Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

3D Formation Control of Multiple Torpedo-type Underactuated AUVs ⋆

Ji-Hong Li * Daegil Park * Hyungjoo Kang * Gun Rae Cho *

* *Korea Institute of Robotics and Technology Convergence, Jigok-Ro 39, Nam-Gu, Pohang 37666, Republic of Korea (e-mail: {jhli5, daegilpark, hjkang, sandman}@kiro.re.kr).*

Abstract: This paper considers the formation problem for a group of torpedo-type AUVs (autonomous underwater vehicles). For each vehicle, there are only three control inputs available for the vehicle's 6-DOF motion in the water. So this is a typical underactuated system. For these underactuated multi-agent system, we propose a sort of virtual structure based formation scheme. Virtual structure is a graph with each node taken as virtual leader for each specific agent vehicle. And for the vehicle's motion control, a sort of path following scheme is used to force the vehicle to follow the virtual leader's trajectory. Proposed formation scheme can guarantee the exponential following in the spherical coordinate frame, and some of simulation studies are carried out to demonstrated this kind of following performance.

Keywords: Multi-agent system, Formation control, Underactuated system, Path following, Marine systems, Lyapunov stability.

1. INTRODUCTION

Throughout the past three decades, formation control has been one of the most intense research topic in the control community. Reynolds (1987) introduced a distributed behavioural model for flocks of birds, herds of land animals, and schools of fishes. This model can be summarized as three heuristic rules: flock centering, collision avoidance and velocity matching. In the formation algorithm, each dynamic agent was modelled as particle system - a simple double-integrator system. This kind of agent model has been inherited in most of the following research works (Leonard and Fiorelli, 2001; Olfati-Saber and Murray, 2002; Fiorelli et al., 2006; Do, 2007). In addition to the particle system, nonlinear models were applied for underwater vehicles (Dunbar and Murray, 2002) and for wheel robots with terminal constraints (Fax and Murray, 2004), in both of which the nonlinear dynamics were fully actuated. Recently, underactuated nonlinear models were frequently considered in the formation control works (Li and Lee, 2008; Li et al., 2009; Cui et al., 2010; Li et al., 2019; Gao and Guo, 2019). In these literatures, despite the control targets were underwater flying vehicles, formation schemes only on the horizontal 2D plane were discussed. In this paper, we extend the previous works (Li and Lee, 2008; Li et al., 2009) to the 3D space.

In this paper, we assume that there is a surface vessel, which is equipped with a fully integrated underwater positioning and communication system, covers all the agent vehicles in the group. The vessel can simultaneously track all the vehicles and communicate with them. On the other hand, we assume that there is not direct communication link between any of two vehicles. In our previous works (Li and Lee, 2008; Li et al., 2009), the formation geometry was constructed by forcing each agent vehicle to maintain certain distances with other vehicles. To do so, every vehicle had to know all other vehicles' position information. In practice, this might significantly increase the acoustic communication loads between the vehicles and surface vessel and moreover can cause severe time-delay in the position update. Under this consideration, in this paper, we propose a sort of virtual structure formation scheme as mentioned in Oh et al. (2015). The surface vessel designs a virtual structure, which is a graph with each node taken as virtual leader for specific agent vehicle, and then transmits the vehicle's tracked position and its specific virtual leader's motion information to each agent. The potential function for the vehicles formation consists of two parts: one is for each vehicle to follow its virtual leader, and the second is for obstacle avoidance. This potential function is more simple than the ones used in Li and Lee (2008) and Li et al. (2009), where the formation potential contained three components.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 AUV Model

Usually, for most of underwater vehicles, their gravity centers are designed to be (much) lower than the buoyancy centers, and this kind of mechanism can guarantee the vehicles' roll dynamic stability in the low speed motion. Under this consideration, in this paper, we consider the following 5DOF vehicle kinematics and dynamics (Fossen, 2011; Li et al., 2016)

⋆ This work was supported by the project No. 17-CM-RB-16 titled "Development of Multi-sensor Fusion based AUV's Terminal Guidance and Docking Technology," funded by the Agency for Defense Development (ADD) in the South Korea.

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \\ \dot{\theta}_i \\ \dot{\psi}_i \end{bmatrix} = \begin{bmatrix} c\theta_i c\psi_i & -s\psi_i & s\theta_i c\psi_i & 0 & 0 \\ c\theta_i s\psi_i & c\psi_i & s\theta_i s\psi_i & 0 & 0 \\ -s\theta_i & 0 & c\theta_i & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1/c\theta_i \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ q_i \\ r_i \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix} \dot{u}_i \\ \dot{v}_i \\ \dot{w}_i \\ \dot{q}_i \\ \dot{r}_i \end{bmatrix} = \begin{bmatrix} f_{ui} \\ f_{vi} \\ f_{wi} \\ f_{qi} \\ f_{ri} \end{bmatrix} + \begin{bmatrix} b_{ui} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_{qi} & 0 \\ 0 & 0 & b_{ri} \end{bmatrix} \begin{bmatrix} \tau_{ui} \\ \tau_{qi} \\ \tau_{ri} \end{bmatrix}, \quad (2)$$

where $s(\cdot) = sin(\cdot)$ and $c(\cdot) = cos(\cdot)$; $(x_i, y_i, z_i), i = 1, \cdots, n$ is the ith vehicle's position in the navigation frame, θ_i and ψ_i each denotes the pitch and yaw angle; (u_i, v_i, w_i) is the velocity vector and q_i and r_i are pitch and yaw angular rates, all in the ith vehicle's body-fixed frame; $f_{ai}(\cdot) \in C^1$ with $a \in \{u, v, w, q, r\}$ indicates the ith vehicle's nonlinear hydrodynamics including damping, inertial, Coriolis, and gravitational terms each in the surge, sway, heave, pitch, and yaw dynamics; surge force τ_{ui}, pitch moment τ_{qi}, and yaw moment τ_{ri} are three only available control inputs with nonzero constant gains b_{ui}, b_{qi}, and b_{ri}.

