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Design of Scalable Controllers for Power Systems ⋆

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Abstract: This paper studies scalable control of power systems, *i.e.,* control with the constraint that controllers of all generators are the same. This control framework is useful to reduce the cost of constructing large-scale power systems because we can obtain controllers of all generators merely by designing one controller. The problem addressed here is to find the same controllers stabilizing an equilibrium point of the resulting feedback system and improving the performance in terms of the time response. As a solution to this problem, we present controllers to uniformly increase the damping forces of generators. We then show that an equilibrium point of the resulting feedback system is stable under certain conditions. In addition, we present a design method of the controller gain for improving the performance of the resulting feedback system in terms of the time response.

Keywords: Power systems, scalable control, stability, output feedback control, Lyapunov equation.

1. INTRODUCTION

Control of power systems has received considerable attention in the field of power engineering. In particular, many renewable energy sources with uncertainties have been recently introduced by environmental concerns [Shah et al. (2015)], and thus power system control will become increasingly important.

So far, many studies on power system control have been conducted. For example, there are studies on controller design based on frequency responses [Dysko et al. (2010)], nonlinear observers [Mahmud et al. (2012)], hybrid control [Zhang et al. (2019)], and sliding mode control [Huerta et al. (2019)]. On the other hand, in the existing studies, researchers have implicitly assumed that controllers of generators are *separately* designed and implemented. Such an assumption leads to the increase of time and effort required for controller design and implementation if the number of generators is large. For example, in the method proposed in [Dysko et al. (2010)], controllers are designed by a step-by-step procedure; thus, when applying this method to 100 generators (controllers), the following 100 steps are necessary:

1) designing controller 1,
2) designing controller 2 based on controller 1,

⋮

100) designing controller 100 based on controllers $1, 2, \ldots,$ 99.

This is a problem to be solved because power systems are generally large-scale systems.

Thus, this paper considers *scalable* control of power systems. More precisely, we consider power system control under the constraint that controllers of all generators are the *same*. By imposing this constraint, we can obtain controllers of all generators merely by designing one controller, and do not have to implement a different controller in each generator. This reduces time and effort spent to design and implement controllers.

This paper aims to establish a framework of scalable control of power systems. To this end, under the above constraint, we consider a design problem of controllers such that an equilibrium point of the resulting feedback system is stable and the performance in terms of the time response is improved. For this problem, this paper makes the following two contributions:

1) As a solution to the design problem, we present scalable controllers to uniformly increase the damping force of each generator. The difficulty of our problem is that a straightforward approach based on linear state feedback control [Franklin et al. (2010)] is not available due to the above scalability constraint. However, by focusing on the damping forces of generators, we can obtain a solution to the problem. We then show that if the original (*i.e.,* uncontrolled) system has a stable equilibrium point, the stability is preserved by the proposed scalable controllers.
2) We present a design method of the controller gain for improving the performance of the resulting feedback system in terms of the time response. In this method, we seek a gain minimizing a quadratic performance index by a linear search. It is difficult to directly solve the minimization problem of the performance index because the product of variables appears in its constraint. However, by focusing on the fact that the

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gain is common in all the controllers and by using a linear search, we can find an appropriate gain.

Note that this paper is based on our preliminary version [Izumi et al. (2019)]. The preliminary version has focused on the stability of an equilibrium point of the resulting feedback system. Meanwhile, this paper considers not only the stability but also the performance in terms of the time response, and presents a design method of the controller gain for achieving good performance.

Notation. Let $\mathbb{R}$ and $\mathbb{R}_+$ be the real number field and the set of positive real numbers, respectively. We use 0 to represent both the zero scalar and the zero vector. The n-dimensional vector of ones is represented by 1_n, *i.e.,* $1_n := [1\ 1\ \cdots\ 1]^\top$. The $n \times n$ identity matrix and the $n \times m$ zero matrix are represented by I_n and $0_{n\times m}$, respectively. For the numbers $x_1, x_2, \ldots, x_n \in \mathbb{R}$, let $\mathrm{diag}(x_1, x_2, \ldots, x_n)$ be the diagonal matrix whose i-th diagonal element is x_i. We denote by $\mathrm{tr}(M)$ the trace of the matrix M. Finally, $\mathsf{E}(x)$ represents the expectation of the random variable x.

