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Robust Fuzzy Attitude Stabilization of Networked Spacecraft under Actuator Saturation and Persistent Disturbance ⋆

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Abstract: This paper is concerned with event-triggered robust attitude control of networked spacecraft with actuator saturation and persistent bounded disturbance via Takagi-Sugeno (T-S) fuzzy approach. Based on aperiodic sampling, a discrete-time event-triggered scheme avoiding Zeno phenomenon naturally is adopted to reduce communication burden within network. Based on the T-S fuzzy model of spacecraft, an event-triggered fuzzy controller subject to saturation constraint is established to perform robust attitude stabilization of spacecraft. Furthermore, in comparison with the most often used H_∞ performance, L_∞-gain performance is employed to handle the persistent bounded disturbances. By virtue of convex optimization method, the problem of controller synthesis is formulated in terms of a set of linear matrix inequalities (LMIs). Finally, simulation results are provided to prove the effectiveness of the fuzzy control scheme.

Keywords: Spacecraft control, Attitude control, Actuator saturation, External disturbance.

1. INTRODUCTION

The networked spacecraft with fractionated or plug-and-play architecture, which breaks down a spacecraft into physically independent and standardized modules that interact with each other through wireless communication, has gained a growing amount of interest during the past few decades (Mathieu and Weigel (2006); Lyke (2012)). In comparison with the traditional spacecraft with specific structure, the networked architecture offers more flexibility, maintainability, and reconfigurability for spacecraft design and manufacture (Wu (2015); Wang et al. (2019)). Considering that signal transmission is conducted by virtue of wireless communication in networked spacecraft, it is of real importance to design a control strategy capable of obtaining satisfactory performance with less communication resource.

Event-triggered control technique has been widely viewed as an effective solution to networked control problem. In the event-triggered mechanism, a prescribed condition is set to determine the transmission of the data sampled by sensor, hence the “unnecessary” signal transmissions are considerably reduced (Heemels et al. (2012)). Recently, an increasing number of research studies have been conducted for spacecraft attitude control with event-triggered mechanism. To mention a few, Wu et al. (2018), Wang et al. (2019), and Xu et al. (2019) investigated the problems of event-triggered attitude stabilization, event-triggered attitude tracking, and event-triggered attitude synchronization for networked spacecrafts, respectively. However, in these achievements the problem of actuator saturation is not considered, which reduces the applicability of the control laws. Moreover, the event-triggered schemes involved in above studies are all continuous-time ones, which requires continuous measurements and extra operations for avoiding Zeno phenomenon. In fact, a discrete-time event-triggered mechanism with sampled measurements is more suitable for digital platforms in practice. When further taking into account the factors of sampling jitters, data dropouts, and noisy environment, the discrete measurements obtained by a digital sensor are more likely to be aperiodic. Therefore, the investigation of an aperiodic event-triggered attitude control strategy subject to actuator saturation is a challenging problem to be resolved.

On the other hand, T-S fuzzy model has been extensively recognized as a powerful framework to handle complex nonlinear systems (Tanaka and Wang (2001)). In recent years, some research efforts have been made on the T-S fuzzy modeling and control of spacecraft. For example, based on T-S fuzzy models, Sun et al. (2017), Xu et al. (2017), and Sendi and Ayoubi (2018) proposed various fuzzy control strategies for flexible spacecraft attitude control. It should be mentioned that, in the studies above, the effects of external disturbances were suppressed by using H_∞ performance. However, the disturbances affecting spacecraft are persistent and bounded in practice. The H_∞ performance is appropriate for tackling the energy-bounded disturbances rather than the persistent bounded disturbances. Obviously, it is a challenging work to attenuating the persistent bounded disturbances during spacecraft attitude control based on T-S fuzzy approach.

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Motivated by the discussions above, this paper is dedicated to developing an aperiodic event-triggered control law to perform attitude stabilization of networked spacecraft with actuator saturation and persistent bounded disturbance via T-S fuzzy approach. Rather than using traditional continuous-time or periodic event-triggered conditions, a more robust and applicable one, aperiodic event-triggered condition, is employed to investigated attitude control of networked spacecraft, where the Zeno phenomenon is avoided naturally due to discrete measurements. By characterizing nonlinear attitude kinematics and dynamics of spacecraft with T-S fuzzy model, a fuzzy attitude control law is constructed according to aperiodic event-triggered mechanism. The problem of actuator saturation is tackled with utilizing anti-windup design approach. Furthermore, the L_∞-gain performance is ensured for the closed-loop system to attenuate the persistent bounded disturbances. Finally, with the aid of convex optimization technique, the problem of controller synthesis is formulated in terms of a set of LMIs.

Notations: In this paper, the superscripts "-1" and "T" for matrix $\boldsymbol{W}$ denote the inverse and transpose of matrix $\boldsymbol{W}$, respectively. $\boldsymbol{W}+\boldsymbol{W}^T$ is simplified as $[\boldsymbol{W}]_s$. The sets of $n\times m$ real matrices, n-dimensional vectors, and non-negative integers are denoted by $\mathtt{R}^{n\times m}$, $\mathtt{R}^n$, and $\mathtt{N}$, respectively. $\boldsymbol{W}>0$ ($\boldsymbol{W}<0$) implies that matrix $\boldsymbol{W}$ is positive (negative) definite. diag$\{\cdots\}$ is a diagonal matrix. $\iota_{\min}(\boldsymbol{W})$ is the minimum eigenvalue of $\boldsymbol{W}$. $\boldsymbol{I}$ ($\boldsymbol{0}$) is a suitably dimensioned identity (zero) matrix. For vector $\boldsymbol{f}(t)$, its Euclidean norm and ∞-norm are denoted by $\|\boldsymbol{f}(t)\|=\sqrt{\boldsymbol{f}^T(t)\boldsymbol{f}(t)}$ and $\|\boldsymbol{f}(t)\|_\infty=\sup_{t\geq 0}\|\boldsymbol{f}(t)\|$, respectively. $L_\infty=\{\boldsymbol{f}(t)\,|\,\|\boldsymbol{f}(t)\|_\infty<\infty\}$ denotes the linear vector space consisting of ∞-norm-bounded vectors. The skew-symmetric matrix of $\boldsymbol{f}(t)$ is denoted by $\boldsymbol{f}^\times(t)$. The symmetric term in a matrix is denoted by "$\star$". $\overline{a,b}$ represents the set of $\{a,a+1,\cdots,b\}$ where a and b are two integers ensuring $a\leq b$.

2. MODELING AND PROBLEM FORMULATION

The kinematic and dynamic equations of a rigid spacecraft are expressed as (Sidi (1997))

$$\dot{q}_0(t)=-\frac{1}{2}\boldsymbol{q}^T(t)\boldsymbol{\omega}(t) \tag{1}$$

$$\dot{\boldsymbol{q}}(t)=\frac{1}{2}\left(q_0(t)\boldsymbol{I}+\boldsymbol{q}^\times(t)\right)\boldsymbol{\omega}(t) \tag{2}$$

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}(t)=-\boldsymbol{\omega}^\times(t)\boldsymbol{J}\boldsymbol{\omega}(t)+\mathrm{sat}\left(\boldsymbol{u}(t)\right)+\boldsymbol{\tau}(t) \tag{3}$$

where quaternions $q_0(t)$ and $\boldsymbol{q}(t)=[q_1(t)\ q_2(t)\ q_3(t)]^T$ satisfy $\boldsymbol{q}^T(t)\boldsymbol{q}(t)+q_0^2(t)=1$; $\boldsymbol{\omega}(t)=[\omega_1(t)\ \omega_2(t)\ \omega_3(t)]^T$ is the vector of angular velocity; $\boldsymbol{J}>0$ is the inertia matrix; $\boldsymbol{\tau}(t)$ is the persistent disturbance bounded by a positive constant $\overline{\tau}$, implying $\|\boldsymbol{\tau}(t)\|\leq\overline{\tau}$; $\mathrm{sat}(\boldsymbol{u}(t))=[\mathrm{sat}(u_1(t))\ \ \mathrm{sat}(u_2(t))\ \ \mathrm{sat}(u_3(t))]^T$ is the vector of saturated control torques,

$$\mathrm{sat}\left(u_s(t)\right)=\mathrm{sign}\left(u_s(t)\right)\min\left\{|u_s(t)|,\overline{u}_s\right\}, \tag{4}$$

where $\overline{u}_s>0$ ($s\in\overline{1,3}$) are saturation levels.

