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A kinematic parameter estimator applied to a bi-directional vehicle

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Abstract: This paper deals with the online estimation of the geometric and kinematic parameters of a wheeled mobile robot, with the objective of its precise navigation. To do this, the implementation of an algorithm for estimating these parameters is proposed. It is based on a quadratic criterion to be minimized, which is a function of the difference between the measured and estimated robot pose. The estimator inputs external measurements of robot positions and speeds and proprioceptive inertial and odometer measurements, and outputs an estimate of model parameters. Its expression does not depend on the position of the robot relative to the path to be followed and its possible tracking errors, nor on the speed of the tracking. The experimental implementation carried out on a real bi-directional container truck under realistic operating conditions has shown its performance.

Keywords: Automotive system identification and modelling; Autonomous Mobile Robots.

1. INTRODUCTION

One of the main challenges of mobile wheeled robots is their precise navigation along a trajectory. It depends among other things on their type of wheels and their configurations (see Tzafestas (2014) for an overview of wheels and chassis). This has an impact on the models of the robot to be considered and their parameters. Possible models are, for example, sensor models, actuator models, kinematic models, dynamic models or flexibility models. And an accurate identification of a robot model from measurements is very useful, either for state observation (Carvalho Filho et al. (2019)) or for the implementation of a model-based control of the robot (see Fig. 1). State observers can use Bayesian estimation theory tools such as the Extended Kalman Filter (EKF) or the Particle Filter (Thrun et al. (2005) or Siegwart and Nourbakhsh (2004)), while many model-based mobile robot controllers (Frank (2018)) are available, such as model reference adaptive control (Ashoorirad et al. (2006)) or model predictive control (MPC) (Rossiter (2003), Lenain et al. (2004)).

The state reconstructor or estimator is a system whose inputs are the inputs and outputs of the real process and whose output is an estimate of the state of that process. The process model can be linear or non-linear, continuous or discrete, deterministic or stochastic. The case of noisy systems, where random phenomena occur, is referred to filters such as EKF. For example, we can cite the case of an EKF-based kinematic parameter estimation for a passenger vehicle (Brunker et al. (2017)). Other deterministic methods have been proposed in the past. Non-linear model estimation can be done with the Gauss-Newton method (Norton (1986)) which ensures quadratic convergence. In that case, the estimation is done iteratively by minimizing the sum of the squares of errors between the measurement outputs and the outputs predicted by the current theoretical model. Since input measurement noise can lead to bias errors, this constraint can be processed with total least squares (Van Huffel and Vandewalle (1991)). Less accurate, the screw axis measurement (Hollerbach and Wampler (1996)) is a class of methods which measure the joint axes in the form of lines in space. One of these methods is the circle point analysis (Mooring et al. (1991)) for the moving of joints one by one to generate a circle around a measurement point. Other approaches use the Jacobian matrix (Bennett et al. (1992)), or even rely on the control theory. One example is the estimation of slip parameters using the sliding-mode control theory (Song et al. (2009)), based on the vehicle kinematic model and measurements.

Here, the deterministic case is investigated for the observation of the parameters of a discretized non-linear model of a mobile robot, for its implementation on the embedded computer of a real robotic system. The proposed new approach is similar to those mentioned above in that a quadratic criterion is minimized. Nevertheless, it has the advantage of having an expression that does not depend on trajectory tracking performance, and its expression is sufficiently general to be able to propose the estimation of any type of geometric or kinematic parameter of any type of wheeled robot model. The block diagram Fig. 1 gives an overview of the classic architecture of the control system (similar to the one of Cariou et al. (2009)).

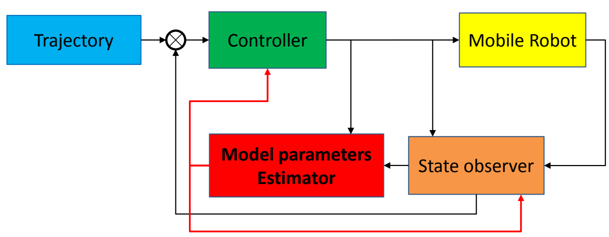

Fig. 1. Control block diagram

The Controller and the State observer are based on a kinematic model of the vehicle. The function of the Model parameters Estimator is to correct the parameters of this model online, from the controllers' control outputs and the state observer's position and speed outputs.

In particular, this parameter estimation makes it possible to compensate for wheel-ground sliding phenomena with dynamics dependent on the position measurement period, but also to estimate the value of parameters that are difficult to measure directly with precision, such as the effective wheel diameter (required for longitudinal odometry), steering angle offsets, or even the wheelbase.

