t_2) \\ V^B(t_2) \\ V^C(t_2) \end{bmatrix} = \begin{bmatrix} C_A & 0 & 0 \\ 0 & C_B & 0 \\ 0 & 0 & C_c \end{bmatrix} \begin{bmatrix} V^A(t_1) \\ V^B(t_1) \\ V^C(t_1) \end{bmatrix} \quad (17)$$

and $C_A \neq C_B \neq C_C \neq 0$ where C_A, C_B and C_C are complex valued constants, then $\text{Rank}(\begin{bmatrix} \Psi_V(t_1) \\ \Psi_V(t_2) \end{bmatrix}) = 5$

Proof. Given the definition in (11) to define

$$\boldsymbol{\Psi_V} = \begin{bmatrix} \Psi_V(t_1) \\ \Psi_V(t_2) \end{bmatrix} \quad (18)$$

and consider a vector $U = [u_1\ u_2\ u_3\ u_4\ u_5\ u_6]^T$ which spans the right nullspace of $\boldsymbol{\Psi_V}$ which implies

$$\boldsymbol{\Psi_V}(\lambda U) = 0, \lambda \in \mathbb{C} \quad (19)$$

Since $\boldsymbol{\Psi_V}$ in (19) is a sparse matrix, dropping the elements from $\boldsymbol{\Psi_V}$ which are zero will result in

$$\boldsymbol{\psi_V} U_1 = 0,\ \boldsymbol{\psi_V} U_2 = 0,\ \boldsymbol{\psi_V} U_3 = 0 \quad (20)$$

where

$$\boldsymbol{\psi_V} = \begin{bmatrix} V^A(t_1) & V^B(t_1) & V^C(t_1) \\ C_A V^A(t_1) & C_B V^B(t_1) & C_C V^C(t_1) \end{bmatrix} \quad (21)$$

and $U_1 = [u_1\ u_2\ u_3]^T$, $U_2 = [u_2\ u_4\ u_5]^T$, $U_3 = [u_3\ u_5\ u_6]^T$. Since $\text{rank}(\boldsymbol{\psi_V}) = 2$, only a unique vector spans the right nullspace of $\boldsymbol{\psi_V}$ which implies

$$U_2 = \alpha U_1,\ U_3 = \beta U_1, \text{ with } \alpha, \beta \in \mathbb{C}. \quad (22)$$

From (22), it can be observed that the vector U is unique. Since there is a single vector which spans the right nullspace of $\boldsymbol{\Psi_V}$ and Proposition 3 holds true. □

Remark 4. It is clear that if $C_A = C_B = C_C = 0$ then for $\boldsymbol{\Psi_V}$ in (18), $\text{rank}(\boldsymbol{\Psi_V}) = 3$.

From Proposition 3 it can be inferred that even 2 measurements of a 3 phase vector are not sufficient to estimate the values of the 6 dimensional θ_Y.

Proposition 5. Consider $\boldsymbol{\Psi_V}$ given by

$$\boldsymbol{\Psi_V} = \begin{bmatrix} \Psi_V(t_1) \\ \Psi_V(t_2) \\ \Psi_V(t_3) \end{bmatrix} \quad (23)$$

such that $\text{rank}(\Psi_V(t_3)) = 3$. If $\text{rank}(\boldsymbol{\psi_V}) = 3$ where $\boldsymbol{\psi_V}$ is given as

$$\boldsymbol{\psi_V} = \begin{bmatrix} V^A(t_1) & V^B(t_1) & V^C(t_1) \\ V^A(t_2) & V^B(t_2) & V^C(t_2) \\ V^A(t_3) & V^B(t_3) & V^C(t_3) \end{bmatrix} \quad (24)$$

then $\text{Rank}(\boldsymbol{\Psi_V}) = 6$

Proof. Consider a vector $U = [u_1\ u_2\ u_3\ u_4\ u_5\ u_6]^T$ which spans the right nullspace of $\boldsymbol{\Psi_V}$ which implies $\boldsymbol{\Psi_V}(\lambda U) = 0$, $\lambda \in \mathbb{C}$. Let $U_1 = [u_1\ u_2\ u_3]^T$, $U_2 = [u_2\ u_4\ u_5]^T$, $U_3 = [u_3\ u_5\ u_6]^T$. Then $\boldsymbol{\psi_V} U_1 \neq 0$, $\boldsymbol{\psi_V} U_2 \neq 0$, $\boldsymbol{\psi_V} U_3 \neq 0$ since $\text{rank}(\boldsymbol{\psi_V}) = 3$. This implies $\Psi_V(t_1)U \neq 0$, $\Psi_V(t_2)U \neq 0$ and $\Psi_V(t_3)U \neq 0$. Hence there exists no vector in the right nullspace of $\boldsymbol{\Psi_V}$. Therefore, $\text{Rank}(\boldsymbol{\Psi_V}) = 6$. □

The above proposition indicates that if three measurements of the three phase line voltage $V(t_1)$, $V(t_2)$ and $V(t_3)$ are linearly independent then $\text{rank}(\boldsymbol{\Psi_V}) = 6$ where $\boldsymbol{\Psi_V}$ is given in (23).

