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Control of an automated wheelchair

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Abstract: Due to the world's aging population, the development of affordable and easy to use wheelchairs is becoming a priority. In this study, the control of an automated wheelchair is proposed. The model equations are derived from the Euler-Lagrange equations, then a descriptor model is formulated. Next, a Takagi-Sugeno descriptor model with a limited number of rules is derived. The control and observation of the model is studied using the delayed non-quadratic Lyapunov function. The closed loop stability is proven using the separation theorem. Lastly, simulation results are given and discussed.

Keywords: *Wheelchair, Takagi Sugeno, Descriptor, Control, non-quadratic delayed Lyapunov function.*

1 INTRODUCTION

According to the World Health Organization, 10% of the world's population has disabilities and 10% of them or 65 million people use a wheelchair (World Health Organization, 2010). A manual wheelchair has two large wheels the user pushes for propulsion and two casters wheels in front for stability. Thanks to the recent advance in the electronics it is possible to transform a manual wheelchair into an electric wheelchair at a reasonable cost. This transformation is realized by replacing the original push wheels with motorized ones. The hubs in the original wheels are replaced by electric motors that are powered by a battery pack, Figure 1. The new motors will propel the wheelchair reducing fatigue of the user. Such assistance kits already exist and Autonomad Mobility, partner of the project, produces and sells such kits under the patent 10,252,638 (United States Patent No. US10252638B2, 2019).

In some situations, the caster wheels will obstruct the wheelchair from passing over small obstacles or holes, a solution to this problem is to balance the wheelchair on its larger wheels so that the caster wheels will no longer obstruct the wheelchair. The purpose of this paper is to develop a control law that will swing the wheelchair from the grounded position to a self-balancing position at equilibrium.

The dynamic model of the wheelchair can be easily derived using Euler Lagrange equations. In order to derive a model suitable for the control design, the resulting dynamic model is discretized. Takagi Sugeno models represent the dynamics of a nonlinear systems over a compact set of the systems state space (Tanaka & Wang, 2004). Basically, it consists of linear sub-models blended using nonlinear membership function. Then, the wheelchair model is formulated as non-linear descriptor model. Since some of the states in the state vector cannot be measured, an observer is needed. The system stabilisation and observation are then studied using parameter depending Lyapunov functions (Ding, Sun, & Yang, 2006; T. M. Guerra & Vermeiren, 2004) (Lee, Park, & Joo, 2011) in a delayed framework (Lendek, Guerra, & Lauber, 2015). A separation principle given is directly issued from the classical approach (Yoneyama, Nishikawa, Katayama, & Ichikawa, 2001) and allows the observer and controller gains to be computed separately while ensuring the overall closed loop stability.

Figure 1: Self-balancing wheelchair

2 WHEELCHAIR MODELLING

In order to derive the equations of the wheelchair dynamics, it is suggested to lump the wheelchair and the body into a single moving pendulum as depicted in Figure 2. The pendulum angle with respect to the vertical axis is denoted by ψ and the position of the pendulum pivot by x. The wheel angular position is $\vartheta = r^{-1}x$ with r the wheel radius.

$\vartheta_e = xr^{-1} - \psi$ is the relative position between the wheel and the chassis. It is measured by an encoder.

The overall objective is to control the pendulum in the upper equilibrium position $\psi = 0$. For the wheelchair under consideration the behaviour of the wheels with the embedded motors powered by the battery pack are assumed identical. The control unit, located at one of the armrests, contains an inertial measurement unit (3 accelerometers and 3 gyroscopes). Thanks to the anti-tippers and the caster wheels, ψ is restricted between -0.4 and $0.4\, rd$.

The dynamics of the system are defined from mechanical equations derived using Lagrange Mechanics. Table 1 describes all the system parameters.

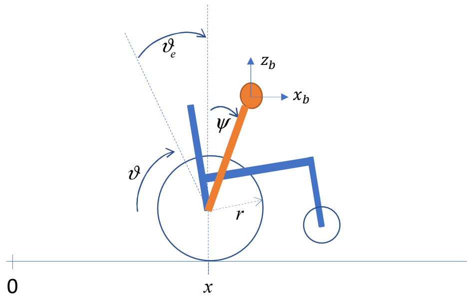

Figure 2. Schematic of the wheelchair. The considered pendulum is depicted in orange.

