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An Optimal Control Method to Coordination of Pricing and Advertising for a Supply Chain: The Consignment Mode ⋆

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Abstract: This paper addresses the channel coordination problem under the consignment mode based on the optimal control method. Considering the impacts from the advertising and platform goodwill onto the market demand, the equilibrium strategies regarding the pricing and the advertising are computed within the centralized and decentralized settings by applying the optimal control theory. Moreover, this paper compares the optimal control strategies in the two decision structures, where the comparison shows that the channel players invest more in advertising under the centralized case only when the share of the revenue exceeds a certain threshold. In addition, a contract scheme including the platform use fees and the cooperative advertising is proposed, which leads to profit Pareto improvements for the channel members. Finally, some simulations are provided to illustrate the impacts of the systems parameters onto the platform goodwill and coordination.

Keywords: Consignment mode; Advertising strategies; Optimal Control; Coordination mechanism; Platform goodwill.

1. INTRODUCTION

The consignment contract is a specific contract form, in which the manufacturer is allowed to sell their products directly to customer by renting the retailer's shelf space. Moreover, the manufacturer and the retailer can share the profits according to a pre-arranged sharing rule for each unit of the product sold Adida and Ratisoontorn (2011); Hu et al. (2017); Wu et al. (2020). In recent years, with the rapid developments of Internet, the consignment selling has successful applications in some online marketplaces (e.g. eBay.com, Amazon.com) Hu et al. (2014). Generally, each member in the supply chain tries to make economic profitability by improving the product demand. For this purpose, it is well recognized that the advertising has played a crucial role due to its efficiency in attracting consumers Xie and Neyret (2009). As such, the advertising strategies have been discussed for marketing channel in the environment of consignment sale Wu et al. (2016); Jin et al. (2015). In Wu et al. (2016), an optimal advertising effort level has been given for the vendor-managed consignment inventory (VMCI), and the effect from consumer return onto the advertising strategy has been revealed. In Jin et al. (2015), the advertising problem in a supply chain has been addressed by considering the capital constraint, where it has revealed that the combination of a consignment agreement with the manufacturer's promotion right is beneficial to all channel members. On the other hand, the advertising problems in the dynamic setting are also considered by applying the optimal control theory Giovann et al. (2019); Burattoa et al. (2019); Lu et al. (2019). In Giovann et al. (2019), the benefits of a cooperative advertising have been explored by taking the dynamic effects of advertising into account. Subsequently, the dynamic model of cooperative advertising scheme has been established in Burattoa et al. (2019), where the effects of the price and the quality on the goodwill have been considered and a manufacturer's cooperative advertising scheme can always enhance the channel profits.

Coordinating a supply chain is an important and challenging issue since each chancel player independently maximizes its own profit. Up to now, few results have addressed the coordination problem for supply chain under the consignment mode Li et al. (2009); Chen et al. (2010, 2011). In Chen et al. (2011), a novel mechanism with a two-part slotting fee has been proposed, which can effectively improve the profitability of the channel members in the static setting. On the other hand, some methods have been proposed to deal with the coordination problems for the supply chain under the consignment mode from the dynamic viewpoint Chen (2013); Wang et al. (2016); Wu et al. (2018). For example, the bilateral participation

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contract has been designed in Wang et al. (2016) for the mobile platforms, where a differential equation and the game theory have been used to reveal the relationship between app developers and the platform owner. In Wu et al. (2018), a state-dependent contract has been designed for the supply chain with the demand dependent on the goodwill. Compared with existing results, it is worthwhile to point out that few schemes are available to handle the effects from the advertising effort and the consignment platform goodwill onto the demand. Hence, there is a need to provide an efficient contract to coordinate the supply chain with improved requirements of the channel performance, which motivates the current topic.