Remark 1. Here, we assume that the Euler angle takes the value within $[-\pi,\pi]$ such that quaternion $q_0(t)$ is obtained by $q_0(t)=\sqrt{1-\|\boldsymbol{q}(t)\|^2}$.

In addition, it is true that

$$\mathrm{sat}\left(\boldsymbol{u}(t)\right)=\boldsymbol{u}(t)-\varpi\left(\boldsymbol{u}(t)\right) \tag{5}$$

where $\varpi\left(\boldsymbol{u}(t)\right)=[\varpi\left(u_1(t)\right)\ \varpi\left(u_2(t)\right)\ \varpi\left(u_3(t)\right)]^T$ and

$$\varpi\left(u_s(t)\right)=\begin{cases}u_s(t)-\overline{u}_s, & \text{if}\quad u_s(t)>\overline{u}_s\\ 0, & \text{if}\quad u_s(t)\in[-\overline{u}_s,\overline{u}_s]\\ u_s(t)+\overline{u}_s, & \text{if}\quad u_s(t)<-\overline{u}_s\end{cases} \tag{6}$$

where $s\in\overline{1,3}$.

According to T-S fuzzy approach (Tanaka and Wang (2001); Xu et al. (2017)), the T-S fuzzy description of the considered spacecraft (1)-(3) is expressed as

Model Rule i: **IF** $\rho_1(t)$ **is** H_1^i **and** $\rho_2(t)$ **is** $H_2^i\ \cdots\ \rho_g(t)$ **is** H_g^i, **THEN**

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\mathrm{sat}(\boldsymbol{u}(t))+\boldsymbol{B}_i\boldsymbol{\tau}(t)\\ \boldsymbol{y}(t)=\boldsymbol{C}_i\boldsymbol{x}(t)\end{cases} \tag{7}$$

where $i\in\overline{1,\lambda}$, λ is the number of fuzzy rules under consideration, and

$$\boldsymbol{A}_i=\begin{bmatrix}-\boldsymbol{J}^{-1}\boldsymbol{x}_{\omega i}^\times(t)\boldsymbol{J} & 0\\ \frac{1}{2}\sqrt{1-\|\boldsymbol{x}_{qi}(t)\|^2\boldsymbol{I}} & -\frac{1}{2}\boldsymbol{x}_{\omega i}^\times(t)\end{bmatrix},\ \boldsymbol{B}_i=\begin{bmatrix}\boldsymbol{J}^{-1}\\ 0\end{bmatrix},$$

$\boldsymbol{C}_i=\boldsymbol{I}$, $\boldsymbol{x}(t)=[x_1(t)\ x_2(t)\ x_3(t)\ x_4(t)\ x_5(t)\ x_6(t)]^T\triangleq[\omega_1(t)\ \omega_2(t)\ \omega_3(t)\ q_1(t)\ q_2(t)\ q_3(t)]^T$, $\boldsymbol{x}_\omega(t)=[\omega_1(t)\ \omega_2(t)\ \omega_3(t)]^T$, $\boldsymbol{x}_q(t)=[q_1(t)\ q_2(t)\ q_3(t)]^T$, $\rho_s(t)$ ($s\in\overline{1,g}$) are premise variables, and H_s^i ($s\in\overline{1,g}$) are fuzzy sets.

Based on fuzzy blending, the global T-S fuzzy model is stated as

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\sum_{i=1}^{\lambda}\zeta_i\left(\boldsymbol{\rho}(t)\right)\left\{\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\mathrm{sat}\left(\boldsymbol{u}(t)\right)+\boldsymbol{B}_i\boldsymbol{\tau}(t)\right\}\\ \boldsymbol{y}(t)=\sum_{i=1}^{\lambda}\zeta_i\left(\boldsymbol{\rho}(t)\right)\boldsymbol{C}_i\boldsymbol{x}(t)\end{cases} \tag{8}$$

where $\zeta_i\left(\boldsymbol{\rho}(t)\right)=\frac{\chi_i(\boldsymbol{\rho}(t))}{\sum_{j=1}^{\lambda}\chi_j(\boldsymbol{\rho}(t))}$, $\chi_i(\boldsymbol{\rho}(t))=\prod_{j=1}^{g}H_j^i(\rho_j(t))$, $\zeta_i\left(\boldsymbol{\rho}(t)\right)\in[0,1]$, $\sum_{i=1}^{\lambda}\zeta_i\left(\boldsymbol{\rho}(t)\right)=1$, and $\boldsymbol{\rho}(t)=[\rho_1(t),\rho_2(t),\cdots,\rho_g(t)]^T$.

With the event-triggered scheme located in sensor to controller channel, the discrete measurements acquired by sensor will be checked by the event-triggered condition and sent to controller when the condition is satisfied. In this work, we define the triggering instants as t_k ($k\in\mathtt{N}$) meeting $\bigcup_{k\in\mathtt{N}}[t_k,t_{k+1})=[0,+\infty)$ and define the aperiodic sampling instants existing in $[t_k,t_{k+1})$ as t_k^l ($l\in\mathtt{N}$) satisfying

$$0<\underline{h}\leq t_k^{l+1}-t_k^l\triangleq h_{l+1}\leq\overline{h}, \tag{9}$$

where $t_k^0\triangleq t_k$, $\underline{h}$ and $\overline{h}$ are two constants. Therefore, we have $[t_k,t_{k+1})=\bigcup\Phi_l$ with $\Phi_l=[t_k^l,t_k^l+h_{l+1})$. Inspired by Peng and Zhang (2015), the aperiodic event-triggered condition is designed as

$$t_{k+1}=t_k+\sum_{i=1}^{l-1}h_i+\inf_l\left\{h_l|\boldsymbol{e}(t_k^l)^T\boldsymbol{\Upsilon}\boldsymbol{e}(t_k^l)\geq\nu(t_k)\boldsymbol{x}^T(t_k)\boldsymbol{\Upsilon}\boldsymbol{x}(t_k)\right\}, \tag{10}$$

where $t_k^l=t_k+\sum_{i=1}^{l}h_i$, $\boldsymbol{e}(t_k^l)=\boldsymbol{x}(t_k^l)-\boldsymbol{x}(t_k)$, $\nu(t_k)$ is updated by $\nu(t_{k+1})=\max\{\nu(t_k)(1-(2\mu_1/\pi)\mathrm{atan}[\mu_2\times$

$(\|\boldsymbol{x}(t_{k+1})-\boldsymbol{x}(t_k)\|-\mu_3)]),\underline{\nu}\}$ where $\mu_s\geq 0$ $(s\in\overline{1,3})$ are design parameters, $\underline{\nu}\in[0,1)$ denotes the lower bound of $\nu(t_k)$ and $\nu(0)=\underline{\nu}$, $\boldsymbol{\Upsilon}$ is a weighting matrix.

