Stability and Stabilizability of TS-based Interconnected Nonlinear Systems ⋆

Souhail TIKO * **Fouad MESQUINE** * **Ahmed EL HAJJAJI** **

* *UCAM, FSSM, Physics Department, Marrakesh, Morocco (e-mail: souhail.tiko@gmail.com, mesquine@uca.ac.ma).*
** *University of Jules-Vernes Picaride, France (e-mail: Ahmed.Hajjaji@u-picardie.fr)*

Abstract: This paper aims to establish stability conditions for large scale nonlinear systems with unknown bounded interconnection terms. These conditions are then used to design of stabilizing controllers for such systems. Takagi-Sugeno (TS) fuzzy modeling is used to describe the nonlinear interconnected systems. Further, the drawbacks of using slack variables by adding null term and the Lyapunov fuzzy function in terms of increasing conservatism are highlighted. Unlike the continuous time TS fuzzy systems, the null product technique, when applied to continuous-time TS fuzzy large-scale systems (CFLSS) leads to more conservative results. Numerical examples are presented to illustrate our theoretical findings.

Keywords: Interconnected nonlinear systems, Fuzzy models, Large scale systems, Stability analysis, Stabilization design, Lyapunov methods, Fuzzy Lyapunov functions, LMI's.

1. INTRODUCTION

Recently, large-scale systems, with interconnected subsystems, such as power systems, industrial processes, and communication networks have been continuously studied in Šiljak (1991), Jiang (2004), Zečević and Šiljak (2010). Besides nonlinearities, some problems such as high dimension, controller structural constraints and unknown information of the interconnection terms are the main challenges in the control of the large-scale systems. Hence, control design for such systems is a difficult task. To overcome these problems and to control more efficiently, the decentralized control technique is widely constructed for each subsystem of the large-scale system instead of the entire system. Further, the Takagi-Sugeno fuzzy modeling is one of the most efficient nonlinear control techniques used for large-scale nonlinear systems.
In fact, the Takagi-Sugeno fuzzy model Takagi et al. (1985) has been widely applied in the stability analysis and/or control synthesis of nonlinear systems. Accordingly, large-scale interconnected systems based on TS fuzzy models are extensively investigated Tseng and Chen (2001), Hsiao et al. (2005), Wang (2005), Wang et al. (2005), Tseng (2008). Most of studies of the large-scale systems focus their works on known interconnection terms. Whereas, Stanković et al. (2007, 2009) deal with the unknown bounded interconnection problem. Hence, the problem of the maximum interconnection bound has been studied in Koo et al. (2013); Yang et al. (2016); Koo et al. (2016); Kim et al. (2018). However, few studies, to the authors knowledge, solve the unknown interconnection problem. Consequently, the first motivation of this paper is to contribute in this sense. On the other hand, different Lyapunov function families are utilized extensively for the TS fuzzy models, such as common quadratic Lyapunov functions and fuzzy Lyapunov functions Tanaka et al. (2003). Generally, the use of the fuzzy Lyapunov function (FLF) to obtain stabilization conditions cannot be simply expressed in LMI's forms. To overcome this problem, a systematic approach to reduce conservativeness in stability analysis and control design for continuous-time TS fuzzy systems is presented in Mozelli et al. (2009). The proposed approach is based on defining fuzzy Lyapunov function and introducing the null product technique. This last introduces slack matrix variables for decoupling the Lyapunov matrices and the system matrices. As a result, stability and stabilization conditions are converted into LMIs differently from the methods presented in Tanaka et al. (2003).

Usually, when relying on the FLF, for continuous-time TS fuzzy systems (CFS), to design stability and stabilization conditions, the null product technique given in Mozelli et al. (2009) is one of the most efficient techniques Faria et al. (2013); Xie et al. (2015, 2016). The extension to the case of continuous-time TS Fuzzy Large Scale Systems (CFLSS) is obtained in El Hajjaji et al. (2016); Benzaouia et al. (2016); Kim et al. (2018). However, the impact of the null product technique on stability and stabilization conditions has not been studied until now for CFLSS. Whence, detailed and comparative study is needed for such systems with unknown interconnection. Accordingly, the aim of this work is first to extend stability and stabilizability conditions obtained for fuzzy systems to the case of interconnected nonlinear systems. Second,

