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Switched control of a three-phase AC–DC power converter ⋆

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Abstract: In this paper, a new switched control strategy for three-phase AC-DC power converters, also known as controlled rectifiers, is proposed. More specifically, a switching function is determined to command the converter switches at each instant of time assuring global asymptotic tracking of a desired reference trajectory, which is related to a constant output voltage and sinusoidal phase currents. This is generally a desired situation for AC-DC power converters as it can ensure unitary power factor and rectified steady-state DC voltage simultaneously. The proposed switching function must also assure a guaranteed performance cost. The design conditions are based on a Lyapunov function, which is dependent on the electrical angle, and are expressed in terms of linear matrix inequalities (LMIs). Simulation results put in evidence the effectiveness of the proposed methodology and motivate future related works.

Keywords: AC-DC power converter, switched nonlinear systems, guaranteed cost, linear matrix inequalities

1. INTRODUCTION

Switched control strategies in the power electronics domain constitute a topic that is drawing attention from scientists and engineers in the last years. The ability to address switching events directly instead of adopting an averaged model (and pulse-width modulation) allows to tackle the stability problem from a more rigorous theoretical perspective, as well as, to better study the dynamic behaviour in low switching-frequency operation. The classical DC-DC power converters can be modelled as switched affine systems, which are characterized by presenting several equilibrium points composing a region of great interest in the state space. For this class of systems the literature presents some results regarding global asymptotic stability and guaranteed performance, see Bolzern and Spinelli (2004); Patino et al. (2009); Deaecto et al. (2010); Egidio et al. (2017). In these references, control strategies govern the system state trajectories towards a chosen equilibrium point by adequately designing a switching function that selects one of the available affine subsystems (often called system modes) to be activated at each instant of time. For general switched systems theory, the reader can refer to the books Liberzon (2003) and Sun and Ge (2011).

Besides the DC-DC power converters, systems composed of alternating current circuits are equally important. However, their analysis and control design in the context of switched systems theory have received less investigation to date. The difficulty comes from the fact that the time-varying nature of AC currents leads to a new class of switched affine systems, characterized by the presence of sinusoidal functions in its dynamic model, which make the problem more difficult to be handled. In this context, the authors in Scharlau et al. (2013) have proposed a state-dependent switching rule to command the switches of a three-phase inverter feeding a squirrel-cage induction motor in order to regulate the shaft rotational velocity using an approach based on auxiliary reference frames. More recently, authors in Egidio et al. (2019) have treated the same problem, but controlling a three-phase permanent magnet synchronous machine from a novel methodology, which considers directly the nonlinear system model, without using any auxiliary reference frame. This control methodology assures asymptotic stability of an equilibrium point of interest composed of phase currents and a constant rotational velocity in a single control loop. Dealing with DC-AC power converter, reference Sanchez et al. (2019) has proposed a control law based on the hybrid dynamic system theory, where the main goal is to track a sinusoidal reference trajectory assuring a minimum dwell time and guaranteeing practical stability. For AC-DC power converters there are only few results using techniques based on the switched control theory. See for instance the recent reference Hadjeras et al. (2019), where a hybrid control law is proposed for a three-level Neutral Point Clamped (NPC) converter, working as a rectifier, in order to regulate the output DC voltage.

In this paper, our main contribution is to design a state-dependent switching function able to control the output DC voltage of a three-phase bidirectional AC-DC power converter, also known as a controlled rectifier, without using any auxiliary reference frame or modulation strategies and in a single control loop. The switching function must command the inverter switches in order to assure global

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asymptotic tracking of a desired reference composed of sinusoidal phase currents and a constant output voltage. The conditions are based on a Lyapunov function dependent on the input voltage electrical angle and take into account the minimization of a guaranteed performance cost. Moreover, they are described in terms of linear matrix inequalities (LMIs) and can be solved without difficulty using off-the-shelf optimization tools. The theory is validated by means of simulation results concerning an AC-DC converter borrowed from Bouafia et al. (2009) and a theoretical comparison with averaged model techniques is carried out.

The notation is standard. For real vectors or matrices, $(')$ refers to their transpose. For symmetric matrices, $(\bullet)$ denotes each of their symmetric blocks. The symbols $\mathbb{R}$ and $\mathbb{N}$ denote the sets of real and natural numbers, respectively. For a symmetric matrix, $X > (<)\ 0$ denotes a positive (negative) definite matrix. The unit simplex Λ is composed of all nonnegative vectors $\lambda \in \mathbb{R}^N$, such that $\sum_{j\in\mathbb{K}} \lambda_j = 1$. The convex combination of matrices $\{X_1, \cdots, X_N\}$ is denoted by $X_\lambda = \sum_{i=1}^{N} \lambda_i X_i$, $\lambda \in \Lambda$. The Hermitian operator is given as $\mathrm{He}\{X\} = X + X'$ for any real square matrix X.