Table 1 : Models parameters

Parameter	Description
M_b	Mass of the wheelchair frame and human body
J_b	Inertia of the wheelchair frame and human body (rotating about the wheel axel)
M_w	Mass of each wheel
J_w	Inertia of each wheel (rotating about the wheel axis)
r	Radius of each wheel
g	Acceleration of gravity
l	Distance to the center of mass from wheel axel
μ_{wg}	Viscous friction between wheel & ground
μ_m	Viscous friction in the motor (gears & bearings)
K_t	Motor torque constant

The wheelchair is comprised of two subsystems: (i) The wheel and axis assembly and (ii) the pendulum which is composed of all the parts rotating around the wheel axel. For each subsystem $i \in \{1,2\}$, the kinetic energy is denoted by T_i, the potential energy by V_i, the dissipated power by P_i and the generalised force is γ_i. The generalised coordinate vector is $q = (x, \psi)$.

Under the no-slip assumption, the wheel angle is related to the wheelchair position $x = r\vartheta$. The body position (x_b, z_b) is given as $x_b = x + l\sin(\psi)$, $z_b = l\cos(\psi)$. The two sub-system energy and power are then computed:

$$T_1 = 2\left(\tfrac{1}{2} M_w \dot{x}^2 + \tfrac{1}{2} J_w \dot{\vartheta}^2\right) \tag{1}$$

$$U_1 = 0 \tag{2}$$

$$P_1 = 2\left(\tfrac{1}{2}\mu_{wg}\dot{x}^2\right) \tag{3}$$

$$\gamma_1 = \tfrac{K_t}{r} I \tag{4}$$

$$T_2 = \tfrac{1}{2} M_b\left(\dot{x}_b^{\,2} + \dot{z}_b^{\,2}\right) + \tfrac{1}{2} J_b \dot{\psi}^2 \tag{5}$$

$$U_2 = M_b g z_b \tag{6}$$

$$P_2 = 2\left[\tfrac{1}{2}\mu_m\left(\dot{\vartheta}^2 - \dot{\psi}^2\right)\right] \tag{7}$$

$$\gamma_2 = -K_t I \tag{8}$$

Let L be the Lagrangian of the whole system and P the total dissipated power:

$$L = T_1 + T_2 - U_1 - U_2 \tag{9}$$

$$P = P_1 + P_2 \tag{10}$$

The system dynamic is given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial P}{\partial \dot{q}_i} = \gamma_i \quad for \quad i = 1,2 \tag{11}$$

The dynamics of the system is then written as a descriptor representation with $X = (\dot{x}, \dot{\psi}, x, \psi)^T$ the state vector:

$$E(\psi(t), \dot{\psi}(t))\dot{X}(t) = A(\psi(t), \dot{\psi}(t))X(t) + BI(t) \tag{12}$$

And he measured output is $y(t) = (\vartheta_e(t), \dot{\psi}(t), \psi(t))^T$:

$$y(t) = CX(t) \tag{13}$$

With $E(\cdot) = \begin{pmatrix} M_b + 2M_w + \frac{2J_w}{r^2} & M_b l\cos(\Psi(t)) & 0 & 0 \\ M_b l\cos(\Psi(t)) & M_b l^2 + J_b & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

$$A(\cdot) = \begin{pmatrix} a_{00} & a_{01} & 0 & 0 \\ a_{10} & a_{11} & 0 & a_{13} \\ \multicolumn{2}{c}{I_{2x2}} & \multicolumn{2}{c}{0_{2x2}} \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & \frac{1}{r} & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

With $a_{00} = -2\mu_w - \frac{2\mu_m}{r^2}$, $a_{01} = \frac{2\mu_m}{r} + M_b l\sin(\Psi(t))\dot{\Psi}(t)$, $a_{10} = 2\mu_m r^{-1}$, $a_{11} = -2\mu_m$ $a_{13} = M_b g l \sin c\left(\frac{\Psi(t)}{\pi}\right)$.