Considering (10), the event-triggered fuzzy controller $\boldsymbol{u}(t)$ is constructed as

Controller Rule i: **IF** $\rho_1(t_k)$ **is** H_1^i **and** $\rho_2(t_k)$ **is** $H_2^i\cdots$ $\rho_g(t_k)$ **is** H_g^i, **THEN**

$$\boldsymbol{u}(t)=\boldsymbol{F}_i\left(\boldsymbol{x}(t_k^l)-\boldsymbol{e}(t_k^l)\right),\ t\in\Phi_l \tag{11}$$

where $\boldsymbol{F}_i$ $(i\in\overline{1,\lambda})$ are the gain matrices.

The global event-triggered fuzzy controller is stated as

$$\boldsymbol{u}(t)=\sum_{i=1}^{\lambda}\zeta_i\left(\boldsymbol{\rho}(t_k)\right)\boldsymbol{F}_i\left(\boldsymbol{x}(t_k^l)-\boldsymbol{e}(t_k^l)\right),\ t\in\Phi_l \tag{12}$$

Substituting (12) into (8) and noting (5), the closed-loop system is stated as

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\displaystyle\sum_{i=1}^{\lambda}\sum_{j=1}^{\lambda}\zeta_i\left(\boldsymbol{\rho}(t)\right)\zeta_j\left(\boldsymbol{\rho}(t_k)\right)\{\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\boldsymbol{F}_j\\ \qquad\times\left(\boldsymbol{x}(t_k^l)-\boldsymbol{e}(t_k^l)\right)-\boldsymbol{B}_i\varpi\left(\boldsymbol{u}(t)\right)+\boldsymbol{B}_i\boldsymbol{\tau}(t)\}\\ \boldsymbol{y}(t)=\displaystyle\sum_{i=1}^{\lambda}\zeta_i\left(\boldsymbol{\rho}(t)\right)\boldsymbol{C}_i\boldsymbol{x}(t),\quad t\in\Phi_l\end{cases} \tag{13}$$

For simplicity, $\sum_{i=1}^{\lambda}\zeta_i\left(\boldsymbol{\rho}(t)\right)$ and $\sum_{i=1}^{\lambda}\zeta_i\left(\boldsymbol{\rho}(t_k)\right)$ are denoted by $\sum_{i=1}^{\lambda}\zeta_i$ and $\sum_{i=1}^{\lambda}\zeta_{ik}$ in the sequel, respectively.

In this study, the aim is to find the event-triggered fuzzy controller (12) which ensures

- The closed-loop system (13) is exponentially stable when $\boldsymbol{\tau}(t)=0$;
- The L_∞-gain performance given below is guaranteed to suppress $\boldsymbol{\tau}(t)$ under zero initial conditions,
$$\sup_{\boldsymbol{\tau}(t)\in L_\infty}\frac{\|\boldsymbol{y}(t)\|_\infty}{\|\boldsymbol{\tau}(t)\|_\infty}<\gamma \tag{14}$$
where $\gamma>0$ is a scalar.

3. EVENT-TRIGGERED CONTROL STRATEGY

For the convenience of expression, in system (13) we denote $\boldsymbol{x}(t)\in\mathtt{R}^n$, $\boldsymbol{u}(t)\in\mathtt{R}^m$, $\varpi\left(\boldsymbol{u}(t)\right)\in\mathtt{R}^m$, $\boldsymbol{\tau}(t)\in\mathtt{R}^m$, and $\boldsymbol{y}(t)\in\mathtt{R}^w$.

Moreover, we define the set $\mathtt{E}\left(\boldsymbol{U},1\right)$ which reads

$$\mathtt{E}\left(\boldsymbol{U},1\right)=\left\{\boldsymbol{x}(t)\in\mathtt{R}^n|\boldsymbol{x}^T(t)\boldsymbol{U}\boldsymbol{x}(t)\leq 1\right\} \tag{15}$$

where $\boldsymbol{U}$ is a positive definite matrix, and define the set $\mathtt{G}$ which reads

$$\mathtt{G}=\left\{\boldsymbol{x}(t)\in\mathtt{R}^n|\left|\sum_{i=1}^{\lambda}\zeta_{ik}\left(\boldsymbol{F}_{is}-\boldsymbol{N}_{is}\right)\boldsymbol{x}(t_k^l)\right|\leq\overline{u}_s\right\} \tag{16}$$

where $\boldsymbol{N}_i$ $(i=\overline{1,\lambda})$ are general matrices, $\boldsymbol{F}_{is}$ and $\boldsymbol{N}_{is}$ denote the sth row of $\boldsymbol{F}_i$ and $\boldsymbol{N}_i$, respectively $(s\in\overline{1,m})$.

According to Da Silva and Tarbouriech (2005), the following inequality holds for any $\boldsymbol{x}(t)\in\mathtt{G}$,

$$\varpi^T\left(\boldsymbol{u}(t)\right)\boldsymbol{M}\left(\varpi\left(\boldsymbol{u}(t)\right)-\sum_{i=1}^{\lambda}\zeta_{ik}\boldsymbol{N}_i\boldsymbol{x}(t_k^l)\right)\leq 0, \tag{17}$$

where $\boldsymbol{M}>0$ is a diagonal matrix.

Based on the discussion above, the event-triggered fuzzy control scheme is gvien in the theorem below.

Theorem 1. Take into account the spacecraft system (13). Given positive constants ξ, γ, ϕ, $\overline{\tau}$, $\underline{\nu}$, and $\overline{u}_s$ $(s\in\overline{1,m})$, if there exist appropriately dimensioned matrices $\boldsymbol{U}>0$, $\boldsymbol{Y}>0$, $\boldsymbol{\Upsilon}>0$, $\boldsymbol{Q}_i$ $(i\in\overline{1,3})$, $\boldsymbol{L}_i$ $(i\in\overline{1,2})$, $\boldsymbol{F}_i$, $\boldsymbol{N}_i$ $(i\in\overline{1,\lambda})$, and diagonal matrix $\boldsymbol{M}>0$ such that the following inequalities hold,

$$\sum_{i=1}^{\lambda}\sum_{j=1}^{\lambda}\zeta_i\zeta_{jk}\boldsymbol{\Lambda}_{ijr}(h_{l+1})<0, \tag{18}$$

$$\begin{bmatrix}\overline{u}_s^2/\left(1+\xi\overline{\tau}^2/\phi\right)\boldsymbol{U} & \boldsymbol{F}_{is}^T-\boldsymbol{N}_{is}^T\\ \star & \boldsymbol{I}\end{bmatrix}\geq 0, \tag{19}$$

$$\begin{bmatrix}\phi\boldsymbol{U} & 0 & \boldsymbol{C}_i^T\\ \star & (\gamma-\xi)\boldsymbol{I} & 0\\ \star & \star & \gamma\boldsymbol{I}\end{bmatrix}>0 \tag{20}$$

where $i,j\in\overline{1,\lambda}$, $s\in\overline{1,m}$, $r\in\overline{1,2}$, and