2. PROBLEM FORMULATION

Consider the three-phase AC-DC power converter, given in Figure 1, which is based on a three-phase controlled rectifier with an inductive input filter and an output capacitor feeding a resistive load. By means of Kirchoff's voltage and current laws, its dynamic model is described by the following switched nonlinear system

$$\dot{x}(t) = A_{\sigma(t)}x(t) + b(\theta(t)), \quad x(0) = x_0 \tag{1}$$

where $x(t) = [i_\phi(t)' \;\; v_o(t)]' \in \mathbb{R}^4$ is the state vector, $i_\phi(t) = [i_a(t)\ i_b(t)\ i_c(t)]' \in \mathbb{R}^3$ are input phase currents and $v_o(t) \in \mathbb{R}$ is the output voltage. The switching signal $\sigma(t) \in \mathbb{K} = \{1, \cdots, 7\}$ is responsible for selecting one of the seven possible subsystems at each instant of time. System matrices are given by

$$A_\sigma = \begin{bmatrix} -(R_L/L)I & -(1/L)S_\sigma \\ (1/C)S_\sigma' & -1/(R_oC) \end{bmatrix}, b(\theta) = \begin{bmatrix} (1/L)v_\phi \\ 0 \end{bmatrix} \tag{2}$$

where R_L and L are the resistance and inductance of each coupling inductor, C is the dc-link capacitance, R_o is the load resistance, $v_\phi = v_m f(\theta)$ is the input voltage, v_m is the peak phase-to-neutral voltage and the vector function $f(\theta) = [f_a(\theta)\ f_b(\theta)\ f_c(\theta)]' \in \mathbb{R}^3$ is defined by

$$f_a(\theta) = \sin(\theta) \tag{3}$$
$$f_b(\theta) = \sin(\theta - 2\pi/3) \tag{4}$$
$$f_c(\theta) = \sin(\theta - 4\pi/3) \tag{5}$$

Vectors $S_i \in \mathbb{R}^3$, $i \in \mathbb{K}$, take values according to Table 1. In this table, the converter switch s_i, $i = \{1, 2, 3\}$ is 1

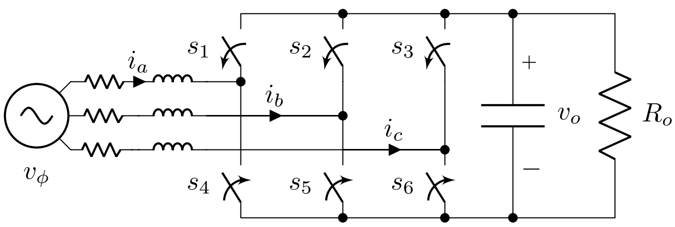

Fig. 1. Three-phase AC-DC power converter

Table 1. Switching function σ, switches state and vector S_i

σ	s_1	s_2	s_3	S_σ'
1	0	0	1	[-1/3 -1/3 2/3]
2	0	1	0	[-1/3 2/3 -1/3]
3	0	1	1	[-2/3 1/3 1/3]
4	1	0	0	[2/3 -1/3 -1/3]
5	1	0	1	[1/3 -2/3 1/3]
6	1	1	0	[1/3 1/3 -2/3]
7	1 0	1 0	1 0	[0 0 0]

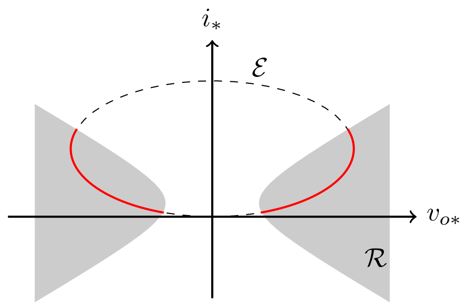

Fig. 2. Representation of X_e (in red) as $\mathcal{E} \cap \mathcal{R}$.

when it is closed and 0 when it is open. Moreover, each pair (s_1, s_4), (s_2, s_5) and (s_3, s_6) is alternately commanded, as for instance, when $s_1 = 1$, $s_4 = 0$ and *vice-versa*. A time-varying angular parameter $\theta(t)$ represents the input voltage electrical angle and is assumed to respect $\theta(t) = \omega t + \theta_0$ with constant angular frequency ω. In this paper, this parameter is considered to be measurable or adequately estimated.

Our main goal is to design a state dependent switching function $\sigma(t) = u(x(t), \theta(t))$ with $u(x, \theta) : \mathbb{R}^4 \times \mathbb{R} \to \mathbb{K}$ capable of orchestrating the switching events of this system, bringing $v_o(t)$ to a reference value v_{o*} chosen by the designer. In order to operate in an unitary power factor situation, phase currents $i_\phi(t)$ must track a sinusoidal reference $i_{\phi*}(\theta(t)) = i_* f(\theta(t))$, synchronized with the source phase voltages. The control problem also allows for the minimization of an upper bound for the quadratic cost

$$\mathcal{J} = \int_0^\infty r\|i_\phi(t) - i_{\phi*}(t)\|^2 + (v_o(t) - v_{o*})^2 dt \tag{6}$$

with the weight $r \geq 0$ chosen by the designer.