$$B = \begin{pmatrix} \frac{K_t}{r} & -K_t & 0 & 0 \end{pmatrix}^T.$$

3 DISCRETE TAKAGI SUGENO DESCRIPTOR

Notations: As usual and when there is no ambiguity, we note $X_k = X(kT_s)$ where $T_s = 0.05\,s$ is the sampling period. Considering a premisses vector $z_k \in \mathbb{R}^z$ being sampled and any matrices Y, Y_i of appropriate size, we note:

$Y_v = \sum_{i=1}^{n_v} v_i(z_k) Y_i$, $Y_h = \sum_{i=1}^{n_h} h_i(z_k) Y_i$. The nonlinear functions $v_i \;\; i \in \{1, \ldots n_v\}$ and $h_i \;\; i \in \{1, \ldots n_h\}$ hold the convex sum property: $v_i(z_k) \in [0,1]$, $h_i(z_k) \in [0,1]$, $\sum_{i=1}^{n_v} v_i(z_k) = 1$ and $\sum_{i=1}^{n_h} h_i(z_k) = 1$. Extra samples can be used via the following notation: $Y_{v-} = \sum_{i=1}^{n_v} v_i(z_{k-1}) Y_i$, $Y_{hh-} = \sum_{i=1}^{n_h}\sum_{i=1}^{n_h} h_i(z_k) h_j(z_{k-1}) Y_{ij}$ or any combinations of them.

In order to design the discrete time controller, using the Euler approximation scheme and the dynamics given by equation (12) can be approximated as:

$$E_d(\psi_k, \dot{\psi}_k) X_{k+1} \approx A_d(\psi_k, \dot{\psi}_k) X_k + B_d I_k \tag{14}$$

with $A_d(\cdot) = (E(\cdot) + T_s A(\cdot))$, $E_d(\cdot) = E(\cdot)$, $B_d = T_s B$.

Takagi Sugeno models along with the sector nonlinearity approach allows representing exactly a nonlinear system over a compact subset of the state space (Ohtake, Tanaka, & Wang, 2001; Tanaka & Wang, 2004).

Remark: It is important to notice that, even if $E_d(\psi_k, \dot{\psi}_k)$ is invertible whatever are ψ_k and $\dot{\psi}_k$ we want to keep the descriptor form of (14) for its qLPV (Takagi-Sugeno) representation (Taniguchi, Tanaka, Ohtake, & Wang, 2001). This is due to the fact that using $E_d^{-1}(\cdot)$ will end with $E_d^{-1}(\cdot) B_d$ inducing more complex LMI constraints as the number of vertices of the closed-loop will increase from n to n^2, see the discussion in (Estrada-Manzo, Lendek, Guerra, & Pudlo, 2015).

Overall, there are 3 nonlinearities present in the matrices of the non-linear discrete model (14): $\cos(\psi_k)$, $\mathrm{sinc}\left(\frac{\psi_k}{\pi}\right)$ and $\sin(\psi_k)\dot{\psi}_k$. Therefore, let us define a compact set of the variables used in the nonlinearities, i.e.:

$$\Omega_x = \left\{(\psi_k, \dot{\psi}_k) \in \mathbb{R}^2, |\psi_k| \le 0.4rd, |\dot{\psi}_k| \le 10\, rd/s\right\} \tag{15}$$

Applying straightforwardly the nonlinear sector approach leads to a perfect representation of the model (14) in Ω_x using $n_r = 2^3 = 8$ vertices. Nevertheless, following some previous works (T.-M. Guerra, Bernal, & Blandeau, 2018), it is possible to reduce the number of vertices to $n_r = 2^2 = 4$. This is due the nature of both functions $\cos(\psi_k)$ and $\mathrm{sinc}\left(\frac{\psi_k}{\pi}\right)$ that nearly coincide on the consider interval $[-0.4, 0.4]$ of the compact set Ω_x. Effectively, considering that the Taylor's expansions of both functions are: $\cos(\psi_k) = 1 - \frac{\psi_k^2}{2} + o(\psi_k^2)$ and $\mathrm{sinc}\left(\frac{\psi_k}{\pi}\right) = 1 - \frac{\psi_k^2}{6} + o(\psi_k^2)$ it is easy to show that:

$$\left|\mathrm{sinc}\left(\frac{\psi_k}{\pi}\right) - \cos(\psi_k)\right| < 3.8\% \tag{16}$$

