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H_∞ interactive controller design for teaching purposes ⋆

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Abstract: H_∞-based controller design is one of the most powerful methodologies for controller design in the frequency domain. Unfortunately, its use requires advanced knowledge of control theory. The use of interactive tools allows to a non-specialized user to use these techniques and to learn the fundamental concepts behind the H_∞ theory. In this work a graphical and completely interactive tool used to introduce students in H_∞ control is presented.

Keywords: Automatic Control Education, Closed-loop loop-shaping, H_∞ control, Mixed sensitivity, Interactive design.

1. INTRODUCTION

Automatic control is a transversal subject explained in most engineering studies and other careers (Arevalo et al., 2020). Unfortunately, the time that different degrees devote to this subject is quite limited, and in recent years the theory of control has undergone many significant advances. For all this, and in order to train students in advanced techniques it is important to look for ways to transmit knowledge quickly and efficiently.

Controller design in the frequency domain is one of the most interesting approaches and offers the designer a wide range of possibilities and tools. On many occasions students learn algorithms as recipes but do not internalize important concepts. The use of interactive loop-shaping (Diaz et al., 2017) has been proposed to overcome these problems. The open-loop transfer function is shaped but having in mind the closed-loop transfer functions which are the relevant ones. To address this problem interactive closed-loop loop-shaping has been proposed (Diaz et al., 2019). In this approach the controller is defined interactively in order to met the required specifications. This approach has two main drawbacks. The first is that it is not so straightforward to obtain the optimal solution, the second one is that shaping one transfer function is not so difficult, but simultaneous shaping more than one transfer function with just one degree of freedom (i.e. the controller) is not so simple. Many constraints exist, and sometimes designers impose constraints which might not be achievable (Boulet and Duan, 2007; Seron, 2010).

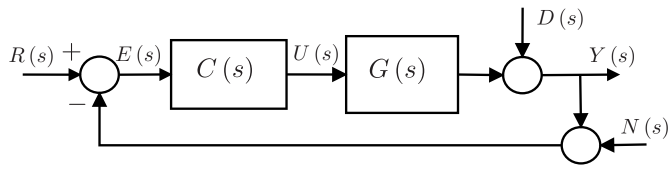

Figure 1. Closed-loop control system block diagram.

Some years ago there was an important research effort to develop H_∞ optimal theory. This theory allows to automatically determine the controller which minimizes the H_∞ norm of a transfer function relating a given input with a given output. Although related numerical methods are provided in most popular software packages like MATLAB (Balas et al., 2019) it is not much used in preliminary courses because it has a sophisticated mathematical formulation and in some cases it is not so straightforward to use (Llana and Almansa, 2016). However, used in the appropriate manner H_∞ optimal theory can be thought as a way to shape closed-loop transfer functions. Nowadays it is possible to integrate the numerical methods which address the H_∞ problem in interactive tools which help the students to use them without needing to get in deep on the numerical methods required to obtain the control. This allows to start obtaining controllers from the very beginning and focusing on relevant control objectives.

This paper describes the minimal theoretical contents that students need to know to use the H_∞ theory as a way to perform closed-loop loop-shaping. Besides, the paper describes the graphical and interactive software tool that has been developed to address this. The paper

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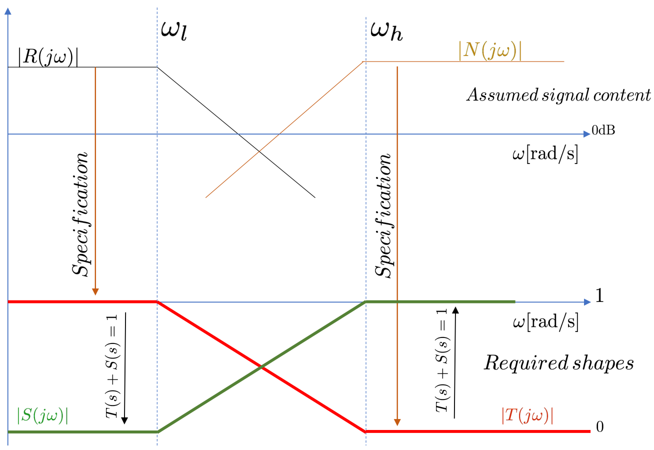

Figure 2. Signal frequency decomposition and desired frequency responses.

is organized as follows: in section 2 basic concepts to define the design objectives and how to write them in terms of an H_∞ optimization problem are introduced, section 3 contains basic hints about the numerical methods required to obtain the optimal controller, section 4 give a brief description of the interactive tool that has been developed, section 5 presents an example developed with the interactive tool. Finally section 6 summarizes some conclusions and future works.

