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Computationally Efficient Model Predictive Control for Real Time Implementation experimentally applied on a Hydraulic Differential Cylinder

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Abstract: In the field of model predictive control (MPC), the computational effort of solving the optimization problem considering long prediction horizon is often challenging, making the implementation in real time infeasible. On the other side, a longer prediction horizon leads to better control performance. Therefore, a trade off between computational effort and control performance should accurately be achieved for the selection of prediction horizons within model predictive algorithms. This paper presents a new scheme for model predictive control (structured MPC) considering a fixed set of rules for assumed control structures simplifying the optimization problem and therefore reducing the computational effort of MPC while maintaining advantages of adaptive and optimal behavior due to MPC in combination with evaluation using prediction horizons. The suitable control input is defined considering a performance measure. Experimental results using a hydraulic differential cylinder test rig validate the advantages of the introduced approach for real time application of MPC in comparison to standard controllers.

1. INTRODUCTION AND PROBLEM DEFINITION

Model predictive control (MPC) is an advanced control technique which has been applied successfully in many practical applications. This controller type uses an optimizer for the optimal control evaluating over a future time horizon based upon a mathematical or input-output model of the process. Model predictive control has influenced the directions of development of industrial control systems (Yan and Wang (2012)).
If the system is linear and can be represented by a linear model, the constrains are linear and the cost function is a quadratic cost function, then linear time-invariant MPC can be considered as an adequate controller. In this case a convex optimization problem has to be solved and the cost function has a single global optimum. If the system to be controlled is a nonlinear system but can be approximated by linear models, adaptive and gain-scheduled MPC controllers can be used. Both controllers work based on multiple linearized model, each representing the nonlinear function around its operating point.
The adaptive MPC uses the updated internal system model at each time step (Kim (2010)). It is worth mentioning that adaptive MPC approaches are used when the structure of optimization problem is not changed across different operating conditions over the prediction horizon. Otherwise gain-scheduled MPC should be used. In gain-scheduled MPC the system model is linearized at the operating points of interest. For each operating point a linear MPC controller is designed off-line (Chisci et al. (2003)) considering the optimization procedure. Each controller operates independent from the other ones and may have different number of constrains. It is noteworthy that in this approach an algorithm should be designed to switch between the predefined MPC controllers for different operating conditions. Although, using of independent MPC controller approaches is an advantage of gain-scheduled MPC. This approach requires more computational memory than adaptive MPC.
Model predictive control is typically characterized by a high calculation effort which significantly depends on the optimization algorithm. Determination of optimum control parameters in real time is the core of MPC approach. Suitable selection of the prediction horizon is required to ensure the suitable performance in steady state. However, the computational effort of solving the optimization problem considering prediction procedure is often challenging, making the implementation in real time impractical because the optimization must be solved at each time step. Therefore, a trade off between computational complexity and performance should accurately be done for the selection of prediction horizon length.
Several methodologies are proposed to overcome the high computational effort of optimization procedure focusing on real-time applications. One of the proposed approach is denoted as 'move blocking strategy' (Cagienard et al. (2007)) which tries to divide the prediction horizon to two parts, the first interval with a small sample time and the second one with a large sample time, considering that high-resolution sampling of the plant is mainly required near the present time step. The second methodology is based on extrapolation (Geyer et al. (2011)) while the third one

is based on event-based horizon (Geyer and Mastellone (2012)). In (Ma et al. (2011)) a periodic moving window blocking strategy is used to reduce the computational time for constrained optimization problem for the operation of building cooling systems although the process is slow itself. Accordingly, most of the real time application of MPC is related to the slow process with limited dynamics. Considering systems with slow dynamics, the well-known MPC with high computational load can be successfully implemented in real time. In the case of fast dynamical behavior implementation of MPC will be challenging. In addition to all aforementioned approaches to run MPC faster, several methods are proposed for example model order reduction, shorter prediction and control horizons, and reduced number of constrains. For fast application with a very small sample time, the explicit MPC can be used which reduces the run time by precomputing the optimal solution off-line (Bumroongsri and Kheawhom (2012)). Another option to guarantee the worst-case execution time of MPC controller is to use suboptimal solution for optimization procedure (Zeilinger et al. (2011)).