From (16), applying the sector nonlinear approach to $\cos(\psi_k)$ on $[-0.4, 0.4]$ uses the functions $w_1(\psi_k) = \frac{1-\cos(\psi_k)}{1-\cos(2)}$ and $w_2(\psi_k) = 1 - w_1(\psi_k)$. Now it is possible to write $\mathrm{sinc}\left(\frac{\psi_k}{\pi}\right)$ with these 2 functions of the sector from $\cos(\psi_k)$:

$$\left.\mathrm{sinc}\left(\frac{\psi_k}{\pi}\right)\right|_{app} = w_{1app}(\psi_k) \times \cos(0.4) + \left(1 - w_{1app}(\psi_k)\right) \times 1 \tag{17}$$

With: $w_{1app}(\psi_k) = \lambda_f w_1(\psi_k) + \lambda_0$.

At the end the following model is obtained with 4 vertices:

$$\begin{aligned} \sum_{i=1}^{2} v_i(z_k) E_i x_{k+1} &= \sum_{i=1}^{4} h_i(z_k) A_i x_k + B u_k \\ y_k &= C x_k \end{aligned} \tag{18}$$

Or in a more compact form using the aforementioned notation:

$$\begin{aligned} E_v x_{k+1} &= A_h x_k + B u_k \\ y_k &= C x_k \end{aligned} \tag{19}$$

At last notice that considering the general problem of finding an observer and a controller for (19) in a single LMI constraints problem is not feasible as the general problem is not convex. Nevertheless, as the premises are measured it is possible to derive a separation principle extending the work of (Yoneyama et al. 2001).

4 FEEDBACK CONTROL DESIGN

Step 1 corresponds to define a feedback controller that stabilizes the system (19) supposing the state is perfectly known. Consider the following state feedback law:

$$u_k = K_{hh^- v} G_{h^- v}^{-1} x_k \tag{20}$$

With: $K_{hh^-v} = \sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{2} h_i(z_k)h_j(z_{k-1})v_k(z_k)K_{ijk}$

The regularity of $G_{h^-v}^{-1}$ will be discussed further on. The closed-loop writes:

$$E_v x_{k+1} = \left(A_h + BK_{hh^-v}G_{h^-v}^{-1}\right)x_k \tag{21}$$

Or equivalently as an equality constraint:

$$\begin{bmatrix} A_h + BK_{hh^-v}G_{h^-v}^{-1} & -E_v \end{bmatrix}\begin{bmatrix} x_k \\ x_{k+1} \end{bmatrix} = 0 \tag{22}$$

Proposition 1: the discrete descriptor model (19) is asymptotically stabilized by the state feedback controller (20) if there exists matrices $P_i = P_i^T$, G_{jk}, K_{ijk} and F_{ij}, $i,j \in \{1,\ldots 4\}$, $k \in \{1,2\}$ such that the following LMI constraints:

$$\begin{bmatrix} -G_{jk} - G_{jk}^T + P_j & (*) & 0 \\ A_i G_{jk} + BK_{ijk} & -E_k F_{ij} - F_{ij}^T E_k^T & (*) \\ 0 & F_{ij} & -P_i \end{bmatrix} < 0 \tag{23}$$

Are satisfied for all i,j,k.

Proof: We use therein delayed parameter dependent Lyapunov functions introduced in (Lendek et al., 2015) and extended to descriptor form in (Estrada-Manzo et al., 2015)

$$V(x_k) = x_k^T P_{h^-}^{-1} x_k \tag{24}$$

Its variation over one sample can be written as:

$$\Delta V(e_k) = \begin{bmatrix} x_k \\ x_{k+1} \end{bmatrix}^T \begin{bmatrix} -P_{h^-}^{-1} & 0 \\ 0 & P_h^{-1} \end{bmatrix}\begin{bmatrix} x_k \\ x_{k+1} \end{bmatrix} < 0 \tag{25}$$

Using the so-called Finsler's lemma (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994), (25) holds under equality constraint (22) if and only if:

$$\mathcal{M}\begin{bmatrix} A_h + BK_{hh^-v}G_{h^-v}^{-1} & -E_v \end{bmatrix} + (*) + \begin{bmatrix} -P_{h^-}^{-1} & 0 \\ 0 & P_h^{-1} \end{bmatrix} < 0 \tag{26}$$