2. CLOSED-LOOP LOOP-SHAPING: BASIC IDEAS

2.1 System Definition

In this work, the closed-loop control scheme shown in Figure 1 will be assumed. The closed-loop system is composed of a plant $G(s)$, connected in negative feedback with a controller $C(s)$. Most relevant signals in the closed-loop system correspond to the output, $Y(s)$, the control action, $U(s)$ and the error, $E(s)$. Finally, the inputs are the reference, $R(s)$, the output disturbances $D(s)$ and the measurement noise, $N(s)$.

All these signals are related through the following set of equations:

$$\begin{bmatrix} Y(s) \\ E(s) \\ U(s) \end{bmatrix} = \begin{bmatrix} T(s) & -S(s) & -T(s) \\ S(s) & -S(s) & -S(s) \\ C(s)S(s) & -C(s)S(s) & C(s)S(s) \end{bmatrix} \begin{bmatrix} R(s) \\ D(s) \\ N(s) \end{bmatrix}, \tag{1}$$

where $L(s) = C(s)G(s)$ is the open-loop transfer function, $S(s) = \frac{1}{1+L(s)}$ is the sensitivity function and $T(s) = \frac{L(s)}{1+L(s)}$ is the complementary sensitivity function.

2.2 Constraints

Before analyzing the required transfer function shapes it is convenient to consider to which constraints they are subject to. As it is easy to see, all transfer functions depend on the controller; and the controller is the only degree of freedom the designer has (Figure 1). Consequently if ones changes one of the transfer function, all transfer functions will be affected. This is particularly clear in the relation between $T(s)$ and $S(s)$: $T(s) + S(s) = 1$.

Another important constraint, is what it is known as the *sensitivy integrals* or the *waterbed phenomena* (Skogestad and Postlethwaite, 2005; Costa-Castelló and Dormido, 2015), which for the sensitivity function states that if $S(s) = \frac{1}{1+L(s)}$ is a stable transfer function, then:

$$\int_0^\infty \ln |S(j\omega)| d\omega = -\kappa\frac{\pi}{2} + \pi \sum_k^{n_{nmp}} p_k,$$

where $p_k \in \mathbb{C}^+$ are the unstable poles of $L(s)$, n_{nmp} are the number of unstable poles of $L(s)$ and $\kappa = \lim_{s\to\infty} sL(s)$. For $L(s)$ being relative degree one or more, $S(s)$ is relative degree 0 with $\lim_{s\to\infty} S(s) = 1$.

An interesting relationship between the open-loop and the closed-loop transfer function is that the minimal distance from $L(j\omega)$ to the point $-1 + j0$ in the Nyquist plot can be computed as:

$$\begin{aligned} d(-1, L(j\omega)) &= \inf_\omega |-1 - L(j\omega)| = \inf_\omega |1 + L(j\omega)| \\ &= \left[\sup_\omega \frac{1}{|1 + L(j\omega)|}\right]^{-1} = \|S(s)\|_\infty^{-1}. \end{aligned}$$

All these characteristics are intrinsic to linear control systems and cannot be avoided by selecting a controller.

2.3 Specifications

In order to determine the desired form for a transfer function, it is necessary to see what relationship the different signals have between each other. These can be summarized as follows:

- The output, $Y(s)$, must track the reference, $R(s)$.
- The output, $Y(s)$, must be insensitive to disturbances, $D(s)$, and measuring noise, $N(s)$.