$\boldsymbol{\Lambda}_{ijr}(h_{l+1})=\boldsymbol{\Xi}_{ij}+\boldsymbol{\Omega}_r(h_{l+1}),\boldsymbol{\Omega}_1(h_{l+1})=h_{l+1}\boldsymbol{R}_1^T\boldsymbol{Y}\boldsymbol{R}_1,$
$\boldsymbol{\Omega}_2(h_{l+1})=h_{l+1}\boldsymbol{R}_2^T\boldsymbol{L}e^{\phi\overline{h}}\boldsymbol{Y}^{-1}\boldsymbol{L}^T\boldsymbol{R}_2,\ \boldsymbol{L}=\left[\boldsymbol{L}_1^T\ \boldsymbol{L}_2^T\right]^T,$
$\boldsymbol{R}_1=\left[\boldsymbol{0}\ \boldsymbol{I}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right],\ \boldsymbol{R}_2=\begin{bmatrix}\boldsymbol{I}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{I}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\end{bmatrix},$
$\boldsymbol{\Xi}_{ij}^{(11)}=[\boldsymbol{Q}_1\boldsymbol{A}_i]_s+\phi\boldsymbol{U}+[\boldsymbol{L}_1]_s,\boldsymbol{\Xi}_{ij}^{(12)}=-\boldsymbol{Q}_1+\boldsymbol{A}_i^T\boldsymbol{Q}_2^T-$ $\boldsymbol{L}_1+\boldsymbol{L}_2^T,\boldsymbol{\Xi}_{ij}^{(13)}=\boldsymbol{Q}_1\boldsymbol{B}_i\boldsymbol{F}_j+\boldsymbol{A}_i^T\boldsymbol{Q}_3^T-\boldsymbol{L}_1+\boldsymbol{L}_2^T,\boldsymbol{\Xi}_{ij}^{(14)}=$ $-\boldsymbol{Q}_1\boldsymbol{B}_i\boldsymbol{F}_j,\boldsymbol{\Xi}_{ij}^{(15)}=-\boldsymbol{Q}_1\boldsymbol{B}_i,\boldsymbol{\Xi}_{ij}^{(16)}=\boldsymbol{Q}_1\boldsymbol{B}_i,\boldsymbol{\Xi}_{ij}^{(22)}=$ $-[\boldsymbol{Q}_2]_s,\boldsymbol{\Xi}_{ij}^{(23)}=-\boldsymbol{Q}_3^T+\boldsymbol{Q}_2\boldsymbol{B}_i\boldsymbol{F}_j,\boldsymbol{\Xi}_{ij}^{(24)}=-\boldsymbol{Q}_2\boldsymbol{B}_i\boldsymbol{F}_j,$ $\boldsymbol{\Xi}_{ij}^{(25)}=-\boldsymbol{Q}_2\boldsymbol{B}_i,\boldsymbol{\Xi}_{ij}^{(26)}=\boldsymbol{Q}_2\boldsymbol{B}_i,\boldsymbol{\Xi}_{ij}^{(33)}=-[\boldsymbol{L}_2]_s+\underline{\nu}\boldsymbol{\Upsilon}$ $+[\boldsymbol{Q}_3\boldsymbol{B}_i\boldsymbol{F}_j]_s,\boldsymbol{\Xi}_{ij}^{(34)}=-\boldsymbol{Q}_3\boldsymbol{B}_i\boldsymbol{F}_j-\underline{\nu}\boldsymbol{\Upsilon},\boldsymbol{\Xi}_{ij}^{(35)}=-\boldsymbol{Q}_3\boldsymbol{B}_i$ $+\boldsymbol{N}_j^T\boldsymbol{M},\boldsymbol{\Xi}_{ij}^{(36)}=\boldsymbol{Q}_3\boldsymbol{B}_i,\boldsymbol{\Xi}_{ij}^{(44)}=(\underline{\nu}-1)\boldsymbol{\Upsilon},\boldsymbol{\Xi}_{ij}^{(55)}=$ $-2\boldsymbol{M},\boldsymbol{\Xi}_{ij}^{(66)}=-\xi\boldsymbol{I},$

then the control goals can be achieved by employing event-triggered fuzzy controller (12) within local region $\mathtt{E}\left(\boldsymbol{U},1\right)$.

Proof. For the purpose of demonstrating the theorem, the following Lyapunov functional $V(t)$ is constructed for system (13),

$$V(t)=V_1(t)+V_2(t,t_k^l),\quad t\in\Phi_l \tag{21}$$

where $V_1(t)=\boldsymbol{x}(t)^T\boldsymbol{U}\boldsymbol{x}(t)$ and

$$V_2(t,t_k^l)=(t_k^l+h_{l+1}-t)\int_{t_k^l}^{t}e^{-\phi(t-\alpha)}\dot{\boldsymbol{x}}^T(\alpha)\boldsymbol{Y}\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha$$

By differentiating $V(t)$, we can obtain

$$\dot{V}_1(t)=2\boldsymbol{x}^T(t)\boldsymbol{U}\dot{\boldsymbol{x}}(t)+\phi\boldsymbol{x}(t)^T\boldsymbol{U}\boldsymbol{x}(t)-\phi V_1(t) \tag{22}$$

$$\dot{V}_2(t,t_k^l) = (t_k^l + h_{l+1} - t)\dot{\boldsymbol{x}}(t)\boldsymbol{Y}\dot{\boldsymbol{x}}(t) - \int_{t_k^l}^{t} e^{-\phi(t-\alpha)}\dot{\boldsymbol{x}}^T(\alpha) \times \boldsymbol{Y}\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha - \phi(t_k^l + h_{l+1} - t)\int_{t_k^l}^{t} e^{-\phi(t-\alpha)}\dot{\boldsymbol{x}}^T(\alpha)\boldsymbol{Y} \times \dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha \leq (t_k^l + h_{l+1} - t)\dot{\boldsymbol{x}}(t)\boldsymbol{Y}\dot{\boldsymbol{x}}(t) - \int_{t_k^l}^{t} e^{-\phi\overline{h}}\dot{\boldsymbol{x}}^T(\alpha) \times \boldsymbol{Y}\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha - \phi V_2(t,t_k^l) \leq (t_k^l + h_{l+1} - t)\dot{\boldsymbol{x}}^T(t)\boldsymbol{Y}\dot{\boldsymbol{x}}(t) + (t-t_k^l)\boldsymbol{\varphi}^T(t)\boldsymbol{L}e^{\phi\overline{h}}\boldsymbol{Y}^{-1} \times \boldsymbol{L}^T\boldsymbol{\varphi}(t) + 2\boldsymbol{\varphi}^T(t)\boldsymbol{L}\left(\boldsymbol{x}(t) - \boldsymbol{x}(t_k^l)\right) - \phi V_2(t,t_k^l) \tag{23}$$

where $\boldsymbol{\varphi}(t) = \left[\boldsymbol{x}^T(t)\ \boldsymbol{x}^T(t_k^l)\right]^T$.

In addition, it is also follows that

$$2\sum_{i=1}^{\lambda}\sum_{i=j}^{\lambda}\zeta_i\zeta_{jk}\left\{\left[\boldsymbol{x}^T(t)\boldsymbol{Q}_1 + \dot{\boldsymbol{x}}^T(t)\boldsymbol{Q}_2 + \boldsymbol{x}^T(t_k^l)\boldsymbol{Q}_3\right] \times \left[\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{F}_j\boldsymbol{x}(t_k^l) - \boldsymbol{B}_i\boldsymbol{F}_j\boldsymbol{e}(t_k^l) - \boldsymbol{B}_i\varpi\left(\boldsymbol{u}(t)\right) + \boldsymbol{B}_i\boldsymbol{\tau}(t) - \dot{\boldsymbol{x}}(t)\right]\right\} = 0 \tag{24}$$

Considering the condition (10), when t_k^l is not the triggering instant, it is true that

$$0 < -\boldsymbol{e}^T(t_k^l)\boldsymbol{\Upsilon}\boldsymbol{e}(t_k^l) + \underline{\nu}\left[\boldsymbol{x}(t_k^l) - \boldsymbol{e}(t_k^l)\right]^T\boldsymbol{\Upsilon}\left[\boldsymbol{x}(t_k^l) - \boldsymbol{e}(t_k^l)\right] \tag{25}$$