In this paper, the aim is to handle the channel coordination problem within the consignment selling format, where the effects of the manufacturer's advertising and the retailer's platform goodwill are simultaneously considered. In order to achieve this objective, the equilibrium solutions regarding the pricing and advertising for the benchmark and for decentralized scenarios are characterized and the corresponding equilibrium solutions under different decision scenarios are compared. Subsequently, based on the above equilibrium solutions, a novel contract including the cooperative advertising and the platform use fees is given to coordinate the whole channel. Finally, the effect caused by key parameters onto the optimal profits and coordination is discussed. The rest of the paper is organized as follows. Section 2 develops the optimal control model in a dynamic setting. Sections 3-4 discuss the optimal equilibrium solutions in the centralized and decentralized scenarios, and include the relationship between equilibrium solutions. Subsequently, a novel contract is proposed in Section 5. Finally, the simulations and the conclusions are provided in Section 6-7, respectively.

2. PROBLEM DESCRIPTION AND BASIC MODEL

We consider a setting, in which the retailer operates an online sale platform and the manufacturer sells the product directly to consumers in the sale platform by the consignment contract. In particular, c_M denotes the manufacturer's constant production cost and c_R depicts the retailer' unit handling cost for selling the product to the market. For convenience of subsequent analysis, define $c = c_M + c_R$ and $m = \frac{c_R}{c}$. Under the consignment contract, the manufacturer pays the retailer ϕ share of the selling price as the commission for each unit sold. It should be noted that the platform goodwill can effectively attract consumers to buy the product.

In order to get more benefits, the retailer can improve the platform goodwill by various means. Generally, the advertisement is a very important mean and a necessary tool for building the brand. Let the retailer's platform goodwill represented by a state variable $G(t)$ and the retailer's advertising investment over time t be denoted by $U_R(t) \geq 0$. Then, the retailer's platform goodwill over time is described by the following equation:

$$\dot{G}(t) = \rho U_R(t) - \delta G(t),\ G(0) = G_0, \tag{1}$$

In (1), the parameter ρ represents the platform goodwill sensitivity to the retailer's advertising investment, δ denotes the diminishing rate, and G_0 depicts the initial retailer's platform goodwill.

Note that both the platform goodwill and the advertising effort have positive effects on the market demand. Thus, we assume that the market demand $D(t)$ satisfies the following equation:

$$D(t) = (\alpha - \beta p(t))(\eta U_M(t) + \gamma G(t)), \tag{2}$$

where $p(t)$ is the product's retail price, $U_M(t)$ is the manufacturer's advertising effort, α is the base market size, β reflects the price elasticity, η and γ reflect the effectiveness of the manufacturer's advertising effort and the retailer's platform goodwill on market size.

Subsequently, the advertising expenditures $C(U_R(t))$ and $C(U_M(t))$ are assumed as

$$C(U_R(t)) = \frac{1}{2}kU_R^2(t),\ C(U_M(t)) = \frac{1}{2}kU_M^2(t), \tag{3}$$

where k is the advertising cost factor.

Given an infinite time horizon, the optimization functions regarding the manufacturer and the retailer are considered:

$$\begin{aligned} J_M = \int_0^{+\infty} & e^{-rt}[((1-\phi)p(t) - c_M)(\alpha - \beta p(t)) \\ & \times(\eta U_M(t) + \gamma G(t)) - C(U_M(t))]dt, \end{aligned} \tag{4}$$

$$\begin{aligned} J_R = \int_0^{+\infty} & e^{-rt}[(\phi p(t) - c_R)(\alpha - \beta p(t))(\eta U_M(t) \\ & +\gamma G(t)) - C(U_R(t))]dt, \end{aligned} \tag{5}$$

with $r > 0$ being a discount rate. Next, the optimization function of total channel is the sum of the optimization functions of the manufacturer and the retailer, i.e.,

$$\begin{aligned} J_C = \int_0^{+\infty} & e^{-rt}[(p(t) - c_M - c_R)(\alpha - \beta p(t))(\eta U_M(t) \\ & +\gamma G(t)) - C(U_R(t)) - C(U_M(t))]dt. \end{aligned} \tag{6}$$

In what follows, there are two aspects to be achieved. Firstly, the optimal equilibrium strategies of the retailer and the manufacturer under both the centralized and decentralized scenarios are given. Secondly, a coordination mechanism is presented for the concerned supply chain.