Adopting the auxiliary state variable $\xi(t) = x(t) - x_e(\theta(t))$ with the equilibrium trajectory $x_e(\theta(t)) = [i_{\phi*}(\theta(t))' \;\; v_{o*}]'$, we can write the equivalent switched nonlinear system

$$\dot{\xi}(t) = A_{\sigma(t)}\xi(t) + \ell_{\sigma(t)}(\theta(t)), \quad \xi(0) = \xi_0 \tag{7}$$

with $\ell_i(\theta) = A_i x_e(\theta) + b(\theta) - \dot{x}_e(\theta)$, $i \in \mathbb{K}$, and $\xi_0 = x_0 - x_e(\theta_0)$. This permits to tackle the trajectory tracking problem

$$\lim_{t\to\infty} \|x(t) - x_e(\theta(t))\| = 0 \tag{8}$$

for the original system (1) by assuring global asymptotic stability of the origin $\xi = 0$ for the equivalent system (7).

As it will be clear afterwards, the set of equilibrium pairs (i_*, v_{o*}) defining the trajectories $x_e(\theta)$ is given by

$$X_e = \mathcal{E} \cap \mathcal{R} \tag{9}$$

with the ellipse $\mathcal{E}$ and the region $\mathcal{R}$ given, respectively, as follows

$$\mathcal{E} = \{(i_*, v_{o*}) \in \mathbb{R}^2 : R_L i_*^2 - v_m i_* + 2v_{o*}^2/(3R_o) = 0\} \quad (10)$$

$$\mathcal{R} = \{(i_*, v_{o*}) \in \mathbb{R}^2 : (v_m - R_L i_*)^2 + (L\omega i_*)^2 \le v_{o*}^2/3\} \quad (11)$$

An illustration of one possible set X_e is depicted in Figure 2. From the definition of $\mathcal{E}$ it can be concluded that a necessary condition on v_{o*} for the existence of an i_* such that $(i_*, v_{o*}) \in X_e$ is that $|v_{o*}| \le v_m\sqrt{\frac{3R_o}{8R_L}} = \bar{v}_{o*}$, which has been determined by considering a non-negative discriminant for the second order equation of i_* presented in (10). Hence, a pair $(i_*, v_{o*}) \in X_e$ can be obtained by providing the desired v_{o*} to solve the second order equation in (10) for i_*, providing up to two candidate pairs (i_*, v_{o*}) each of which can be tested to $(i_*, v_{o*}) \in \mathcal{R}$. At this point some trigonometric properties must be presented and discussed.

2.1 Trigonometric properties

Consider the vector function $f(\theta)$ defined by identities (3)-(5). Trigonometric relations allow us to conclude that $e'f(\theta) = 0,\ \forall\theta \in \mathbb{R}$, with $e = [1\ 1\ 1]'$. Hence, the vector function $f(\theta)$ belongs to a plane in $\mathbb{R}^3$ perpendicular to the vector e, formally defined as

$$\Pi = \{v \in \mathbb{R}^3\ :\ v'e = 0\} \quad (12)$$

Consequently, along a trajectory of $\theta(t)$, the time derivative of $f(\theta)$ also belongs to the same plane Π and is given by $\dot{f}(\theta) = \omega g(\theta)$ with $g(\theta) = [g_a(\theta)\ g_b(\theta)\ g_c(\theta)]' \in \mathbb{R}^3$ and

$$g_a(\theta) = \cos(\theta) \quad (13)$$

$$g_b(\theta) = \cos(\theta - 2\pi/3) \quad (14)$$

$$g_c(\theta) = \cos(\theta - 4\pi/3) \quad (15)$$

where $e'g(\theta) = 0$. Moreover, it also follows that

$$f(\theta)'f(\theta) = 3/2, \quad f(\theta)'g(\theta) = 0, \quad g(\theta)'g(\theta) = 3/2 \quad (16)$$

The matrix

$$R(\theta) = \begin{bmatrix} f(\theta) & g(\theta) & 0 \\ 0 & 0 & \sqrt{3/2} \end{bmatrix} \quad (17)$$

will be extensively adopted throughout this paper and some of its properties have to be highlighted. Firstly, from (16), we have that

$$R(\theta)'R(\theta) = (3/2)I \quad (18)$$

Additionally, it is also true that $\dot{g}(\theta) = -\omega f(\theta)$ for all $\theta \in \mathbb{R}$, allowing to demonstrate that

$$\dot{R}(\theta(t)) = R(\theta(t))\Omega \quad (19)$$

with the skew-symmetric matrix

$$\Omega = \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (20)$$

Even though these properties resemble those from rotation matrices, notice that $R(\theta)$ is not a square matrix. Finally, for an arbitrary diagonal matrix $D = \mathrm{diag}(d_1 I, d_2)$ we also have that

$$DR(\theta) = R(\theta)V'DV \quad (21)$$

with the full rank matrix

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (22)$$

3. MAIN RESULTS

The next lemma is of great importance to obtain the design conditions based on LMIs, which assure global asymptotic stability of the origin $\xi = 0$ of system (7).