To achieve this, $S(s) \approx 0$, which implies $T(s) \approx 1$, although this would be appropriate for the reference and the disturbance it would not be from the noise point of view. To overcome this problem, hypotheses are often made about the frequency content of the different signals. Thus, it is assumed that references and disturbances only have relevant frequency components in the low frequency range, while noise has them in the high frequency range. This induces a frequency decomposition and indicates the range of values for the frequency responses of $S(s)$ and $T(s)$ (Figure 2).

Desired specifications can be graphically visualized in different diagrams and different transfer functions. Figure 4 shows the specifications for the sensitivity function in the Bode diagram while Figure 5 shows the specifications for the sensitivity function in the polar diagram.

2.4 Specifications in the H_∞ framework

Although it would be possible to define an optimization problem based defined by the line segments shown in Figure 4, this would lead to a non-convex optimization problem. Therefore, the constraints on the sensitivity function are defined by a transfer function of the form:

$$|S(j\omega)| < \frac{1}{|W_e(j\omega)|}, \forall\omega, \tag{2}$$

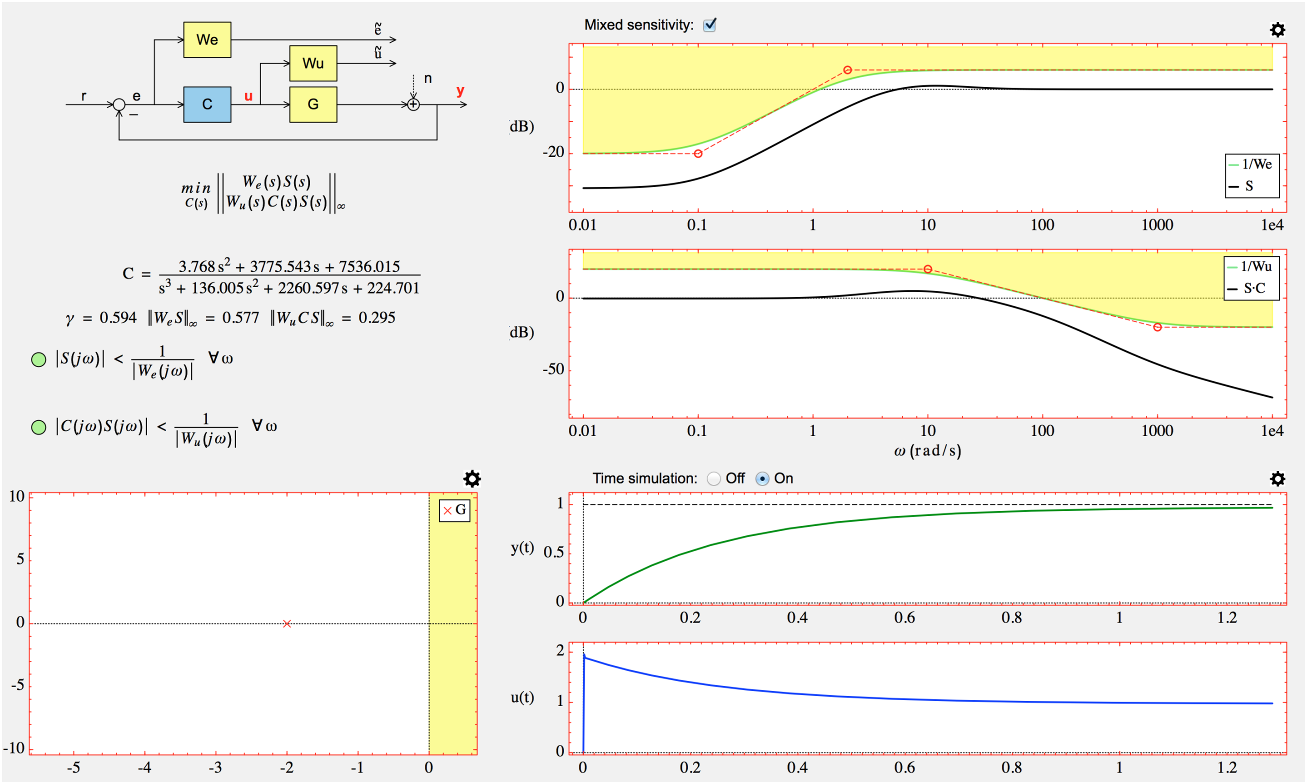

Figure 3. Interactive application main view

where $W_e(s)$ is a transfer function used to define a bound over $|S(j\omega)|$. (2) can be transformed in the following

$$\|S(s)W_e(s)\|_\infty < 1.$$

In order to obtain a controller which guarantees (2), usually the Linear Fractional Transformation (LFT) can be used. Given the LFT shown in Figure 6, it is possible to numerically obtain the controller $K(s)$ such that the H_∞ norm between $W(z)$ and $Z(z)$ is minimized.

