Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Attitude Stabilization of Quadrotor with Input Time Delay

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Abstract: A predictor feedback control for attitude stabilization of quadrotors with input time delay has been proposed in this paper by representing the attitude using rotation matrices to avoid the singularities and ambiguities associated with Euler angles and quaternions. The closed loop system is shown to be asymptotically stable with respect to a norm defined in the text. The norm has been defined in terms of states and past control efforts and hence explicitly results in Lyapunov Krasovskii functional for the system. A cascade of PDE-ODE system and the concept of transport delay has been used in the proof.

Keywords: Input-Time Delay, Lyapunov-Krasovskii functional, Geometric Control, Quadrotors.

1. INTRODUCTION

Control of unmanned aerial vehicles are very challenging due to their highly nonlinear dynamics, external disturbances and parameter variations with flight conditions. Several linear and nonlinear control techniques have been proposed in the literature for stabilization and tracking control of quadrotors. In Bouabdallah et al. [2004], PID and LQR control techniques are applied on a simplified linear model for stabilization of a quadrotor. Since then nonlinear control approaches such as feedback linearization have been employed to achieve better performance. Since the dynamics is underactuated and any combination of outputs for input output feedback linearization results in unstable zero dynamics, dynamic extension to feedback linearization has been attempted in literature. A dynamic feedback extension of the control input is employed in Mistler et al. [2001] to develop the controller. While compensating gyroscopic and Coriolis effects, a novel feedback controller is proposed in Tayebi and McGilvray [2004].

Backstepping and sliding mode control methodologies are reported in Bouabdallah and Siegwart [2005], Madani and Benallegue [2006], Xu and Ozguner [2006, 2008] for stabilization of quadrotor dynamics. Resorting to advantages of sliding mode control such as insensitivity to modeling errors, parametric uncertainties and other disturbances such a controller has been proposed in Xu and Ozguner [2006, 2008]. All of the above reported literature either utilize Euler angles or quaternions for representing attitude of the quadrotor. To remove the singularities associated with Euler angles and ambiguities with quaternions, rotation matrices have been used to develop controllers in Lee et al. [2010, 2012], Lee [2013], Mellinger and Kumar [2011]. In Lee et al. [2010] nonlinear tracking controller has been developed directly on SE(3) which is almost global. Robust Lee et al. [2012], Lee et al. [2013] and adaptive robust Lee [2013] variant of this controller have also been proposed in the literature.

Time delays are ubiquitous in physical systems and engineering applications. The literature on control of quadrotors with input delay is very limited. In Liu et al. [2016b], a robust attitude controller was designed for multiple input multiple output uncertain quadrotors considering parametric uncertainties, external disturbances and input time delays. A similar robust controller considering parametric uncertainties, unmodeled uncertainties and input as well as state delays is designed in Liu et al. [2016a] Liu et al. [2017].

The contributions of this article are mentioned below. Firstly, the article proposes a stabilizing controller for quadrotors with input time delay using rotation matrices for attitude dynamics. It is well known that Euler angles exhibit singularity when representing attitude of a rigid body whereas the quaternions exhibit sign ambiguities. Hence rotation matrices have been used to represent the attitude of the quadrotors. There are very few references in literature compensating input delay in a quadrotor and this article is an attempt to bridge that gap. A predictor feedback has been developed to compensate for input time delay in rotation dynamics of the quadrotor. The method does not make any approximations on input delay which is prevalent in current literature on input delay compensation. The predictor based method has the advantage of yielding an expression for Lyapunov Krasovskii functional in the process of designing the controller and we do not have to assume such a functional beforehand. The asymptotic stability of the closed loop system is proved based on a norm defined with respect to current states and past control efforts.

The paper is organized as follows. Section 2 of the paper presents the problem formulation and various configuration errors. Some definitions used in this paper are given in Section 3. Section 4 presents the proposed controller along with the proof. Numerical simulation results are given in Section 5.