This architecture ensuring an online correction of the model parameters, this will make it easier to manage high speed variations.

Thereafter, the kinematic model of a vehicle with steered front and rear axles is designed in section 2 for observation and control. The model chosen is horizontal plane - roll and pitch are not taken into account. Then, an estimator of the parameters of this model is designed in section 3. This estimator is applied to particular cases of parameters in section 4. Then, test results are presented in section 5 before the conclusion section 6.

2. KINEMATIC MODEL

The kinematic model used for the estimator is described in this section. The plan model of a bi-steered vehicle is chosen, it includes the usual case of the car. However, any other type of model could have been considered, the method remaining the same.

2.1 Equations of motion

The kinematic model is computed by using the Lie group formalism, described in detail for example by Murray et al. (1994). This allows more clarity in equations writing.

In the following, The frame $(\mathbf{f})$ is fixed to the vehicle front wheel centre with its longitudinal axis in the wheel steering direction. The same applies to frame $(\mathbf{r})$. $(\mathbf{0})$ is the world frame and $(\mathbf{m})$ is the vehicle body frame. $(\mathbf{c}_f)$ and $(\mathbf{c}_r)$ are the front and rear reference frames on the path. Notations are the following. ${}^j\mathbf{T}_i^k$ is the twist at the origin of frame (i) with respect to frame (j), expressed in frame (k). For simplicity, (k) is not specified if $k = i$. Notations are the same for the pose matrix ${}^j\mathbf{H}_i^k$. ${}^j\mathbf{Ad}_i$ is the adjoint matrix, for transporting frame (i) into frame (j). Notations are the same for ${}^j\mathbf{R}_i$, ${}^j\mathbf{p}_i$ and scalars.

In the Fig. 2, $\alpha_{f,r}$ are the front and rear steering angles, $\delta_{f,r}$ are the front and rear sliding angles and $\beta_{f,r} = \alpha_{f,r} + \delta_{f,r}$. Then, kinematic equations are defined in both frames $(\mathbf{f})$ and $(\mathbf{r})$ by the following twists:

$$ {}^o\mathbf{T}_f = \begin{bmatrix} \dot{\theta}_m + \dot{\beta}_f \\ v_f \\ 0 \end{bmatrix}, \ {}^o\mathbf{T}_r = \begin{bmatrix} \dot{\theta}_m + \dot{\beta}_r \\ v_r \\ 0 \end{bmatrix} \tag{1} $$

And twists of frames $(\mathbf{f})$ and $(\mathbf{r})$ relative to the vehicle body and expressed in these frames are written as follows:

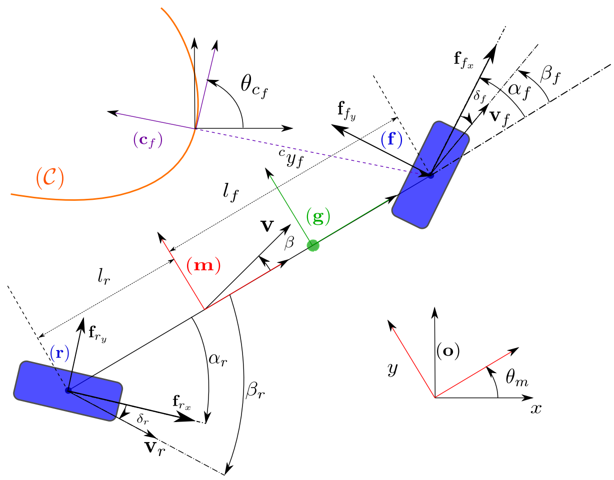

Fig. 2. Horizontal plane model of a bi-directional vehicle relative to its reference trajectory

$$ {}^m\mathbf{T}_f = \begin{bmatrix} \dot{\beta}_f \\ 0 \\ 0 \end{bmatrix}, \ {}^m\mathbf{T}_r = \begin{bmatrix} \dot{\beta}_r \\ 0 \\ 0 \end{bmatrix} \tag{2} $$

The twists defined by (1) are not independent. The combination of (1) and (2) allows to link the longitudinal velocities v_f and v_r in order to obtain the following expression of the yaw velocity $\dot{\theta}_m$ of the mobile:

$$ {}^o\mathbf{T}_m = {}^m\mathbf{Ad}_f \left({}^o\mathbf{T}_f - {}^m\mathbf{T}_f \right) = {}^m\mathbf{Ad}_r \left({}^o\mathbf{T}_r - {}^m\mathbf{T}_r \right) \tag{3} $$

with:

- ${}^m\mathbf{Ad}_{f,r} = \begin{bmatrix} 1 & 0_{1\times 2} \\ -{}^m\mathbf{p}_{\perp f,r} & {}^m\mathbf{R}_{f,r} \end{bmatrix}$;
- ${}^m\mathbf{p}_{f,r}$, the vectors $\mathbf{mf}$ and $\mathbf{mr}$ expressed in the mobile frame $(\mathbf{m})$;
- ${}^m\mathbf{p}_{\perp f,r}$, the vector obtained by a $\frac{\pi}{2}$ rotation of the vector ${}^m\mathbf{p}_{f,r}$;
- ${}^m\mathbf{R}_{f,r}$, a rotation matrix defining the direction of frames respectively $(\mathbf{f})$ and $(\mathbf{r})$ in the $(\mathbf{m})$ frame.

From the above equations, the following two equations are obtained:

$$ \begin{cases} \dot{\theta}_m &= v_f \sigma_f \\ v_r &= v_f \phi_f^v \end{cases} \tag{4} $$

with following variables:

- $l = l_f - l_r$ (l_f and l_r are signed with respect to the $(\mathbf{m})$ frame)
- $\sigma_f = \frac{\sin(\beta_f - \beta_r)}{l \cos \beta_r}$
- $\phi_f^v = \frac{\cos \beta_f}{\cos \beta_r}$

Similarly, taking v_r, $\dot{\beta}_f$ and $\dot{\beta}_r$ as independent variables leads to:

$$ \begin{cases} \dot{\theta}_m &= v_r \sigma_r \\ v_f &= v_r \phi_r^v \end{cases} \tag{5} $$

with following variables:

- $\sigma_r = \frac{\sin(\beta_f - \beta_r)}{l \cos \beta_f}$
- $\phi_r^v = \frac{\cos \beta_r}{\cos \beta_f}$

2.2 Estimation models

For the estimation, twists and positions of (**f**) and (**r**) frames are used. Differently from previous works (e.g. Cariou et al. (2009), Lenain et al. (2004) or Lenain et al. (2006)), the parameters estimation does not depend on a path. The major advantage is that it is not disturbed in case of significant path tracking error.

3. MODEL PARAMETERS ESTIMATOR ALGORITHM

The objective of this section is to define an estimator of geometric and kinematic parameters, based on the kinematic model defined in the previous section and by defining the variation of the parameter to be estimated such that it minimizes a quadratic criterion that is a function of the difference between the measured and estimated robot pose. For the estimator design, measured proprioceptive state variables are considered, such as:

- ω_f and ω_r, the averages of speeds of rotation of the front and rear wheel axles;
- α_f and α_r, the front and rear steering angles;
- $\dot{\alpha}_f$ and $\dot{\alpha}_r$, the front and rear steering velocities.

These odometry measurements are received from the vehicle at the sampling period dt. Inertial data could also have been considered. In practice, the controller's steering speed outputs are often used because they avoid measurement noise.

For the exteroceptive variables measurements, the following variables are assumed to be available:

- ${}^o\mathbf{p}_{m_{mes}}$, the measured position of frame (**m**) with respect to the reference frame (**o**);
- ${}^o\theta_{m_{mes}}$, the measured mobile robot orientation with respect to the reference frame (**o**).

The state observation is based on equation (3) and on the pose ${}^o\mathbf{H}_m = \begin{bmatrix} {}^o\theta_m \\ {}^o\mathbf{p}_m \end{bmatrix}$ of the mobile. Observed speeds and positions of (**f**) and (**r**) frames, are given by the following relationships:

$$\begin{cases} {}^o\hat{\mathbf{T}}_{f,r} &= {}^m\widehat{\mathbf{Ad}}_{f,r}^{-1}\,{}^o\hat{\mathbf{T}}_m + {}^m\hat{\mathbf{T}}_{f,r} \\ {}^o\hat{\mathbf{H}}_{f,r} &= {}^o\hat{\mathbf{H}}_m + \begin{bmatrix} \hat{\beta}_{f,r} \\ l_{f,r}\begin{bmatrix} \cos\hat{\theta}_m \\ \sin\hat{\theta}_m \end{bmatrix} \end{bmatrix} \end{cases} \quad (6)$$

These equations are established by considering the following properties of the mathematical expected value: $E(a\mathbf{X}+b\mathbf{Y}) = aE(\mathbf{X}) + bE(\mathbf{Y})$ with a and b real values; and $E(\mathbf{X}Y) = E(\mathbf{X})E(\mathbf{Y})$ only if $\mathbf{X}$ and $\mathbf{Y}$ are independent variables, which is the case here, $\beta_{f,r}$ being independent from $\dot{\theta}_m$ and $\mathbf{v}_m$.