3. THE PRICING AND ADVERTISING STRATEGIES WITHIN THE CENTRALIZED SCENARIO

Considering the centralized scenario, two channel players are integrated as a whole and the optimal control strategies are designed to maximize the objective functional of the whole supply chain.

Now, we are ready to solve and obtain the equilibrium strategies within the centralized scenario.

Theorem 1. Within the centralized scenario, the equilibrium strategies are represented by:

$$p^{C*}(t) = \frac{\alpha + \beta c}{2\beta}, \tag{7}$$

$$U_M^{C*}(t) = \frac{\eta(\alpha - \beta c)^2}{4k\beta}, \tag{8}$$

$$U_R^{C*}(t) = \frac{\rho\gamma(\alpha-\beta c)^2}{4k\beta(r+\delta)}. \quad (9)$$

The time path of platform goodwill is

$$G_C^*(t) = (G_0 - G_C^S)e^{-\delta t} + G_C^S, \quad (10)$$

where $G_C^S = \frac{\gamma\rho^2(\alpha-\beta c)^2}{4\delta k\beta(r+\delta)}$ represents the platform goodwill in the steady state case. Moreover, the optimal profit of total channel is given as follows:

$$J_C^* = \frac{\eta^2(\alpha-\beta c)^4}{32kr\beta^2} + \frac{\gamma(\alpha-\beta c)^2}{4\beta(r+\delta)}G_0 + \frac{\gamma^2\rho^2(\alpha-\beta c)^4}{32kr\beta^2(r+\delta)^2}. \quad (11)$$

Proof. Within the centralized scenario, the optimization problem for the total channel can be given by

$$\begin{aligned} &\max\ J_C \\ &\text{s.t.}\ \ (1) \end{aligned} \quad (12)$$

For the optimization problem (12), the Maximum principle the utilized and the current-value Hamiltonian for the total supply chain is constructed as follows:

$$H_C = (p(t)-c)(\alpha-\beta p(t))(\eta U_M(t)+\gamma G(t)) - \frac{1}{2}kU_R^2(t)) - \frac{1}{2}kU_M^2(t + \mu_C(t)(\rho U_R(t) - \delta G(t)). \quad (13)$$

with $\mu_C = \mu_C(t)$ being the costate variable.

The necessary conditions for optimal control strategies can be expressed as:

$$\frac{\partial H_C}{\partial p} = (\eta U_M(t)+\gamma G(t))(-2\beta p + \alpha + \beta c) = 0, \quad (14)$$

$$\frac{\partial H_C}{\partial U_M} = -kU_M + \eta(p(t)-c)(\alpha-\beta p(t)) = 0, \quad (15)$$

$$\frac{\partial H_C}{\partial U_R} = -kU_R + \rho\mu_C = 0, \quad (16)$$

$$\dot{\mu}_C = r\mu_C - \frac{\partial H_C}{\partial G} = (r+\delta)\mu_C - \gamma(p(t)-c)(\alpha-\beta p(t)). \quad (17)$$

Notice that the equations (14)-(16) imply

$$p(t) = \frac{\alpha+\beta c}{2\beta}, \quad (18)$$

$$U_M = \frac{\eta(\alpha-\beta c)^2}{4k\beta} \quad (19)$$

$$U_R = \frac{\rho}{k}\mu_C. \quad (20)$$

Substituting (18) into equation (17), we obtain

$$\dot{\mu}_C = (r+\delta)\mu_C - \frac{\gamma(\alpha-\beta c)^2}{4\beta}. \quad (21)$$

By solving the above equation (21) and according to the condition $\lim_{t\to\infty} e^{-rt}\mu_C(t) = 0$, one has

$$\mu_C = \frac{\gamma(\alpha-\beta c)^2}{4\beta(r+\delta)}. \quad (22)$$

Substituting (22) into equation (20) yields

$$U_R = \frac{\rho\gamma(\alpha-\beta c)^2}{4k\beta(r+\delta)}. \quad (23)$$

Finally, substituting (18)-(19) and (23) into (1) and (6), the time path of retailer's platform goodwill and the optimal profit of total channel can be given, which ends the proof.