Hydraulic cylinders are broadly used in different industrial aspects, e.g., heavy machines, cranes, and robots. The motion behavior of hydraulic differential cylinders can be defined by nonlinear differential equations with strong nonlinear behavior (Jelali and Kroll (2012)). Motion control of power trains, drive trains, or even actuators has been in the focus of several scientific and industrial efforts of the last decade. Advanced control approaches are based on knowledge (models resp.) about the system to be controlled, realized by mathematical models (e.g., sets of differential equations). To control such a system the fast dynamics of the system has to be considered in design procedure. The controller should be able to realize the required input in real time.

In the context of this paper, the approach of MPC is modified: firstly a fixed controller scheme is selected and secondly the optimization task is simplified to significantly reduce the computational effort. A more detailed description in combination with simulation results is given in (Bakhshande et al. (2019)). Main purpose of this contribution is to reduce the computational effort and complexity of MPC to make the implementation in real time possible. A hydraulic differential test rig is considered to validate the advantages of proposed approach.

This paper is organized as follows. In Section 2 the standard MPC considering different optimization methodologies is introduced. In Section 3, the new MPC control strategy is outlined in detail. In Section 4 the hydraulic cylinder system used for validation of the proposed approach is introduced. In Section 5 the proposed approach is implemented on the hydraulic system to evaluate the well-known and proposed approaches, individual effects, and obtainable results. Finally, conclusions are drawn in Section 6.

2. PRELIMINARIES

2.1 Model predictive control (MPC)

Although a number of different model predictive control algorithms have been developed over the years, the main idea (i.e., the explicit application of a process model, the receding horizon and optimization of a cost function) is always the same (Ławryńczuk (2007)). Accordingly the cost function

$$J(k) = \sum_{r=1}^{N2} ||\boldsymbol{y}^{ref}(k+r|k) - \boldsymbol{y}(k+r|k)||^2 + \rho \sum_{r=0}^{N_u-1} ||\Delta\boldsymbol{u}(k+r|k)||^2, \tag{1}$$

is minimized by an optimization algorithm considering input vector $\boldsymbol{u}(k)$, prediction horizon N_2, and control horizon N_u. Equation (1) is the algebraical description of the core idea of MPC. In every time instant k, the cost function J has to be minimized. All components of this function will be introduced in the sequel.

For each sampling instant k, a set of values which is going to be controlled in a certain future horizon, exists. This horizon is called control horizon, described with symbol N_u. The expression of controlled variable $\boldsymbol{u}(k)$, also called control signal, in control horizon N_u is

$$\boldsymbol{u}(k) = \begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+N_u-1|k) \end{bmatrix}. \tag{2}$$

Accordingly

$$\Delta\boldsymbol{u}(k) = \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+N_u-1|k) \end{bmatrix} \tag{3}$$

$$= \begin{bmatrix} u(k|k) - u(k-1|k) \\ u(k+1|k) - u(k|k) \\ \vdots \\ u(k+N_u-1|k) - u(k+N_u-2|k) \end{bmatrix}, \tag{4}$$

is calculated to minimize the differences between the predicted values of the outputs $\boldsymbol{y}$ (or states) and the reference trajectory $\boldsymbol{y}^{ref}$ over the prediction horizon as shown (1).

The $\rho > 0$ value denoted in (1) bounds the changing rate of inputs $\Delta\boldsymbol{u}$ directly. By increasing the ρ value the increment of input value is decreased and consequently the controller becomes slower.

In (1), N_2 denotes the predictive horizon, in which the future outputs are predicted using system model while

$$\boldsymbol{y}(k) = \begin{bmatrix} y(k|k) \\ y(k+1|k) \\ \vdots \\ y(k+N_2-1|k) \end{bmatrix}, \tag{5}$$

denotes the prediction of the outputs for the future sampling calculated at the current sampling instant k using a dynamic model of the process.