4. BALANCE CALIBRATION

4.1 Phasor Calibration

The assumption that a circuit is balanced at time t given in (16) may be violated due to unbalanced loading of the circuit. In addition, (16) may also not hold in case of calibration or measurement errors in the Potential Transformers (PTs) used to measure the actual voltage. To allow calibration of voltages to satisfy (16), the complex voltages V^B and V^C can be scaled with respect to V^A so that the negative sequence voltage (error) V_e at time t given by

$$V_e(t) = V^A(t) + V^B(t) + V^C(t) \tag{25}$$

is minimized. This is done by a multiplication similar to (17) and given by

$$\begin{bmatrix} V^A(t_1) \\ \hat{V}^B(t_1) \\ \hat{V}^C(t_1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_1^B & 0 \\ 0 & 0 & C_1^C \end{bmatrix} \begin{bmatrix} V^A(t_1) \\ V^B(t_1) \\ V^C(t_1) \end{bmatrix} + \begin{bmatrix} 0 \\ C_0^B \\ C_0^C \end{bmatrix} \tag{26}$$

where the scaling on V^A is normalized and $\hat{V}^B(t_1)$ and $\hat{V}^C(t_1)$ are the calibrated estimates of $V^B(t_1)$ and $V^C(t_1)$. Minimization of the calibration error

$$\epsilon(t, \theta_{\boldsymbol{V^B}}, \theta_{\boldsymbol{V^C}}) = V^A(t) + \hat{V}^B(t) + \hat{V}^C(t) \tag{27}$$

as a function of the calibration parameters

$$\theta_{\boldsymbol{V^B}} = \left[C_1^B\ C_0^B\right]^T \quad \theta_{\boldsymbol{V^C}} = \left[C_1^C\ C_0^C\right]^T \tag{28}$$

can also be done via a LS minimization problem

$$\frac{1}{N} \sum_{t=p}^{N+p-1} \epsilon(t, \theta_{\boldsymbol{V^B}}, \theta_{\boldsymbol{V^C}}) \epsilon^*(t, \theta_{\boldsymbol{V^B}}, \theta_{\boldsymbol{V^C}}) \tag{29}$$

It should be noted that the minimization in (28) is formulated over a moving time horizon to allow for continued adjustment of the calibration over a longer a time horizon of N samples. The time horizon of N samples is much longer than the number of samples used for the recursive admittance estimation to still facilitate fast detection of admittance changes.

The parameters C_1^B, C_1^C, C_0^B and C_0^C can be estimated by considering the following vectorized data

$$\begin{aligned} \boldsymbol{V^B} &= C_1^B \boldsymbol{V^A} \times e^{j2\pi/3} + C_0^B \times \boldsymbol{1_N} \\ \boldsymbol{V^C} &= C_1^C \boldsymbol{V^A} \times e^{-j2\pi/3} + C_0^C \times \boldsymbol{1_N} \end{aligned} \tag{30}$$

where

$$\boldsymbol{V^i} = \begin{bmatrix} V^i(t_1) \\ V^i(t_2) \\ \cdots \\ V^i(t_N) \end{bmatrix}, \ \forall i \in \{A, B, C\}$$

and $\boldsymbol{1_N}$ is a column matrix of size $1 \times N$ with each element being 1. A regression equation similar to (10) of the format

$$\begin{aligned} \boldsymbol{V^B} &= \left[\boldsymbol{V^A} \times e^{j2\pi/3}\quad \boldsymbol{1_N}\right] \theta_{\boldsymbol{V^B}} + \epsilon(t, \theta_{\boldsymbol{V^B}}) \\ \boldsymbol{V^C} &= \left[\boldsymbol{V^A} \times e^{-j2\pi/3}\quad \boldsymbol{1_N}\right] \theta_{\boldsymbol{V^C}} + \epsilon(t, \theta_{\boldsymbol{V^C}}) \end{aligned} \tag{31}$$