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we focus on the fact that restrictive stability conditions for fuzzy systems obtained using quadratic Lyapunov function, may be non restrictive in the case of large scale nonlinear systems. Especially, when important terms of interconnection are concerned. A comparison is performed on numerical examples to confirm our claim.
The remainder of the paper is given as follows. First, some notations and acronyms used along the paper are specified. The studied problem is formulated and the fuzzy model of the nonlinear systems is recalled in Section II. Section III, is devoted to some preliminaries useful in the sequel. Section IV begins by extending stability conditions to the case of large scale systems with unknown interconnection using quadratic Lyapunov function and fuzzy Lyapunov function. Feasibility comparison on the designed conditions is given on a numerical example. Further, and as a second step, the stabilization conditions are derived for both cases and comparison is performed on a numerical example where the PDC controllers are derived. The drawback of the null product method, in the control law design of large scale systems, is pointed out on the example. Section V concludes the paper with some remarks.

1.1 Notations and Acronyms

For a symmetric matrix A, $A > 0$ denotes A positive definite. $Sym(A)$ stands for $A + A^T$. To reduce space use, the following acronyms will be used throughout the paper: **TS** Takagi Sugeno, **LSS** Large Scale System, **CFLSS** Continuous-time Takagi-Sugeno Fuzzy Large Scale Systems, **CFS** continuous-time Takagi-Sugeno fuzzy systems, **QLF** Quadratic Lyapunov Function, **FLF** fuzzy Lyapunov function.

2. PROBLEM FORMULATION

Consider a continuous-time large scale system composed of J interconnected nonlinear subsystems. The state evolution of each nonlinear subsystem may be written as follows:

$$\dot{x}_i(t) = g_i\big(x_i(t), u_i(t)\big) + h_i(x(t)), i = 1, \dots, J \quad (1)$$

where $g_i\big(x_i(t), u_i(t)\big)$ is a piecewise continuous vector function, $x_i(t) \in \mathbb{R}^{n_{ix}}$ the state vector, $u_i(t) \in \mathbb{R}^{n_{iu}}$ the input vector, for the i^{th} subsystem. Further, $x(t) = [x_1(t)^T x_2(t)^T \dots x_J(t)^T]^T$ notes the state of the entire system and $h_i(x)$ the interconnection term traducing the effect of other subsystems $j \neq i$ on subsystem i. Due to its excellent approximation, TS modeling is used to represent each nonlinear subsystem. To this end, one can define, for the i^{th} subsystem, r_i rules and s fuzzy sets $F^l_{ik}, k = 1, 2, \dots, s$ leading to the following:

$$S_i \begin{cases} \mathit{Rule}\ \ l\ :\mathit{IF}\ \ z_{i1}(t)\ \mathit{is}\ F^l_{i1}\ \mathit{and}\dots\ z_{is}(t)\ \mathit{is}\ F^l_{is} \\ \mathit{THEN}\ \ \dot{x}_i(t) = A^l_i x_i(t) + B^l_i u_i(t) + h_i(x(t)) \end{cases} \quad (2)$$

with $l = 1, 2, \dots, r_i$, and A^l_i, B^l_i real matrices of appropriate dimensions. $z_{ik}(t), k = 1, 2, \dots, s$ are the premise variables.
Using the singleton fuzzyfier to the IF-THEN rule (2), product inference and the center-average defuzzification, the state of the i^{th} subsystem may be written as:

$$\begin{aligned} \dot{x}_i(t) &= \sum_{l=1}^{r_i} \mu^l_i(z_i(t))\Big(A^l_i x_i(t) + B^l_i u_i(t) + h_i(x(t))\Big) \\ &=: A_i(\mu_i)x_i(t) + B_i(\mu_i)u_i(t) + h_i(x(t)) \end{aligned} \quad (3)$$

where $\mu^l_i(z_i(t))$ is a fuzzy membership function in the i^{th} subsystem such that for $l = 1, 2, \dots, r_i$.:

$\mu^l_i(z_i(t)) \geq 0$ and $\sum_{l=1}^{r_i} \mu^l_i(z_i(t)) = 1$. The interconnection term in our case is assumed bounded but unknown to generalize the case where it is given. Hence, the following assumption will be taken into account:

Assumption 1. Stanković et al. (2007) Stanković et al. (2009) Koo et al. (2013) Vector functions $h_i(x)$, $i = 1, \dots, J$ are unknown but satisfy quadratic inequality:

$$h^T_i(x)h_i(x) \leq \alpha^2_i x^T (H_i)^T H_i x$$

where α^2_i are bound scalars of the interconnection terms, and H_i are constant matrices with appropriate dimension.