For estimates of parameters, two successive state estimates of ${}^o\mathbf{H}_{f,r}$ are considered ${}^o\hat{\mathbf{H}}_{f,r_k}$ and ${}^o\hat{\mathbf{H}}_{f,r_{k+n}}$, synchronous with measures (n number of sampling steps between two measures). If the odometry is correct, the change between these two estimates is given by:

$$\left[{}^o\hat{\mathbf{H}}_{f,r_{k+n}}\right] = \left[{}^o\hat{\mathbf{H}}_{f,r_k}\right] \prod_{1\leq j\leq n} \mathrm{e}^{dt\left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right]} \quad (7)$$

where ${}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}$ is the odometry twist calculated at the $k+j$ step. It is important to note that, in general, the product of matrix exponentials is not equal to the exponential of the sum of these matrices, and in particular: $e^{dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right]} \neq e^{dt(\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right] + \left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right])}$.

The expression of ${}^o\hat{\mathbf{H}}_{f,r_{k+n}}$ can also be written as follows:

$${}^o\hat{\mathbf{H}}_{f,r_{k+n}} = \begin{pmatrix} {}^o\mathbf{R}_{f,r_k}\mathbf{R}_{(\omega_k dt)}\cdots\mathbf{R}_{(\omega_{k+j}dt)}\cdots \\ {}^o\mathbf{p}_{f,r_k} + {}^o\mathbf{R}^k_{f,r_k}\mathbf{p}_{k+1} + \cdots \end{pmatrix}.$$

Based on a variation of a parameter μ to identify, the variation of ${}^o\hat{\mathbf{H}}_{f,r_{k+n}}$ expressed in frames (**f**) and (**r**) by the equation (7), may be written as:

$$\frac{\partial}{\partial\mu}\left[{}^o\hat{\mathbf{H}}_{f,r_{k+n}}\right] = \frac{\partial}{\partial\mu}\left[{}^o\hat{\mathbf{H}}_{f,r_k}\right] \prod_{1\leq j\leq n} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right]} +$$
$$\left[{}^o\hat{\mathbf{H}}_{f,r_k}\right] \sum_{1\leq i\leq n} \Big(\prod_{1\leq j\leq i-1} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right]} \frac{\partial}{\partial\mu} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]}$$
$$\prod_{i+1\leq j\leq n} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right]}\Big)$$

The first term of the right member is null since μ is varying only after the step k. Then, by rearranging the first product:

$$\frac{\partial}{\partial\mu}\left[{}^o\hat{\mathbf{H}}_{f,r_{k+n}}\right] = \left[{}^o\hat{\mathbf{H}}_{f,r_k}\right] \prod_{1\leq j\leq n} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right]}$$
$$\sum_{1\leq i\leq n} \left(\prod_{i+1\leq j\leq n} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right]} \right)^{-1} e^{-dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]}$$
$$\frac{\partial}{\partial\mu} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]} \left(\prod_{i+1\leq j\leq n} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+j}_{k+j_{odo}}\right]} \right)$$

Consequently:

$$\frac{\partial}{\partial\mu}\left[{}^o\hat{\mathbf{H}}_{f,r_{k+n}}\right] = \left[{}^o\hat{\mathbf{H}}_{f,r_{k+n}}\right] \sum_{1\leq i\leq n} \Bigg[{}^{k+i}\mathbf{Ad}^{-1}_{k+n}$$
$$\left(e^{-dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]} \frac{\partial}{\partial\mu} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]} \right)^{\mathrm{v}} \Bigg]$$

with the exponent v meaning the vector form. The following first order approximation is given:

$$e^{-dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]} \frac{\partial}{\partial\mu} e^{dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]} \approx$$
$$\left(\mathbf{Id} - dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]\right) \frac{\partial}{\partial\mu} \left(\mathbf{Id} + dt\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]\right)$$
$$\approx dt \frac{\partial}{\partial\mu} \left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right]$$

As a result:

$$\left[{}^o\hat{\mathbf{H}}_{f,r_{k+n}}\right]^{-1} \frac{\partial}{\partial\mu}\left[{}^o\hat{\mathbf{H}}_{f,r_{k+n}}\right] \approx$$
$$dt \sum_{1\leq i\leq n} \left[{}^{k+i}\mathbf{Ad}^{-1}_{k+n} \left(\frac{\partial}{\partial\mu}\left[{}^o\hat{\mathbf{T}}^{k+i}_{k+i_{odo}}\right] \right)^{\mathrm{v}} \right]$$