can be used to find calibration coefficient estimates

$$\begin{aligned} \hat{\theta}_{\boldsymbol{V^B}} &= \left(\frac{1}{N} \boldsymbol{\Phi}^*_{\boldsymbol{V^A}} \boldsymbol{\Phi}_{\boldsymbol{V^A}}\right)^{-1} \frac{1}{N} \boldsymbol{\Phi}_{\boldsymbol{V^A}} \boldsymbol{V^B} \\ \hat{\theta}_{\boldsymbol{V^C}} &= \left(\frac{1}{N} \boldsymbol{\Phi}^*_{\boldsymbol{V^A}} \boldsymbol{\Phi}_{\boldsymbol{V^A}}\right)^{-1} \frac{1}{N} \boldsymbol{\Phi}_{\boldsymbol{V^A}} \boldsymbol{V^C} \end{aligned} \tag{32}$$

where $\boldsymbol{\Phi}_{\boldsymbol{V^A}} = \left[\boldsymbol{V^A} \times e^{-j2\pi/3}\quad \boldsymbol{1_N}\right]$ and minimize the LS error in (13). Equations similar to (26), (31) and (32) are used to estimate the calibrated phase B and C currents, $\hat{I}^B$ and $\hat{I}^C$.

4.2 Illustration of Phasor Calibration

For the illustration of the need to calibrate voltage phasor data, experimental data sampled at 60 Hz from a 3 phase distribution circuit located in Southern California is used to estimate the calibration coefficient in (28). The circuit has a format similar to Fig. 1, where voltage differences between 2 buses, and a branch current between the same buses is used for estimation of admittance. A plot of the normalized three phase voltage of the distribution circuit is given in Fig. 2 in the normalized units of (Vu). The bottom figure shows the negative sequence

$$V_{neg}(t) = V^A(t) + \hat{V}^B(t) + \hat{V}^C(t) \tag{33}$$

that ideally should be 0 if the circuit is balanced and voltages are calibrated. It could be observed from the experimental data voltages at each phase are not the same, while also a large perturbation in the negative voltage $V_{neg}(t)$ is observed during the experiment.

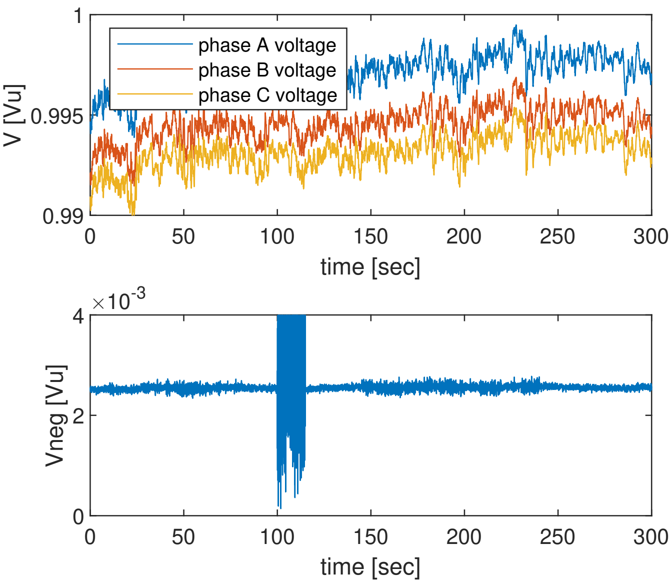


Fig. 2. Three phase uncalibrated RMS voltage difference (top) and the negative sequence of voltage (bottom) sampled at 60 Hz over 300 seconds.

After an initial window of N data points, the calibration parameters are estimated every $1/60^{th}$ of a second. The calibration parameters are then used to find the calibrated phase voltages and currents using (26). A plot of the calibrated normalized three phase voltage of the distribution circuit is given in Fig. 3 in the normalized units of (Vu). It is clear that $V_{neg}(t)$ is indeed close to zero, except during the time of a large perturbation in the negative voltage $V_{neg}(t)$ providing a clear indicator of the circuit being unbalanced.