The problem addressed below is to extend fuzzy techniques stability and stabilizability to large scale or interconnected nonlinear systems. QLF and FLF are used to derive new less restrictive conditions. For this last, the null product introducing slack variables and decoupling system matrices from Lyapunov ones is used. A comparison between the obtained conditions is performed. Hence, it is shown that, contrary to TS fuzzy systems the FLF-based conditions are not always less restrictive than QLF-based conditions for large scale systems.

3. PRELIMINARIES

The following technical lemmas will be employed to develop the results below.

Lemma 1. Petersen (1987) For all matrices $X, Y \in \mathbb{R}^{v \times w}$ and $\varepsilon > 0$, the following inequality holds true:

$$X^T Y + Y^T X \leq \varepsilon X^T X + \frac{1}{\varepsilon} Y^T Y.$$

Lemma 2. Assume Assumption 1 holds true, and let $H_i = [H_{i1}\ H_{i2}\ \cdots\ H_{iJ}]$ where H_{ij} has n_{jx} columns; then the following inequality holds true:

$$\sum_{i=1}^{J} h^T_i(x)h_i(x) \leq \sum_{i=1}^{J}\sum_{j=1}^{J} J\alpha^2_j\ x^T_i (H_{ji})^T H_{ji} x_i.$$

Proof. The interconnection satisfies Assumption 1, that is

$$\begin{aligned} \sum_{i=1}^{J} h^T_i(x)h_i(x) &\leq \sum_{i=1}^{J} \alpha^2_i x^T (H_i)^T H_i x \\ &= \sum_{i=1}^{J} \alpha^2_i \sum_{j=1}^{J}\sum_{k=1}^{J} x^T_j (H_{ij})^T H_{ik} x_k. \end{aligned}$$

Based on Lemma 1, it is easy to obtain

$$\sum_{i=1}^{J} h_i^T(x)h_i(x) \leq \sum_{i=1}^{J}\alpha_i^2\sum_{j=1}^{J}\sum_{k=1}^{J}\frac{1}{2}\big(x_j^T(H_{ij})^T H_{ij}x_j + x_k^T(H_{ik})^T H_{ik}x_k\big)$$
$$=\sum_{i=1}^{J}\sum_{j=1}^{J} J\alpha_i^2\ x_j^T(H_{ij})^T H_{ij}x_j$$
$$=\sum_{i=1}^{J}\sum_{j=1}^{J} J\alpha_j^2\ x_i^T(H_{ji})^T H_{ji}x_i.$$

Similar inequality as presented above have been used in Koo et al. (2013); Kim et al. (2018); Yang et al. (2016); Koo et al. (2016) to estimate unknown interconnection terms but without scalar J. However, the equality "$x^T(H_i)^T H_i x = \sum_{j=1}^{J} x_j^T(H_{ij})^T H_{ij}x_j$" , used in these works, may be questionable.

4. MAIN RESULTS

Two stability conditions are now derived from fuzzy techniques applied to the case of large scale nonlinear systems. The first is based on the **QLF** and the second on the **FLF** with the null product technique. Comparison is performed on a numerical example. Further, stabilization conditions are deduced. Design of stabilizing controllers is performed in both cases to show the applicability and the performance of the proposed techniques. Numerical simulations comparing the two approaches are also presented.

4.1 Stability conditions for the CFLSS using the QLF:

New stability condition based on the **QLF** is presented for **CFLSS** in the following Theorem.

Theorem 3. If there exist matrices $P_i = P_i^T > 0$, such that for $l = 1, \dots, r_i$ $i = 1, \dots, J$

$$\begin{bmatrix} Sym(P_iA_i^l) + \sum_{j=1}^{J} J\alpha_j^2\ (H_{ji})^T H_{ji} & * \\ P_i & -\mathbb{I} \end{bmatrix} < 0 \qquad (4)$$

then the **CFLSS** with $u(t) = 0$ in (3) is asymptotically stable.