Furthermore, it can be inferred from (19) that

$$\left|\sum_{i=1}^{\lambda}\zeta_{ik}\left(\boldsymbol{F}_{is} - \boldsymbol{N}_{is}\right)\boldsymbol{x}(t)\right|^2 \leq \frac{\overline{u}_s^2}{1 + \frac{\xi\overline{\tau}^2}{\phi}}\boldsymbol{x}^T(t)\boldsymbol{U}\boldsymbol{x}(t), \tag{26}$$

which indicates that $\mathtt{E}(\boldsymbol{U},1) \subseteq \mathtt{G}$. Therefore, according to inequality (17), we have

$$\dot{V}(t) \leq \dot{V}(t) - 2\varpi^T(\boldsymbol{u}(t))\boldsymbol{M}\varpi(\boldsymbol{u}(t)) + 2\varpi^T(\boldsymbol{u}(t))\sum_{j=1}^{\lambda}\zeta_{jk}\boldsymbol{M}\boldsymbol{N}_j\boldsymbol{x}(t_k^l) \tag{27}$$

Substituting (22)-(25) into (27), one obtains

$$\dot{V}(t) + \phi V(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\zeta_i\zeta_{jk}\boldsymbol{\psi}^T(t)\left\{\frac{t_k^l + h_{l+1} - t}{h_{l+1}}\boldsymbol{\Lambda}_{ij1}(h_{l+1}) + \frac{t - t_k^l}{h_{l+1}}\boldsymbol{\Lambda}_{ij2}(h_{l+1})\right\}\boldsymbol{\psi}(t) + \xi\boldsymbol{\tau}^T(t)\boldsymbol{\tau}(t) \tag{28}$$

where

$$\boldsymbol{\psi}(t) = [\boldsymbol{x}^T(t)\ \dot{\boldsymbol{x}}^T(t)\ \boldsymbol{x}^T(t_k^l)\ \boldsymbol{e}^T(t_k^l)\ \varpi^T(\boldsymbol{u}(t))\ \boldsymbol{\tau}^T(t)]^T.$$

Under the condition of (18), inequality (28) will result in

$$\dot{V}(t) \leq -\phi V(t) + \xi\boldsymbol{\tau}^T(t)\boldsymbol{\tau}(t) \tag{29}$$

Integrating (29) over $[t_k^l, t)$ and noting $\lim_{t\to t_k^{l-}} V(t) \geq V(t_k^l)$, it is easy to find that

$$V(t) \leq e^{-\phi(t-t_k^l)}V(t_k^l) + \xi\int_{t_k^l}^{t} e^{-\phi(t-\alpha)}\boldsymbol{\tau}^T(\alpha)\boldsymbol{\tau}(\alpha)\mathrm{d}\alpha \leq e^{-\phi(t-t_k)}V(t_k) + \xi\int_{t_k}^{t} e^{-\phi(t-\alpha)}\boldsymbol{\tau}^T(\alpha)\boldsymbol{\tau}(\alpha)\mathrm{d}\alpha \leq \cdots \leq e^{-\phi t}V(0) + \xi\int_{0}^{t} e^{-\phi(t-\alpha)}\boldsymbol{\tau}^T(\alpha)\boldsymbol{\tau}(\alpha)\mathrm{d}\alpha \tag{30}$$

Moreover, from (21), we have

$$V(t) \geq \boldsymbol{x}^T(t)\boldsymbol{U}\boldsymbol{x}(t),\ t \in \Phi_l \tag{31}$$

When $\boldsymbol{\tau}(t) = 0$, by substituting (31) into (30), one obtains

$$\|\boldsymbol{x}(t)\|^2 \leq e^{-\phi t}\frac{V(0)}{\iota_{\min}(\boldsymbol{U})} \tag{32}$$

Hence, system (13) is exponentially stable when $\boldsymbol{\tau}(t) = 0$.

In the sequel, the aim is to demonstrate the L_∞-gain performance for system (13). Firstly, from (30) and (31), we can further know that

$$\boldsymbol{x}^T(t)\boldsymbol{U}\boldsymbol{x}(t) \leq V(0) + \frac{\xi}{\phi}\|\boldsymbol{\tau}(t)\|_\infty^2 \tag{33}$$

Therefore, when $\boldsymbol{x}(0) \in \mathtt{E}(U,1)$, it is found that $\boldsymbol{x}^T(t)\boldsymbol{U}\boldsymbol{x}(t) \leq 1 + \xi\overline{\tau}^2/\phi$. Due to the existence of (26), it is known that $\boldsymbol{x}(t) \in \mathtt{G}$ in the scenario of $\boldsymbol{\tau}(t) \neq 0$.

Noting zero initial conditions, one has

$$\boldsymbol{x}^T(t)\boldsymbol{U}\boldsymbol{x}(t) \leq \frac{\xi}{\phi}\|\boldsymbol{\tau}(t)\|_\infty^2 \tag{34}$$

Additionally, (20) leads to

$$\boldsymbol{y}^T(t)\boldsymbol{y}(t) < \gamma\phi\boldsymbol{x}^T(t)\boldsymbol{U}\boldsymbol{x}(t) + (\gamma^2 - \gamma\xi)\boldsymbol{\tau}^T(t)\boldsymbol{\tau}(t) \tag{35}$$

Then, with combining (34) and (35), we have

$$\|\boldsymbol{y}(t)\|_\infty^2 < \gamma^2\|\boldsymbol{\tau}(t)\|_\infty^2, \tag{36}$$

which means $\sup_{\boldsymbol{\tau}(t)\in L_\infty}\frac{\|\boldsymbol{y}(t)\|_\infty}{\|\boldsymbol{\tau}(t)\|_\infty} < \gamma$. The proof is therefore complete.

The problem of controller synthesis is solved in the following theorem.

Theorem 2. Take into account the spacecraft system (13). Given the positive constants ξ, γ, ϕ, $\overline{\tau}$, $\underline{\nu}$, $\overline{u}_s$ ($s \in \overline{1,m}$), ε_i ($i \in \overline{1,3}$), κ_i ($i \in \overline{1,\lambda}$), and $h_{l+1} \in \{\underline{h}, \overline{h}\}$, if there exist appropriately dimensioned matrices $\widehat{\boldsymbol{U}} > 0$, $\widehat{\boldsymbol{Y}} > 0$, $\widehat{\boldsymbol{\Upsilon}} > 0$, $\widehat{\boldsymbol{Q}}$, $\widehat{\boldsymbol{L}}_i$ ($i \in \overline{1,2}$), $\widehat{\boldsymbol{F}}_i$, $\widehat{\boldsymbol{N}}_i$, $\boldsymbol{P}_{ijr} > 0$, $\boldsymbol{G}_{ijr} > 0$, $\boldsymbol{K}_{ijr} = \boldsymbol{K}_{jir}^T$, $\boldsymbol{K}_{i(j+\lambda)r} = \boldsymbol{K}_{(j+\lambda)ir}^T$ ($i,j \in \overline{1,\lambda}$, $r \in \overline{1,2}$), and diagonal matrix $\boldsymbol{M} > 0$ such that the inequalities exhibited below are feasible,

$$\boldsymbol{P}_{ijr} + \boldsymbol{P}_{jir} - \boldsymbol{G}_{ijr} - \boldsymbol{G}_{jir} \leq \boldsymbol{K}_{ijr} + \boldsymbol{K}_{jir}, \tag{37}$$

$$\widehat{\boldsymbol{\Lambda}}_{ijr}(h_{l+1}) - 2\boldsymbol{P}_{ijr} + 2\boldsymbol{G}_{ijr} + \sum_{s=1}^{\lambda}\kappa_s\left(\boldsymbol{P}_{isr} + \boldsymbol{P}_{sir} + \boldsymbol{G}_{isr} + \boldsymbol{G}_{sir}\right) \leq \boldsymbol{K}_{i(j+\lambda)r} + \boldsymbol{K}_{(j+\lambda)ir} \tag{38}$$