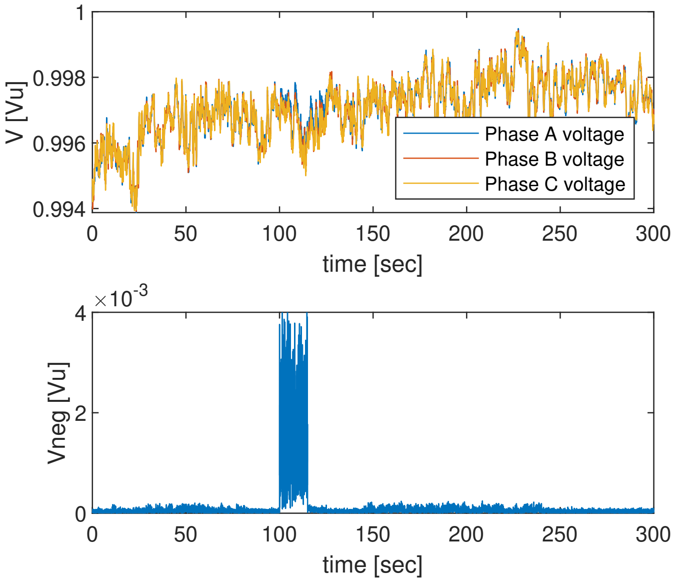


Fig. 3. Three phase calibrated RMS voltage difference (top) and the negative sequence of voltage (bottom) sampled at 60 Hz over 300 seconds.

5. ADMITTANCE ESTIMATION

5.1 Validation

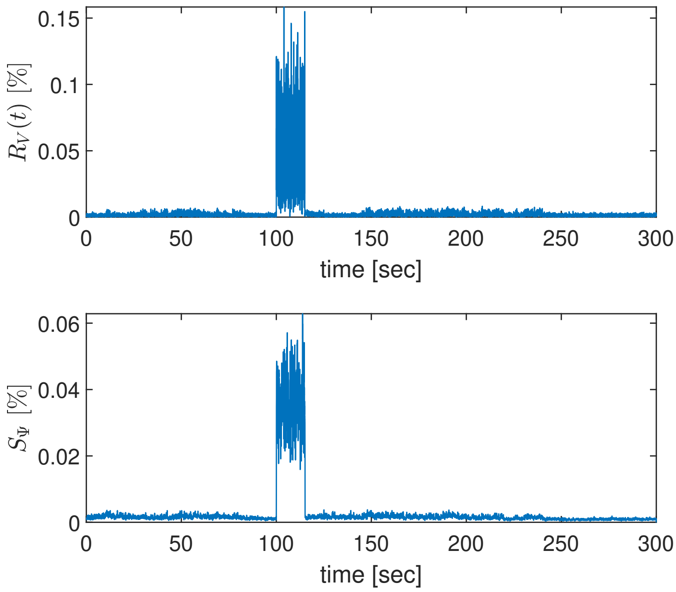
Although Fig. 3 now shows the balanced voltage signals, much harder to observe is the presence of an actual voltage disturbance that temporarily unbalanced the circuit. As explained in section 3.2, for estimation of the three phase admittance, it is necessary to have an excitation in the voltage signal that temporary violates the balanced situation by by having voltage relation given in (17).
One way to detect the temporary unbalance of voltage signals is to plot the normalized signal

$$R_V(t) = \frac{|V_{neg}(t)|}{|V_{pos}(t)|} \times 100\% \tag{34}$$

where V_{neg} is given in (33) and $V_{pos}(t) = V^A(t) + \hat{V}^B(t) \times e^{-j2\pi/3} + \hat{V}^C(t) \times e^{j2\pi/3}$. The signal $R_V(t)$ is used to see what the relative effect of V_{neg} is to V_{pos}. The plots in Fig. 4 show that when the magnitude of $R_V(t)$ is significantly different from 0, the rank of the regressor $\boldsymbol{\Psi_V}$ as in (23) is 6. This change in rank is caused when the phase voltages are not balanced. In practice, the size of the regressor $\boldsymbol{\Psi_V}$ is chosen to be larger than 3. For fast detection of line admittance changes, the size of the regressor $\boldsymbol{\Psi_V}$ in (11) should be chosen to be as small as possible for early detection of faults and changes in admittance. For the results of Fig. 4, $\boldsymbol{\Psi_V}$ has size 10. The fact that $\boldsymbol{\Psi_V}$ in (23) has rank 6 instead of the default rank 5 by Proposition 5 can be validated by evaluating the ratio

$$S_{\boldsymbol{\Psi}} = \frac{\underline{\sigma}(\boldsymbol{\Psi_V})}{\overline{\sigma}(\boldsymbol{\Psi_V})} \times 100\% \tag{35}$$

where $\underline{\sigma}(\boldsymbol{\Psi_V})$ is the smallest singular value of $\boldsymbol{\Psi_V}$ and $\overline{\sigma}(\boldsymbol{\Psi_V})$ is the largest singular value in $\boldsymbol{\Psi_V}$, as done in Fig. 4 at the bottom.



cm.