Proof. Consider the **QLF** $V_1(x,t)$ given by:

$$V_1(t) = \sum_{i=1}^{J} x_i(t)^T P_i x_i(t)$$

Its time-derivative is:

$$\dot{V}_1(t) = \sum_{i=1}^{J}\big\{\dot{x}_i^T(t)P_ix_i(t) + x_i^T(t)P_i\dot{x}_i(t)\big\}$$
$$= \sum_{i=1}^{J}\big\{x_i^T Sym\big(P_iA_i(\mu_i)\big)x_i + h_i^T(x)P_ix_i + x_i^TP_ih_i(x)\big\}. \qquad (5)$$

Applying Lemma 1 to (5) produces

$$\dot{V}_1(t) \leq \sum_{i=1}^{J}\big\{x_i^T\big(Sym\big(P_iA_i(\mu_i)\big) + P_iP_i\big)x_i + h_i^T(x)h_i(x)\big\} \qquad (6)$$

Applying Lemma 2 to (6) yields

$$\dot{V}_1(t) \leq \sum_{i=1}^{J} x_i^T\big\{Sym\big(P_iA_i(\mu_i)\big) + P_iP_i + \sum_{j=1}^{J} J\alpha_j^2\ (H_{ji})^T H_{ji}\big\}x_i \qquad (7)$$
$$= \sum_{i=1}^{J}\sum_{l=1}^{r_i}\mu_i^l x_i^T\Phi_i^l x_i. \qquad (8)$$

where

$$\Phi_i^l = Sym\big(P_iA_i^l\big) + P_iP_i + \sum_{j=1}^{J} J\alpha_j^2\ (H_{ji})^T H_{ji}$$

From (8), if conditions $P_i > 0$ and $\Phi_{ij}^l < 0$ are satisfied then, $\dot{V}_1 < 0$ which implies the asymptotic stability of **CFLSS** (3). By applying Schur complement to $\Phi_i^l < 0$, one obtains (4).

Remark 1. If LMIs (4) are feasible with maximum scalar bounds of interconnection $\bar{\alpha}_j$. then they are feasible for all $\alpha_j \leq \bar{\alpha}_j$. In fact, one can notice that for all $\alpha_j < \bar{\alpha}_j$

$$\alpha_j^2(H_{ji})^T H_{ji} < \bar{\alpha}_j^2(H_{ji})^T H_{ji}$$

and hence feasibility is obtained for all α_j's less than $\bar{\alpha}_j$.

4.2 Stability conditions for the CFLSS using the FLF:

Consider the fuzzy Lyapunov function (**FLF**):

$$V_2(t) = \sum_{i=1}^{J}\sum_{l=1}^{r_i}\mu_i^l(x_i(t))x_i^T(t)P_i^lx_i(t),$$
$$=: x_i^T(t)P_i(\mu_i)x_i(t) \qquad (9)$$

The use of the **FLFs** always invokes the following problems:

(1) Stability conditions have a large number of inequalities as well as stabilization conditions.
(2) In general case when using the PDC control law, the stabilization conditions are expressed in terms of bilinear matrix inequalities (BMIs) .

The above problems have been solved by introducing the null product technique in Mozelli et al. (2009) for **CFS** as

$$\sum_{i}^{J} 2[x_i^TM_{1i} + \dot{x}_i^TM_{2i}]\times[-\dot{x}_i + A_i(\mu_i)x_i + h_i(x)] = 0 \quad (10)$$

Hence, Slack matrices are added to decouple the Lyapunov matrices from system matrices. As a result, stability and stabilization conditions have been written as linear matrix inequalities (LMIs), a by-product of this technique is the reduction in the number of LMIs. The following theorem presents the stability condition extension to **CFLSS** based on the **FLF** with the null product (10). Let $\phi_i^\rho(\rho = 1, 2, ..., r_i)$ be given scalars to satisfy $|\ \dot{\mu}_i^\rho\ | \leqslant \phi_i^\rho$.

Theorem 4. If there exist matrices $P_i^l = {P_i^l}^T > 0$, $\Lambda_i^l = {\Lambda_i^l}^T$, M_{1i} and M_{2i} such that for $l, \rho = 1, \dots, r_i; i = 1, \dots, J$

$$P_i^\rho + \Lambda_i^l > 0 \qquad (11)$$

$$\begin{bmatrix} \sum_{\rho=1}^{r_i}\phi_i^\rho(P_i^\rho+\Lambda_i^l)+Sym(M_{1i}A_i^l) & & & \\ +2\sum_{j=1}^{J}J\alpha_j^2\ (H_{ji})^T H_{ji} & * & * & * \\ P_i^l-M_{1i}^T+M_{2i}A_i^l & -Sym(M_{2i}) & * & * \\ M_{1i}^T & 0 & -\mathbb{I} & * \\ 0 & M_{2i}^T & 0 & -\mathbb{I} \end{bmatrix} < 0 \tag{12}$$

then **CFLSS** with $u(t)=0$ in (3) is asymptotically stable.