4. THE PRICING AND ADVERTISING STRATEGIES WITHIN THE DECENTRALIZED SCENARIO

Under the decentralized scenario, the two channel players will maximize their own profit independently. The retailer firstly offers the advertising strategy U_R, and then the manufacturer decides the selling price $p(t)$ and the advertising strategy U_M based on the advertising strategy U_R.

Theorem 2. Within the decentralized scenario, the optimal control strategies of the two channel players can be represented by:

$$p^{D*}(t) = \frac{(1-\phi)\alpha + \beta c_M}{2\beta(1-\phi)}, \quad (24)$$

$$U_M^{D*}(t) = \frac{\eta[(1-\phi)\alpha - \beta c_M]^2}{4k\beta(1-\phi)}, \quad (25)$$

$$U_R^{D*}(t) = \frac{\rho\gamma A}{4k\beta(r+\delta)(1-\phi)^2}, \quad (26)$$

where $A = [\phi((1-\phi)\alpha + \beta c_M) - 2\beta(1-\phi)c_R][(1-\phi)\alpha - \beta c_M]$. The time path of platform goodwill is:

$$G_D^*(t) = (G_0 - G_D^S)e^{-\delta t} + G_D^S, \quad (27)$$

where $G_D^S = \frac{\gamma\rho^2 A}{4\delta k\beta(r+\delta)(1-\phi)^2}$ represents the platform goodwill in the steady state case. Moreover, the optimal profits for two channel players can be given as follows:

$$J_M^{D^*} = \frac{\eta^2((1-\phi)\alpha-\beta c_M)^4}{32kr\beta^2(1-\phi)^2} + \frac{\gamma((1-\phi)\alpha-\beta c_M)^2}{4\beta(r+\delta)(1-\phi)}G_0 + \frac{\gamma\delta((1-\phi)\alpha-\beta c_M)^2}{4r\beta(1-\phi)(r+\delta)}G_D^S, \quad (28)$$

$$J_R^{D^*} = \frac{A\eta^2((1-\phi)\alpha-\beta c_M)^2}{16kr\beta^2(1-\phi)^3} - \frac{\rho^2\gamma^2A^2}{32kr\beta^2(r+\delta)^2(1-\phi)^4} + \frac{\gamma A G_0}{4\beta(r+\delta)(1-\phi)^2} + \frac{\gamma\delta A G_D^S}{4\beta r(r+\delta)(1-\phi)^2}. \quad (29)$$

Proof. The proof of this theorem can be fulfilled by conducting two steps. Firstly, the manufacturer's optimal control strategies are given. Accordingly, the following optimization problem of the manufacturer is considered:

$$\begin{aligned} &\max J_M \\ &\text{s.t.}\ \ (1) \end{aligned} \quad (30)$$

The Hamiltonian associated with the optimal control problem in (30) is given by:

$$H_M = ((1-\phi)p(t) - c_M)(\alpha - \beta p(t))(\eta U_M(t) + \gamma G(t)) \\ -\frac{1}{2}kU_M^2(t) + \mu_M(t)(\rho U_R(t) - \delta G(t)), \quad (31)$$

where $\mu_M(t)$ represents the costate variable. The necessary conditions for optimal strategies are:

$$\frac{\partial H_M}{\partial p} = (\eta U_M(t) + \gamma G(t))(-2\beta(1-\phi)p \\ +\alpha(1-\phi) + \beta c_M) = 0, \quad (32)$$

$$\frac{\partial H_M}{\partial U_M} = -kU_M + \eta((1-\phi)p(t) - c_M) \times \\ (\alpha - \beta p(t)) = 0, \quad (33)$$

$$\dot{\mu}_M = r\mu_M - \frac{\partial H_M}{\partial G} = (r+\delta)\mu_M \\ -\gamma((1-\phi)p(t) - c_M)(\alpha - \beta p(t)). \quad (34)$$