2. PROBLEM FORMULATION

2.1 Dynamic Model

The attitude dynamics of a quadrotor with input delay is represented below:

$$\begin{aligned} \dot{R} &= R\hat{\Omega} \\ I\dot{\Omega} &= -\Omega \times I\Omega + U(t-D) \end{aligned} \tag{1}$$

$R \in SO(3)$: Rotation matrix from body reference frame to inertial reference frame.
Ω : angular velocity of rigid body w.r.t inertial frame.
$\hat{(\cdot)}$: map from $\mathbb{R}^3$ to $\mathfrak{so}(3)$, space of skew symmetric matrices i.e. if $\Omega = [b_1 \;\; b_2 \;\; b_3]^T$, then

$$\hat{\Omega} = \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix}$$

I : Moment of Inertia.
τ : external torque input in the body frame.
D : known input delay.

2.2 Configuration Error

The angular velocity error is given by, $e_\Omega = \Omega$. The error function on $\mathsf{SO}(3)$ Bullo and Lewis [2004] is chosen to be :

$$\Psi = \frac{1}{2}\text{tr}(I - R)$$

The properties of the error function used in this paper can be recalled from Invernizzi and Lovera [2017] but is mentioned below for the sake of completeness.
(1) Ψ is locally positive definite about $R = I_{3\times3}$.
(2) The time derivative of Ψ is

$$\frac{d}{dt}\Psi = (\text{skew}(R)^\vee)^T e_\Omega = e_R^T \Omega$$

where $(\cdot)^\vee$ denotes map from $\mathfrak{so}(3)$ to $\mathbb{R}^3$ and e_R is attitude error vector. Here,

$$e_R = \text{skew}(R)^\vee = \frac{1}{2}(R - R^T) \tag{2}$$

(3) For $\Psi < \psi < 2$, it is locally quadratic

$$h_1||e_R||^2 \le \Psi \le h_2||e_R||^2 \tag{3}$$

$$h_1 = \frac{1}{2}, \quad h_2 = \frac{1}{(2-\psi)}$$

(4) The time derivative of the attitude error vector e_R is given by

$$\dot{e}_R = E(R)e_\Omega$$

$$\text{where} \quad E(R) = \frac{1}{2}(\text{tr}(R)I_{3\times3} - R^T)$$

The norm of $\dot{e}_R$ satisfies the following inequality and will be used in the stability analysis Invernizzi and Lovera [2017]

$$||\dot{e}_R|| \le \frac{3}{\sqrt{2}}||\Omega||$$

The problem of stabilizing the attitude of a quadrotor without input delay is not trivial but the problem has been tackled in several papers. In this article we will present a method of stabilizing the attitude in presence of input delay.

3. SOME DEFINITIONS

3.1 Forward Completeness

A system

$$\dot{x} = f(x, u) \tag{4}$$

with a locally Lipschitz vector field $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is said to be forward complete if, for every initial condition $x(0) = \zeta$ and every measurable locally essentially bounded input signal $u : \mathbb{R}_+ \to \mathbb{R}$, the corresponding solution is defined for all $t \ge 0$ i.e., the maximal interval of existence of solutions is $T_{max} = +\infty$.

***Theorem** 1.* Krstic [2009] System (4) is forward complete if and only if there exist a nonnegative-valued, radially unbounded, smooth function $V : \mathbb{R}^n \to \mathbb{R}_+$ and a class-$\mathcal{K}_\infty$ function σ such that

$$\frac{\partial V(x)}{\partial x} f(x, u) \le V(x) + \sigma(|u|)$$

3.2 Input to State Stability

The nonlinear system

$$\dot{x} = f(t, x, u) \tag{5}$$

is said to be input to state stable(ISS) Khalil [2002], Krstic et al. [1995] if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any initial state $x(t_0)$ and any bounded input $u(t)$, the solution exists for all $t \ge t_0$ and satisfies

$$||x(t)|| \le \beta(||x(t_0)||, t - t_0) + \gamma(sup_{t_0 \le \tau \le t}||u(\tau)||)$$