For torpedo-type AUV, there is only one thruster providing surge force, and therefore to activate the vehicle's pitch and yaw dynamics, it has to possess a considerable forward speed. On the other hand, if $\theta_i = \pm\pi/2$, $\dot{\psi}_i = r/cos\theta_i$ is not defined and therefore there is a singularity problem. Under these considerations, in this paper, we make the following assumption on the vehicles' motion.

Assumption 1. In (1) and (2), $u_i > 0$ and $|\theta_i| < \pi/2$.

Remark 1. For the convenience of discussion, in this paper we do not include any of uncertainty terms in the vehicle's dynamics (2). Indeed, known bounded or even unknown bounded uncertainty terms still can be properly handled using appropriate control methods (Li et al., 2016, and references therein).

2.2 Spherical Coordinate Transformation

For torpedo-type AUVs, the sway and heave forces are unavailable, and therefore the most difficulty in the path tracking is how to properly handle these sway and heave dynamics. To tackle this problem, in this paper, we introduce the following spherical coordinate transformation in the vehicle's body-fixed frame.

$$u_{li} = \sqrt{u_i^2 + v_i^2 + w_i^2}, \;\; \theta_{li} = \theta_i + \theta_{ai}, \;\; \psi_{li} = \psi_i + \psi_{ai}, \; (3)$$

where $\theta_{ai} = -atan(w_i/\sqrt{u_i^2 + v_i^2})$ and $\psi_{ai} = atan(v_i/u_i)$.

Since $u_i > 0$, θ_{ai} and ψ_{ai} are both well defined in the domain $(-\pi/2, \; \pi/2)$.

Using the spherical coordinate $(u_{li}, \; \theta_{li}, \; \psi_{li})$, the ith vehicle's kinematics can be rewritten as

$$\begin{aligned} \dot{x}_i &= u_{li} cos\theta_{li} cos\psi_{li}, \\ \dot{y}_i &= u_{li} cos\theta_{li} sin\psi_{li}, \\ \dot{z}_i &= -u_{li} sin\theta_{li}, \end{aligned} \quad (4)$$

For the vehicle's path $(x_i(t), \; y_i(t), \; z_i(t))$, it can be embodied by $(u_i(t), \; v_i(t), \; w_i(t), \; \theta_i(t), \; \psi_i(t))$ through the kinematics (1), or by the spherical coordinate $(u_{li}(t), \; \theta_{li}(t), \; \psi_{li}(t))$ using (4). In this paper, we apply the latter method.

Given a reference path $(x_{di}(t), \; y_{di}(t), \; z_{di}(t))$, the range error is defined as following

$$r_{ei} = \sqrt{x_{ei}^2 + y_{ei}^2 + z_{ei}^2}, \quad (5)$$

where $x_{ei} = x_{di} - x_i, \; y_{ei} = y_{di} - y_i, \; z_{ei} = z_{di} - z_i$.

Using (4), the time derivative of range error can be expanded as

$$\begin{aligned} \dot{r}_{ei} = \; & u_{ldi}[cos\theta_{ldi} cos\theta_{bi} cos(\psi_{ldi} - \psi_{bi}) + sin\theta_{ldi} sin\theta_{bi}] \\ & - u_{li}\{cos\theta_{li} cos\theta_{bi}[cos(\psi_{li} - \psi_{bi}) - 1] \\ & + cos(\theta_{li} - \theta_{bi})\}, \end{aligned} \quad (6)$$

where θ_{bi} and ψ_{bi} are defined as

$$\theta_{bi} = -atan\left(z_{ei}/\sqrt{x_{ei}^2 + y_{ei}^2}\right), \quad \psi_{bi} = atan2\,(y_{ei}, x_{ei}). \quad (7)$$

2.3 Control Problem

Given the range error dynamics as (6), the trajectory tracking control problem is that: properly steer the vehicle so as to $r_{ei} \to c_i$ with $c_i > 0$ constant, while keeping the vehicle's attitude $(\theta_{li}, \; \psi_{li})$ matching with certain desired one $(\theta_{li}^D, \; \psi_{li}^D)$. If $(\theta_{li}^D, \; \psi_{li}^D)$ is chosen as $(\theta_{ldi}, \; \psi_{ldi})$, then it is the path tracking problem; and if $(\theta_{li}^D, \; \psi_{li}^D) = (\theta_{bi}, \; \psi_{bi})$, then it becomes the path following case (Li, 2016, and references therein).

In this paper, we apply the path following scheme in the vehicles' formation. Substituting $(\theta_{li}, \; \psi_{li}) = (\theta_{li}^D, \; \psi_{li}^D) = (\theta_{bi}, \; \psi_{bi})$ into (6), we have

$$\dot{r}_{ei} = u_{ldi} A_i - u_{li}, \quad (8)$$

where $A_i = cos\theta_{ldi} cos\theta_{bi} cos(\psi_{ldi} - \psi_{bi}) + sin\theta_{ldi} sin\theta_{bi}$.

Proposition 1. Consider (8). If u_{li} is chosen as

$$u_{li} = u_{li}^D = u_{ldi} A_i + k_{Ri}(r_{ei} - c_i), \quad (9)$$

with $k_{Ri}, c_i > 0$ design parameters, then we can guarantee the exponential convergence of $r_{ei} \to c_i$.