Fig. 4. Evaluation of R_V in (34) (top) and $S_{\boldsymbol{\Psi}}$ in (35) at 60 Hz over 300 seconds.

To validate the admittance estimation, a simulation with a known admittance matrix

$$Y = \begin{bmatrix} 1.000+0.010i & 0.010-0.012i & -0.200+0.030i \\ 0.010-0.012i & 1.100+0.400i & 0.300-0.027i \\ -0.200+0.030i & 0.300-0.027i & 0.900+0.300i \end{bmatrix}$$

is used to generate the phase currents $I = YV + I_n$, where I_n is an independent white noise perturbation on the current measurement. Using the phase voltages and the simulated phase currents, $\boldsymbol{I}$ and $\boldsymbol{\Psi_V}$ given in (11) are used to estimate the parameter vector $\hat{\theta}_Y$ using (13). The resulting estimated parameter $\hat{\theta}_Y$ is given by

$$\hat{\theta}_Y = \begin{bmatrix} 1.0000+0.0100i \\ 0.0100-0.0120i \\ -0.2000+0.0300i \\ 1.0894+0.3980i \\ 0.2964-0.0371i \\ 0.9109+0.2912i \end{bmatrix} \tag{36}$$

and closely resembles the imposed admittance values.

5.2 Experimental Results

For the experimental results, the data of one of the branch currents of the network in Fig. 1 is used along with the data of the calibrated voltage differences. The data is collected from a distribution circuit in southern California. Similar to the procedure done for balancing the three phase voltages in (28), (30) and (32), the three phase currents are balanced and a figure of the normalized balanced current and the negative sequence current is depicted in Fig. 5

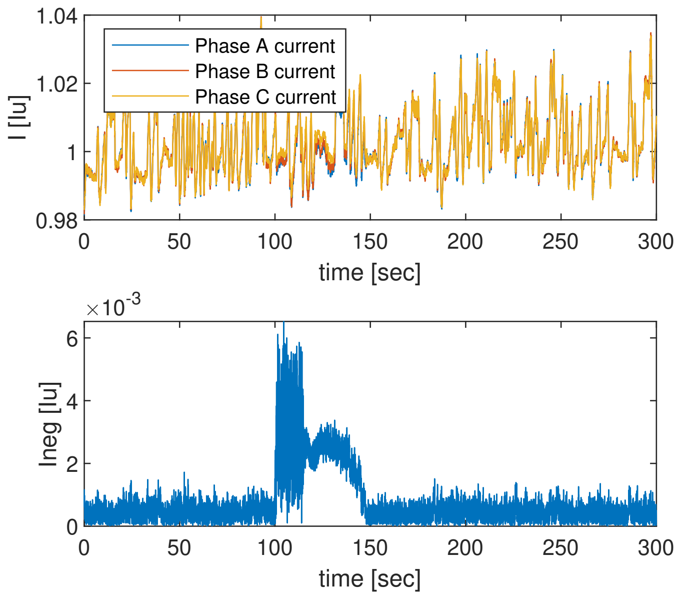


Fig. 5. Three phase calibrated RMS current (top) and the negative sequence of current (bottom) sampled at 60 Hz over 300 seconds.

The admittance parameter θ_Y given in (7) is estimated as

$$\hat{\theta}_Y = \begin{bmatrix} 0.0034 - 0.0004i \\ 0.0001 - 0.0003i \\ -0.0000 + 0.0001i \\ 0.0039 - 0.0003i \\ -0.0001 + 0.0003i \\ 0.0034 - 0.0001i \end{bmatrix} \quad (37)$$

The parameters now indicate the effective admittance seen between the voltage nodes and the branch currents in the network. The values are consistent with the impedance information obtained from the actual lines of the network. More importantly, both phase admittance and phase-to-phase admittance were estimated during the time period at which there was a presence of an actual voltage disturbance that temporarily unbalanced the circuit.

6. CONCLUSION AND FUTURE RESEARCH

A least squares approach for calibrating voltage and current measurements is combined with a recursive Least Squares (LS) approach to estimate the three phase admittance of physical line connections between buses in a power network. Specific conditions on the excitation of the voltage signals are formulated to guarantee a unique solution to the LS optimization to estimate the three phase line admittance. The excitation conditions for estimation are monitored in real-time via a singular value decomposition of the regressor matrix used in the LS optimization. Simulation and experimental results on actual synchrophasor data validate the estimation results. Future work will focus on developing a real-time fault detection and fault identification algorithm from the estimated admittance variations.

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