***Theorem** 2.* Khalil [2002] Let $V : [0, \infty) \times R^n \to R$ be a continuously differentiable function such that

$$\alpha_1(||x||) \le V(t, x) \le \alpha_2(||x||)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \le -W_3(x), \quad \forall \; ||x|| \ge \rho(||u||) > 0$$

$\forall (t, x, u) \in [0, \infty) \times R^n \times R^m$, where α_1, α_2 are class $\mathcal{K}_\infty$ functions, ρ is a class $\mathcal{K}$ function, and $W_3(x)$ is a continuous positive definite function on R^n. Then the system 5 is input to state stable with $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho$.

3.3 Norms for PDE state variables

Since the PDE state variable $u(x, t)$ is a function of two arguments, x and t, taking a norm in one of the variables, for example, in x, makes the norm a function of the other variable i.e.

$$||u(t)||_{L_2[0,D]} = \Big(\int_0^D u^2(x, t) \Big)^{\frac{1}{2}}$$

or

$$||u(t)||_{L_\infty[0,D]} = \text{sup}_{x \in [0,D]}|u(x, t)|$$

4. CONTROLLER DESIGN

***Proposition** 1.* The controller given below asymptotically stabilizes system (1) for D=0.

$$\kappa(\Omega) = -e_R - k_\Omega \Omega + \Omega \times I\Omega$$

Proof This can be proved from Invernizzi and Lovera [2017], Lee et al. [2010]

The delay in the system (1) can be modeled by the following first-order hyperbolic PDE, also referred to as the "transport PDE" :

$$u_t(x,t) = u_x(x,t)$$
$$u(D,t) = U(t)$$

The solution to this equation is

$$u(x,t) = U(t + x - D),$$

and therefore the output

$$u(0,t) = U(t - D)$$

gives the delayed input. The system (1) can be modeled by the following ODE-PDE cascade known as plant system

$$I\dot{\Omega} = -\Omega \times I\Omega + u(0,t)$$
$$u_t(x,t) = u_x(x,t)$$
$$u(D,t) = U(t) \tag{6}$$

Let us define the following direct and inverse backstepping transformation

$$w(x,t) = u(x,t) - \kappa(p(x,t)) \tag{7}$$
$$u(x,t) = w(x,t) + \kappa(\pi(x,t)) \tag{8}$$

Then the plant system (6) can also be represented by the following ODE-PDE cascade known as target system :

$$I\dot{\Omega} = -e_R - k_\Omega \Omega + w(0,t)$$
$$w_t(x,t) = w_x(x,t)$$
$$w(D,t) = 0 \tag{9}$$

The predictor variables are represented by the following differential equations with appropriate initial conditions given below :

$$\dot{R} = R\widehat{p(x,t)}$$
$$Ip_x(x,t) = -p(x,t) \times Ip(x,t) + u(x,t)$$
$$p(0,t) = \Omega(t) \tag{10}$$

and

$$\dot{R} = R\widehat{\pi(x,t)}$$
$$I\pi_x(x,t) = -e_R - k_\pi \pi(x,t) + w(x,t)$$
$$\pi(0,t) = \Omega(t) \tag{11}$$

The p-system (10) and the π-system (11) are used to generate the plant-predictor and the target predictor in the following manner :

$$P(t) = p(D,t),$$
$$\Pi(t) = \pi(D,t).$$

***Proposition** 2.*

$$p_t(x,t) = p_x(x,t),$$
$$\pi_t(x,t) = \pi_x(x,t).$$

Proof This can be proved by noting that $u(x,t)$ and $w(x,t)$ are functions of only one variable, $x+t$, and therefore so are $p(x,t)$ and $\pi(x,t)$ based on the ODEs (10) and (11). This implies that