$$\begin{bmatrix} \boldsymbol{W}_{11r} & \boldsymbol{W}_{12r} \\ \star & \boldsymbol{W}_{11r} \end{bmatrix} < 0, \tag{39}$$

$$\begin{bmatrix} \overline{u}_s^2/\left(1 + \xi\overline{\tau}^2/\phi\right)\widehat{\boldsymbol{U}} & \widehat{\boldsymbol{F}}_{is}^T - \widehat{\boldsymbol{N}}_{is}^T \\ \star & \boldsymbol{I} \end{bmatrix} \geq 0, \tag{40}$$

$$\begin{bmatrix} \phi\widehat{\boldsymbol{U}} & 0 & \widehat{\boldsymbol{Q}}\boldsymbol{C}_i^T \\ \star & (\gamma - \rho)\boldsymbol{I} & 0 \\ \star & \star & \gamma\boldsymbol{I} \end{bmatrix} > 0 \tag{41}$$

where

$$\boldsymbol{W}_{11r} = \begin{bmatrix} \boldsymbol{K}_{11r} & \cdots & \boldsymbol{K}_{1\lambda r} \\ \vdots & \ddots & \vdots \\ \boldsymbol{K}_{\lambda 1r} & \cdots & \boldsymbol{K}_{\lambda\lambda r} \end{bmatrix},$$

$$\boldsymbol{W}_{12r} = \begin{bmatrix} \boldsymbol{K}_{1(\lambda+1)r} & \cdots & \boldsymbol{K}_{1(2\lambda)r} \\ \vdots & \ddots & \vdots \\ \boldsymbol{K}_{\lambda(\lambda+1)r} & \cdots & \boldsymbol{K}_{\lambda(2\lambda)r} \end{bmatrix},$$

$$\widehat{\boldsymbol{\Lambda}}_{ijr}(h_{l+1}) = \widehat{\boldsymbol{\Xi}}_{ij} + \widehat{\boldsymbol{\Omega}}_r(h_{l+1}), \widehat{\boldsymbol{\Omega}}_1(h_{l+1}) = h_{l+1}\boldsymbol{R}_1^T\widehat{\boldsymbol{Y}}\boldsymbol{R}_1,$$

$$\boldsymbol{\Omega}_2(h_{l+1}) = h_{l+1}\boldsymbol{R}_2^T\widehat{\boldsymbol{L}}e^{\phi\bar{h}}\widehat{\boldsymbol{Y}}^{-1}\widehat{\boldsymbol{L}}^T\boldsymbol{R}_2,\ \widehat{\boldsymbol{L}} = \left[\widehat{\boldsymbol{L}}_1^T\ \widehat{\boldsymbol{L}}_2^T\right]^T,$$

$$\boldsymbol{R}_1 = [\,\boldsymbol{0}\ \boldsymbol{I}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\,],\ \boldsymbol{R}_2 = \begin{bmatrix} \boldsymbol{I}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0} \\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{I}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0} \end{bmatrix},$$

$$\widehat{\boldsymbol{\Xi}}_{ij}^{(11)} = [\varepsilon_1\boldsymbol{A}_i\widehat{\boldsymbol{Q}}^T]_s + \phi\widehat{\boldsymbol{U}} + [\widehat{\boldsymbol{L}}_1]_s, \widehat{\boldsymbol{\Xi}}_{ij}^{(12)} = -\varepsilon_1\widehat{\boldsymbol{Q}}^T + \varepsilon_2 \times \widehat{\boldsymbol{Q}}\boldsymbol{A}_i^T - \widehat{\boldsymbol{L}}_1 + \widehat{\boldsymbol{L}}_2^T, \widehat{\boldsymbol{\Xi}}_{ij}^{(13)} = \varepsilon_1\boldsymbol{B}_i\widehat{\boldsymbol{F}}_j + \varepsilon_3\widehat{\boldsymbol{Q}}\boldsymbol{A}_i^T - \widehat{\boldsymbol{L}}_1 + \widehat{\boldsymbol{L}}_2^T, \widehat{\boldsymbol{\Xi}}_{ij}^{(14)} = -\varepsilon_1\boldsymbol{B}_i\widehat{\boldsymbol{F}}_j, \widehat{\boldsymbol{\Xi}}_{ij}^{(15)} = -\varepsilon_1\boldsymbol{B}_i, \widehat{\boldsymbol{\Xi}}_{ij}^{(16)} = \varepsilon_1\boldsymbol{B}_i,$$

$$\widehat{\boldsymbol{\Xi}}_{ij}^{(22)} = -[\varepsilon_2\widehat{\boldsymbol{Q}}]_s, \widehat{\boldsymbol{\Xi}}_{ij}^{(23)} = -\varepsilon_3\widehat{\boldsymbol{Q}} + \varepsilon_2\boldsymbol{B}_i\widehat{\boldsymbol{F}}_j, \widehat{\boldsymbol{\Xi}}_{ij}^{(24)} = -\varepsilon_2 \times \boldsymbol{B}_i\widehat{\boldsymbol{F}}_j, \widehat{\boldsymbol{\Xi}}_{ij}^{(25)} = -\varepsilon_2\boldsymbol{B}_i, \widehat{\boldsymbol{\Xi}}_{ij}^{(26)} = \varepsilon_2\boldsymbol{B}_i, \widehat{\boldsymbol{\Xi}}_{ij}^{(33)} = -[\widehat{\boldsymbol{L}}_2]_s + \underline{\nu}\widehat{\boldsymbol{\Upsilon}} + [\varepsilon_3\boldsymbol{B}_i\widehat{\boldsymbol{F}}_j]_s, \widehat{\boldsymbol{\Xi}}_{ij}^{(34)} = -\varepsilon_3\boldsymbol{B}_i\widehat{\boldsymbol{F}}_j - \underline{\nu}\widehat{\boldsymbol{\Upsilon}},$$

$$\widehat{\boldsymbol{\Xi}}_{ij}^{(35)} = -\varepsilon_3\boldsymbol{B}_i + \widehat{\boldsymbol{N}}_j^T\boldsymbol{M}, \widehat{\boldsymbol{\Xi}}_{ij}^{(36)} = \varepsilon_3\boldsymbol{B}_i,$$

$$\widehat{\boldsymbol{\Xi}}_{ij}^{(44)} = (\underline{\nu} - 1)\widehat{\boldsymbol{\Upsilon}}, \widehat{\boldsymbol{\Xi}}_{ij}^{(55)} = -2\boldsymbol{M}, \widehat{\boldsymbol{\Xi}}_{ij}^{(66)} = -\xi\boldsymbol{I},$$

then there exists event-triggered fuzzy controller (12) with $\boldsymbol{F}_i = \widehat{\boldsymbol{F}}_i\widehat{\boldsymbol{Q}}^{-T}$ such that the control goals can be achieved within local region $\mathtt{E}(\boldsymbol{U}, 1)$.