Lemma 1. Let scalars $(\kappa, \eta, \alpha, \beta, \gamma, \mu, \nu, \rho)$ of the structured positive definite matrices

$$T_I = \begin{bmatrix} \kappa I & 0 \\ 0 & \eta \end{bmatrix}, \quad T_R = \begin{bmatrix} \alpha & \beta & \mu \\ \beta & \gamma & \nu \\ \mu & \nu & \rho \end{bmatrix} \quad (23)$$

be given. The inequality

$$T_I - R(\theta)T_R R(\theta)' > 0 \quad (24)$$

holds for every $\theta \in \mathbb{R}$ if and only if

$$J'T_I J - T_R > 0 \quad (25)$$

holds for $J = \sqrt{2/3}V$.

Proof: Firstly, notice that performing the Schur Complement Lemma in (24) with respect to T_R and multiplying both sides of the result by $\mathrm{diag}(I, T_R)$, we obtain

$$\begin{bmatrix} \kappa I & \bullet & \bullet & \bullet & \bullet \\ 0 & \eta & \bullet & \bullet & \bullet \\ \alpha f(\theta)' + \beta g(\theta)' & \sqrt{3/2}\mu & \alpha & \bullet & \bullet \\ \beta f(\theta)' + \gamma g(\theta)' & \sqrt{3/2}\nu & \beta & \gamma & \bullet \\ \mu f(\theta)' + \nu g(\theta)' & \sqrt{3/2}\rho & \mu & \nu & \rho \end{bmatrix} > 0 \quad (26)$$

Now, multiplying the second row and column by $\sqrt{2/3}$ and applying once more the Schur Complement Lemma, but now with respect to κ, we obtain

$$\begin{bmatrix} 2\eta/3 & \bullet & \bullet & \bullet \\ \mu & \alpha & \bullet & \bullet \\ \nu & \beta & \gamma & \bullet \\ \rho & \mu & \nu & \rho \end{bmatrix} - \kappa^{-1} \begin{bmatrix} 0 \\ \alpha f(\theta)' + \beta g(\theta)' \\ \beta f(\theta)' + \gamma g(\theta)' \\ \mu f(\theta)' + \nu g(\theta)' \end{bmatrix} \begin{bmatrix} 0 \\ \alpha f(\theta)' + \beta g(\theta)' \\ \beta f(\theta)' + \gamma g(\theta)' \\ \mu f(\theta)' + \nu g(\theta)' \end{bmatrix}' =$$

$$= \begin{bmatrix} 2\eta/3 & \bullet & \bullet & \bullet \\ \mu & \alpha & \bullet & \bullet \\ \nu & \beta & \gamma & \bullet \\ \rho & \mu & \nu & \rho \end{bmatrix} - \frac{3}{2}\kappa^{-1} \begin{bmatrix} 0 & 0 \\ \alpha & \beta \\ \beta & \gamma \\ \mu & \nu \end{bmatrix} \begin{bmatrix} 0 & 0 \\ \alpha & \beta \\ \beta & \gamma \\ \mu & \nu \end{bmatrix}' > 0 \quad (27)$$

where the equality follows from the identities (16). Finally, performing the Schur Complement Lemma in (27) with respect to κ, rearranging rows and columns and applying once more the Schur Complement Lemma with respect to T_R we have (25), concluding the proof. □

This lemma provides a tool to efficiently verify that a matrix function as given in (24) is positive definite for all $\theta \in \mathbb{R}$ by evaluating the LMI (25). Its importance will be clear in the proof of our main theorem.

To present globally asymptotically stabilizing design conditions, let us adopt a parameter dependent Lyapunov function

$$v(\xi, \theta) = \xi' P(\theta)\xi \quad (28)$$

with the positive definite matrix

$$P(\theta) = P_I - R(\theta)P_R R(\theta)' \quad (29)$$

where

$$P_I = \begin{bmatrix} pI & 0 \\ 0 & q \end{bmatrix} > 0 \quad (30)$$

and $P_R > 0$ are to be determined.

Evaluating the time derivative of $v(\xi, \theta)$ along trajectories $\xi(t)$ and $\theta(t)$, we obtain

$$\dot{v}(\xi, \theta) = \xi' W_\sigma(\theta)\xi + 2\xi' P(\theta)\ell_\sigma(\theta) \quad (31)$$

with

$$W_\sigma(\theta) = A_\sigma' P(\theta) + P(\theta)A_\sigma + \dot{P}(\theta) \tag{32}$$

The next theorem presents sufficient conditions for guaranteeing that $\dot{v}(\xi,\theta) < 0$ for all $\xi \neq 0$.