Proof. Consider FLF (9), its time-derivative is:

$$\dot{V}_2(t)=\sum_{i=1}^{J}\{\dot{x}_i^T P_i(\mu_i)x_i+x_i^T(t)P_i(\mu_i)\dot{x}_i+x_i^T\dot{P}_i(\mu_i)x_i\}.$$

by applying null product (10) it follows that:

$$\begin{aligned}\dot{V}_2(t)=\sum_{i=1}^{J}\{&\dot{x}_i^T P_i(\mu_i)x_i+x_i^T(t)P_i(\mu_i)\dot{x}_i+x_i^T\dot{P}_i(\mu_i)x_i\\ &+2[x_i^T M_{1i}+\dot{x}_i^T M_{2i}]\times[-\dot{x}_i+A_i(\mu_i)x_i+h_i(x)]\}.\end{aligned} \tag{13}$$

As (5)-(7), one obtains:

$$\begin{aligned}\dot{V}_2(t)\le\sum_{i=1}^{J}\{&\dot{x}_i^T P_i(\mu_i)x_i+x_i^T(t)P_i(\mu_i)\dot{x}_i+x_i^T\dot{P}_i(\mu_i)x_i\\ &-2x_i^T M_{1i}\dot{x}_i-2\dot{x}_i^T M_{2i}\dot{x}_i+2x_i^T M_{1i}A_i(\mu_i)x_i\\ &+2\dot{x}_i^T M_{2i}A_i(\mu_i)x_i+x_i^T M_{1i}M_{1i}^T x_i\\ &+\dot{x}_i^T M_{2i}M_{2i}^T\dot{x}_i+2\sum_{j=1}^{J}J\alpha_j^2\ x_i^T(H_{ji})^T H_{ji}x_i\}\\ \le\sum_{i=1}^{J}\sum_{l=1}^{r_i}&\mu_i^l\xi_i^T\Theta_i^l\xi_i.\end{aligned} \tag{14}$$

where

$$\xi_i^T=\begin{bmatrix}x_i^T & \dot{x}_i^T\end{bmatrix}$$

$$\Theta_i^l=\begin{bmatrix} \sum_{\rho=1}^{r_i}\phi_i^\rho(P_i^\rho+\Lambda_i^l) & \\ Sym(M_{1i}A_i^l)+M_{1i}M_{1i}^T & * \\ +2\sum_{j=1}^{J}J\alpha_j^2\ (H_{ji})^T H_{ji} & \\ P_i^l-M_{1i}^T+M_{2i}A_i^l & M_{2i}M_{2i}^T-Sym(M_{2i}) \end{bmatrix}$$

From (14), $\dot{V}_2<0$ implies the asymptotic stability of the **CFLSS** (3) if the stability conditions of $P_i^l>0$, $P_i^\rho+\Lambda_i^l>0$ and $\Theta_i^l<0$ are satisfied. By applying Schur complement to $\Theta_i^l<0$, one obtains (12).

Remark 2. The use of the FLF, for CFLSS, involves the bound on derivatives of the membership function as it is always the case with CFS.

4.3 Stability example

Consider the fuzzy large-scale system composed of three subsystems. The three rules of the interconnected system are as follows ($i=1,\ldots,3$):

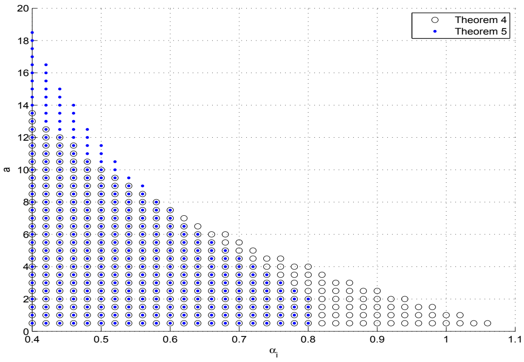

Fig. 1. Stability region for different α_i

$$S_i\begin{cases}Rule\ 1: IF\ x_{i1}(t)\ is\ F_{i1}^1\ \ THEN\ \dot{x}_i=A_i^1x_i+h_i(x)\\ Rule\ 2: IF\ x_{i1}(t)\ is\ F_{i1}^2\ \ THEN\ \dot{x}_i=A_i^2x_i+h_i(x)\end{cases}$$