Proof. Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\left(r_{ei} - c_i\right)^2. \quad (10)$$

Differentiating (10) and substituting (8) and (9) into it, we can get $\dot{V} = (r_{ei} - c_i)(u_{ldi} A - u_{li}) = -k_{Ri}(r_{ei} - c_i)^2 = -2k_{Ri} V$. □

For ith vehicle, the trajectory tracking control objective in this paper is to force the vehicle to follow $(u_{li}^D, \; \theta_{li}^D, \; \psi_{li}^D)$, instead of directly tracking $(u_{ldi}, \; \theta_{ldi}, \; \psi_{ldi})$. Consequently, the vehicle's path following model can be expressed in the form of spherical coordinate $(u_{li}, \; \theta_{li}, \; \psi_{li})$ as following

$$\begin{aligned} \dot{r}_{ei} &= u_{ldi} A_i - u_{li}\left[cos\theta_{li} cos\theta_{bi}(cos\psi_{lei} - 1) + cos\theta_{lei}\right], \\ \dot{\theta}_{lei} &= \dot{\theta}_{li}^D - \dot{\theta}_{ai} - q_i, \\ \dot{\psi}_{lei} &= \dot{\psi}_{li}^D - \dot{\psi}_{ai} - r_i sec\theta_i, \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{u}_{li} &= f_{uli} + b_{ui} cos\theta_{ai} cos\psi_{ai} \tau_{ui}, \\ \dot{q}_i &= f_{qi} + b_{qi} \tau_{qi}, \\ \dot{r}_i &= f_{ri} + b_{ri} \tau_{ri}, \end{aligned} \quad (12)$$

where $\theta_{lei} = \theta_{li}^D - \theta_{li} = \theta_{bi} - \theta_{li}$, $\psi_{lei} = \psi_{li}^D - \psi_{li} = \psi_{bi} - \psi_{li}$, and $f_{uli} = f_{ui} cos\theta_{ai} cos\psi_{ai} + f_{vi} cos\theta_{ai} sin\psi_{ai} + f_{wi} sin\theta_{ai}$.

3. FORMATION RULES

Consider a group of n AUVs, all of which have the same kinematics and dynamics as (1) and (2). There is a surface vessel, which is equipped with a fully integrated underwater positioning and communication device such that can simultaneously track all the vehicles and also can exchange information with each of them. Moreover, there isn't communication link between any of two vehicles. For this kind of marine vehicles group, we propose a sort of virtual structure formation scheme as in Oh et al. (2015). The surface vessel designs a virtual structure, which is a sort of graph with each node taken as virtual leader for specific vehicle. The formation rules can be summarized as: *virtual leader following* and *obstacle avoidance.*

3.1 Virtual Leader Following

As aforementioned, given the ith vehicle's virtual leader information $(x_{di}(t),\ y_{di}(t),\ z_{di}(t))$, which can be embodied using $(u_{ldi}(t),\ \theta_{ldi}(t),\ \psi_{ldi}(t))$ through (4), the control objective is to force the vehicle to follow $(u_{li}^D(t),\ \theta_{li}^D(t),\ \psi_{li}^D(t))$ instead of directly tracking $(u_{ldi}(t),\ \theta_{ldi}(t),\ \psi_{ldi}(t))$. For this reason, the corresponding *Leader Following* potential function is chosen as following

$$V_{VLF} = V_\alpha + V_\beta, \tag{13}$$

with

$$V_\alpha = \frac{1}{2}\sum_{i=1}^{n} \gamma_R (r_{ei} - c_i)^2, \quad V_\beta = \frac{1}{2}\sum_{i=1}^{n} \big[\gamma_u (u_{li}^D - u_{li})^2 + \gamma_\theta (\theta_{li}^D - \theta_{li})^2 + \gamma_\psi (\psi_{li}^D - \psi_{li})^2\big],$$

where u_{li}^D is chosen as (9), and $\theta_{li}^D = \theta_{bi}$, $\psi_{li}^D = \psi_{bi}$; $\gamma_u, \gamma_\theta, \gamma_\psi > 0$ are weighting factors.

3.2 Obstacle Avoidance

For each AUV, there is a FLS (forward looking sonar) mounted in front of it. Using this FLS, the vehicle can detect the obstacle(s) around it. In this paper, we only consider the position fixed obstacles and apply the same obstacle model as in Li et al. (2009). For each detected obstacle block, it is modeled as a point from which to the vehicle is the shortest distance. Vehicle's obstacle avoidance is guided by the following potential function

$$V_{OA} = \sum_{i=1}^{n} \sum_{q\in\Omega} f_p(||P_{i,q} - P_i||, a_\gamma, b_\gamma), \tag{14}$$

where $f_p(\xi, a, b)$ is a *smooth potential function* which is defined in *Definition 1* in Li et al. (2009) with $0 < a < b$ design parameters; $P_{i,q}$ is the modeled obstacle point detected by the ith vehicle and P_i is the vehicle's position; $\Omega(P_i, t)$ is a subgroup of obstacles included in the ith vehicle's sonar covered 3D space up to time $t > 0$, and $||\cdot||$ denotes the vector Euclidean norm.

Remark 3. For *smooth potential function* $f_p(\xi, a, b)$, if $a = b$, then $f_p = 0$ (Li et al., 2009). From obstacle avoidance point of view, there is no need to have cohesion between the vehicle and the obstacles. With this consideration, in (11), we set $b_\gamma = a_\gamma$.

Remark 4. If there are multiple obstacles around the vehicle, then it is well known that the smooth potential function as (11) cannot always guarantee the global minimum. In case of local minimum, which means the vehicle is trapped by the obstacles, there may be several counter plans applicable in practice. One is that the surface vessel, which is monitoring all the vehicles in near the real-time, can design a new virtual leader for this trapped vehicle and guide the vehicle to escape the local minimum. Also, the vessel simply command the trapped vehicle up to the surface and recover it, while the other vehicles are still in their missions. How to deal with the local minimum problem in practice is out of the scope of this paper.

Without loss of generality, in this paper we make the following assumptions.