Thus, from (32) and (33), it follows that

$$p(t) = \frac{(1-\phi)\alpha + \beta c_M}{2\beta(1-\phi)}, \quad (35)$$

$$U_M(t) = \frac{\eta[(1-\phi)\alpha - \beta c_M]^2}{4k\beta(1-\phi)}. \quad (36)$$

Secondly, the retailer's control strategy is discussed. The following optimal control problem related to the retailer can be discussed:

$$\max J_R \\ \text{s.t. } (1) \quad (37)$$

Similarly, substituting (35)-(36) into equation (37) leads to the advertising effort of the retailer:

$$U_R(t) = \frac{\rho\gamma[\phi((1-\phi)\alpha - \beta c_M) - 2\beta(1-\phi)c_R]}{4k\beta(r+\delta)(1-\phi)^2} \times \\ [(1-\phi)\alpha - \beta c_M], \quad (38)$$

Finally, substituting (35)-(36) and (38) into (4)-(5) and (1), the time path of retailer's platform goodwill and the optimal profits of the channel member are obtained.

Furthermore, the following proposition can be obtained by comparing the optimal control strategies for the centralized system with the equilibrium strategies for the decentralized system.

Proposition 1: The optimal control strategies under the centralized and decentralized settings have the following relationship:

(1) $p^{C*}(t) \le p^{D*}(t)$, if $\phi \ge m$,

(2) $U_M^{C*}(t) > U_M^{D*}(t)$, if
$$1 - \left(\frac{\alpha - \beta c + \sqrt{(\alpha-\beta c)^2 + 4\alpha\beta c_M}}{2\alpha}\right)^2 < \phi < 1,$$

(3) $U_R^{C*}(t) > U_R^{D*}(t)$.

Remark 1: Proposition 1 shows that, when $\phi \ge max\left\{m, 1 - \left(\frac{\alpha-\beta c+\sqrt{(\alpha-\beta c)^2+4\alpha\beta c_M}}{2\alpha}\right)^2\right\} = m$, all channel member will invest more advertising efforts simultaneously and set lower retail price under the centralized case.

5. THE DESIGN OF COORDINATION CONTRACT

Note that the centralized decision structure often results in higher profits, it is necessary to design a contract mechanism such that the all channel players choose the same strategies as in the centralized decision structure. To achieve this purpose, a cooperative advertising and platform use fees (CA-PUF) contract is introduced. Under the proposed CA-PUF contract, the retailer shares the manufacturer's advertising cost and also charges the manufacturer platform use fees.

Let n be the retailer's advertising participation rate and $f(G) = MG(t) + L$ be the platform use fees, where M and L are constants. Based on this CA-PUF contract, the profit functions of the manufacturer and the retailer are expressed as

$$J_M^{CP} = \int_0^{+\infty} e^{-rt}[((1-\phi)p(t) - c_M)D(t) \\ -(1-n)C(U_M(t)) - MG(t) - L]dt, \quad (39)$$

$$J_R^{CP} = \int_0^{+\infty} e^{-rt}[(\phi p(t) - c_R)D(t) - C(U_R(t)) \\ -nC(U_M(t)) + MG(t) + L]dt. \quad (40)$$

Similar to the proof of Theorem 2, the equilibrium strategies under the CA-PUF contract can be obtained readily.