$$p_t(x,t) = p_x(x,t),$$
$$\pi_t(x,t) = \pi_x(x,t).$$

***Proposition** 3.* The plant system (10) is forward complete

Proof Let us consider the following nonnegative-valued, radially unbounded, smooth function

$$J = \frac{1}{2} p(x,t)^T I p(x,t) + \Psi$$

Then,

$$\frac{1}{2}\lambda_{min}(I)||p||^2 + h_1||e_R||^2 \leq J \leq \frac{1}{2}\lambda_{max}(I)||p||^2 + h_2||e_R||^2$$

and

$$\begin{aligned} \dot{J} &= p^T u + e_R^T p \\ &\leq ||p||^2 + \frac{1}{2}||e_R||^2 + \frac{1}{2}||u||^2 \\ &\leq \frac{1}{min(\frac{1}{2}\lambda_{min}(I), h_1)} J + \frac{1}{2}||u||^2 \\ &= J(p(x,t)) + \sigma(||u||) \end{aligned}$$

Hence from Section 3.1 system (10) is forward complete.

***Proposition** 4.* The target system (11) is input to state stable.

Proof Let us consider the following Lyapunov function

$$L = \pi(x,t)^T I \pi(x,t) + \Psi + cIe_R^T \pi(x,t) \tag{12}$$

Then,

$$\lambda_{min}(\mathcal{W}_1)||z_R||^2 \leq L \leq \lambda_{max}(\mathcal{W}_2)||z_R||^2$$

where

$$\mathcal{W}_1 = \begin{bmatrix} \frac{1}{2}\lambda_{min}(I) & \frac{c}{2}\lambda_{min}(I) \\ \frac{c}{2}\lambda_{min}(I) & h_1 \end{bmatrix} \quad \mathcal{W}_2 = \begin{bmatrix} \frac{1}{2}\lambda_{max}(I) & \frac{c}{2}\lambda_{max}(I) \\ \frac{c}{2}\lambda_{max}(I) & h_2 \end{bmatrix}$$

and $z_R = [||e_R|| \;\; ||\pi||]$. The time derivative of (12) is

$$\begin{aligned} \dot{L} &\leq -k_\pi||\pi||^2 + \pi^T w - c||e_R||^2 - ck_\pi e_R^T \pi + ce_R^T w \\ &+ \frac{3c}{\sqrt{2}}||\pi||^2 \\ &\leq -\lambda_{min}(A)||z_R||^2 + \frac{1+c}{2}||w||^2 \\ &\leq -\lambda_{min}(A)||z_R||^2 \;\; \forall \;\; ||z_R|| \geq \sqrt{\frac{1+c}{2\lambda_{min}(A)}}||w|| \end{aligned}$$

$$A = \begin{bmatrix} \frac{1}{2}c & \frac{ck_\pi}{2} \\ \frac{ck_\pi}{2} & k_\pi - \frac{1}{2} - \frac{3c}{\sqrt{2}} \end{bmatrix}$$

Hence from Section 3.2 system (11) is input-to-state stable.

***Proposition** 5.* Consider the closed loop system

$$I\dot{\Omega} = -\Omega \times I\Omega + U(t-D)$$
$$U(t) = -e_R - k_\Omega P(t) + P(t) \times IP(t)$$
$$P(t) = \int_{t-D}^{t} (-P(\theta) \times IP(\theta) + U(\theta))d\theta + \Omega(t), \quad t \geq 0$$
$$P(\theta) = \int_{-D}^{\theta} (-P(\sigma) \times IP(\sigma) + U(\sigma))d\sigma + \Omega(0), t \in [-D, 0]$$

with an initial condition $\Omega_0 = \Omega(0)$ and $U_0(\theta) = U(\theta)$, $\theta \in [-D, 0]$. Then there exists a function $\beta \in \mathcal{KL}$ such that

$$||e_R(t)||^2 + ||\Omega(t)||^2 + \int_{t-D}^{t} ||U(\theta)||^2 d\theta \leq$$
$$\beta(||e_R(0)||^2 + ||\Omega(0)||^2 + \int_{-D}^{0} ||U(\theta)||^2 d\theta, t)$$