Where

$$A_1^1=\begin{bmatrix}-3 & 0.6\\ 0.2 & -2\end{bmatrix}, A_1^2=\begin{bmatrix}-1 & 0.4\\ -0.2 & -1\end{bmatrix}, A_2^1=\begin{bmatrix}-2 & 1\\ 0.2 & -a\end{bmatrix}$$

$$A_2^2=\begin{bmatrix}-1 & 1\\ -0.6 & -1\end{bmatrix}, , A_3^1=\begin{bmatrix}-3 & 0.5\\ 0.1 & -1\end{bmatrix}, A_3^2=\begin{bmatrix}-2 & 0.4\\ -a-5 & -3\end{bmatrix}$$

$h_i(x)$ satisfy the assumption 1, with

$$H_1=-H_2=H_3=\begin{bmatrix}0.1 & 0 & 0.1 & 0 & 0.1 & -0.2\\ 0 & 0 & 0 & 0 & 0.1 & 0.2\end{bmatrix},\ \phi_i^\rho=1.$$

Feasibility is tested for different values of a and different bounding terms α_i. Bounding terms are taken such that $\alpha_1=\alpha_2=\alpha_3$. Figure 1 shows the obtained results. In fact, Figure 1 confirms Remark 1 about interconnection bounds. Further, for small values of α_i, Theorem 4 provides larger regions of feasibility than Theorem 3, as generally expected for Fuzzy systems. In opposite, for large values of α_i the feasibility region obtained by Theorem 3 becomes more important than that obtained by Theorem 4. Therefore stability conditions by QLF seems less conservative than that obtained by FLF even if the null product technique is used. In fact, Theorem 4 has a maximum bounds of interconnection bigger than Theorem 3 for values of $a>8$, but for $a<8$ it's the opposite.

Hence, it is clear that the use of the null product technique to introduce slack matrices increases the effect of the interconnection term for **CFLSS**. That's why this technique leads to more conservative results in general cases in contrary with **CFS**.

Remark 3. According to Koo et al. (2013); Yang et al. (2016); Koo et al. (2016); Kim et al. (2018), the feasibility is tested just by the guarantee of the maximum interconnection bound, which is not enough to judge the effectiveness of the results. For a fair judgment and to the authors knowledge, for the first time, the feasibility is tested for different values of a and different bounding terms α_i in Figure 1.

4.4 stabilization conditions

In this section, the decentralization concept and PDC approach are employed. Fuzzy controllers share the same fuzzy rules and sets of the **CFLSS**

$$u_i(t)=\sum_{l=1}^{r_i}\mu_i^l K_i^l x_i(t)=:K_i(\mu_i)x_i(t) \tag{15}$$

The i^{th} subclosed-loop system is

$$\dot{x}_i(t) = \big(A_i(\mu_i) + B_i(\mu_i)K_i(\mu_i)\big)x_i(t) + h_i(x(t) \quad (16)$$

Based on the two procedures proposed above for stability, new stabilization conditions are enunciated in the sequel. Solving the proposed LMI's enables one to design the stabilizing controllers for closed-loop asymptotic stability. First, the **QLF** is used in the following Theorem for **CFLSS**.

Theorem 5. If there exist symmetric matrices Q_i, and any matrices Y_i^l such that LMI's below hold true, then the closed-loop CFLSS (16) is asymptotically stable. Further, decentralized PDC fuzzy controllers (15) are given by the local state feedback gains $K_i^m = Y_i^m Q_i^{-T}$ for $l, m = 1, \ldots, r_i$; $i = 1, \ldots, J$:

$$Q_i > 0 \quad (17)$$
$$\Psi_i^{ll} < 0 \quad (18)$$
$$\Psi_i^{lm} + \Psi_i^{ml} < 0 \qquad l < m \quad (19)$$

with

$$\Psi_i^{lm} = \begin{bmatrix} Sym(A_i^l Q_i + B_i^l Y_i^m) + \mathbb{I} & * \\ W_i Q_i & -\dfrac{1}{J}\mathbb{I} \end{bmatrix},$$

$$Q_i^{-1} = P_i \text{ and } W_i = [\alpha_1 H_{1i}^T \cdots \alpha_j H_{ji}^T \cdots \alpha_J H_{Ji}^T]^T.$$