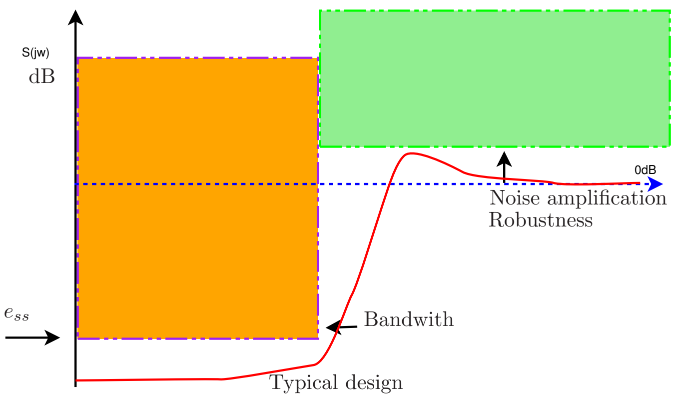

Figure 4. Bode plot Sensitivity function specifications.

The LFT formulation can be used by using the scheme shown in Figure 7, with $Z(s) = \widetilde{E}(s)$, $W(s) = R(s)$, $V(s) = E(s)$, and $X(s) = U(s)$.

Although minimizing $\|S(s)W_e(s)\|_\infty$ produces excellent results over $S(s)$ and $T(s)$. In some cases the transfer function relating the noise with the control action, $C(s)S(s)$. To avoid this, it is necessary to include additional constraints over the $|C(j\omega)S(j\omega)|$. To do this, the mixed sensitivity formulation (Skogestad and Postlethwaite, 2005) is used. The LFT formulation can be used by using the scheme shown in Figure 8, with $Z(s) = \begin{bmatrix} \widetilde{E}(s) \\ \widetilde{U}(s) \end{bmatrix}$, $W(s) = R(s)$, $V(s) = E(s)$, and $X(s) = U(s)$. This approach usually offers very satisfactory results.

3. NUMERICAL SOLVING

The H_∞ norm of a SISO stable transfer function corresponds to the maximum of the frequency response gain. For the MIMO case the concept is exactly the same but one should look for it in the Singular Value Decomposition plot. Computing the H_∞ norm using the transfer function gain (or the maximum SVD value) is not so straight forward (iterative methods are required) and it is a non-convex problem.

Similarly the solution of the H_∞ optimal controller was not an easy task and was a hot research topic for many years. Doyle et al. (1989) come up with a solution. In Doyle et al. (1989) two very important elements were introduced:

- A Riccatti equation, depending on a free parameter γ, which fulfillment implies that the H_∞ norm of the system under study is lower than γ.
- A Riccatti equation, depending on a free parameter γ, which fulfillment implies that the controller makes the closed-loop system H_∞ norm lower than γ. Additionally, the analytic expression for a controller which makes the closed-loop system H_∞ norm lower than γ is provided.

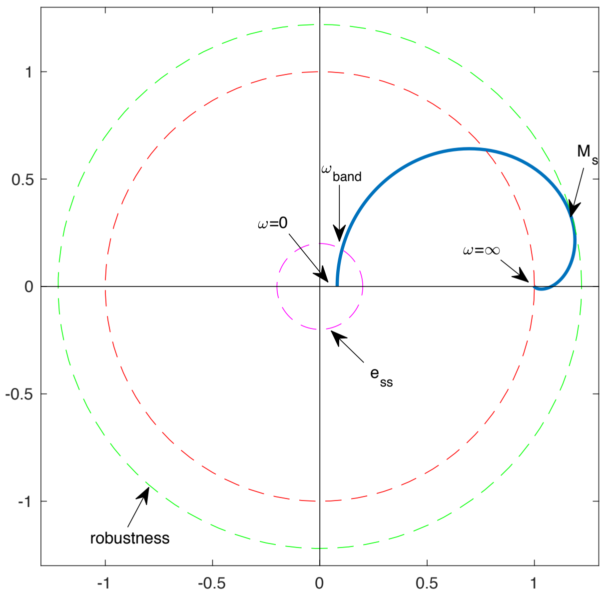

Figure 5. Sensitivity function specifications over the polar diagram.