Note that the prediction horizon N_2 is always larger than the control horizon N_u. Consequently, the predicted output vector

$$[\boldsymbol{y}(k+1|k),\ \boldsymbol{y}(k+2|k),\ ...,\ \boldsymbol{y}(k+N_u|k)]^T, \tag{6}$$

is generated by input vector

$$[\boldsymbol{u}(k|k),\ \boldsymbol{u}(k+1|k),\ ...,\ \boldsymbol{u}(k+N_u-1|k)]^T, \tag{7}$$

in the control horizon. In the time interval from $k+N_u$ to $k+N_2-1$, inputs are considered without changes, which means outputs from $\boldsymbol{y}(k+N_u+1|k)$ to $\boldsymbol{y}(k+N_2|k)$ in time interval $[k+N_u+1,\ k+N_2]$ are all evaluated by $\boldsymbol{u}(k+N_u-1|k)$ based on the prediction results.
The main idea of MPC can be concluded as the minimization of difference between future output and reference trajectory using the prediction results. To achieve this objective, the optimization algorithm plays a very important role.
For every continuous instant k, the present plant output $y(k|k)$ is sent to the controller and replaced by the predicted output by system model in last step $y(k|k-1)$ and then stored in controller as time delay value. Afterwards the optimized input set will be founded by an optimization algorithm (e.g. genetic algorithm). More specifically, genetic algorithm generates a population of input sets and uses system model to evaluate the predicted outputs, then calculates the cost function separately and selects the input set which leads to the minimum J value.

2.2 OPTIMIZATION

Optimization is one of the most essential part in MPC. Most algorithms like gradient descent, are looking along gradient descent line to find the local minimum. To use this kind of line searching methods, the Jacobi and Hessian matrix of cost function should be determined at first (Ławryńczuk (2007)). However, evaluating these two matrices is a challenging task for a neural network-based MIMO system.
In computer science and operations research, a genetic algorithm (GA) is a meta-heuristic inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (EA). Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover, and selection. Instead of finding local minimum in line searching algorithm, genetic algorithm is able to find the global minimum (Dasgupta and Michalewicz (2013)).
Beside genetic algorithm there are various optimization approaches like particle swarm optimization, pattern search optimization, Levenberg–Marquardt algorithm, nonlinear constrained optimization, etc. The computational effort and complexity of MPC is related to the computational time of optimization procedure. According to the available literature and to the best of authors knowledge the MPC approach is primarily used in the process industry (Qin and Badgwell (2003)) where the computational effort is not a main challenge because the process is slow itself and calculation of the optimal solution can be done in real time.

3. STRUCTURED MPC: NEW DESIGN FOR REAL-TIME APPLICATION OF MPC APPROACH

As mentioned before, MPC implementation is computationally demanding because prediction in combination with online optimization is required for improved performance of the control. From the other side, calculation effort for solving the optimization problem is significantly determined by the size of prediction horizon. In this section, a new strategy is proposed to achieve long prediction horizons well within achievable levels of computational effort.

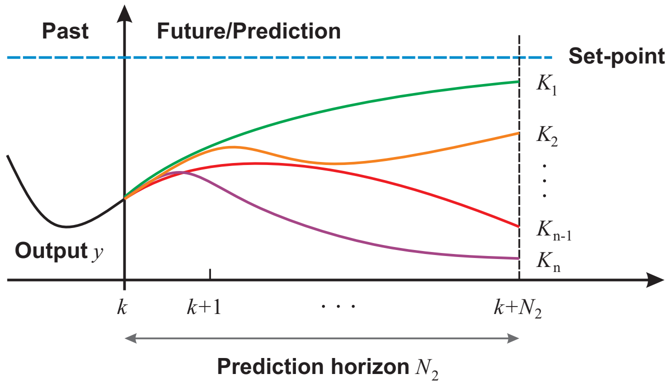

Fig. 1. Schematic diagram of the selection approach at time step k

In the context of this contribution, the approach of direct adaptive control is used to reduce the computational effort according to the objectives. The system model is used within adaptation mechanism to evaluate the suitability of control parameters and to identify the best possible value. Consequently, the time consuming optimization of cost function is replaced by a set of control rules (according to a given and therefore fixed controller structure) and selection of control parameters is done considering the MPC procedure. In the proposed structured MPC no explicitly integrated optimization is required.
The main idea is to use output feedback control in combination with MPC approach. The schematic diagram of the selection approach at the time step k is shown in figure 1. The future output values of the system is calculated over a prediction horizon by using the mathematical model and considering different input values $u(t)$ based on the output feedback control