Proof. To demonstrate the theorem, we define the matrices as follows: $\widehat{\boldsymbol{Q}} = \boldsymbol{Q}^{-1}$, $\boldsymbol{Q}_1 = \varepsilon_1\boldsymbol{Q}$, $\boldsymbol{Q}_2 = \varepsilon_2\boldsymbol{Q}$, $\boldsymbol{Q}_3 = \varepsilon_3\boldsymbol{Q}$, $\widehat{\boldsymbol{U}} = \widehat{\boldsymbol{Q}}\boldsymbol{U}\widehat{\boldsymbol{Q}}^T$, $\widehat{\boldsymbol{Y}} = \widehat{\boldsymbol{Q}}\boldsymbol{Y}\widehat{\boldsymbol{Q}}^T$, $\widehat{\boldsymbol{L}}_1 = \widehat{\boldsymbol{Q}}\boldsymbol{L}_1\widehat{\boldsymbol{Q}}^T$, $\widehat{\boldsymbol{L}}_2 = \widehat{\boldsymbol{Q}}\boldsymbol{L}_2\widehat{\boldsymbol{Q}}^T$, $\widehat{\boldsymbol{\Upsilon}} = \widehat{\boldsymbol{Q}}\boldsymbol{\Upsilon}\widehat{\boldsymbol{Q}}^T$, $\widehat{\boldsymbol{F}}_j = \boldsymbol{F}_j\widehat{\boldsymbol{Q}}^T$, $\widehat{\boldsymbol{N}}_j = \boldsymbol{N}_j\widehat{\boldsymbol{Q}}^T$, $\boldsymbol{\Delta}_1 = \mathrm{diag}\left\{\widehat{\boldsymbol{Q}}, \widehat{\boldsymbol{Q}}, \widehat{\boldsymbol{Q}}, \widehat{\boldsymbol{Q}}, \boldsymbol{I}, \boldsymbol{I}\right\}$, $\boldsymbol{\Delta}_2 = \mathrm{diag}\left\{\widehat{\boldsymbol{Q}}, \widehat{\boldsymbol{Q}}, \widehat{\boldsymbol{Q}}, \widehat{\boldsymbol{Q}}, \boldsymbol{I}, \boldsymbol{I}, \widehat{\boldsymbol{Q}}\right\}$, $\boldsymbol{\Delta}_3 = \mathrm{diag}\left\{\widehat{\boldsymbol{Q}}, \boldsymbol{I}\right\}$, $\boldsymbol{\Delta}_4 = \mathrm{diag}\left\{\widehat{\boldsymbol{Q}}, \boldsymbol{I}, \boldsymbol{I}\right\}$. According to the Corollary 2 in Arino and Sala (2008), (37)-(41) can be achieved by implementing congruent transformations for (18)-(20) using $\boldsymbol{\Delta}_1$, $\boldsymbol{\Delta}_2$, $\boldsymbol{\Delta}_3$, and $\boldsymbol{\Delta}_4$, respectively. The proof is therefore complete.

4. ILLUSTRATIVE EXAMPLE

To prove the effectiveness of the proposed control method, simulation results are shown in this section. It is assumed that the aperiodic event-triggered mechanism (10) is implemented in the sensor module which interacts with controller module through wireless communication. The inertia matrix $\boldsymbol{J}$ and external disturbance $\boldsymbol{\tau}(t)$ are set as

$$\boldsymbol{J} = \begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{bmatrix} (\mathrm{kg} \cdot \mathrm{m}^2)$$

$$\boldsymbol{\tau}(t) = \begin{bmatrix} -1.5 + 2\cos(0.06t) - 0.5\cos(0.2t) \\ 2 + 1.5\sin(0.06t) - 1\cos(0.2t) \\ -1.5 + 2\sin(0.06t) - 1.5\sin(0.2t) \end{bmatrix} \times 10^{-3}\,(\mathrm{N} \cdot \mathrm{m})$$

The operating regions of system are defined as $\omega_i(t) \in [-1\mathrm{rad/s}, 1\mathrm{rad/s}]$ and $q_i(t) \in [-0.56, 0.56]$ $(i \in \overline{1,3})$. System state $\boldsymbol{x}(t)$ is chosen as the premise variable $\boldsymbol{\rho}(t)$. The T-S fuzzy model of the spacecraft under consideration is stated as

Model Rule 1: **IF** $\boldsymbol{\omega}(t)$ **is about** $[0\ 0\ 0]^T$ **and** $\boldsymbol{q}(t)$ **is about** $[0\ 0\ 0]^T$, **THEN**

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{B}_1\mathrm{sat}\,(\boldsymbol{u}(t)) + \boldsymbol{B}_1\boldsymbol{\tau}(t)$$

Model Rule 2: **IF** $\boldsymbol{\omega}(t)$ **is about** $[0\ 0\ 0]^T$ **and** $\boldsymbol{q}(t)$ **is about** $[0.56\ 0.56\ 0.56]^T$, **THEN**

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_2\boldsymbol{x}(t) + \boldsymbol{B}_2\mathrm{sat}\,(\boldsymbol{u}(t)) + \boldsymbol{B}_2\boldsymbol{\tau}(t)$$

Model Rule 3: **IF** $\boldsymbol{\omega}(t)$ **is about** $[0\ 0\ 0]^T$ **and** $\boldsymbol{q}(t)$ **is about** $[-0.56\ -0.56\ -0.56]^T$, **THEN**

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_3\boldsymbol{x}(t) + \boldsymbol{B}_3\mathrm{sat}\,(\boldsymbol{u}(t)) + \boldsymbol{B}_3\boldsymbol{\tau}(t)$$

Model Rule 4: **IF** $\boldsymbol{\omega}(t)$ **is about** $[0.5\ 0.5\ 0.5]^T$ **and** $\boldsymbol{q}(t)$ **is about** $[0\ 0\ 0]^T$, **THEN**

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_4\boldsymbol{x}(t) + \boldsymbol{B}_4\mathrm{sat}\,(\boldsymbol{u}(t)) + \boldsymbol{B}_4\boldsymbol{\tau}(t)$$

Model Rule 5: **IF** $\boldsymbol{\omega}(t)$ **is about** $[-0.5\ -0.5\ -0.5]^T$ **and** $\boldsymbol{q}(t)$ **is about** $[0\ 0\ 0]^T$, **THEN**

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_5\boldsymbol{x}(t) + \boldsymbol{B}_5\mathrm{sat}\,(\boldsymbol{u}(t)) + \boldsymbol{B}_5\boldsymbol{\tau}(t)$$

Model Rule 6: **IF** $\boldsymbol{\omega}(t)$ **is about** $[1\ 1\ 1]^T$ **and** $\boldsymbol{q}(t)$ **is about** $[0.56\ 0.56\ 0.56]^T$, **THEN**

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_6\boldsymbol{x}(t) + \boldsymbol{B}_6\mathrm{sat}\,(\boldsymbol{u}(t)) + \boldsymbol{B}_6\boldsymbol{\tau}(t)$$

Model Rule 7: **IF** $\boldsymbol{\omega}(t)$ **is about** $[-1\ -1\ -1]^T$ **and** $\boldsymbol{q}(t)$ **is about** $[-0.56\ -0.56\ -0.56]^T$, **THEN**

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_7\boldsymbol{x}(t) + \boldsymbol{B}_7\mathrm{sat}\,(\boldsymbol{u}(t)) + \boldsymbol{B}_7\boldsymbol{\tau}(t)$$

Additionally, using the membership functions H_j^i $(i \in \overline{1,7}$, $j \in \overline{1,6})$ similar to the ones given in Xu et al. (2017), defining $\xi = 0.08$, $\phi = 0.05$, $\overline{u}_s = 2\mathrm{Nm}$ $(s \in \overline{1,3})$, $\varepsilon_i = 1$ $(i \in \overline{1,3})$, $\overline{\tau} = 0.01$, $\kappa_i = 0.12$ $(i \in \overline{1,7})$, $\underline{h} = 0.001$, $\overline{h} = 0.1$, $\underline{\nu} = 0.2$, $\mu_1 = 10$, $\mu_2 = 1$, $\mu_3 = 0.02$, and resolving the LMIs presented in Theorem 2 to find the minimized γ by Matlab LMI Toolbox, the performance index $\gamma_{\min} = 0.1616$ is obtained. Concurrently, the weighting matrix $\boldsymbol{\Upsilon}$ of aperiodic event-triggered condition (10) and feedback gain matrices $\boldsymbol{F}_i$ $(i \in \overline{1,7})$ of controller (12) are also calculated out, which are all presented in Appendix A.