Proof The proof will be completed in four steps :

4.1 Step 1

Since the plant system is forward complete from Proposition 3 we can write

$$\dot{J}(p(x,t)) \leq J(p(x,t)) + \frac{1}{2}||u(x,t)||^2$$

Therefore,

$$J(p(x,t)) \leq e^x J(p(0,t)) + \frac{1}{2}\int_0^x e^{x-\zeta}||u(\zeta,t)||^2 d\zeta$$
$$= e^x J(\Omega(t)) + \frac{1}{2}\int_0^x e^{x-\zeta}||u(\zeta,t)||^2 d\zeta$$
$$= e^x J(\Omega(t)) + \frac{1}{2}(e^x - 1)\sup_{0\leq\zeta\leq x}||u(\zeta,t)||^2$$

From Proposition 3 we get

$$\frac{1}{2}\lambda_{min}(I)||p||^2 + h_1||e_R||^2 \leq e^x(\frac{1}{2}\lambda_{max}(I)||\Omega||^2 + h_2||e_R||^2)$$
$$+ \frac{1}{2}(e^x - 1)\sup_{0\leq\zeta\leq x}||u(\zeta,t)||^2$$
$$\implies min\left(\frac{1}{2}\lambda_{min}(I), h_1\right)(||p||^2 + ||e_R||^2) \leq$$
$$e^D(max\left(\frac{1}{2}\lambda_{max}(I), h_2\right)(||\Omega||^2 + ||e_R||^2))$$
$$+ \frac{1}{2}(e^D - 1)\sup_{0\leq\zeta\leq x}||u(\zeta,t)||^2$$

Therefore we have

$$||p(x,t)||^2 + ||e_R(x,t)||^2 \leq$$
$$\rho_1(||e_R||^2 + ||\Omega(t)||^2 + \sup_{0\leq\zeta\leq x}||u(\zeta,t)||)$$

for some $\rho_1 \in \mathcal{K}_\infty$. Defining the L_2 norm in x as

$$||l(t)||_{L_2[0,D]} = \left(\int_0^D ||l(x,t)||^2 dx\right)^{\frac{1}{2}}$$

we can write

$$(||p(t)|| + ||e_R(t)||)_{L_2[0,D]} \leq$$
$$\rho_1(||e_R(t)||^2 + ||\Omega(t)||^2 + ||u(t)||_{L_2[0,D]})$$

From Proposition 1

$$||\kappa(p)|| \leq ||e_R|| + k_p||p|| + \lambda_{max}(I)||p||^2 = \rho_2(||p||)$$

for $\rho_2 \in \mathcal{K}_\infty$. Considering (10) with (7) as output map we can conclude :

$$||\Omega(t)||^2 + ||e_R(t)||^2 + ||w(t)||_{L_2[0,D]} \leq$$
$$\rho_3(||e_R||^2 + ||\Omega(t)||^2 + ||u(t)||_{L_2[0,D]})$$

for some $\rho_3 \in \mathcal{K}_\infty$.

4.2 Step 2

Since the target system is input-to-state stable we get from Proposition 4

$$||\pi(x,t||) \leq \beta_1(||\pi(0,t)||,t) + \gamma_1\left(\underset{0\leq\zeta\leq x}{sup}||w(x,t)||\right)$$
$$||\pi(x,t||) \leq \beta_1(||\Omega(t)||,t) + \gamma_1\left(\underset{0\leq\zeta\leq x}{sup}||w(x,t)||\right)$$

for some class $\mathcal{KL}$ function β_1 and class $\mathcal{K}$ function γ_1. Similar to Step 1 taking a L_2-norm on both sides we get :

$$(||\pi(t)|| + ||e_R(t)||)_{L_2[0,D]} \leq$$
$$\beta_1(||e_R(t)||^2 + ||\Omega(t)||^2, 0) + \gamma_1(||w(t)||_{L_2[0,D]})$$