2. PROBLEM FORMULATION

Consider a power system Σ with n generators.

The dynamics of generator i ($i \in \{1, 2, \ldots, n\}$) is described by the swing equation

$$M_i\ddot{\delta}_i(t) = P_{mi} - P_{ei}(\delta(t)) - D_i\dot{\delta}_i(t) + u_i(t), \quad (1)$$

where $\delta_i(t) \in \mathbb{R}$ is the phase angle of the generator voltage, $\delta(t) \in \mathbb{R}^n$ represents the phase angles of all the generator voltages, *i.e.,* $\delta(t) := [\delta_1(t)\ \delta_2(t)\ \cdots\ \delta_n(t)]^\top$, $u_i(t) \in \mathbb{R}$ is the control input, and $M_i \in \mathbb{R}_+$, $P_{mi} \in \mathbb{R}$, and $D_i \in \mathbb{R}_+$ are the inertia constant, the mechanical input, and the damping coefficient, respectively. The variable $P_{ei}(\delta(t)) \in \mathbb{R}$ is the electrical output expressed as

$$P_{ei}(\delta(t)) := \sum_{j=1}^{n} E_iE_jB_{ij}\sin(\delta_i(t) - \delta_j(t)), \quad (2)$$

where $E_i \in \mathbb{R}_+$ is the generator voltage and $B_{ij} \in \mathbb{R}$ is the susceptance between generators i and j. In general, $B_{ij} = B_{ji}$ holds for every $(i, j) \in \{1, 2, \ldots, n\}^2$. Note in (2) that the power system Σ is assumed to be lossless.

We suppose that a controller K_i is embedded in each generator i. This is of the form

$$K_i : u_i(t) = f(\delta_i(t), \dot{\delta}_i(t)), \quad (3)$$

where $\delta_i(t)$ and $\dot{\delta}_i(t)$ are the inputs, $u_i(t)$ is the output, and $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a function. In (3), f does not have the subscript i; that is, f is assumed to be the same for the controllers K_i ($i = 1, 2, \ldots, n$). This implies that all the controllers have the same structure, and in this sense, K_i ($i = 1, 2, \ldots, n$) are scalable controllers.

Then, we consider the following problem.

Problem 1. For the (lossless) power system Σ, find scalable controllers $K_1, K_2, \ldots, K_n$ (*i.e.,* a function f) such that the resulting feedback system

(a) has an asymptotically stable equilibrium point,
(b) achieves good performance in terms of the time response compared to the original system (*i.e.,* the system with $u_i(t) \equiv 0$ for every $i \in \{1, 2, \ldots, n\}$).

Problem 1 cannot be solved by directly using linear state feedback control. In fact, by regarding all the generators as a plant and designing the feedback gain, we obtain a gain matrix whose elements are generally different from each other; thus, the resulting controllers cannot be expressed as (3). This fact makes the problem challenging.

3. STABILIZATION BY SCALABLE CONTROLLERS

We first consider (a) in Problem 1. More precisely, we present scalable controllers such that an equilibrium point of the resulting feedback system is guaranteed to be stable.

3.1 Proposed Controllers

As mentioned in the previous section, the straightforward approach using linear state feedback control is not available for Problem 1. Hence, we focus on damping forces, which are known as a fundamental characteristic of generators, and consider controllers based on the forces.

Based on this idea, we propose the following solution to Problem 1:

$$f(\delta_i(t), \dot{\delta}_i(t)) := -k\dot{\delta}_i(t), \quad (4)$$

where $k \in \mathbb{R}_+$ is the controller gain. In (4), k is the same for the controllers K_i ($i = 1, 2, \ldots, n$) because the subscript i is not attached to k. Therefore, this solution results in scalable controllers. The proposed controllers given by (3) and (4) play the role of uniformly increasing the damping forces of all the generators. In fact, substituting (3) and (4) into (1) yields

$$M_i\ddot{\delta}_i(t) = P_{mi} - P_{ei}(\delta(t)) - (D_i + k)\dot{\delta}_i(t) \quad (5)$$

for every $i \in \{1, 2, \ldots, n\}$, which means that the damping coefficient of each generator i increases from D_i to $D_i + k$ by the proposed controllers.

For the proposed scalable controllers, the following result is obtained.