And:

$$\left(\left[{}^{o}\hat{\mathbf{H}}_{f,r_{k+n}}\right]^{-1}\frac{\partial}{\partial\mu}\left[{}^{o}\hat{\mathbf{H}}_{f,r_{k+n}}\right]\right)^{\mathrm{v}}\approx dt\sum_{1\leq i\leq n}{}^{k+i}\mathbf{Ad}_{k+n}^{-1}\frac{\partial}{\partial\mu}{}^{o}\hat{\mathbf{T}}_{k+i_{odo}}^{k+i} \quad (8)$$

A criterion C to be minimized is then considered:

$$\mathrm{Min}_{\mu}C = \frac{1}{2}\left\|\left\{\left[{}^{o}\hat{\mathbf{H}}_{f,r_{k+n}}\right]^{-1}\left(\left[{}^{o}\mathbf{H}_{f,r_{mes}}\right]-\left[{}^{o}\hat{\mathbf{H}}_{f,r_{k+n}}\right]\right)\right\}^{\mathrm{v}}\right\|^{2}$$

To simplify the notations, it is rewritten as follows:

$$\mathrm{Min}_{\mu}C = \frac{1}{2}\left\|\left([\mathbf{H}]^{-1}\delta[\mathbf{H}]\right)^{\mathrm{v}}\right\|^{2}$$

And its derivative is calculated:

$$\begin{aligned}\frac{\partial}{\partial\mu}\left([\mathbf{H}]^{-1}([\mathbf{H}_{mes}]-[\mathbf{H}])\right) &= \frac{\partial}{\partial\mu}\left([\mathbf{H}]^{-1}[\mathbf{H}_{mes}]\right)\\ &= -[\mathbf{H}]^{-1}\frac{\partial[\mathbf{H}]}{\partial\mu}[\mathbf{H}]^{-1}[\mathbf{H}_{mes}]\\ &= -[\mathbf{H}]^{-1}\frac{\partial[\mathbf{H}]}{\partial\mu}\left(\mathbf{Id}+[\mathbf{H}]^{-1}\delta[\mathbf{H}]\right)\end{aligned}$$

For a small variation of μ:

$$\frac{\partial}{\partial\mu}\left([\mathbf{H}]^{-1}\delta[\mathbf{H}]\right)\delta\mu = -[\mathbf{H}]^{-1}\frac{\partial[\mathbf{H}]}{\partial\mu}\delta\mu-[\mathbf{H}]^{-1}\frac{\partial[\mathbf{H}]}{\partial\mu}[\mathbf{H}]^{-1}\delta[\mathbf{H}]\delta\mu$$

The second term of the right member is considered null for a second order variation.
Therefore, the variation of criterion C for a variation of μ is:

$$\delta_{\mu}C = \left([\mathbf{H}]^{-1}\delta[\mathbf{H}]\right)^{\mathrm{v}t}\frac{\partial}{\partial\mu}\left([\mathbf{H}]^{-1}\delta[\mathbf{H}]\right)^{\mathrm{v}}\delta\mu\approx -\left([\mathbf{H}]^{-1}\delta[\mathbf{H}]\right)^{\mathrm{v}t}\left([\mathbf{H}]^{-1}\frac{\partial[\mathbf{H}]}{\partial\mu}\right)^{\mathbf{v}}\delta\mu$$

To guarantee a negative or null value of this derivative, $\delta\mu$ is chosen to be:

$$\begin{aligned}\delta\mu &= +l_{\mu}\left([\mathbf{H}]^{-1}\frac{\partial[\mathbf{H}]}{\partial\mu}\right)^{\mathrm{v}t}\left([\mathbf{H}]^{-1}\delta[\mathbf{H}]\right)^{\mathrm{v}}\\ &= +l_{\mu}\left([\mathbf{H}]^{-1}\frac{\partial[\mathbf{H}]}{\partial\mu}\right)^{\mathrm{v}t}\left(\left[{}^{o}\hat{\mathbf{T}}_{f,r_{k+n}mes}^{f,r_{k+n}}\right]\right)^{\mathrm{v}}\end{aligned}$$

where l_{μ} is a positive factor.
This last equality is clarified:

$$[\mathbf{H}]^{-1}\delta[\mathbf{H}] = \begin{pmatrix}\mathbf{R}^{t} & -\mathbf{R}^{t}\mathbf{p}\\ \mathbf{0}_{1\mathrm{x}2} & 1\end{pmatrix}\begin{pmatrix}\delta\mathbf{R} & \delta\mathbf{p}\\ \mathbf{0}_{1\mathrm{x}2} & 0\end{pmatrix}$$