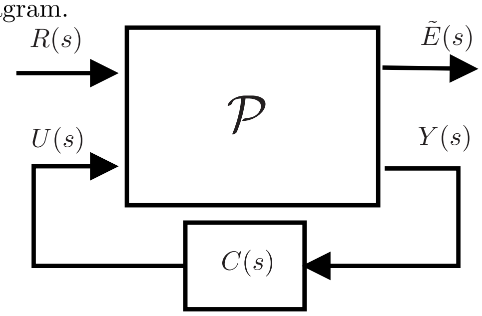

Figure 6. LFT problem formulation

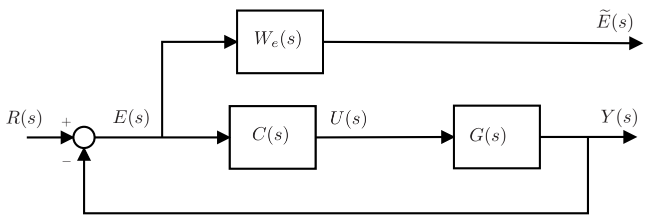

Figure 7. Control scheme used to minimize $\|W_e(s)S(s)\|_\infty$ in the LFT framework.

Although these methods do not allow obtaining either the H_∞ norm nor the H_∞ optimal controller directly, they can be used to obtain both using bisection methods.

Unfortunately, the optimal controller described in Doyle et al. (1989) requires the plant to fulfill certain conditions that are not usually met. In (Safanov et al., 1989) a transformation that allows to put almost any plant in the form required by (Doyle et al., 1989) was introduced. This makes it easier for the method to be used widely

All these methods have been implemented in Sysquake (Piguet, 2018), which directly supports Riccati equations solving. Obtaining the optimal controller takes much less than one second for low-order systems.

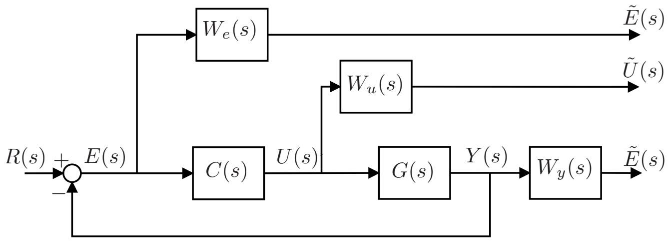

Figure 8. Mixed sensitivity LFT formulation.

4. INTERACTIVE TOOL

Figure 3 shows the main window of the interactive application designed to interactively apply the design methodology described in section 2.4. In the left-lower part of the application, an interactive pole-zero map used to define the plant can be seen. In the left-upper part of the application, the scheme used to define the LFT problem is shown(Figure 7 or Figure 8). In the left part, the optimization problem, the solution controller and the values of $\gamma = \left| \begin{matrix} W_e(s)S(s) \\ W_e(s)S(s) \end{matrix} \right|_\infty$, $\|W_e(s)S(s)\|_\infty$ and $\|W_u(s)C(s)S(s)\|_\infty$ can be seen (other information such as the parameters of $W_e(s)$ and $W_u(s)$ can also be seen in this area depending on the concrete mode). Finally, two colored bullets are shown to indicate if the specification on $S(s)$ and $C(s)S(s)$ are fulfilled (shown in green) or not (shown in red).

The right upper diagram allows to interactively define the specification, $W_e(s)$, over the sensitivity function. Specifications defines a forbidden region (shown in yellow). The value of the controller, $C(s)$, is automatically updated when the specification changes and consequently the value of $S(s)$ is automatically updated.