Theorem 1. For the (lossless) power system Σ, assume that the original system has an asymptotically stable equilibrium point. Let $K_1, K_2, \ldots, K_n$ be given by (3) and (4). Then, the equilibrium point remains asymptotically stable for every $k \in \mathbb{R}_+$.

Sketch of Proof. Let us introduce the function $V(\delta, \dot{\delta}) := U(\delta, \dot{\delta}) - U_e$, where $U(\delta, \dot{\delta})$ is an energy function for the original system, defined as

$$U(\delta, \dot{\delta}) := \frac{1}{2}\sum_{i=1}^{n} M_i\dot{\delta}_i^2 - \sum_{i=1}^{n} P_{mi}\delta_i - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E_iE_jB_{ij}\cos(\delta_i - \delta_j) \quad (6)$$

and U_e is the value of $U(\delta, \dot{\delta})$ at the asymptotically stable equilibrium point. Then, the following three facts and an extension of the Lyapunov's stability theorem (*e.g.,* Corollary 4.1 in [Khalil (2002)]) prove the theorem.

(i) The locations of the equilibrium points of the power system Σ are the same as those of the original system for every $k \in \mathbb{R}_+$.
(ii) Consider the equilibrium point that is asymptotically stable in the original system. Then, there exists a set

$\mathbb{D}$ satisfying the following two conditions for every $k \in \mathbb{R}_+$: (ii–a) $\mathbb{D}$ contains the equilibrium point but does not contain the others; (ii–b) $V(\delta, \dot{\delta})$ is positive definite on $\mathbb{D}$ when the equilibrium point is considered as the origin.

(iii) For every $k \in \mathbb{R}_+$, the time derivative of $V(\delta, \dot{\delta})$ is negative semidefinite, and no solution can stay identically in the points in $\mathbb{D}$ satisfying $\dot{V}(\delta, \dot{\delta}) = 0$ other than the above equilibrium point.

$\square$

Theorem 1 means that if there exists an asymptotically stable equilibrium point in the original system, the stability is preserved by the proposed scalable controllers for every $k \in \mathbb{R}_+$.

3.2 Example

Consider the power system Σ with $n := 3$. Based on [Sauer and Pai (1998)], let $M_1 := 0.125\,\mathrm{s}^2$, $M_2 := 0.0340\,\mathrm{s}^2$, $M_3 := 0.0160\,\mathrm{s}^2$, $P_{m1} := 1.00\,\mathrm{pu}$, $P_{m2} := 1.00\,\mathrm{pu}$, $P_{m3} := -2.00\,\mathrm{pu}$, $D_1 := 0.0531\,\mathrm{s}$, $D_2 := 0.0265\,\mathrm{s}$, $D_3 := 0.00531\,\mathrm{s}$, $E_1 := 1.05\,\mathrm{pu}$, $E_2 := 1.05\,\mathrm{pu}$, $E_3 := 1.02\,\mathrm{pu}$, and

$$B := \begin{bmatrix} -2.99 & 1.51 & 1.23 \\ 1.51 & -2.72 & 1.01 \\ 1.23 & 1.01 & -2.37 \end{bmatrix},$$

where $B \in \mathbb{R}^{3\times 3}$ is the matrix whose (i,j)-element is B_{ij} in the unit of pu. In this case, the (original) system has an asymptotically stable equilibrium point at $[\delta^\top \ \dot{\delta}^\top]^\top = [0.862 \ 0.911 \ -0.102 \ 0^\top]^\top$, which satisfies the assumption in Theorem 1. We further let K_1, K_2, and K_3 be given by (3) and (4) with $k := 0.1$.

Fig. 1 shows the time evolution of $\delta(t)$ for $[\delta^\top(0) \ \dot{\delta}^\top(0)]^\top := [1.2 \ 0 \ 0.5 \ 0^\top]^\top$, where each line corresponds to each element of $\delta(t)$. We see that $\delta(t)$ converges to that at the equilibrium point. Fig. 2 shows the time evolution of $V(\delta(t), \dot{\delta}(t))$. It turns out that $V(\delta(t), \dot{\delta}(t))$ is nonnegative and monotonically decreases as time goes on. These results validate Theorem 1.