Considering $\mathcal{M} = \begin{bmatrix} 0 & F_{hh^-}^{-1} \end{bmatrix}^T$ and using the property of congruence with the full rank matrix $diag\begin{bmatrix} G_{h^-v}^T, & F_{hh^-}^T \end{bmatrix}$ leads to (25) holds if:

$$\begin{bmatrix} -G_{h^-v}^T P_{h^-}^{-1} G_{h^-v} & (*) \\ A_h G_{h^-v} + BK_{hh^-v} & -E_v F_{hh^-} - F_{hh^-}^T E_v^T + F_{hh^-}^T P_h^{-1} F_{hh^-} \end{bmatrix} < 0 \tag{27}$$

Now using the following well-known property $-G_{h^-v}^T P_{h^-}^{-1} G_{h^-v} \le -G_{h^-v} - G_{h^-v}^T + P_{h^-}$ on the first entry of (27) and a Schur's complement on the last one gives:

$$\begin{bmatrix} -G_{h^-v} - G_{h^-v}^T + P_{h^-} & (*) & 0 \\ A_h G_{h^-v} + BK_{hh^-v} & -E_v F_{hh^-} - F_{hh^-}^T E_v^T & (*) \\ 0 & F_{hh^-} & -P_h \end{bmatrix} < 0 \tag{28}$$

From which it is possible to state the following proposition that corresponds to a special case of (Estrada-Manzo et al., 2015).

Remark: there is no relaxation to use as the input matrix B is constant. From the first entry of we deduce from (28) that: $G_{h^-v} + G_{h^-v}^T > P_{h^-} > 0$ which guarantees the regularity of G_{h^-v}.

5 OBSERVER DESIGN AND GLOBAL STABILITY

We use therein delayed parameter dependent Lyapunov functions following the work of (Estrada-Manzo et al., 2015) Consider the following observer:

$$\begin{aligned} E_v \hat{x}_{k+} &= A_h \hat{x}_k + Bu_k + H_{h^-}^{-1} L_{hh^-v}\left(y_k - \hat{y}_k\right) \\ \hat{y}_k &= C\hat{x}_k. \end{aligned} \tag{29}$$

The regularity of $H_{h^-}^{-1}$ will be discussed thereafter. Thus let us define the state error $e_k = x_k - \hat{x}_k$ and the discrete Lyapunov function with P_{h-} a definite positive matrix:

$$V(e_k) = e_k^T P_{h-} e_k > 0 \tag{30}$$

The state error dynamic is therefore described via:

$$E_v e_k^+ = \left(A_h - H_{h^-}^{-1} L_{hh^-v} C\right)e_k \tag{31}$$

That can be transformed as the following equality constraint:

$$\begin{bmatrix} A_h - H_{h^-}^{-1} L_{hh^-v} C & -E_v \end{bmatrix}\begin{bmatrix} e_k \\ e_k^+ \end{bmatrix} = 0 \tag{32}$$

The variation of (30) on one sample writes:

$$\Delta V(e_k) = \begin{bmatrix} e_k \\ e_{k+} \end{bmatrix}^T \begin{bmatrix} -P_{h-} & 0 \\ 0 & P_h \end{bmatrix}\begin{bmatrix} e_k \\ e_{k+} \end{bmatrix} < 0 \tag{33}$$

Using the so-called Finsler's lemma (Boyd et al., 1994), (33) holds under equality constraint (32) if and only if:

$$\mathcal{M}\begin{bmatrix} A_h - H_{h^-}^{-1} L_{hh^-v} C & -E_v \end{bmatrix} + (*) + \begin{bmatrix} -P_{h-} & 0 \\ 0 & P_h \end{bmatrix} < 0 \tag{34}$$

And using $\mathcal{M} = \begin{bmatrix} 0 & H_{h-} \end{bmatrix}^T$ it follows:

$$\begin{bmatrix} -P_{h-} & (*) \\ H_{h-}A_h - L_{hh^-v}C & -H_{h-}E_v - E_v^T H_{h-}^T + P_h \end{bmatrix} < 0 \tag{35}$$