Proof. consider (7) of Theorem 3 and change terms $A_i(\mu_i)$ by subclosed-loop form $A_i(\mu_i) + B_i(\mu_i)K_i(\mu_i)$. Follow steps (5)- (7). $\dot{V}_1(t) < 0$ holds true if conditions of $P_i > 0$, $\bar{\Psi}_i^{ll} < 0$ and $\bar{\Psi}_i^{lm} + \bar{\Psi}_i^{ml} < 0$ are satisfied, where

$$\bar{\Psi}_i^{lm} = Sym\big(P_i(A_i^l + B_i^l K_i^m)\big) + P_i P_i + \sum_{j=1}^{J} J\alpha_j^2\ (H_{ji})^T H_{ji}$$

pre- and post-multiply $P_i > 0$, $\bar{\Psi}_i^{ll} < 0$ and $\bar{\Psi}_i^{lm} + \bar{\Psi}_i^{ml} < 0$ by P_i^{-1}, the inequalities (17), (18) and (19) are obtained.

Stabilization conditions based on the **FLF** and the null product (10) are obtained for **CFLSS** in the following Theorem:

Theorem 6. Let $\eta_i > 0$ ($i = 1, 2,\ , r_i$) be given scalars. If there exist symmetric matrices $\bar{Q}_i^l$ and any matrices $\bar{Y}_i^l$, R_i such that LMIs below hold true, then the closed-loop **CFLSS** (16) is asymptotically stable. Further, decentralized PDC (15) are given by local state feedback gains $K_i^m = \bar{Y}_i^m R_i^{-T}$ for $l, m, \rho = 1, \ldots, r_i$; $i = 1, \ldots, J$:

$$\bar{Q}_i^l > 0 \quad (20)$$
$$\bar{Q}_i^\rho + \bar{\Lambda}_i^l > 0 \quad (21)$$
$$\Xi_i^{ll} < 0 \quad (22)$$
$$\Xi_i^{lm} + \Xi_i^{ml} < 0 \qquad l < m \quad (23)$$

with

$$\Xi_i^{lm} = \begin{bmatrix} \begin{matrix}\sum_{\rho=1}^{r_i} \phi_i^\rho(\bar{Q}_i^\rho + \bar{\Lambda}_i^l) \\ +Sym(A_i^l R_i^T + B_i^l \bar{Y}_i^m) + \mathbb{I}\end{matrix} & * & * \\ \begin{matrix}\bar{Q}_i^l - R_i \\ +\eta_i(A_i^l R_i^T + B_i^l \bar{Y}_i^m)\end{matrix} & \begin{matrix}\eta_i^2 \mathbb{I} \\ -\eta_i Sym(R_i)\end{matrix} & * \\ W_i R_i^T & 0 & -\dfrac{1}{2J}\mathbb{I} \end{bmatrix},$$

$W_i = [\alpha_1 H_{1i}^T \cdots \alpha_j H_{ji}^T \cdots \alpha_J H_{Ji}^T]^T$, $M_i^{-1} = R_i$,
$\bar{Q}_i^l = R_i P_i^l R_i^T$, $\bar{\Lambda}_i^l = R_i P_i^l R_i^T$.

Proof. Change terms $A_i(\mu_i)$ by subclosed-loop form $A_i(\mu_i) + B_i(\mu_i)K_i(\mu_i)$, with the particular case of $M_{1i} = M_i, M_{2i} = \eta_i M_i$ in condition (12). following steps (13)-(14) leads to $\dot{V}_2(t) < 0$ if the conditions of $P_i^l > 0$, $P_i^\rho + \Lambda_i^l > 0$, $\bar{\Xi}_i^{ll} < 0$ and $\bar{\Xi}_i^{lm} + \bar{\Xi}_i^{ml} < 0$ hold true, with

$$\bar{\Xi}_i^{lm} = \begin{bmatrix} \begin{matrix}\sum_{\rho=1}^{r_i} \phi_i^\rho(P_i^\rho + \Lambda_i^l) \\ +Sym(M_i(A_i^l + B_i^l K_i^m)) + M_i M_i^T \\ +2\sum_{j=1}^{J} J\alpha_j^2\ (H_{ji})^T H_{ji}\end{matrix} & * \\ P_i^l - M_i^T + \eta_i M_i(A_i^l + B_i^l K_i^m) & \begin{matrix}\eta_i^2 M_i M_i^T \\ -\eta_i Sym(M_i)\end{matrix} \end{bmatrix}$$

Pre- and post-multiply $P_i^l > 0$ and $P_i^\rho + \Lambda_i^l > 0$ by the non singular matrices M_i^{-1} and M_i^{-T}, respectively, and pre- and post-multiply $\bar{\Xi}_i^{ll} < 0$ and $\bar{\Xi}_i^{lm} + \bar{\Xi}_i^{ml} < 0$ by $diag(M_i^{-1}, M_i^{-1})$ and $diag(M_i^{-T}, M_i^{-T})$, respectively, inequalities (20), (21), (22) and (23) are obtained.