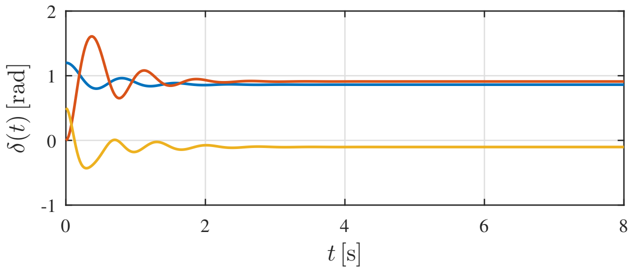

Fig. 1. Time evolution of $\delta(t)$ for $k := 0.1$

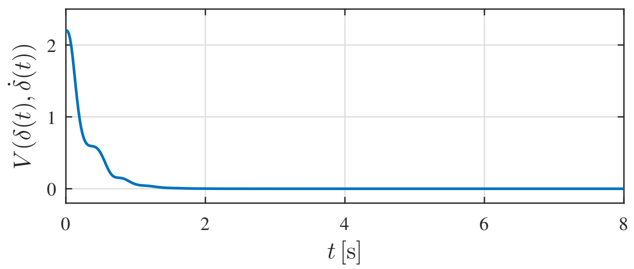

Fig. 2. Time evolution of $V(\delta(t), \dot{\delta}(t))$

4. DESIGN OF CONTROLLER GAIN

In this section, for (b) in Problem 1, we present a design method of the gain k for improving the performance of the resulting feedback system in terms of the time response.

4.1 Preliminary

From $\delta_i(t) - \delta_j(t)$ in (2), we can show that the equilibrium points of the feedback system given by (1), (3), and (4) are invariant under the uniform translation $\delta_e \to \delta_e + c1_n$, where $c \in \mathbb{R}$ is a constant. This means that each equilibrium point is not isolated, which makes a discussion of the deviation from it difficult.

Hence, we focus on the relative phase angles, and introduce the new state variable vector $x(t) := [\delta_1(t) - \delta_n(t) \ \ \delta_2(t) - \delta_n(t) \ \cdots \ \delta_{n-1}(t) - \delta_n(t) \ \ \dot{\delta}_1(t) \ \ \dot{\delta}_2(t) \ \cdots \ \dot{\delta}_n(t)]^\top \in \mathbb{R}^{2n-1}$. Then, using (1) and (2), we can express the state equation of the power system Σ as

$$\dot{x}(t) = \begin{bmatrix} x_n(t) - x_{2n-1}(t) \\ x_{n+1}(t) - x_{2n-1}(t) \\ \vdots \\ x_{2n-2}(t) - x_{2n-1}(t) \\ (1/M_1)(P_{m1} - P_{e1,x}(x(t)) - D_1 x_n(t) + u_1(t)) \\ (1/M_2)(P_{m2} - P_{e2,x}(x(t)) - D_2 x_{n+1}(t) + u_2(t)) \\ \vdots \\ (1/M_n)(P_{mn} - P_{en,x}(x(t)) - D_n x_{2n-1}(t) + u_n(t)) \end{bmatrix}, \quad (7)$$

where $x_i(t)$ ($i \in \{1, 2, \ldots, 2n-1\}$) is the i-th element of $x(t)$ and $P_{ei,x}(x(t))$ ($i \in \{1, 2, \ldots, n\}$) is defined as

$$P_{ei,x}(x(t)) := \begin{cases} E_i E_n B_{in} \sin x_i(t) + \displaystyle\sum_{j \in \{1,2,\ldots,n-1\}} E_i E_j B_{ij} \\ \times \sin(x_i(t) - x_j(t)) \qquad \text{if } i \in \{1, 2, \ldots, n-1\}, \\ -\displaystyle\sum_{j \in \{1,2,\ldots,n-1\}} E_n E_j B_{nj} \sin x_j(t) \qquad \text{if } i = n. \end{cases} \quad (8)$$

In addition, based on Theorem 1, we assume that there exists an asymptotically stable equilibrium point in the original system. We represent this equilibrium point and its i-th element by $x_e \in \mathbb{R}^{2n-1}$ and $x_{ei} \in \mathbb{R}$, respectively.