Theorem 1. Consider system (7) evolving from $\xi(0) = \xi_0$, a non-negative scalar r composing $Q = \mathrm{diag}(rI, 1)$ and a desired pair $(i_*, v_{o*}) \in X_e$ be given. If there exist positive scalars p, q of $P_I = \mathrm{diag}(pI, q)$ and a positive definite matrix P_R satisfying the following LMIs

$$J'P_I J - P_R > 0 \tag{33}$$

$$J'(-Q - 2P_I A_I)J - \Psi > 0, \ \Psi > 0 \tag{34}$$

with

$$\Psi = \mathrm{He}\{P_R\big((3/2)A_R - V'A_I V - \Omega'\big) - V'P_I V A_R\} \tag{35}$$

$$A_I = \begin{bmatrix} -(R_L/L)I & 0 \\ 0 & -1/(R_o C) \end{bmatrix} \tag{36}$$

$$A_R = \frac{\sqrt{6}}{3v_{o*}} \begin{bmatrix} 0 & 0 & -v_d/L \\ 0 & 0 & -\omega i_* \\ v_d/C & L\omega i_*/C & 0 \end{bmatrix} \tag{37}$$

and $v_d = R_L i_* - v_m$, then the switching function $\sigma(t) = u(\xi(t), \theta(t))$ with

$$u(\xi,\theta) = \arg\min_{i\in\mathbb{K}} \xi'(W_i(\theta)\xi + 2P(\theta)\ell_i(\theta)) \tag{38}$$

assures that the origin of (7) is a globally asymptotically stable equilibrium point and that

$$\mathcal{J} < v(\xi_0, \theta_0) \tag{39}$$

Proof: The proof follows from (31), which evaluated along an arbitrary trajectory of system (7), under the switching function $\sigma(t) = u(\xi(t), \theta(t))$, yields

$$\begin{aligned} \dot{v}(\xi,\theta) &= \min_{i\in\mathbb{K}} \xi'W_i(\theta)\xi + 2\xi'P(\theta)\ell_i(\theta) \\ &= \min_{\lambda\in\Lambda} \xi'W_\lambda(\theta)\xi + 2\xi'P(\theta)\ell_\lambda(\theta) \\ &\leq \xi'W_{\lambda_*(\theta)}(\theta)\xi + 2\xi'P(\theta)\ell_{\lambda_*(\theta)}(\theta) \end{aligned} \tag{40}$$

where $\lambda_*(\theta)$ is an arbitrary vector inside Λ. Note that if for each θ there exists $\lambda_*(\theta) \in \Lambda$ such that the inequality $W_{\lambda_*(\theta)}(\theta) < -Q$ is satisfied and $\ell_{\lambda_*(\theta)}(\theta) = 0$, then we have $\dot{v}(\xi,\theta) < 0$ and the origin $\xi = 0$ is globally asymptotically stable. In order to verify that $\ell_{\lambda_*(\theta)}(\theta) = 0$, let us write

$$\ell_{\lambda_*(\theta)}(\theta) = \begin{bmatrix} L^{-1}\big((v_m - R_L i_*)f(\theta) - L\omega i_* g(\theta) - v_{o*}S_{\lambda_*(\theta)}\big) \\ C^{-1}(i_* S'_{\lambda_*(\theta)} f(\theta) - v_{o*}/R_o) \end{bmatrix} \tag{41}$$

Now, observe that the polytope $\mathbb{P}$ formed by vertices $(S_1, \cdots, S_7)$ is a regular hexagon perpendicular to the vector $e = [1\ 1\ 1]'$, whose inscribed circumference has radius $1/\sqrt{2}$, as shown in Figure 3. Hence, $S_{\lambda_*(\theta)} \in \Pi$ can be chosen as any linear combination of $f(\theta)$ and $g(\theta)$ as long as its length does not exceed $1/\sqrt{2}$, given that both $f(\theta), g(\theta) \in \Pi$ for all $\theta \in \mathbb{R}$. Indeed, choosing

$$S_{\lambda_*(\theta)} = \frac{v_m - R_L i_*}{v_{o*}} f(\theta) - \frac{L\omega i_*}{v_{o*}} g(\theta) \tag{42}$$

that makes null the first term of (41), the constraint $S'_{\lambda_*(\theta)} S_{\lambda_*(\theta)} \leq 1/2$ yields the region $\mathcal{R}$, as defined in (11). Moreover, replacing $S_{\lambda_*(\theta)}$ in the second term of (41), it is simple to verify that this element becomes null whenever the pair (i_*, v_{o*}) is chosen as a point of the ellipse $\mathcal{E}$. Hence, choosing $(i_*, v_{o*}) \in X_e$ assures that for each $\theta \in \mathbb{R}$ there exists $\lambda_*(\theta)$ such that $\ell_{\lambda_*(\theta)}(\theta) = 0$. For the same $\lambda_*(\theta)$,

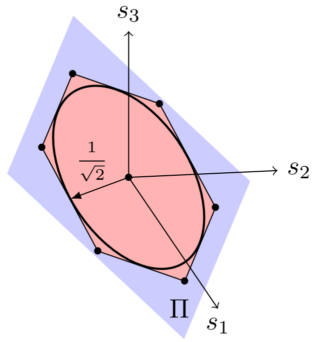

Fig. 3. Graphical representation of polytope $\mathbb{P}$ and inscribed circumference.