4.5 Stabilization example

To compare both obtained conditions in the case of stabilizability, let us consider the fuzzy large-scale system composed of two subsystems:

$$\dot{x}_i = \sum_{l=1}^{2} \mu_i^l(x_{i1}(t))\Big(A_i^l x_i + B_i^l u_i + h_i(x)\Big)$$

where

$$A_1^1 = \begin{bmatrix} 3.6 & 2 \\ -6.5 & -5 \end{bmatrix}, A_1^2 = \begin{bmatrix} -0.4 & 1 \\ -5.8 & -4 \end{bmatrix}, A_2^1 = \begin{bmatrix} 3.6 & -1.6 \\ 6.2 & -4.3 \end{bmatrix}$$

$$A_2^2 = \begin{bmatrix} -10 & -1.2 \\ 6 & -4 \end{bmatrix}, B_1^1 = \begin{bmatrix} -0.45 \\ -3 \end{bmatrix}, B_1^2 = B_2^2 = \begin{bmatrix} -0.4 \\ -3 \end{bmatrix},$$

$$B_2^1 = \begin{bmatrix} -0.15 \\ -3.3 \end{bmatrix},\ \mu_i^1(x_{i1}(t)) = \frac{1}{1 + exp(-2x_{i1})}$$

$\mu_i^2(x_{i1}(t)) = 1 - \mu_i^1(x_{i1}(t))$

interconnection terms satisfy assumption 1, with

$$H_1 = -H_2 = \begin{bmatrix} 0.1 & 0 & 0.1 & -0.2 \\ 0 & 0 & 0.1 & 0.2 \end{bmatrix},\ \alpha_1 = \alpha_2 = 1.4,\ \phi_i^\rho = 1.$$

Theorems 5 and 6 are applied to design stabilizing controllers. Conditions of Theorem 5 are feasible. In opposite, conditions of Theorem 6 are not. This confirms the results for stability conditions. In fact, the use of the null product technique leads to more conservative region. The obtained PDC gains from Theorem 5:

$K_1^1 = [41,8916\ -126,2302]$, $K_1^2 = [9,9183\ -33,0864]$,

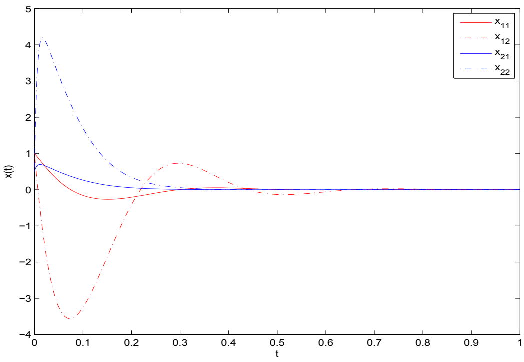

Fig. 2. Closed-loop states evolution using Theorem 5

.

$$K_2^1 = [0,0866\ -2,4895]\,, K_2^2 = [-1,3324\ -1,3933]\,.$$

The states motions of each subsystem with the initial conditions $x_1 = [1\ 1]$ and $x_2 = [0.5\ 0.5]$, using PDC controller given above, are shown in Figure 2.

Through the analysis of the null product technique, the problems of stability analysis and control design based on **FLF** have to be further investigated in the case of Large scale systems.

5. CONCLUSION

In this work, stability and stabilization problems for the continuous-time fuzzy large-scale systems have been studied. Stability analysis shows that for CFLSS, the obtained QLF based condition is less conservative in general case than the one based on FLF. Hence, the disadvantage of the null product technique given in Mozelli et al. (2009) when applied to the FLF for CFLSS has been demonstrated. Unlike the continuous time TS fuzzy systems, using this last leads to more conservative results for CFLSS. Numerical examples and comparison are presented to show the effectiveness of our approach in both cases of stability and controllers design.

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