4.2 Proposed Design Method

For the system (7), we introduce the performance index

$$J := \int_0^\infty (\Delta x^\top(t) Q \Delta x(t) + u^\top(t) R u(t)) dt, \quad (9)$$

where $\Delta x(t) := x(t) - x_e$, $u(t) := [u_1(t) \ u_2(t) \ \cdots \ u_n(t)]^\top$, and $Q \in \mathbb{R}^{(2n-1)\times(2n-1)}$ and $R \in \mathbb{R}^{n\times n}$ are positive definite matrices. This is a standard quadratic performance index, and a smaller value of J indicates better performance. We design the gain k such that J is minimized.

To this end, we linearize the feedback system given by (3), (4), and (7) around $x = x_e$. Using (8), we can linearize the system (7) as

$$\Delta\dot{x}(t) = A\Delta x(t) + Gu(t), \tag{10}$$

where $A \in \mathbb{R}^{(2n-1)\times(2n-1)}$ and $G \in \mathbb{R}^{(2n-1)\times n}$ are defined as

$$A := \left[\begin{array}{ccc|c} & 0_{(n-1)\times(n-1)} & & I_{n-1} \quad -1_{n-1} \\ \hline a_{1,1} & \cdots & a_{1,n-1} & \\ \vdots & \ddots & \vdots & -\mathrm{diag}\left(\frac{D_1}{M_1}, \frac{D_2}{M_2}, \ldots, \frac{D_n}{M_n}\right) \\ a_{n,1} & \cdots & a_{n,n-1} & \end{array}\right], \tag{11}$$

$$G := \left[\begin{array}{c} 0_{(n-1)\times n} \\ \hline \mathrm{diag}\left(\frac{1}{M_1}, \frac{1}{M_2}, \ldots, \frac{1}{M_n}\right) \end{array}\right] \tag{12}$$

for

$$a_{i,j} := \begin{cases} -\frac{1}{M_i}\left(E_iE_nB_{in}\cos x_{ei} + \sum_{j\in\{1,2,\ldots,n-1\}\setminus\{i\}} E_iE_jB_{ij} \times \cos(x_{ei} - x_{ej})\right) & \text{if } i \in \{1,2,\ldots,n-1\},\ i = j, \\ \frac{E_iE_jB_{ij}}{M_i}\cos(x_{ei} - x_{ej}) & \text{if } i \in \{1,2,\ldots,n-1\},\ i \neq j, \\ \frac{E_nE_jB_{nj}}{M_n}\cos x_{ej} & \text{if } i = n. \end{cases} \tag{13}$$

Moreover, by the definitions of $x(t)$ and $\Delta x(t)$ and the fact that $\dot{\delta} = 0$ at the equilibrium point x_e, we obtain $\dot{\delta}(t) = H\Delta x(t)$ for

$$H := \begin{bmatrix} 0_{n\times n-1} & I_n \end{bmatrix}. \tag{14}$$

This, together with (3) and (4), yields

$$u(t) = -kH\Delta x(t). \tag{15}$$

By regarding $H\Delta x(t)$ in (15) as the output of the system (10), we see that the proposed scalable control corresponds to output feedback control [Levine and Athans (1970); Syrmos et al. (1997); Lewis et al. (2012)], where the gain matrix is restricted to the scalar k.

By focusing on this fact and using techniques developed in those studies, we obtain the following theorem.

Theorem 2. Consider the feedback system given by (10) and (15) and the performance index J in (9). Suppose that the initial state $\Delta x(0)$ is a random vector satisfying $\mathsf{E}(\Delta x(0)) = 0$ and $\mathsf{E}(\Delta x(0)\Delta x^\top(0)) = I_{2n-1}$. If there exists a positive definite matrix $P \in \mathbb{R}^{(2n-1)\times(2n-1)}$ such that

$$P(A - kGH) + (A - kGH)^\top P + k^2H^\top RH + Q = 0_{(2n-1)\times(2n-1)}, \tag{16}$$

then

$$\mathsf{E}(J) = \mathrm{tr}(P). \tag{17}$$

Theorem 2 gives a relation between the expectation $\mathsf{E}(J)$ of the performance index J and the gain k when the feedback system is linearized and the initial state $\Delta x(0)$ is regarded as a random vector. The reason for considering the expectation is explained as follows. Since J depends on $\Delta x(0)$ from (9), the minimization of J for a specific $\Delta x(0)$ does not necessarily mean the minimization for all $\Delta x(0)$. Hence, as a form independent of $\Delta x(0)$, $\mathsf{E}(J)$ is employed. Such an approach is typical in the field of output feedback control [Levine and Athans (1970); Syrmos et al. (1997); Lewis et al. (2012)].