Assumption 2. For any given obstacle(s), after a certain period of time, all virtual leaders always move away from the obstacle(s).

Assumption 3. After a period of time, if it finds out that kth vehicle is still trapped by obstacles, then the surface vessel sets the vehicle's u_{lk}^D as $u_{lk}^D = 0$.

4. FORMATION CONTROL DESIGN

In this section, we apply the *virtual structure* concept and propose a formation control algorithm for a group of n underactuated AUVs, each of which has the same kinematics and dynamics as (1) and (2) with *Assumption 1∼ 3.* From (11) and (12), we can see that the vehicle's path following model is in a three-input-three-output second-order strict-feedback form, so the formation problem is solved using general backstepping method (Krstic et al., 1995).

Step 1. As aforementioned, the formation rules consist of *virtual leader following* and *obstacle avoidance.* So, the following Lyapunov function candidate is considered in this step.

$$V_1 = V_\alpha + V_\beta + \gamma_\gamma V_{OA}, \tag{15}$$

where $\gamma_\gamma > 0$ is a weighting factor.

Differentiating (15) and substituting (13) and (14) into it, we have

$$\begin{aligned}
\dot{V}_1 = &\sum_{i=1}^{n} \gamma_R (r_{ei} - c_i)\dot{r}_{ei} \\
&+ \gamma_\gamma \sum_{i=1}^{n}\sum_{q\in\Omega} \frac{\partial f_p(r_{qei}, \alpha_\gamma, \alpha_\gamma)}{\partial r_{qei}} \left[\frac{x_{i,q} - x_i}{r_{qei}}(\dot{x}_{i,q} - \dot{x}_i) \right. \\
&\left. + \frac{y_{i,q} - y_i}{r_{qei}}(\dot{y}_{i,q} - \dot{y}_i) + \frac{z_{i,q} - z_i}{r_{qei}}(\dot{z}_{i,q} - \dot{z}_i)\right] \\
&+ \sum_{i=1}^{n} \Big[\gamma_u (u_{li}^D - u_{li})(\dot{u}_{li}^D - \dot{u}_{li}) + \gamma_\theta (\theta_{li}^D - \theta_{li})(\dot{\theta}_{li}^D - \dot{\theta}_{li}) \\
&+ \gamma_\psi (\psi_{li}^D - \psi_{li})(\dot{\psi}_{li}^D - \dot{\psi}_{li})\Big],
\end{aligned} \tag{16}$$

where $r_{qei} = ||P_{i,q} - P_i||$.

As aforementioned, in this paper we only consider the position fixed obstacle(s). So $\dot{x}_{i,q} = \dot{y}_{i,q} = \dot{z}_{i,q} = 0$ in (16). Furthermore, substituting (11) and (12) into (16), can get

$$\begin{aligned}
\dot{V}_1 =& \gamma_R \sum_{i=1}^{n} (r_{ei} - c_i) \left\{ u_{ldi} A_i - u_{li}^D + u_{li}^D - u_{li} \right. \\
& \left. - u_{li} \left[cos\theta_{li} cos\theta_{bi} (cos\psi_{lei} - 1) + cos\theta_{lei} - 1 \right] \right\} \\
& + \gamma_\gamma \sum_{i=1}^{n} \left[A_{xi} \left(u_{li}^D cos\theta_{li}^D cos\psi_{li}^D - u_{li} cos\theta_{li} cos\psi_{li} \right) \right. \\
& + A_{yi} \left(u_{li}^D cos\theta_{li}^D sin\psi_{li}^D - u_{li} cos\theta_{li} sin\psi_{li} \right) \\
& \left. - A_{zi} \left(u_{li}^D sin\theta_{li}^D - u_{li} sin\theta_{li} \right) \right] \\
& + \sum_{i=1}^{n} \left[\gamma_u u_{lei} (\dot{u}_{li}^D - \dot{u}_{li}) + \gamma_\theta \theta_{lei} (\dot{\theta}_{li}^D - \dot{\theta}_{li}) \right. \\
& \left. + \gamma_\psi \psi_{lei} (\dot{\psi}_{li}^D - \dot{\psi}_{li}) \right] + \Lambda_\gamma, \qquad (17)
\end{aligned}$$

where

$$\Lambda_\gamma = -\gamma_\gamma \sum_{i=1}^{n} \left(A_{xi} \dot{x}_{li}^D + A_{yi} \dot{y}_{li}^D + A_{zi} \dot{z}_{li}^D \right),$$

with $A_{xi} = \sum_{q\in\Omega} \frac{\partial f_p}{\partial r_{qei}} \frac{x_{qei}}{r_{qei}}$, $A_{yi} = \sum_{q\in\Omega} \frac{\partial f_p}{\partial r_{qei}} \frac{y_{qei}}{r_{qei}}$, $A_{zi} = \sum_{q\in\Omega} \frac{\partial f_p}{\partial r_{qei}} \frac{z_{qei}}{r_{qei}}$.