$$u_i = K_i(y(k) - y^{ref}(k)), \tag{8}$$

with gain K_i as a proportional controller. The mean square error (MSE) is calculated to evaluate the control performance over the prediction horizon and select the best K_i as

$$MSE(k) = \frac{1}{N_2}\Sigma_{r=1}^{N_2}(y(k+r|k) - y^{ref}(k+r|k))^2, \tag{9}$$

with prediction horizon N_2. The aim of the proposed structured MPC approach is to find the best predicted path closest to the reference trajectory considering the control structure (8). Therefore, minimum MSE considering different K_i is selected at this step. The corresponding input is used for control and all steps above are repeated.

4. MODEL OF A HYDRAULIC DIFFERENTIAL CYLINDER

In this section the hydraulic cylinder test rig is introduced according to the previous publication (Bakhshande et al. (2020)). A model of a hydraulic differential cylinder with a proportional control valve as shown in Figure 2 (Jelali and Kroll (2012)) is given by

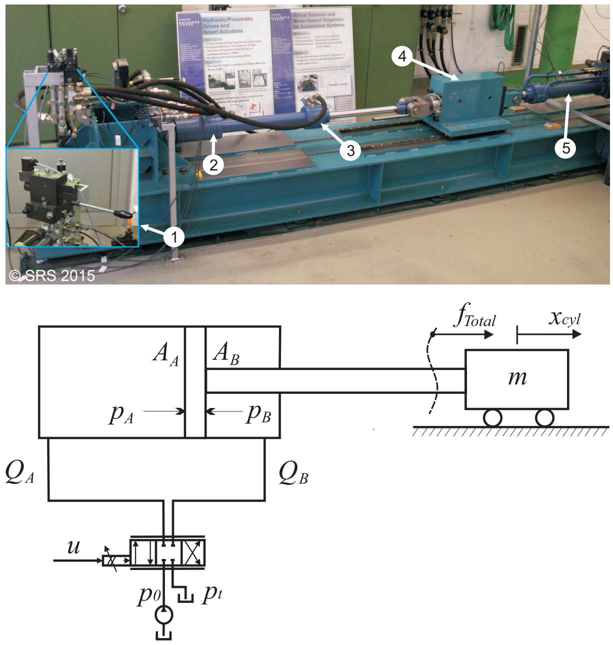

Fig. 2. Sketch and test rig of hydraulic differential cylinder system at the Chair of Dynamics and Control (UDuE), (1) proportional directional control valve, (2) oil supply in chamber A, (3) oil supply in chamber B, (4) moving mass, and (5) load cylinder

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} x_2 \\ \dfrac{1}{m(x_1)}\left[(x_3 - \dfrac{x_4}{\varphi})A_A\right] \\ \dfrac{E_{oil}(x_3)}{V_A(x_1)}(-A_A x_2) \\ \dfrac{E_{oil}(x_4)}{V_B(x_1)}\left(\dfrac{A_A}{\varphi}x_2\right) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{E_{oil}(x_3)}{V_A(x_1)}Q_A(x_3) \\ \dfrac{E_{oil}(x_4)}{V_B(x_1)}Q_B(x_4) \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ d(t) \\ 0 \\ 0 \end{bmatrix}, \tag{10}$$

$$= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u(t) + \boldsymbol{d}(t),$$

$$y(t) = h(\boldsymbol{x}) = x_1(t),$$

with the variant mass

$$m(x_1) = m_{basic} + \rho_{fl}(V_A(x_1) + V_B(x_1)), \tag{11}$$

the volumes V_A, V_B in chambers A and B as

$$V_A(x_1(t)) = V_{cA} + x_1(t)A_A,$$
$$V_B(x_1(t)) = V_{cB} + (H - x_1(t))A_B,\ 0 \le x_1(t) \le H,$$