Considering (11) with (8) as output map we can conclude :

$$||\Omega(t)||^2 + ||e_R(t)||^2 + ||u(t)||_{L_2[0,D]} \leq$$
$$\rho_4(||e_R||^2 + ||\Omega(t)||^2 + ||w(t)||_{L_2[0,D]})$$

4.3 Step 3

Let us consider the target system (9) and prove its exponential stability. Let us consider the following Lyapunov function for that purpose :

$$V(t) = \frac{1}{2}\Omega^T I\Omega + \Psi + cI\Omega^T e_R +$$
$$(c+1)\int_0^D e^{(c+1)x}||w(t,x)||^2$$

V is bounded by

$$min(\lambda_{min}(\mathcal{W}_3), (c+1))(||z_R||^2 + \int_0^D e^{(c+1)x}||w(t,x)||^2)$$
$$\leq V(t) \leq$$
$$max(\lambda_{max}(\mathcal{W}_4)|, c+1)(||z_R||^2 + \int_0^D e^{(c+1)x}||w(t,x)||^2)$$

where

$$\mathcal{W}_3 = \begin{bmatrix} h_1 & \frac{c\lambda_{min}(I)}{2} \\ \frac{c\lambda_{min}(I)}{2} & \frac{\lambda_{min}(I)}{2} \end{bmatrix} \quad \mathcal{W}_4 = \begin{bmatrix} h_2 & \frac{c\lambda_{max}(I)}{2} \\ \frac{c\lambda_{max}(I)}{2} & \frac{\lambda_{max}(I)}{2} \end{bmatrix}$$

Then its time derivative is given by :

$$\dot{V} = -k_\Omega||\Omega||^2 + \Omega^T w(0,t) - c||e_R||^2 - ck_\Omega e_R^T\Omega + ce_R^T$$
$$w(0,t) + cI\Omega_T\dot{e}_R + (c+1)\int_0^D e^{(c+1)x}w(t,x)^T w_t(x,t)dx$$
$$\leq -\frac{c}{2}||e_R||^2 - (k_\Omega - \frac{1}{2} - \frac{3}{\sqrt{2}}c\lambda_{max}(I))||\Omega||^2$$
$$+ \frac{c+1}{2}||w(0,t)||^2 + ck_\Omega||e_R||||\Omega||$$
$$+ (c+1)\int_0^D e^{(c+1)x}w(t,x)^T w_x(x,t)dx$$
$$\leq -\frac{c}{2}||e_R||^2 - (k_\Omega - \frac{1}{2} - \frac{3}{\sqrt{2}}c\lambda_{max}(I))||\Omega||^2$$
$$+ \frac{c+1}{2}||w(0,t)||^2 + ck_\Omega||e_R||||\Omega||$$
$$+ \frac{c+1}{2}\int_0^D e^{(c+1)x}d||w(t,x)||^2$$
$$\leq -\lambda_{min}(B)||z||^2 - (c+1)\int_0^D e^{(c+1)x}||w(t,x)||^2$$

where in the last step integration by parts has been applied and $z = [||e_R|| \ \ ||\Omega||]$ with

$$B = \begin{bmatrix} \frac{c}{2} & \frac{ck_\Omega}{2} \\ \frac{ck_\Omega}{2} & k_\Omega - \frac{1}{2} - \frac{3}{\sqrt{2}}c\lambda_{max}(I) \end{bmatrix}$$

Therefore

$$\dot{V} \leq -\mu V$$

where

$$\mu = \frac{min(\lambda_{min}(B), c+1)}{max(\lambda_{max}(\mathcal{W}_4), c+1)}$$