Substituting (9) into (17) and further expanding it, we can get

$$\begin{aligned}
\dot{V}_1 =& -\sum_{i=1}^{n} k_{Ri} (r_{ei} - c_i)^2 + \gamma_R \sum_{i=1}^{n} (r_{ei} - c_i) \left\{ u_{lei} \right. \\
& \left. + 2u_{li} \left[cos\theta_{li} cos\theta_{bi} sin^2 \frac{\psi_{lei}}{2} + sin^2 \frac{\theta_{lei}}{2} \right] \right\} \\
& + \gamma_\gamma \sum_{i=1}^{n} \left\{ A_{xi} \left[u_{lei} cos\theta_{li}^D cos\psi_{li}^D \right. \right. \\
& - 2u_{li} \left(cos\psi_{li}^D sin \frac{\theta_{li}^D + \theta_{li}}{2} sin \frac{\theta_{lei}}{2} \right. \\
& \left. \left. + \ cos\theta_{li} sin \frac{\psi_{li}^D + \psi_{li}}{2} sin \frac{\psi_{lei}}{2} \right) \right] \\
& + A_{yi} \left[u_{lei} cos\theta_{li}^D sin\psi_{li}^D \right. \\
& - 2u_{li} \left(sin\psi_{li}^D sin \frac{\theta_{li}^D + \theta_{li}}{2} sin \frac{\theta_{lei}}{2} \right. \\
& \left. \left. - \ cos\theta_{li} cos \frac{\psi_{li}^D + \psi_{li}}{2} sin \frac{\psi_{lei}}{2} \right) \right] \\
& \left. - A_{zi} \left(u_{lei} sin\theta_{li}^D + 2u_{li} cos \frac{\theta_{li}^D + \theta_{li}}{2} sin \frac{\theta_{lei}}{2} \right) \right\} \\
& + \sum_{i=1}^{n} \left[\gamma_u u_{lei} (\dot{u}_{li}^D - \dot{u}_{li}) + \gamma_\theta \theta_{lei} (\dot{\theta}_{li}^D - \dot{\theta}_{li}) \right. \\
& \left. + \gamma_\psi \psi_{lei} (\dot{\psi}_{li}^D - \dot{\psi}_{li}) \right] + \Lambda_\gamma, \\
=& -\sum_{i=1}^{n} k_{Ri} (r_{ei} - c_i)^2 \\
& + \sum_{i=1}^{n} \left(\Lambda_{ui} u_{lei} + \Lambda_{\theta i} \theta_{lei} + \Lambda_{\psi i} \psi_{lei} \right) \\
& + \sum_{i=1}^{n} \left[\gamma_u u_{lei} (\dot{u}_{li}^D - f_{uli} - b_{ui} cos\theta_{ai} cos\psi_{ai} \tau_{ui}) \right. \\
& + \gamma_\theta \theta_{lei} (\dot{\theta}_{li}^D - \dot{\theta}_{ai} - \alpha_{qi} + e_{qi}) \\
& \left. + \gamma_\psi \psi_{lei} (\dot{\psi}_{li}^D - \dot{\psi}_{ai} - \alpha_{ri} + e_{ri}) \right] + \Lambda_\gamma, \qquad (18)
\end{aligned}$$

where $\alpha_{qi} = q_i + e_{qi}$ and $\alpha_{ri} = r_i + e_{ri}$ are stabilizing functions (Krstic et al. 1995) for q_i, and r_i, and

$$\begin{aligned}
\Lambda_{ui} =& \gamma_R (r_{ei} - c_i) + \gamma_\gamma \left(A_{xi} cos\theta_{li}^D cos\psi_{li}^D \right. \\
& \left. + \ A_{yi} cos\theta_{li}^D sin\psi_{li}^D - A_{zi} sin\theta_{li}^D \right), \\
\Lambda_{\theta i} =& \frac{sin\theta_{lei}/2}{\theta_{lei}/2} \left[\gamma_R u_{li} (r_{ei} - c_i) sin \frac{\theta_{lei}}{2} \right. \\
& - \gamma_\gamma u_{li} \left(A_{xi} cos\psi_{li}^D sin \frac{\theta_{li}^D + \theta_{li}}{2} + A_{zi} cos \frac{\theta_{li}^D + \theta_{li}}{2} \right. \\
& \left. \left. + \ A_{yi} sin\psi_{li}^D sin \frac{\theta_{li}^D + \theta_{li}}{2} \right) \right], \\
\Lambda_{\psi i} =& \frac{sin\psi_{lei}/2}{\psi_{lei}/2} \left[\gamma_R u_{li} (r_{ei} - c_i) cos\theta_{li} cos\theta_{bi} sin \frac{\psi_{lei}}{2} \right. \\
& - \gamma_\gamma u_{li} cos\theta_{li} \left(A_{xi} sin \frac{\psi_{li}^D + \psi_{li}}{2} \right. \\
& \left. \left. - A_{yi} cos \frac{\psi_{li}^D + \psi_{li}}{2} \right) \right].
\end{aligned}$$

According to (18), the control laws in *Step 1* are chosen as following

$$\tau_{ui} = b_{ui}^{-1} sec\theta_{ai} sec\psi_{ai} \left[\dot{u}_{li}^D - f_{uli} + \gamma_u^{-1} \left(k_{ui} u_{lei} + \Lambda_{ui} \right) \right], \qquad (19)$$

$$\alpha_{qi} = \dot{\theta}_{li}^D - \dot{\theta}_{ai} + \gamma_\theta^{-1} \left(k_{\theta i} \theta_{lei} + \Lambda_{\theta i} \right), \qquad (20)$$

$$\alpha_{ri} = \dot{\psi}_{li}^D - \dot{\psi}_{ai} + \gamma_\psi^{-1} \left(k_{\psi i} \psi_{lei} + \Lambda_{\psi i} \right), \qquad (21)$$

where $k_{ui}, \ k_{\theta i}, \ k_{\psi i} > 0$ are design parameters.

Remark 5. Since $u_i > 0$, it has $\theta_{ai}, \psi_{ai} \in (-\pi/2, \ \pi/2)$, from which we can guarantee that $cos\theta_{ai} cos\psi_{ai} > 0$.