In case that the Mixed sensitivity is active a diagram with the frequency response of $C(s)S(s)$ and its specifications, $W_u(s)$, are shown. Finally, in the right bottom part the output and control action step responses are shown. This view is automatically updated each time the specifications are redefined. The variables that are drawn automatically changed by selecting them over the diagram in the left upper part.

The application has been designed to define interactively the closed-loop specifications, allowing to automatically see the effect of these specifications over the different closed-loop transfer functions and how the specifications from the different transfer function interact between them.

5. EXAMPLE

Let's assume a given plant defined by :

$$G(s) = \frac{1}{(s+1)^3}.$$

This is a relatively simple plant with a monotonous frequency response. For the closed-loop system, the specifications are defined by

$$W_e(s) = \frac{\frac{s}{M_s} + \omega_b}{s + \omega_b \cdot e_{ss}}, \tag{3}$$

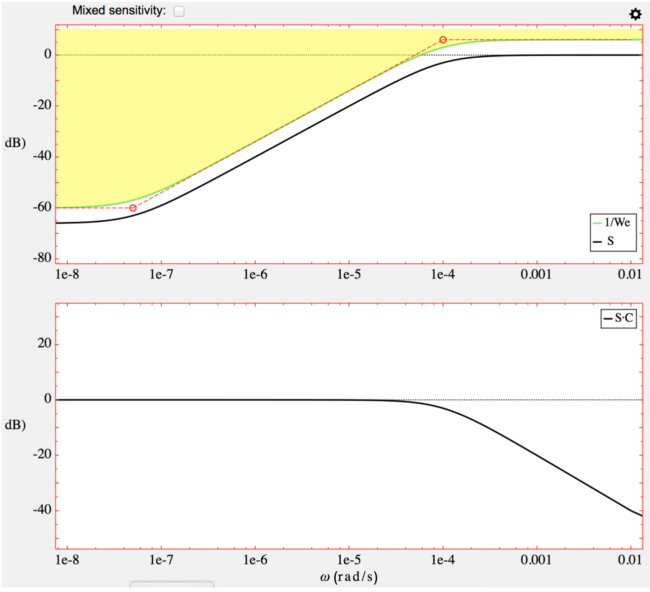

Figure 9. Example I proposed design

with $M_s = 2$, $e_{ss} = 10^{-3}$ and $\omega_b = 5 \cdot 10^{-5}$. The upper part of Figure 9 shows the specifications and the obtained sensitivity function.

This design guarantees a very small error in frequencies up to 10^{-7} rad/s and good robustness margins. Additionally, as it can be seen in the lower part of Figure 9 the frequency response of $C(s)S(s)$ shows a low-pass profile which will extinguish the noise over the control action. Consequently, it is not necessary to include additional constraints.

6. CONCLUSIONS AND FUTURE WORKS

6.1 Conclusions

In this paper a tool which allows to perform H_∞ optimal closed-loop loop-shaping for generic linear plants has been introduced. The specification over the sensitivity function can be interactively defined over the Bode plot, when required additional specification over $C(s)S(s)$ can also be taken into account. Both types of specifications can be interactively and simultaneously analyzed so the trade-off between them can be easily visualized.

The use of this tool helps students to internalize the different concepts and understand the commitments that the designer should always keep in mind. In addition, the tool allows the student to focus on controller design issues without having to invest time in the numerical resolution aspects, which is especially interesting in areas where there are not many class hours.

6.2 Future Works

The authors are currently working to improve the described tool in different directions, most relevant is including the plant uncertainty handling in the tool. H_∞ methodologies are generic so they could take into account this with no difficulties. The effort is focusing on how to visualize in a graphical way this uncertainty and how to represent the trade-off between uncertainty and performance.

H_∞ methodologies generate normally high order controllers, in many cases similar performance could be achieved with lower order controller. A way to check this is reducing the controller order. The authors are working to include order reduction capabilities in the application. Then the comparison between the original controller performance and the new one could be done.

Finally, the authors are working to integrate this tool in the students grading processes. Currently, the applications automatically generates a report summarizing the results achieved by the student. Next step will be including an automatic grading mechanism (Sánchez et al., 2020).

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