Hence there exists a class-$\mathcal{KL}$ function β_2 such that

$$V(t) \leq \beta_2(V(0), t), \quad \forall t \geq 0.$$

Also there exists a function $\beta_4 \in \mathcal{KL}$ such that

$$||e_R(t)||^2 + ||\Omega(t)||^2 + ||w(t)||_{L_\infty[0,D]} \leq$$
$$\beta_4(||e_R(0)||^2 + ||\Omega(0)||^2 + ||w(0)||_{L_\infty[0,D]}, t).$$

4.4 Step 4

From steps 1,2 and 3 one obtains the following result

$$||e_R(t)||^2 + ||\Omega(t)||^2 + ||u(t)||_{L_2[0,D]} \leq$$
$$\rho_4(\beta_4(\rho_3(||e_R(0)||^2 + ||\Omega(0)||^2 + ||u(0)||_{L_2[0,D]}), t)).$$

Then there exists a function $\beta_5 \in \mathcal{KL}$ such that :

$$||e_R(t)||^2 + ||\Omega(t)||^2 + ||u(t)||_{L_2[0,D]} \leq$$
$$\beta_5(||e_R(0)||^2 + ||\Omega(0)||^2 + ||u(0)||_{L_2[0,D]}, t).$$

or

$$||e_R(t)||^2 + ||\Omega(t)||^2 + \int_{t-D}^{t} ||U(\theta)||^2 d\theta \leq$$
$$\beta(||e_R(0)||^2 + ||\Omega(0)||^2 + \int_{-D}^{0} ||U(\theta)||^2 d\theta, t)$$

5. NUMERICAL SIMULATIONS

Numerical simulations were carried out for the system whose moment of inertia is mentioned below.

$I_{xx} = 0.082kg/m^2, I_{yy} = 0.0845kg/m^2, I_{zz} = 0.1377kg/m^2$

The initial conditions for simulation are assumed to be

$$\Omega(0) = [0.2; 0.1; 0.5], \quad \dot{\Omega}(0) = [0; 0; 0],$$

$$R(0) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

The proposed control law is simulated in MATLAB for the quadrotor attitude dynamics with the initial conditions mentioned above. The stabilization performance of the control law is shown in Figs.1-3 for delay D=0.1 sec. It is observed that the quadrotor stabilizes to $(R, \Omega) = (I, 0)$ within 5 sec. The control law proposed in Lee et al. [2010], Invernizzi and Lovera [2017] fails to stabilize the quadrotor in presence of input delay of 0.1 sec while the proposed controller is able to stabilize the quadrotor. Similar stabilization results are obtained with input delay of 0.05 sec, 0.2 sec and 0.3 sec but the simulation results are not given due to repetition as well as lack of space. We can observe that the stabilization results are valid for long input delay as well as small input delay. The results are even robust to variations in inertia matrix and can be demonstrated by repeating the simulation for small deviation in inertia matrix.

6. CONCLUSION

A nonlinear geometric finite-time controller was proposed for attitude stabilization of quadrotors where the attitude is represented using rotation matrices. The predictor method proposed in this paper is able to stabilize the quadrotor in presence of input delay while the methods proposed in the current literature is not able to stabilize the quadrotors in presence of input delay. The proposed method works for wide variation in input delay and is even robust to delay mismatch.