Although we consider finding a gain k minimizing $\mathsf{E}(J)$ using Theorem 2, it is difficult to directly obtain such a k because (16) contains the products of the variables k and P. To overcome this difficulty, we propose a design method based on a linear search for k by focusing on the fact that the design parameter k is scalar. More precisely, we solve (16) for each k in a given range, and adopt a k such that the resulting P is positive definite and $\mathrm{tr}(P)$ is minimum. By fixing k, (16) contains only P as the variable, and thus we can numerically solve it. Moreover, if the search range of k is sufficiently large, we will obtain an optimal k.

Remark 1. As mentioned above, Theorem 2 is based on existing results on output feedback control; in this sense, the theorem is not completely new. Our main contributions here are to show that the proposed scalable control corresponds to constrained output feedback control and to present a design method of the gain k using Theorem 2 and the structure of the proposed scalable controllers.

4.3 Example

Consider again the power system Σ with $n := 3$ discussed in Section 3.2. When describing the system Σ by (7), the system with $u(t) \equiv 0$ has an asymptotically stable equilibrium point x_e at $x = [0.964 \ \ 1.01 \ \ 0^\top]^\top$. We further let K_1, K_2, and K_3 be given by (3) and (4).

Then, we design the gain k using the proposed method. For each k in the interval $[0, 10]$, we solve (16) and calculate $\mathrm{tr}(P)$, where $Q := I_5$, $R := I_3$, and the step size of k is 0.01. As a result, there exists a positive definite P satisfying (16) for every $k \in \{0, 0.01, 0.02, \ldots, 10\}$, and we obtain the relation between k and $\mathrm{tr}(P)$ shown in Fig. 3. From this relation, the (estimated) optimal gain k_{opt} is given by $k_{opt} := 0.91$, for which $\mathrm{tr}(P) = 5.08$ holds. Hence, we obtain $\mathsf{E}(J) = 5.08$ from Theorem 2.

Table 1 summarizes the values of $\mathsf{E}(J)$ and J for $k := 0$, 10, and k_{opt}, where the values of J are obtained by numerically calculating (9) for $x(0) := [0.8 \ \ 0.6 \ \ 0^\top]^\top$ (*i.e.*, $\Delta x(0) := [-0.164 \ \ -0.413 \ \ 0^\top]^\top$). We see that the value of J for $k := k_{opt}$ is smaller than those for $k := 0$ and 10. From this result, we conclude that the proposed design method provides a k such that the resulting feedback system achieves good performance in terms of J.

5. CONCLUSION

This paper has addressed a design problem of scalable controllers for power systems. By focusing on the damping forces of generators, we have presented scalable controllers solving the design problem. It has been shown that the

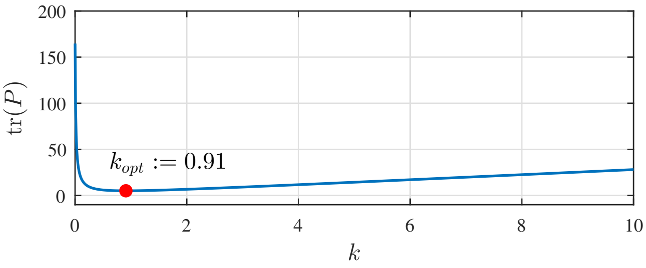

Fig. 3. Relation between k and $\mathrm{tr}(P)$

Table 1. Values of $\mathsf{E}(J)$ and J for several k

k	0	10	k_{opt}
$\mathsf{E}(J)$	165	28.1	5.08
J	12.0	1.64	0.279

proposed scalable controllers preserve the stability of an equilibrium point of the original power system. Moreover, we have presented a design method of the controller gain for improving the performance of the resulting feedback system in terms of the time response. These results are useful to reduce time and effort spent in controller design and implementation for large-scale power systems.

A limitation in this study is that the proposed method is applied only to a simple lossless system with three generators for the illustration purpose. Therefore, in the future, the proposed method should be verified for more practical power systems, *i.e.,* systems with many generators and losses.

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