Proposition 2: considering the discrete descriptor model (19) and the observer (29), the estimation error dynamic is asymptotically stable if there exists matrices $P_i = P_i^T$, L_{ijk} and G_j, $i,j \in \{1,\ldots 4\}$, $k \in \{1,2\}$ such that:

$$\begin{bmatrix} -P_j & (*) \\ H_j A_i - L_{ijk} C & -H_j E_k - E_k^T H_j^T + P_i \end{bmatrix} < 0 \tag{36}$$

Remark: there is no relaxation to use as the output matrix C is constant (see discussion in (Estrada-Manzo et al., 2015)). From the last entry of (35) we deduce: $H_{h-} E_v + E_v^T H_{h-}^T > P_h > 0$, as E_k^{-1} always exists $H_{h^-}^{-1}$ is regular.

Consider now the complete closed-loop which corresponds to replace the control law (20) by $u_k = K_{hh^-v} G_{h^-v}^{-1} \hat{x}_k$ to get:

$$\begin{bmatrix} E_v & 0 \\ 0 & E_v \end{bmatrix} \begin{bmatrix} x_{k+1} \\ e_{k+1} \end{bmatrix} = \begin{bmatrix} A_h + BK_{hh^-v} G_{h^-v}^{-1} & -BK_{hh^-v} G_{h^-v}^{-1} \\ 0 & A_h - H_{h^-}^{-1} L_{hh^-v} C \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix}$$

A direct extension from (Yoneyama et al. 2001) separation principle can be given. Consider the Lyapunov function:

$$V_k = \begin{bmatrix} x_k \\ e_k \end{bmatrix}^T \begin{bmatrix} P_{ch^-}^{-1} & 0 \\ 0 & \lambda P_{oh^-} \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix} > 0 \tag{37}$$

With $\lambda > 0$ a free parameter and $P_{ch^-}^{-1}$ ensures that the state closed-loop without observer (21) is GAS and P_{oh^-} ensures the convergence of the state error (31), i.e. it exists $\gamma_c > 0$ and $\gamma_o > 0$ such that:

$$(*) E_v^{-1} P_{ch^-}^{-1} \left(A_h + BK_{hh^-v} G_{h^-v}^{-1} \right) - P_{ch}^{-1} \leq -\gamma_c I < 0 \tag{38}$$

$$(*) E_v^{-1} P_{oh^-} \left(A_h - H_{h^-}^{-1} L_{hh^-v} C \right) e_k - P_{oh} \leq -\gamma_o I < 0 \tag{39}$$

Considering the variation of (37) gives:

$$(*) \begin{bmatrix} P_{ch^-}^{-1} & 0 \\ 0 & \lambda P_{oh^-} \end{bmatrix} \begin{bmatrix} E_v^{-1} \left(A_h + BK_{hh^-v} G_{h^-v}^{-1} \right) & -E_v^{-1} BK_{hh^-v} G_{h^-v}^{-1} \\ 0 & E_v^{-1} \left(A_h - H_{h^-}^{-1} L_{hh^-v} C \right) \end{bmatrix}$$

$$-\begin{bmatrix} P_{ch}^{-1} & 0 \\ 0 & \lambda P_{oh} \end{bmatrix} < 0 \tag{40}$$

Using (38) and (39), (40) is satisfied if:

$$\begin{bmatrix} -\gamma_c I & \varphi(\cdot) \\ (*) & -\lambda \gamma_o I \end{bmatrix} < 0 \tag{41}$$

With: $\varphi(\cdot) = \left(A_h + BK_{hh^-v} G_{h^-v}^{-1} \right)^T E_v^{-T} P_{ch^-}^{-1} E_v^{-1} BK_{hh^-v} G_{h^-v}^{-1}$. Now, using a Schur's complement on (41) renders:

$$-\lambda \gamma_o I + \gamma_c \varphi^T (\cdot) \varphi(\cdot) < 0 \tag{42}$$

Considering that all the gains of $\varphi(\cdot)$ are bounded, it is easy to show that it exists a fixed $\gamma > 0$ such that: $\varphi^T(\cdot)\varphi(\cdot) < \gamma I$, thus (40) holds if: $-\lambda \gamma_o + \gamma_c \gamma < 0$. Therefore; it always exists a $\lambda > \frac{\gamma_c \gamma}{\gamma_o}$ that ensures the global stability of the closed-loop.