the disturbance $d(t)$ as

$$d(t) = \frac{f_{Total}(t)}{m(x_1)}, \tag{12}$$

the hydraulic flows

$$Q_A(x_3(t)) = \begin{cases} B_\nu sgn(p_0 - x_3(t))\sqrt{|\, p_0 - x_3(t)\,|}, & u \ge 0 \\ B_\nu sgn(x_3(t) - p_t)\sqrt{|\, x_3(t) - p_t\,|}, & u < 0 \end{cases}$$

$$Q_B(x_4(t)) = \begin{cases} -B_\nu sgn(x_4(t) - p_t)\sqrt{|\, x_4(t) - p_t\,|}, & u \ge 0 \\ -B_\nu sgn(p_0 - x_4(t))\sqrt{|\, p_0 - x_4(t)\,|}, & u < 0 \end{cases}$$

with

$$B_\nu = \frac{Q_N}{\sqrt{0.5\Delta p_N}}, \tag{13}$$

and the bulk modulus of elasticity

$$E_{oil}(p) = \frac{1}{2}E_{oil,max}\log_{10}(90\frac{p}{p_{max}} + 3).$$

The input $u(t)$ is the electrical current which is limited as $-I_{max} \le u(t) \le I_{max}$. The flow characteristic of the valve is assumed to be proportional. No internal and external leakage effects are considered. The friction of spool, piston, and cart are neglected in the modeling of cylinder valve (Jelali and Kroll (2012)). The variables and constants are defined in Table 1.

Table 1. Definition of parameters and variables

Variable	Physical meaning	Value (Unit)
$x_1(t) = x_{cyl}(t)$	Displacement of the mass cart	- (m)
$x_2(t) = \dot{x}_{cyl}(t)$	Velocity of the mass cart	- (m/s)
$x_3(t) = p_A(t)$	Pressure in chamber A	- (pa)
$x_4(t) = p_B(t)$	Pressure in chamber B	- (pa)
$f_d(t)$	External force acting on the piston	- (N)
m_{basic}	Basic mass of the cart	279.6 (kg)
ρ_{fl}	Density of the hydraulic oil	870 (kg/mm^3)
p_0	Supply pressure	8×10^6 (pa)
p_t	Tank pressure	5×10^5 (pa)
A_A	Cylinder piston area	3117.2 (mm^2)
A_B	Cylinder ring area	1526.8 (mm^2)
$\varphi = \frac{A_A}{A_B}$	Area ratio	2.042 (-)
$E_{oil,max}$	Max. bulk modulus of elasticity	1.8×10^9 (pa)
p_{max}	Max. supply pressure	2.8×10^7 (pa)
V_{cA}	Pipeline and dead volume (A)	198.6 (cm^3)
V_{cB}	Pipeline and dead volume (B)	297.8 (cm^3)
H	Stroke of the cylinder	0.5 (m)
I_{max}	Max. input current	0.63 (A)
Q_N	Nominal valve flow	85 (L/min)
Δp_N	Pressure drop of valve	9×10^5 (pa)

5. RESULTS AND DISCUSSION

To evaluate the performance of proposed approach a criterion

$$C_{criteria} = [\int_0^T e^2(t)dt, \int_0^T u^2(t)dt], \tag{14}$$

considering the input energy $\int_0^T u^2(t)dt$ (Integral Square Input (ISI)) and control error $\int_0^T e^2(t)dt$ (Integral Square

Table 2. Experimental conditions and parameter selection considered for evaluation of the proposed structured MPC approach

	Gain vector	Supply pressure
Case I	$K_1 = [1, 5, 10, 15]$	8×10^6 pa
Case II	$K_2 = [10, 15, 20, 25]$	8×10^6 pa
Case III	$K_3 = [10, 15, 20, 25, 30]$	8×10^6 pa
Case IV	$K_2 = [10, 15, 20, 25]$	4×10^6 pa