At first order:

$$\begin{aligned}\mathbf{R}^{t}\delta\mathbf{R} &= \mathbf{R}^{t}\mathbf{R}_{mes}-\mathbf{Id}_{2\mathrm{x}2}\\ &= \begin{pmatrix}\cos(\delta\theta) & -\sin(\delta\theta)\\ \sin(\delta\theta) & \cos(\delta\theta)\end{pmatrix}-\mathbf{Id}_{2\mathrm{x}2}\\ &\approx \delta\theta\mathbf{J}\end{aligned}$$

with $\mathbf{J} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$. Then:

$$[\mathbf{H}]^{-1}\delta[\mathbf{H}]\approx\begin{pmatrix}\delta\theta\mathbf{J} & \mathbf{R}^{t}\delta\mathbf{p}\\ \mathbf{0}_{1\mathrm{x}2} & 0\end{pmatrix} = [\mathbf{T}_{\delta}]$$

with $\mathbf{T}_{\delta} = \begin{pmatrix}\delta\theta\\ \mathbf{R}^{t}\delta\mathbf{p}\end{pmatrix}$ being ${}^{o}\hat{\mathbf{T}}_{f,r_{k+n}mes}^{f,r_{k+n}}$ here.

Finally, by using equation (8), the following result is obtained:

$$\begin{cases}\delta\mu = l_{\mu}dt\sum_{1\leq i\leq n}\frac{\partial}{\partial\mu}{}^{o}\hat{\mathbf{T}}_{k+i_{odo}}^{k+i^{t}}\;{}^{k+i}\mathbf{Ad}_{k+n}^{-t}\;{}^{o}\hat{\mathbf{T}}_{f,r_{k+n}mes}^{f,r_{k+n}}\\ \hat{\mu}_{+} = \hat{\mu}_{-}+\delta\mu\end{cases} \quad (9)$$

The estimator (9) is applied to some μ parameter cases, namely wheel diameter, wheelbase and sideslip angle, in the following section.

4. PARAMETER ESTIMATES

4.1 Wheel diameter estimate

Estimating the equivalent wheel diameter is equivalent to estimating the longitudinal slippage. For estimating the equivalent wheel diameter, equations (1) are considered. To simplify them, $\hat{\beta}_f$ and $\hat{\beta}_r$ are supposed to be invariant during a sampling step, so odometry twists ${}^{o}\mathbf{T}_{f,r_j}$ are simplified accordingly.

Then, by using equations (4) and (5), and that $v_{f,r_j} = \frac{d_{wheel}}{2}\omega_{f,r_j}$, the partial derivative of the odometry twist ${}^{o}\hat{\mathbf{T}}_{f,r_j}$ with respect to d_{wheel} is given by:

$$\frac{\partial{}^{o}\hat{\mathbf{T}}_{f,r_j}}{\partial d_{wheel}} = \frac{1}{2}\begin{bmatrix}\hat{\omega}_{f,r_j}\hat{\sigma}_{f,r_j}\\ \hat{\omega}_{f,r_j}\\ 0\end{bmatrix} \quad (10)$$

Depending on whether the front or rear wheels of the mobile robot are actuated, the expression with ${}^{o}\hat{\mathbf{T}}_{f_j}$ or ${}^{o}\hat{\mathbf{T}}_{r_j}$ will be used. The Estimator (9) can then be used with a gain $l_{d_{wheel}}$.

4.2 Wheelbase estimate

For estimating the mobile robot wheelbase, odometry twists ${}^{o}\hat{\mathbf{T}}_{f,r_j}$ are simplified as previously, with $\hat{\beta}_f$ and $\hat{\beta}_r$ being supposed to be invariant during a sampling step. And, by using equations (4) and (5), the partial derivative of the odometry twist ${}^{o}\hat{\mathbf{T}}_{f,r_j}$ with respect to l is given by:

$$\frac{\partial{}^{o}\hat{\mathbf{T}}_{f,r_j}}{\partial l} = \begin{bmatrix}-\frac{1}{l}\hat{v}_{f,r_j}\hat{\sigma}_{f,r_j}\\ 0\\ 0\end{bmatrix} \quad (11)$$

The Estimator (9) is then used with a gain l_l.