6 SIMULATION RESULTS

The LMIs and (36) have been solved using the SeDuMi solver (Sturm, 1999). For the observer the following matrices have been obtained, however, the values for the gains are not given in this paper due to confidential issues imposed by the company Autonomad.

$$P_1 = \begin{pmatrix} 4.909 & 7.097 \cdot 10^{-2} & -4.423 \cdot 10^{-1} & 2.744 \cdot 10^{-6} \\ 7.097 \cdot 10^{-2} & 1.602 & -9.839 \cdot 10^{-3} & -4.423 \cdot 10^{-6} \\ -4.423 \cdot 10^{-1} & -9.839 \cdot 10^{-3} & 4.781 \cdot 10^{-2} & -2.744 \cdot 10^{-7} \\ 2.744 \cdot 10^{-6} & -4.423 \cdot 10^{-6} & -2.744 \cdot 10^{-7} & 1.686 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 4.908 & 7.133 \cdot 10^{-2} & -4.423 \cdot 10^{-1} & 3.247 \cdot 10^{-6} \\ 7.133 \cdot 10^{-2} & 1.602 & -9.854 \cdot 10^{-3} & 1.799 \cdot 10^{-6} \\ -4.423 \cdot 10^{-1} & -9.854 \cdot 10^{-3} & 4.780 \cdot 10^{-2} & -4.891 \cdot 10^{-7} \\ 3.247 \cdot 10^{-6} & 1.799 \cdot 10^{-6} & -4.891 \cdot 10^{-7} & 1.686 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 4.910 & 7.165 \cdot 10^{-2} & -4.424 \cdot 10^{-1} & -5.314 \cdot 10^{-7} \\ 7.165 \cdot 10^{-2} & 1.603 & -9.872 \cdot 10^{-3} & -4.357 \cdot 10^{-6} \\ -4.424 \cdot 10^{-1} & -9.872 \cdot 10^{-3} & 4.782 \cdot 10^{-2} & 8.756 \cdot 10^{-8} \\ -5.314 \cdot 10^{-7} & -4.357 \cdot 10^{-6} & 8.756 \cdot 10^{-8} & 1.686 \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 4.909 & 7.209 \cdot 10^{-2} & -4.424 \cdot 10^{-1} & 9.880 \cdot 10^{-7} \\ 7.209 \cdot 10^{-2} & 1.601 & -9.907 \cdot 10^{-3} & -6.711 \cdot 10^{-7} \\ -4.424 \cdot 10^{-1} & -9.907 \cdot 10^{-3} & 4.781 \cdot 10^{-2} & -1.269 \cdot 10^{-7} \\ 9.880 \cdot 10^{-7} & -6.711 \cdot 10^{-7} & -1.269 \cdot 10^{-7} & 1.686 \end{pmatrix}$$

For the controller, the following matrices have been obtained:

$$P_1 = \begin{pmatrix} 6.851 \cdot 10^{-3} & -1.400 \cdot 10^{-2} & -3.679 \cdot 10^{-2} & 5.173 \cdot 10^{-4} \\ -1.400 \cdot 10^{-2} & 3.123 \cdot 10^{-2} & 9.925 \cdot 10^{-2} & -1.985 \cdot 10^{-3} \\ -3.679 \cdot 10^{-2} & 9.925 \cdot 10^{-2} & 3.264 & -1.146 \cdot 10^{-2} \\ 5.173 \cdot 10^{-4} & -1.985 \cdot 10^{-3} & -1.146 \cdot 10^{-2} & 4.357 \cdot 10^{-4} \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 6.851 \cdot 10^{-3} & -1.400 \cdot 10^{-2} & -3.678 \cdot 10^{-2} & 5.173 \cdot 10^{-4} \\ -1.400 \cdot 10^{-2} & 3.123 \cdot 10^{-2} & 9.924 \cdot 10^{-2} & -1.985 \cdot 10^{-3} \\ -3.678 \cdot 10^{-2} & 9.924 \cdot 10^{-2} & 3.264 & -1.146 \cdot 10^{-2} \\ 5.173 \cdot 10^{-4} & -1.985 \cdot 10^{-3} & -1.146 \cdot 10^{-2} & 4.357 \cdot 10^{-4} \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 6.851 \cdot 10^{-3} & -1.400 \cdot 10^{-2} & -3.679 \cdot 10^{-2} & 5.173 \cdot 10^{-4} \\ -1.400 \cdot 10^{-2} & 3.124 \cdot 10^{-2} & 9.925 \cdot 10^{-2} & -1.986 \cdot 10^{-3} \\ -3.679 \cdot 10^{-2} & 9.925 \cdot 10^{-2} & 3.264 & -1.146 \cdot 10^{-2} \\ 5.173 \cdot 10^{-4} & -1.986 \cdot 10^{-3} & -1.146 \cdot 10^{-2} & 4.358 \cdot 10^{-4} \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 6.852\cdot10^{-3} & -1.400\cdot10^{-2} & -3.679\cdot10^{-2} & 5.174\cdot10^{-4} \\ -1.400\cdot10^{-2} & 3.124\cdot10^{-2} & 9.926\cdot10^{-2} & -1.985\cdot10^{-3} \\ -3.679\cdot10^{-2} & 9.926\cdot10^{-2} & 3.264 & -1.146\cdot10^{-2} \\ 5.174\cdot10^{-4} & -1.985\cdot10^{-3} & -1.146\cdot10^{-2} & 4.358\cdot10^{-4} \end{pmatrix}$$

The simulations results are displayed in Figure 3. At time t=1s, the control law is applied to the non-linear system. The observer initial condition is $\hat{X}(0)=(0,0,0,0)^T$ and the wheelchair is initially grounded, so the system initial conditions is $X(0)=(0,0,0.3,0)^T$. The reactive force of the ground being not accounted for in the model, from $t=0\,s$ to $t=1\,s$, the observed state is subject to a steady state error. Nevertheless, once the controller is used, thanks to its fast convergence rate, the state is correctly estimated.

Between $t=1\,s$ and $t=1.4\,s$ the controller applies a large positive control in order to swing the wheelchair to its equilibrium. At $t=1.4\,s$ the vertical position is reached. Between $t=1.4\,s$ and $t=1.9\,s$, a negative control is applied and the wheelchair is leant backward. As a result, the speed is slowly decreased and the wheelchair moves backward toward its initial position $x=0$.

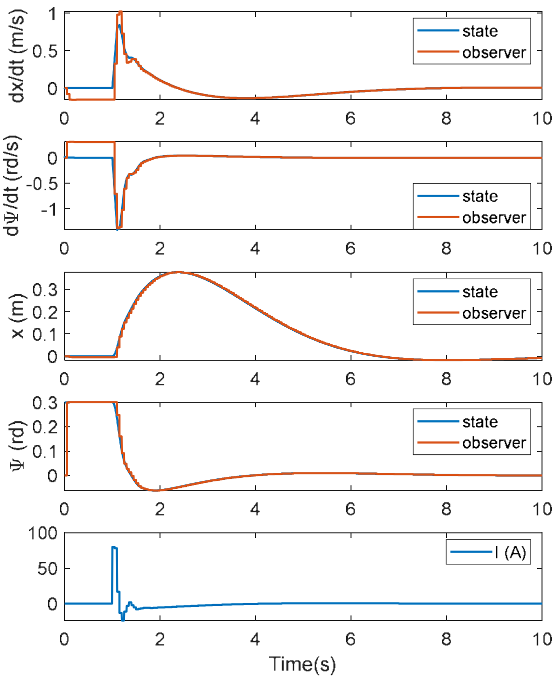

Figure 3: Simulation results

7 CONCLUSION

The design of a controller and observer that allows stabilizing a TS model has been presented. The stability is proven using delayed non-quadratic Lyapunov functions. A separation theorem allows computing the observer and the controller gains separately. The swinging of an automated wheelchair has been presented. The number of non-linearities has been reduced using an approximation introduced in (T.-M. Guerra et al., 2018). The simulation result illustrated the good performances of the observer and the controller. Future work will be devoted to the implementation of the proposed control scheme.

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