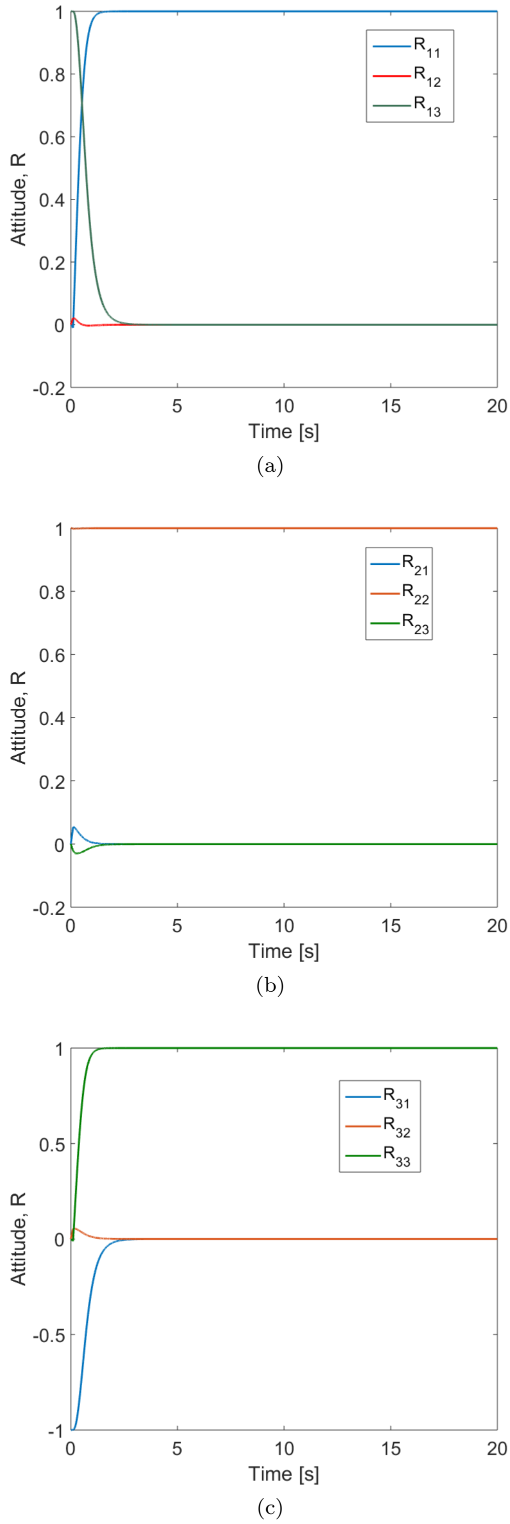

Fig. 1. Attitude Tracking.

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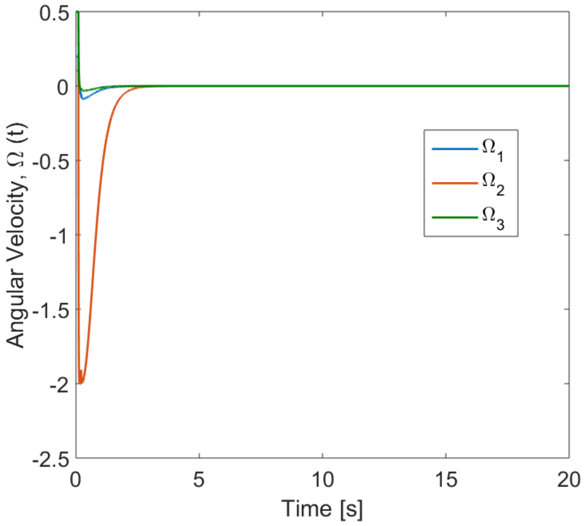

Fig. 2. Angular Velocity stabilization

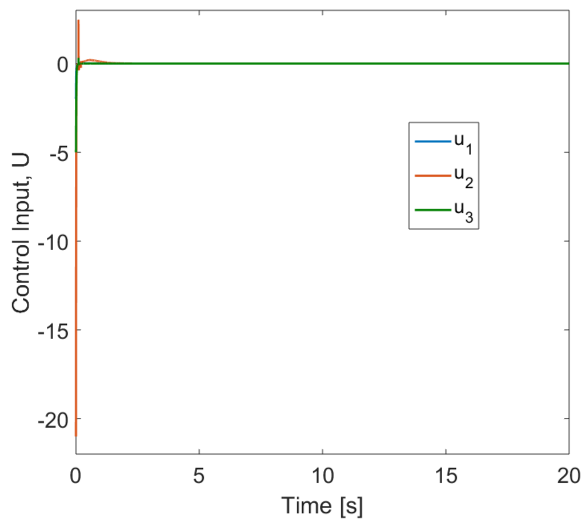

Fig. 3. Control input

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