let us now show that inequality $W_{\lambda_*(\theta)}(\theta) < -Q$ holds for all $\theta \in \mathbb{R}$ whenever inequalities in (34) are satisfied. Firstly, let us rewrite $A_{\lambda_*(\theta)} = A_I - R(\theta)A_R R(\theta)'$ and evaluate $W_{\lambda_*(\theta)}(\theta)$ as

$$W_{\lambda_*(\theta)}(\theta) = 2P_I A_I + R(\theta)\Psi R(\theta)' \tag{43}$$

with Ψ defined in (35) obtained after applying (18), (19), (21) adequately. Now, by applying Lemma 1 in (34) for $T_I = -Q - 2P_I A_I$ and $T_R = \Psi$, and observing that $T_R > 0$, we have that

$$-Q - 2P_I A_I > R(\theta)\Psi R(\theta)' \tag{44}$$

showing that $W_{\lambda_*(\theta)}(\theta) < -Q$. The positive definiteness of $P(\theta)$ is guaranteed by (33) together with Lemma 1, but now with $T_I = P_I$ and $T_R = P_R$. Finally, from (40), we obtain $\dot{v}(\xi,\theta) < -\xi'Q\xi$ for all $\xi \neq 0$, assuring global asymptotic stability of the origin. At last, integrating this last inequality from $t = 0$ up to infinity allows us to determine the upper bound (39), concluding the proof. □

This theorem provides sufficient conditions for the design of a switching function capable of assuring global asymptotic tracking of $x_e(\theta(t))$, bringing the AC-DC converter to a desired output voltage v_{o*} and controlling the input currents to assure unitary power factor operation. It is important to notice that the design conditions are given in terms of LMIs being, therefore, easy-to-solve with readily available tools.

Moreover, a parameter dependent Lyapunov function showed to be well adapted to deal with this class of systems characterized by a dependency on $\theta(t)$. An alternative approach, based on a simple quadratic Lyapunov function $\hat{v}(\xi) = \xi'P\xi$ with a constant $P > 0$ could be considered instead. However, this requires to satisfy the condition

$$A'_{\lambda_*(\theta)}P + PA_{\lambda_*(\theta)} < -Q \tag{45}$$

for all $\theta \in [0, 2\pi)$ by imposing this inequality over a sufficiently fine grid of points. This alternative strategy is not only less computationally efficient but also provides more conservative results in our tests, in spite of the fact that it takes into account more optimization variables. This will be illustrated in Section 4. Finally, notice that the stability conditions in Theorem 1 require only that $A_{\lambda_*(\theta)}$ be Hurwitz stable, making no imposition regarding stability of matrices A_i, $i \in \mathbb{K}$, isolatedly considered, although all of them are also stable. In the sequel, we present some theoretical comparisons with techniques based on an averaged system response and also discussions regarding computational issues.

3.1 Averaged model comparison

Averaged model techniques are extensively used in switched systems such as the proposed converter. Indeed, these techniques rely upon the fact that sufficiently fast switching among subsystems creates an averaged dynamics which governs the state evolution. Surely, this takes into account Fillipov solutions $\xi(t)$ for the system (7) that must satisfy the differential inclusion

$$\dot{\xi}(t) \in \{A_\lambda \xi(t) + \ell_\lambda(\theta(t)) \ : \ \lambda \in \Lambda\} \tag{46}$$

Notice that the origin $\xi = 0$ of this system (7) is an equilibrium point if and only if there exists for each $\theta \in \mathbb{R}$ a $\lambda(\theta) \in \Lambda$ such that $\ell_{\lambda(\theta)}(\theta) = 0$, assuring $\dot{\xi}(t) = 0$. Given the discussions presented in the proof of Theorem 1, we can conclude that, for this particular system, this requirement is fulfilled if and only if $(i_*, v_{o*}) \in X_e$, with X_e given in (9). Hence, the proposed set X_e contains all pairs (i_*, v_{o*}) defining a steady-state response $x_e(\theta(t))$ for (1) attainable by an averaged model strategy.

3.2 Computational issues

From a computational point of view, the evaluation of the proposed switching function (38) is of low complexity, since it can be recast in a simpler equivalent form, that is given in this subsection. Indeed, we have that

$$\begin{aligned} h_i(\xi,\theta) &= \xi'\big(W_i(\theta)\xi + 2P(\theta)\ell_i(\theta)\big) \\ &= \xi'\left(2P(\theta)\Big(A_i(\xi + x_e(\theta)) + b(\theta) - \dot{x}_e(\theta)\Big) + \dot{P}(\theta)\xi\right) \end{aligned}$$

Notice that the dependency on index $i \in \mathbb{K}$ is present only in the term $2\xi' P(\theta) A_i(\xi + x_e(\theta))$, indicating that it is unnecessary to evaluate the remaining ones. Moreover, employing trigonometric identities, we can decompose

$$R(\theta) = G\bar{R}(\theta) \tag{47}$$

with

$$G = \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ -1 & -\sqrt{3} & 0 \\ -1 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{6} \end{bmatrix}, \bar{R}(\theta) = \begin{bmatrix} \sin(\theta) & \cos(\theta) & 0 \\ \cos(\theta) & -\sin(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{48}$$