Theorem 3. Under the CA-PUF contract (denoted by the superscript CP), the equilibrium strategies of all members are described as:

$$p^{CP*}(t) = \frac{(1-\phi)\alpha + \beta c_M}{2\beta(1-\phi)}, \quad (41)$$

$$U_M^{CP*}(t) = \frac{\eta[(1-\phi)\alpha - \beta c_M]^2}{4k\beta(1-n)(1-\phi)}, \quad (42)$$

$$U_R^{CP*}(t) = \frac{\rho\gamma[\phi((1-\phi)\alpha + \beta c_M) - 2\beta(1-\phi)c_R]}{4k\beta(r+\delta)(1-\phi)^2} \times \\ [(1-\phi)\alpha - \beta c_M + M]. \quad (43)$$

From Theorem 3, it can be see that the equilibrium strategies under the CA-PUF contract are dependent on the parameters ϕ, M and n. Once there exists a set of parameter values such that the equilibrium strategies given by (41)-(43) are equal to those given by equations (7)-(9), then the whole channel can be coordinated. Therefore, letting $p_M^{CP*} = p_M^{C*}$, $U_M^{CP*}(t) = U_M^{C*}(t)$ and $U_R^{CP*}(t) = U_M^{C*}(t)$, the following Theorem can be obtained.

Theorem 4. If the contract parameters take the following values:

$$n = \phi = m, \quad (44)$$

$$M = \frac{\gamma(1-m)(\alpha - \beta c)^2}{4\beta}. \quad (45)$$

then the supply chain can be coordinated by the presented CA-PUF contract.

Accordingly, the optimal profits of the manufacturer and the retailer under the proposed contract can be obtained as follows:

$$J_M^{CP^*} = (1-m)J_C^* - \Delta - \frac{L}{r}, \quad (46)$$

$$J_R^{CP^*} = mJ_C^* + \Delta + \frac{L}{r}, \quad (47)$$

where $\Delta = \frac{(1-m)\rho\gamma(\alpha-\beta c)^2}{4rk\beta(r+\delta)} - \frac{MG_0}{r+\delta} - \frac{M\delta G_C^S}{r(r+\delta)}$.

Remark 2: After the coordination of the supply chain, another issue is to improve the economic condition for the manufacturer and the retailer. Therefore, the conditions $J_M^{CP^*} > J_M^{D^*}$ and $J_R^{CP^*} > J_R^{D^*}$ should be satisfied. By solving those two inequalities, we can obtain the fact that all channel members are willing to accept the CA-PUF contract as long as $L_1 < L < L_2$, where $L_1 = r(J_R^{D^*} - mJ_C^* - \Delta)$, $L_2 = r((1-m)J_C^* - J_M^{D^*} - \Delta)$. Moreover, it can be seen that the longer length of interval (L_1, L_2), the coordination capability of the proposed contract stronger is.

6. NUMERICAL ANALYSIS

In this section, some simulations and discussions are given to demonstrate the impact of key parameters onto the coordination capability of the proposed contract and the optimal profits. The basic parameter values are set as follows: $\alpha = 30$, $\beta = 1$, $\eta = 0.75$, $\gamma = 0.5$, $\rho = 0.45$, $\delta = 0.2$, $r = 0.3$, $c = 1$, $m = 0.1$, $\phi = 0.1$, $k = 1$ and $G_0 = 2$. Define $J_D^* = J_R^{D^*} + J_M^{D^*}$. Simulation results are shown in Figs. 1-6.

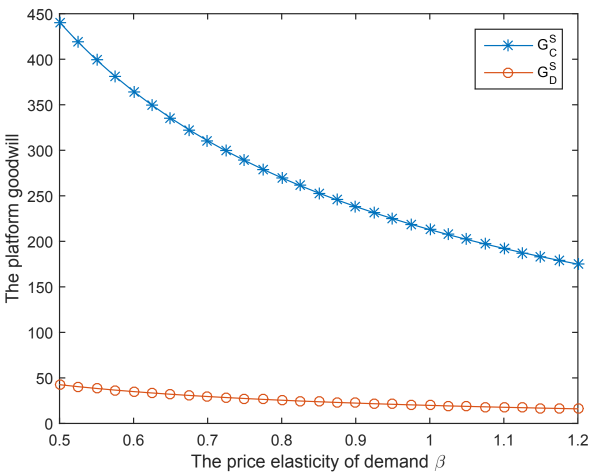

Fig. 1. Impact of β on platform goodwill.