4.3 Sideslip angles estimate

For estimating the sideslip angles, $\hat{\beta}_f$ and $\hat{\beta}_r$ are also supposed to be invariant during a sampling step, and sideslip angles are now taken into account. Thus, odometry twists are rewritten differently from equation (1) as follows:

$${}^{o}\hat{\mathbf{T}}_{f,r_j} = \begin{bmatrix}\hat{\dot{\theta}}_m\\ \hat{v}_{f,r_j}\cos\hat{\delta}_{f,r_j}\\ \hat{v}_{f,r_j}\sin\hat{\delta}_{f,r_j}\end{bmatrix} \quad (12)$$

Considering an actuation of the front wheels by using equations (4), partial derivatives of odometry twists ${}^{o}\hat{\mathbf{T}}_{f,r_j}$ with respect to sliding angles $\delta_{f,r}$ are given by:

$$\begin{cases} \dfrac{\partial^o \hat{\mathbf{T}}_{f_j}}{\partial \delta_f} = \begin{bmatrix} \hat{v}_{f_j} \hat{\psi}^f_{f_j} \\ 0 \\ \hat{v}_{f_j} \end{bmatrix} \\ \dfrac{\partial^o \hat{\mathbf{T}}_{r_j}}{\partial \delta_r} = \begin{bmatrix} \hat{v}_{f_j} \left(-\hat{\psi}^f_{f_j} + \hat{\phi}^r_{f_j} \hat{\sigma}_{f_j} \right) \\ 0 \\ \hat{v}_{f_j} \hat{\phi}^v_{f_j} \end{bmatrix} \end{cases} \tag{13}$$

with:

- $\psi^f_f = \frac{\cos(\beta_f - \beta_r)}{l \cos \beta_r}$;
- $\phi^r_f = \tan \beta_r$;

Considering an actuation of the rear wheels by using equations (5), partial derivatives of odometry twists ${}^o\hat{\mathbf{T}}_{f,r_j}$ with respect to sliding angles $\delta_{f,r}$ are given by:

$$\begin{cases} \dfrac{\partial^o \hat{\mathbf{T}}_{f_j}}{\partial \delta_f} = \begin{bmatrix} \hat{v}_{r_j} \left(-\hat{\psi}^r_{r_j} + \hat{\phi}^f_{r_j} \hat{\sigma}_{r_j} \right) \\ 0 \\ \hat{v}_{r_j} \hat{\phi}^v_{r_j} \end{bmatrix} \\ \dfrac{\partial^o \hat{\mathbf{T}}_{r_j}}{\partial \delta_r} = \begin{bmatrix} \hat{v}_{r_j} \hat{\psi}^r_{r_j} \\ 0 \\ \hat{v}_{r_j} \end{bmatrix} \end{cases} \tag{14}$$

with:

- $\psi^r_r = \frac{-\cos(\beta_f - \beta_r)}{l \cos \beta_f}$;
- $\phi^f_r = \tan \beta_f$;

Terms $\cos \delta_{f,r_j}$ and $\sin \delta_{f,r_j}$ are simplified in equations (13) and (14).

Estimator (9) is then used with the gains l_{δ_f} and l_{δ_r} for the estimation of respectively δ_f and δ_r.

5. CALIBRATION OF A BI-STEERABLE CONTAINER TRUCK

5.1 Description of the test

If the parameter estimator can be used to estimate online parameters during vehicle navigation, it can also be used to estimate steering offsets. This is what has been done for the front and rear directions of an electric prototype container truck, designed for commercial port operations. It is an $18tons$ empty truck, with a $2m$ track and an $11m$ wheelbase. The actuation of the front and rear steering axles is hydraulic, by flow control servovalves. The low-level control is a simple loop proportional to the steering angle error. Since the front wheels are electrically actuated, equations (13) are used.

Proprioceptive sensor data are angular increments of the sensors on both front wheels and the crown steering angle of front and rear axles. Those data are received from the vehicle low-level controller through a CAN bus every $10ms$. Exteroceptive sensors used for this test are a lidar and six pairs of ultrasonic distance sensors ($24ms$ response time, $0.069mm$ resolution, range between $20mm$ and $150mm$). The measured position and orientation of the truck are obtained from the data of these sensors after filtering. This test requires a high degree of accuracy and is performed inside a docking station where containers are placed and deposited by trucks. Outside this docking station, the laser allows the truck to be positioned relative to it. Inside, ultrasound is used to position the truck in relation to the flat walls of this station. The truck moving at low speed, and the ultrasonic sensors being already almost orthogonal to the wall, this allows an accurate millimetric measurement.

Then, the steering sensor calibration process is performed using the sideslip angle estimator while the truck moves in a straight line for $30s$ with zero front and rear steering angle setpoints, at a low speed of $0.25m/s$. Tests are carried out under realistic conditions on asphalt roads.

5.2 Results analysis

The position of the truck is shown in Fig. 3, with a solid black line for the position measured in the centre of the vehicle. The orientation measured at the centre of the vehicle is shown in Fig. 4 in solid black line.