Consider now the matrix $G^\# = (2/3)G'$. Notice that $GG^\# = I - (1/3)\tilde{e}\tilde{e}'$ with $\tilde{e} = [1 \ 1 \ 1 \ 0]'$ and that $\tilde{e}'\xi = \tilde{e}' x_e(\theta) = 0$, assuring the identities

$$GG^\#\xi = \xi, \quad GG^\# x_e(\theta) = x_e(\theta) \tag{49}$$

Defining $\bar{h}_i(\xi,\theta) = \xi' P(\theta) A_i(\xi + x_e(\theta))$, due to the previous discussion, we have that

$$\begin{aligned} \bar{h}_i(\xi,\theta) &= \xi' G^{\#'} G'(P_I A_i - R(\theta) P_R R(\theta)' A_i) GG^\#(\xi + x_e(\theta)) \\ &= \xi' G^{\#'}\left(\mathcal{P}_i - \mathcal{T}(\theta)\mathcal{A}_i\right) G^\#(\xi + x_e(\theta)) \end{aligned}$$

with $\mathcal{P}_i = G' P_I A_i G$, $\mathcal{T}(\theta) = (3/2)\bar{R}(\theta) P_R \bar{R}(\theta)'$ and $\mathcal{A}_i = G' A_i G$. Observe that matrices $\mathcal{P}_i$ and $\mathcal{A}_i$ can be calculated *a priori*. Finally, taking into account that $i_c = -i_a - i_b$, the vectors $\bar{x} = G^\# x$ and $\bar{x}_e(\theta) = G^\# x_e(\theta)$ can be calculated as being

$$\bar{x} = \begin{bmatrix} i_a \\ -(\sqrt{3}/3)(i_a + 2i_b) \\ (\sqrt{6}/3)v_o \end{bmatrix}, \ \bar{x}_e(\theta) = \begin{bmatrix} i_* \sin(\theta) \\ i_* \cos(\theta) \\ (\sqrt{6}/3)v_{o*} \end{bmatrix} \tag{50}$$

leading to an equivalent switching function

$$u(\bar{x},\theta) = \arg\min_{i \in \mathbb{K}} (\bar{x} - \bar{x}_e(\theta))' \left(\mathcal{P}_i - \mathcal{T}(\theta)\mathcal{A}_i\right)\bar{x} \tag{51}$$

which is more adapted for implementation in microcontrollers. Indeed, at each control update the most demanding operations are two trigonometric function evaluation (i.e. $\sin(\theta)$ and $\cos(\theta)$), two 3x3 matrix products for determining $\mathcal{T}(\theta)$ and then 7 evaluations of the expression in (51), which can be efficiently performed for each $i \in \mathbb{K}$.

4. SIMULATION RESULTS

In this simulation, numerical parameters were borrowed from Bouafia et al. (2009) and are given in Table 2. The goal is to bring the output voltage of the AC-DC converter given in Figure 1 to a steady-state value of $v_{o*} = 120$ V whilst operating in unitary power factor. To this end we could verify that, for $i_* = 1.369$ A, the pair $(i_*, v_{o*}) \in X_e$ defines a reachable steady-state trajectory $x_e(\theta) = [i_* f(\theta)' \ v_{o*}]'$. Adopting this equilibrium pair, we have solved the optimization problem

Table 2. System parameters adopted in simulations.

R_o	175 Ω
R_L	0.56 Ω
L	19.5 mH
ω	$2\pi \times 50$ rad/s
C	2.35 mF
v_M	40.825 V

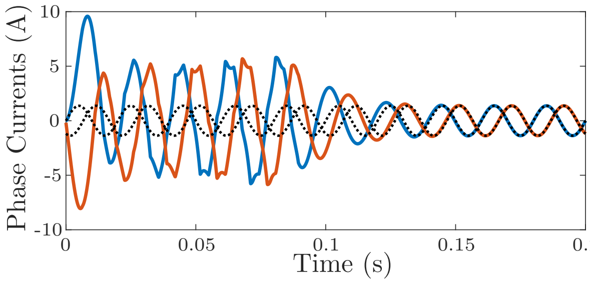

Fig. 4. Phase currents $i_a(t)$ (blue) and $i_b(t)$ (red) and correspondent steady-state references (dashed lines).