Substituting (19)∼(21) into (18), we have

$$\begin{aligned}
\dot{V}_1 =& -\sum_{i=1}^{n} \left[k_{Ri} (r_{ei} - c_i)^2 + k_{ui} u_{lei}^2 + k_{\theta i} \theta_{lei}^2 + k_{\psi i} \psi_{lei}^2 \right. \\
& \left. + \gamma_\theta \theta_{lei} e_{qi} + \gamma_\psi \psi_{lei} e_{ri} \right] + \Lambda_\gamma. \qquad (22)
\end{aligned}$$

Step 2. Rewrite the vehicle's pitch and yaw dynamics in (12) as following

$$\dot{e}_{qi} = \dot{\alpha}_{qi} - f_{qi} - b_{qi} \tau_{qi}, \qquad (23)$$

$$\dot{e}_{ri} = \dot{\alpha}_{ri} - f_{ri} - b_{ri} \tau_{ri}. \qquad (24)$$

In this step, we consider the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2} \left(\gamma_q \sum_{i=1}^{n} e_{qi}^2 + \gamma_r \sum_{i=1}^{n} e_{ri}^2 \right), \qquad (25)$$

where $\gamma_q, \gamma_r > 0$ are design parameters.

Differentiating (25) and substituting (22 (24) into it, we get

$$\begin{aligned}
\dot{V}_2 =& -\sum_{i=1}^{n} \left[k_{Ri} (r_{ei} - c_i)^2 + k_{ui} u_{lei}^2 + k_{\theta i} \theta_{lei}^2 + k_{\psi i} \psi_{lei}^2 \right. \\
& + \gamma_\theta \theta_{lei} e_{qi} + \gamma_\psi \psi_{lei} e_{ri} + \gamma_q e_{qi} \left(\dot{\alpha}_{qi} - f_{qi} - b_{qi} \tau_{qi} \right) \\
& \left. + \gamma_r e_{ri} \left(\dot{\alpha}_{ri} - f_{ri} - b_{ri} \tau_{ri} \right) \right] + \Lambda_\gamma. \qquad (26)
\end{aligned}$$

According to (26), we choose the following control laws

$$\tau_{qi} = b_{qi}^{-1} \left(\dot{\alpha}_{qi} - f_{qi} + \gamma_q^{-1} \left(k_{qi} e_{qi} + \gamma_\theta \theta_{lei} \right) \right), \qquad (27)$$

$$\tau_{ri} = b_{ri}^{-1} \left(\dot{\alpha}_{ri} - f_{ri} + \gamma_r^{-1} \left(k_{ri} e_{ri} + \gamma_\psi \psi_{lei} \right) \right), \qquad (28)$$

where $k_{qi}, k_{ri} > 0$ are design parameters.

Substituting (27) and (28) into (26), have

$$\dot{V}_2 = -\sum_{i=1}^{n} \big[k_{Ri}(r_{ei} - c_i)^2 + k_{ui}u_{lei}^2 + k_{\theta i}\theta_{lei}^2 + k_{\psi i}\psi_{lei}^2 + k_{qi}e_{qi}^2 + k_{ri}e_{ri}^2\big] + \Lambda_\gamma. \quad (29)$$

After a certain period of time, all virtual leaders always move away from the obstacle(s) (*Assumption 2*). So, if the vehicles still follow their corresponding virtual leaders, we have $f_p(r_{qei}, a_\gamma, b_\gamma) = \partial f_P/\partial r_{qei} = 0$. Otherwise, if one or more vehicles are trapped by obstacles, the surface vessel will set corresponding u_{li}^D as zero (*Assumption 3*). As a result, after a certain period of time, we can always guarantee that $\Lambda_\gamma = 0$. Consequently, after that period of time, (26) can be rewritten as

$$\dot{V}_2 = -\sum_{i=1}^{n} \big[k_{Ri}(r_{ei} - c_i)^2 + k_{ui}u_{lei}^2 + k_{\theta i}\theta_{lei}^2 + k_{\psi i}\psi_{lei}^2 + k_{qi}e_{qi}^2 + k_{ri}e_{ri}^2\big] \leq -\lambda V_2, \quad (30)$$

where $\lambda :=$min$\{2k_{Ri}, 2k_{ui}\gamma_u^{-1}, 2k_{\theta i}\gamma_\theta^{-1}, 2k_{\psi i}\gamma_\psi^{-1}, 2k_{qi}\gamma_q^{-1}, 2k_{ri}\gamma_r^{-1}, i = 1, \cdots, n\}$.

Thoerem 1. Consider the formation control of a group of n underactuated AUVs, each of which has the same kinematics and dynamics as (1) and (2) with *Assumption 1∼3.* If the control laws are chosen as (19)∼(21), (27), and (28), then we can guarantee the exponential convergence of $\forall i, r_{ei} \to c_i, \theta_{li} \to \theta_{bi}, \psi_{li} \to \psi_{bi}$.

Remark 6. The design parameters $c_i > 0$ can be chosen arbitrarily small, so as for the AUVs formation also can be arbitrarily close to the given *virtual structure*.

Remark 7. For trapped vehicle(s) by obstacles, after being rescued, we can still control the vehicle(s) to exponentially follow the corresponding leader(s). As for how to rescue the trapped vehicle(s) is out of the scope of this paper.

5. NUMERICAL SIMULATION

In this section, some of simulation studies are carried out to demonstrate the effectiveness of proposed formation scheme for multiple underactuated AUVs. 6-DOF of REMUS AUV model (Prestero, 2001) is applied in the simulation. It is worth to mention that in the simulation, the vehicle's maximum stern and rudder angles are set to 20 degrees, and there is not any restriction added to the vehicle's surge force.