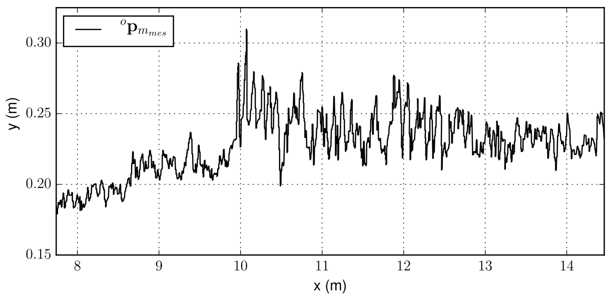

Fig. 3. Truck path tracking

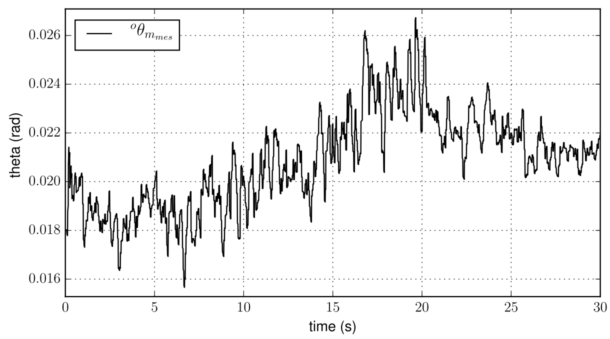

Fig. 4. Measured orientation

It is noticeable that the position is quite noisy with an amplitude of about 2 to $3cm$. This is mainly due to the long length of the truck, which makes it difficult to position it accurately, as any change in the orientation of the vehicle has an impact on its lateral positioning. The walls of the docking station, which are not perfectly flat, can also deteriorate the quality of ultrasound measurements.

As for the measured orientation, although almost zero, it fluctuates slightly between $0.016rad$ ($0.9deg$) and $0.026rad$ ($1.5deg$) with a progressive increase until $20s$ before decreasing again. These fluctuations have a significant influence on the estimation of sideslip angles, with the measured position and orientation terms being used in the expression of the twist ${}^{o}\hat{\mathbf{T}}^{f,r_{k+n}}_{f,r_{k+n}mes}$ used in equation (9).

The sideslip angles are shown in Fig. 5, as a cyan solid line for the front axle and a dotted blue line after filtering, an orange solid line for the rear axle and a red solid line after filtering.

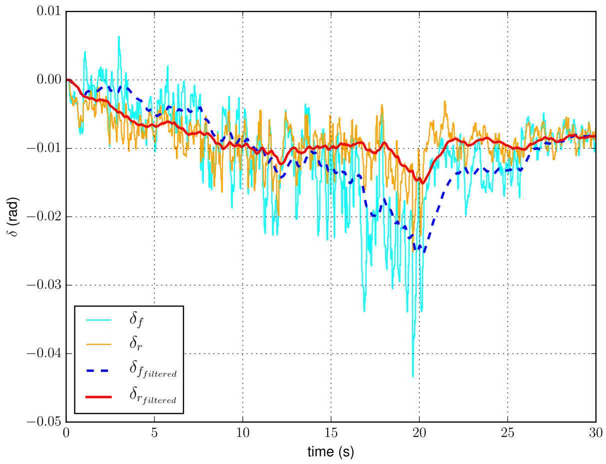

Fig. 5. Sideslip angles

The front and rear sideslip angles take about $10s$ to converge. The value reached is about $0.01rad$ for both axles. Then, a variation that is maximum at $t = 20s$ is observed, then again the curves converge towards the value of $0.01rad$. This clearly shows the dependence of the algorithm on measurement noise and quality, and here in particular on the orientation measurement of the vehicle ${}^{o}\theta_{m_{mes}}$.

6. CONCLUSION

An estimator of the geometric and kinematic parameters of wheeled mobile robots was presented. This estimator was applied to the particular case of a bi-steered vehicle model, although it would also be appropriate for any other type of kinematics, such as the multibody system presented by Lucet and Micaelli (2019). It allows an online update of control or observation models, or a calibration of odometric sensors. This algorithm has the advantage of being independent of the trajectory to be followed, and it is insensitive to tracking errors.

Experimental results have demonstrated its effectiveness in accurately estimating parameters, with a reasonable convergence time of a few seconds. However, it remains sensitive to exteroceptive measuring noises of the position and orientation of the vehicle, which therefore remain to be verified for the correct use of this estimator.

An extension to parameter estimation of a dynamic model is envisaged, although additional difficulties of dependence on the speed of the robot have to be addressed.

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