Setting the initial conditions as $q_0(0) = 0.7477$, $\boldsymbol{q}(0) = [-0.3802\ 0.2124\ 0.5013]^T$, and $\boldsymbol{\omega}(0) = [0\ 0\ 0]^T$, and applying event-triggered fuzzy control law (12) to spacecraft models (1)-(3), the time-domain responses of quaternions, angular velocities, and saturated control inputs are described in Figs. 1-3, respectively. In Figs. 1 and 2, we find that, with using fuzzy controller (12), the quaternions and angular velocities can all be stabilized rapidly and smoothly. In addition, it is observed from Fig. 3 that the control inputs are limited in the range of $[-2\mathrm{Nm}, 2\mathrm{Nm}]$ even in the presence of aperiodic event-triggered condition (10). Hence, it is concluded that the proposed event-triggered controller is effective in stabilizing the networked spacecraft under complex constraints.

5. CONCLUSION

In this work, a robust fuzzy attitude control strategy is developed for a networked spacecraft with actuator satura-

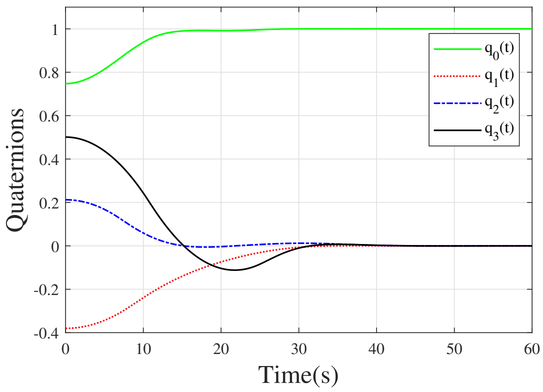

Fig. 1. **Responses of quaternions.**

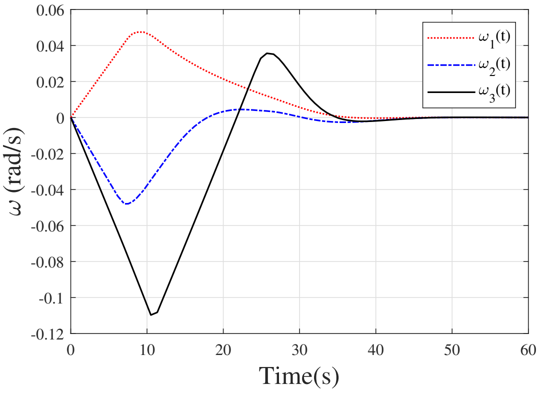

Fig. 2. **Responses of angular velocities.**

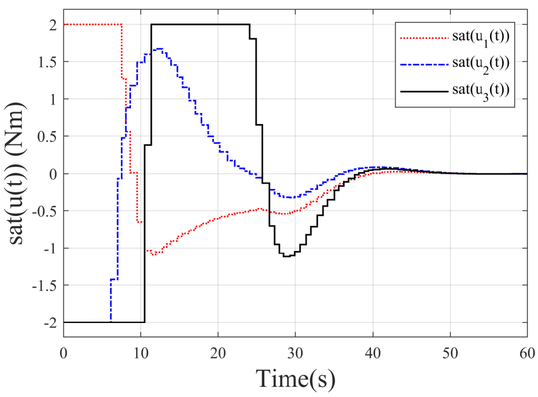

Fig. 3. **Responses of saturated control inputs.**

tion and persistent bounded disturbance under aperiodic event-triggered mechanism. Without the requirement of continuous measurements, a discrete-time event-triggered condition consisting of aperiodically sampled measurements is employed to reduce the communication burden for networked spacecraft. Based on the established fuzzy model of spacecraft, an aperiodic event-triggered fuzzy control law is proposed to carry out exponential attitude stabilization. Meanwhile, the problem of actuator saturation is solved by anti-windup design scheme and the persistent bounded disturbances are attenuated by L_∞-gain performance. The stability and robustness of the resulting closed-loop system are demonstrated strictly.

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Appendix A

The weighting matrix $\boldsymbol{\Upsilon}$ is stated as:

$$\mathbf{\Upsilon} = \begin{bmatrix} 2.8914 & -0.3518 & -0.1452 & 0.3317 & -0.0362 & 0.0542 \\ \star & 1.6962 & -0.0533 & 0.0037 & 0.2039 & 0.0777 \\ \star & \star & 0.9666 & -0.0210 & -0.0422 & 0.1158 \\ \star & \star & \star & 0.4205 & -0.0388 & -0.0319 \\ \star & \star & \star & \star & 0.4071 & -0.0351 \\ \star & \star & \star & \star & \star & 0.4022 \end{bmatrix},$$

The feedback gain matrices $\boldsymbol{F}_i$ $(i \in \overline{1,7})$ are stated as:

$$\boldsymbol{F}_1 = \begin{bmatrix} -157.7479 & 12.8948 & 6.9567 & -35.0656 & 0.0821 & -2.6976 \\ 11.6462 & -117.2082 & 4.5123 & -0.5367 & -37.3799 & -3.5277 \\ 2.7898 & 0.1396 & -85.1616 & 2.4968 & 4.1414 & -40.3453 \end{bmatrix},$$

$$\boldsymbol{F}_2 = \begin{bmatrix} -160.3611 & 11.3954 & 4.6658 & -23.4441 & 0.5155 & -3.8503 \\ 13.9409 & -122.8972 & 2.3392 & -1.4773 & -21.5913 & -5.6948 \\ 7.9967 & 4.2449 & -93.5620 & 1.7209 & 4.0652 & -19.9650 \end{bmatrix},$$

$$\boldsymbol{F}_3 = \begin{bmatrix} -160.1550 & 11.4178 & 4.6782 & -23.4211 & 0.5149 & -3.8407 \\ 13.9519 & -122.7574 & 2.3558 & -1.4739 & -21.5815 & -5.6831 \\ 8.0056 & 4.2671 & -93.4807 & 1.7236 & 4.0662 & -19.9589 \end{bmatrix},$$

$$\boldsymbol{F}_4 = \begin{bmatrix} -157.2247 & 13.2673 & -1.8554 & -34.6536 & 9.7486 & 0.5205 \\ 10.5887 & -112.9959 & 7.4392 & -14.2646 & -36.7005 & -2.2717 \\ 1.7306 & 0.2727 & -83.0726 & 10.2856 & -3.8188 & -40.7727 \end{bmatrix},$$

$$\boldsymbol{F}_5 = \begin{bmatrix} -152.2206 & 13.4019 & 13.4530 & -34.6074 & -9.1900 & -5.9847 \\ 8.3479 & -115.0668 & -1.6433 & 12.7452 & -36.7167 & -5.1645 \\ 1.7797 & -1.5907 & -84.7755 & -5.6095 & 11.8770 & -39.6755 \end{bmatrix},$$

$$\boldsymbol{F}_6 = \begin{bmatrix} -148.2593 & 12.4893 & -6.7283 & -20.6118 & 23.9252 & 7.3831 \\ 5.8268 & -99.5105 & -7.6464 & -35.0864 & -19.6879 & -5.0968 \\ -4.5217 & 8.3651 & -84.7807 & 18.8000 & -14.7530 & -20.6681 \end{bmatrix},$$

$$\boldsymbol{F}_7 = \begin{bmatrix} -139.0046 & 10.1777 & 0.1234 & -22.2481 & -22.0403 & -16.0851 \\ 5.1162 & -105.0553 & -8.5991 & 31.4435 & -17.5781 & -7.2848 \\ 10.9205 & -7.3551 & -86.4632 & -16.6470 & 21.5510 & -17.8586 \end{bmatrix}$$