$$\min_{p,q,P_R} \Big(x_0 - x_e(\theta_0)\Big)' P(\theta_0)\Big(x_0 - x_e(\theta_0)\Big)' \tag{52}$$

subject to (33)-(34) considering $x_0 = 0$ and $\theta_0 = 0$. Moreover, to obtain a fast convergence of $v_o(t)$ towards v_{o*}, we have chosen $r = 0$. This objective function is responsible for minimizing the upper bound (39). An optimal solution was obtained for $p = 6.2576 \times 10^4$, $q = 5.6854 \times 10^3$ and

$$P_R = \begin{bmatrix} 4.1718 \times 10^4 & -0.0082 & -0.0155 \\ -0.0082 & 4.1718 \times 10^4 & -0.0487 \\ -0.0155 & -0.0487 & 3.7902 \times 10^3 \end{bmatrix} \tag{53}$$

assuring an upper bound in (39) of $\mathcal{J} < 1975.32$. For the sake of comparison, solving the analogous problem with the constraint (45), related to a quadratic Lyapunov function $\hat{v}(\xi) = \xi' P \xi$, we have obtained an upper bound $\mathcal{J} < 2965.81$. Simulating the system response from $x_0 = 0$, the obtained state trajectories and the correspondent switching signal are shown in Figure 4, 5 and 6. From these data we can conclude that the proposed switching function was efficient in controlling the converter output voltage towards a constant value of 120 V, under unitary power factor operation. Moreover, as it can be noticed in Figure

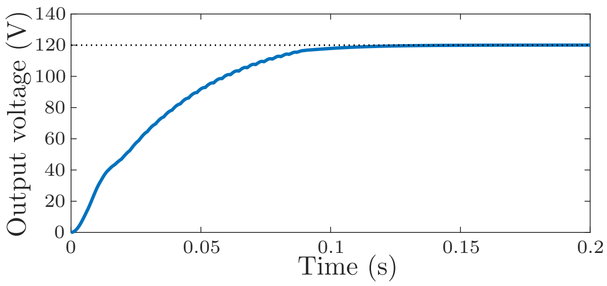

Fig. 5. Output voltage $v_o(t)$ and correspondent steady-state reference (dashed line).

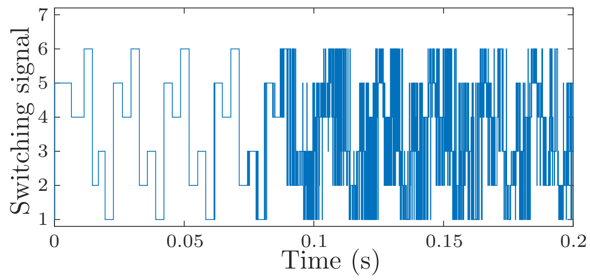

Fig. 6. Switching signal $\sigma(t)$ generated by the proposed switching function.

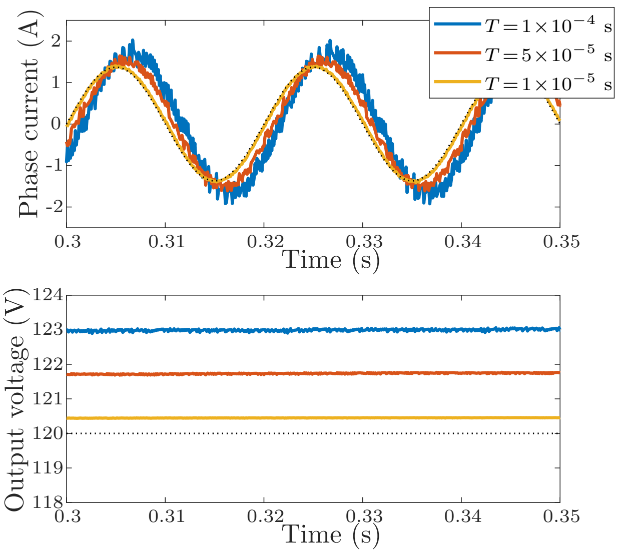

Fig. 7. Steady-state response of current $i_a(t)$ (above) and voltage $v_o(t)$ (below) for several values of T.

6 the switching frequency during the transient response was relatively low, when compared to the steady-state, which may not be observed in control techniques based on averaged models.

Finally, let us investigate the effects of employing a piece-wise constant switching function as

$$\sigma(t) = u(\xi(t_k), \theta(t_k)), \ \forall t \in [t_k, t_{k+1}) \tag{54}$$

where t_k, $k \in \mathbb{N}$, are switching instants respecting $t_0 = 0$ and the switching period $T = t_{k+1} - t_k$. For several values of T the steady-state behaviour of the phase current $i_a(t)$ and output voltage $v_o(t)$ are shown in Figure 7 along with the corresponding references (dotted lines). Notice that the performance is impaired as larger values of switching period T are adopted. This demonstrates the importance of taking into account this aspect into the design step, motivating future works.

5. CONCLUSION

We have presented a novel methodology for controlling an AC-DC converter in a single control loop. This approach takes into account the design of a state-dependent switched function that can bring the system state into some steady state behaviour specified by the designer. To this end, a set of attainable pairs (i_*, v_{o*}) is presented from where an equilibrium trajectory $x_e(\theta(t))$, composed of desired constant voltage and sinusoidal phase currents, is defined and its global asymptotic tracking is assured. The conditions were based on a parameter dependent Lyapunov function and an upper bound for a quadratic cost was guaranteed. The design procedure is written as a convex optimization problem defined by a set of three LMIs and was validated by means of a simulation example.

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