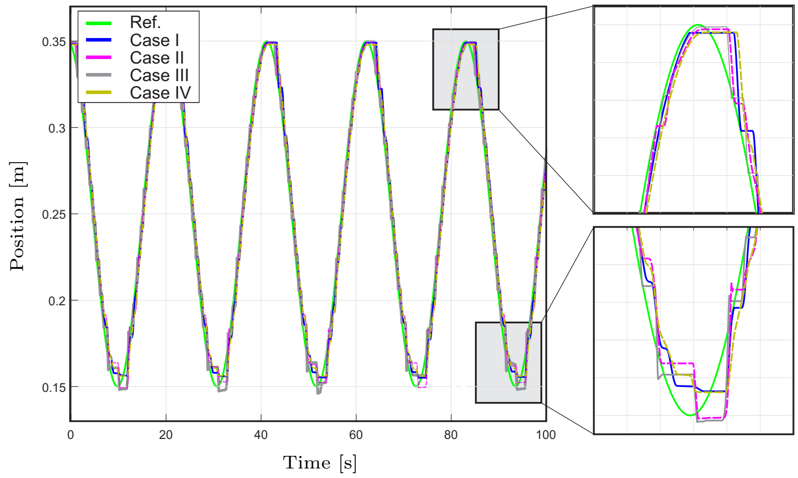

Fig. 3. Position control of hydraulic differential cylinder using structured MPC approach and considering different cases (sinusoidal signal as reference signal)

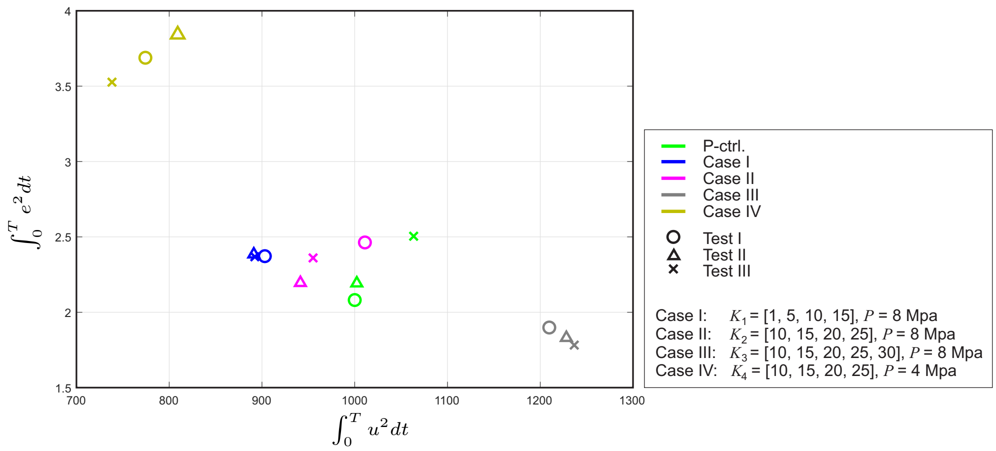

Fig. 4. Comparison of the proposed structured MPC with standard P-controller (The experiment is done and repeated three times (Test I-III) considering same conditions and gains for each controller.)