In the simulation, we consider the 4 vehicles following a virtual structure which is chosen as $\forall i \in \{1, \cdots, 4\}$, $\dot{x}_{di}$=$u_{ld}cos\theta_{ld}cos\psi_{ld}$, $\dot{y}_{di}$=$u_{ld}cos\theta_{ld}sin\psi_{ld}$, $\dot{z}_{di}$=-$u_{ld}sin\theta_{ld}$ with u_{ld}=3m/s, and *if* 0< t <120, θ_{ld}=$\psi_{ld} = 0$, *else*, θ_{ld}=-15deg., $\dot{\psi}_{ld}$=-3/50rad/s. Virtual leaders initial position is set to [50, 30, 18; 50, 0, 1; 65, 0, 27; 35, 0, 27], and the vehicle's initial condition is $X(0)$=[55, 1, 1, 0, 0, $\pi/2$, 0.5, 0, 0, 0, 0, 0; 35, 1, 1, 0, 0, $\pi/2$, 0.5, 0, 0, 0, 0, 0; 95, 1, 1, 0, 0, $\pi/2$, 0.5, 0, 0, 0, 0, 0; 5, 1, 1, 0, 0, $\pi/2$, 0.5, 0, 0, 0, 0, 0] with $X(i, :)$=[x, y, z, ϕ, θ, ψ, u, v, w, p, q, r]. The design parameters are selected as that: $\forall i \in \{1, \cdots, 4\}$, k_{Ri}=1.7, k_{ui}=425, $k_{\theta i}$=$k_{\psi i}$=255, k_{qi}=k_{ri}=42.5, c_i=3, γ_R=50, γ_u=500, γ_θ=γ_ψ=125, γ_γ=0.015, γ_q=γ_r=10. For

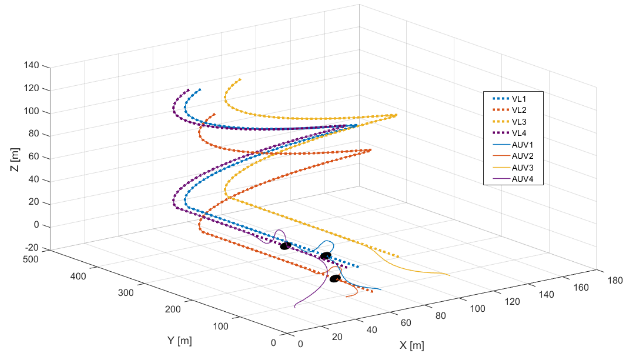

Fig. 1. Virtual structure trajectory and the vehicles formation following with obstacle avoidance.

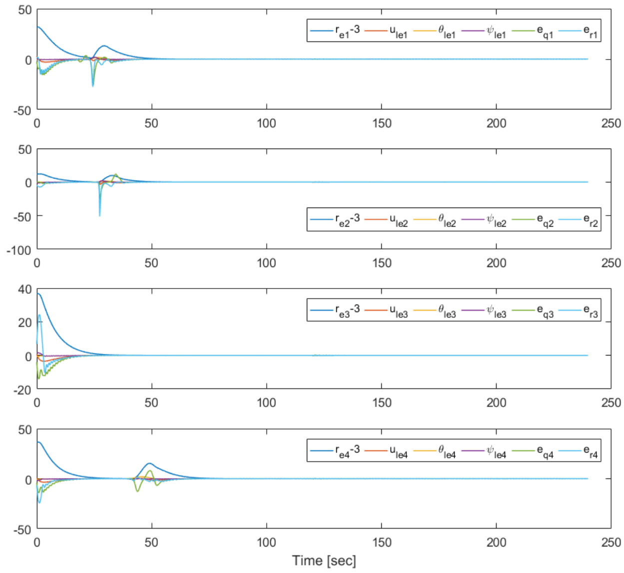

Fig. 2. Exponential convergence of the vehicles' path following errors.

the smooth potential function $f_p(\xi, a_\gamma, b_\gamma)$ in (14), the parameters are set as a_γ=b_γ=16, c_γ=2, and h=0.99.

Simulation results are shown in Fig. 1∼4. Fig. 1 shows the virtual structure trajectory and the vehicles path following with obstacle avoidance. And corresponding path following errors are shown in Fig. 2, from which we can see all these following errors exponentially converge to zero as mentioned in the previous section. In the case of path following, main concern is how to force the range error to converge to the arbitrarily desired value, and the vehicle's desired attitude is directly taken as the orientation angle from the vehicle to the target point such as (θ_{li}^D, ψ_{li}^D)=(θ_{bi}, ψ_{bi}). However, as in the case of trajectory tracking, especially in the path tracking case, the vehicle's motion (u_{li}, θ_{li}, ψ_{li}) is force to exactly track the reference path (u_{ldi}, θ_{ldi}, ψ_{ldi}). So it is interest to investigate the tendency of (θ_{li}^D, ψ_{li}^D)=(θ_{bi}, ψ_{bi}) with the proposed formation scheme in this paper. Fig. 3 shows the convergence of (θ_{li}^D, ψ_{li}^D)=(θ_{bi}, ψ_{bi}) in both cases of the straight and screw lines. And very interestingly, we can see that all vehicles' motions (u_{li}, θ_{li}, ψ_{li}) converge to the given virtual leaders' reference pathes in the spherical coordinate frame. In addition, we also investigate the

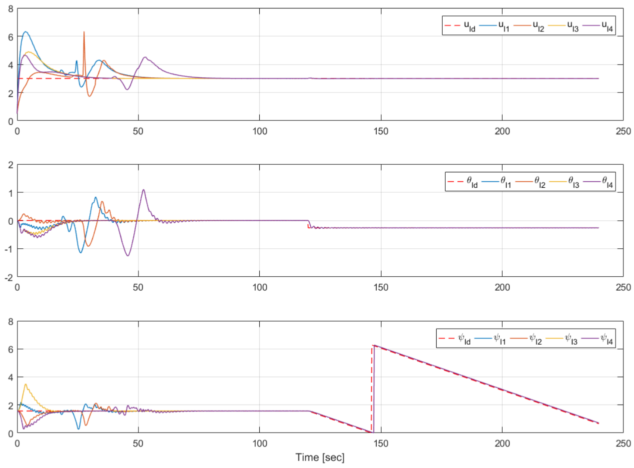

Fig. 3. Convergence of (u_{li}, θ_{li}, ψ_{li}) to (u_{ld}, θ_{ld}, ψ_{ld}).