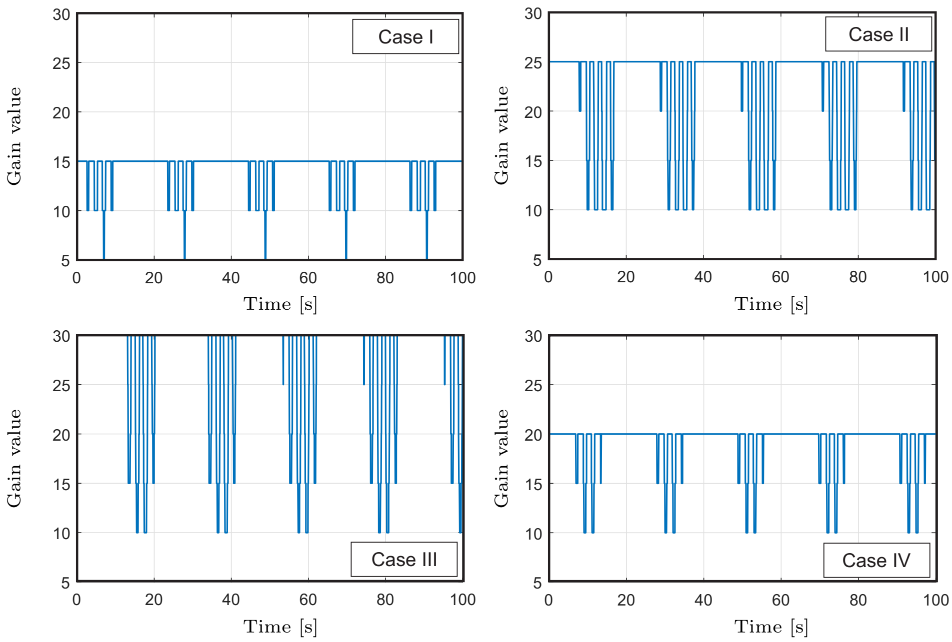

Fig. 5. Gain values selected by the proposed structured MPC approach for Cases I-IV

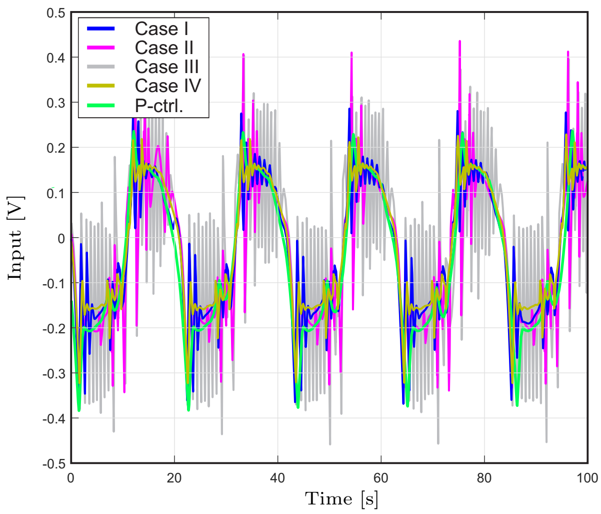

Fig. 6. Input signal for the hydraulic differential cylinder using structured MPC approach and considering different cases (sinusoidal signal as reference signal).

Error (ISE)) is used. The interval length T denotes the time window where the performance is compared. With the same input energy, the result which has lower output error or correspondingly with the same output error which uses less input energy (closer to the origin) provides better performance.

Different cases are considered (Table 2) to compare the effects of gain selection and experimental conditions on the performance of proposed structured MPC. In 'Case I' a gain vector is selected based on the working point and physical limitations regarding hydraulic cylinder. In 'Case II' the gain vector is weighted and increased to see the effects on control performance. In 'Case III' the number of gains are increased. Finally, in 'Case IV' the gain is considered as 'Case II' and the supply pressure is decreased to see the effects of experimental conditions. In figure 3 the tracking results are illustrated for a sinusoidal signal as reference. Gain values selected by the proposed structured MPC approach for Cases I-IV are shown in figure 5. The input signal is shown in figure 6 for all cases. It is evident that the input signal related to Case III is more aggressive compared to other cases.

Criteria (14) is calculated and shown in figure 4 for all cases. For each controller the experiment is done and repeated three times (Test I-III) considering same conditions and gains. Furthermore, the results related to standard P-controller is illustrated. From the results it can be concluded that the proposed structured MPC approach is not only applicable in real time but also produces better tracking error compared with standard approaches (P-Controller) when the vector gain is designed in a suitable way (Case III).

6. CONCLUSIONS

This paper proposes a new MPC approach to reduce the computational effort of MPC for implementation in real time considering systems with fast dynamical behavior. The proposed structured MPC is implemented on a hydraulic differential cylinder test rig considering a non-constant reference value (setpoint). Experimental results show the ability of this controller to achieve suitable tracking performance besides using the advantages of MPC.

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