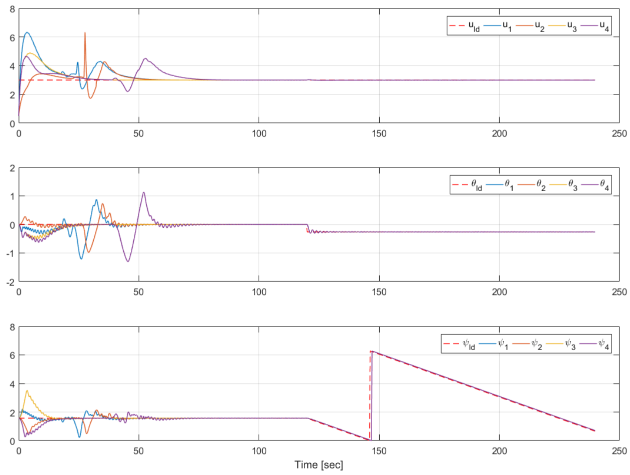

Fig. 4. Convergence of (u_i, θ_i, ψ_i) to (u_{ld}, θ_{ld}, ψ_{ld}).

vehicles' following motions in the cartesian frame, and find that all (u_i, θ_i, ψ_i) have the very similar converge tendency with the given (u_{ld}, θ_{ld}, ψ_{ld}) as shown in Fig. 4.

6. CONCLUSION

In this paper, we have presented a virtual structure based formation control method for a group of underactuated underwater vehicles. Formation potential for each vehicle consists of two parts, one is for virtual leader following and the other one is to obstacle avoidance. According to the underactuated mechanical restricting conditions, for each vehicle's motion control, we have proposed a sort of path following scheme to force the vehicle to follow the given trajectory. Proposed formation scheme can guarantee the exponential convergence of all of following errors in the spherical coordinate frame. Through simulation studies, we have found out that the presented path following method also has the similar tracking performance with the path tracking scheme.

REFERENCES

Cui, R., Ge, S.S., How, B.V., Choo, Y.S. Leader-follower formation control of underactuated autonomous underwater vehicles, *Ocean Engineering*, Vol. 37, Pages 1491–1502, 2010.

Do, K.D. Bounded controller for formation stabilization of mobile agents with limited sensing ranges, *IEEE Transactions on Automatic Control*, Vol. 52, pages 569–576, 2007.

Dunbar, W.B. and Murray, R.M. Model predictive control of coordinated multi-vehicle formations, *Proc. of the 41th IEEE CDC*, pages 4631–4636, Las Vegas, Nevada, December 2002.

Fax, J.A. and Murray, R.M. Information flow and cooperative control of vehicle formation, *IEEE Transactions on Automatic Control*, Vol. 49, pages 1465–1476, 2004.

Fiorelli, E., Leonard, N.E., Bhatta, P., Paley, A., Bachmayer, R., Fatanttoni, D.M. Multi-AUV control and adaptive sampling in Monterey Bay, *IEEE Journal of Oceanic Engineering*, Vol. 31, pages 935–948, 2006.

Fossen, T.I. *Handbook of Marine Craft Hydrodynamics and Motion Control*. John Wiley & Sons. Ltd, 2011.

Gao, Z. and Guo, G. Velocity free leader-follower formation control for autonomous underwater vehicles with line-of-sight range and angle constraints, *Information Sciences* Vol. 486, pages 359–378, 2019.

Krstic, M., Kanellakopoulos, I., Kokotovic, P. *Nonlinear and Adaptive Control Design*. John Wiley & Sons Inc., New York, 1995.

Leonard, N.E. and Fiorelli, E. Virtual leaders, artificial potentials and coordinated control of groups, *Proc. of the 40th IEEE CDC*, pages 2968–2973, Orlando, Florida, December 2001.

Li, J., Du, J., Chang, W.J. Robust time-varying formation control for underactuated autonomous underwater vehicles with disturbances under input saturation, *Ocean Engineering*, Vol. 179, pages 180–188, 2019.

Li, J.H. Path tracking of underactuated ships with general form of dynamics, *International Journal of Control*, Vol. 89, pages 506–517, 2016.

Li, J.H. and Lee, P.M. Stable Schooling for Multiple Underactuated AUVs, *Proc. of 17th IFAC World Congress*, pages 15022–15027, Seoul, Korea, July 2008.

Li, J.H., Jun, B.H., Lee, P.M., Lim, Y.K. Schooling for Multiple Underwater AUVs, *Underwater Vehicles*, In-Tech, Vienna, Austria, pages 295–314, 2009.

Li, J.H., Kang, H., Hong, S.M., Suh, J.H., Li, S. 3D path following control method for torpedo-type AUVs with uncertainty terms in their dynamics, *Proc. of 2016 IEEE International Conference on Robotics and Biomimetics*, pages 1407-1412, Qingdao, China, 2016.

Milne, P.H. *Underwater Acoustic Positioning Systems*. Gulf Publishing Company, 1983.

Oh, K.K., Park, M.C., Ahn, H.S. A survey of multi-agent formation control, *Automatica*, Vol. 53, pages 424–440, 2015.

Olfati-Saber, R. and Murray, R.M. Distributed cooperative control of multiple vehicle formations using structural potential functions, *Proc. of 15th IFAC World Congress*, pages 495-500, Barcelona, Spain, July 2002.

Prestero, T. *Verification of a six-degree of freedom simulation model for the REMUS autonomous underwater vehicles*. Masters Thesis, Department of Ocean Engineering and Mechanical Engineering, MIT, 2001.

Reynolds, C.W. Flocks, herds, and schools : A distributed behavioral model, *Proc. of ACM SIGGRAPH'87*, pages 25–34, Anaheim, California, July 1987.