Figs. 1-3 show that the platform goodwill decreases accompanying with the price elasticity β in both decentralized and centralized settings, which lead to lower channel profit and weaker coordination capability of the contract. Moreover, we can obtain that higher price elasticity β narrows the difference of profits within two decision structures.

Figs. 4-6 depict that the platform goodwill increases accompanying with the its effectiveness γ regardless of the centralization and decentralization settings, which leads to higher channel profit. Besides, we can observe that the length of interval $[L_1, L_2]$ gradually enlarges with the increasing γ, which illustrates that the coordination capability of the contract can be improved by enlarging the effectiveness of the retailer's platform goodwill on the market size. The above simulations have well revealed the coordination ability of the proposed contract and the change trends of the optimal profits.

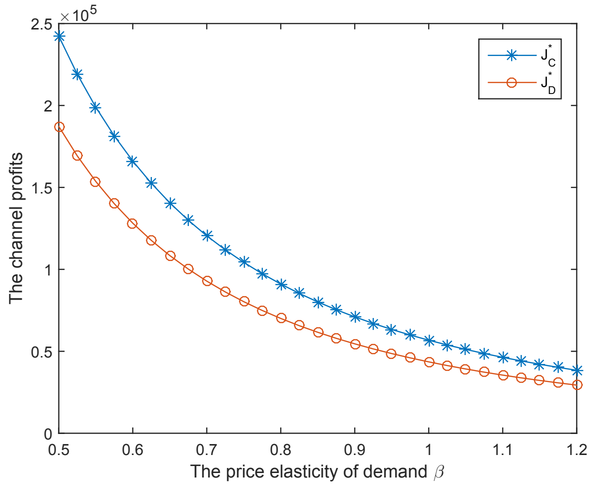

Fig. 2. Impact of β on total profits.

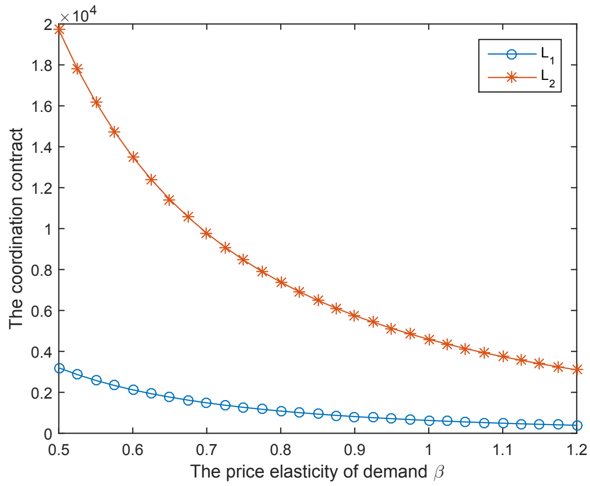

Fig. 3. Impact of β on coordination contract.

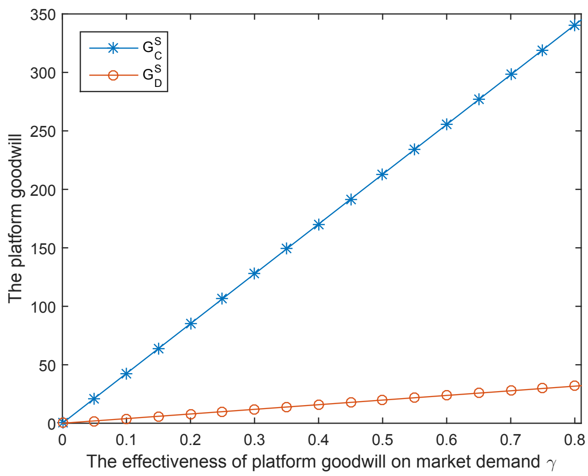

Fig. 4. Impact of γ on platform goodwill.

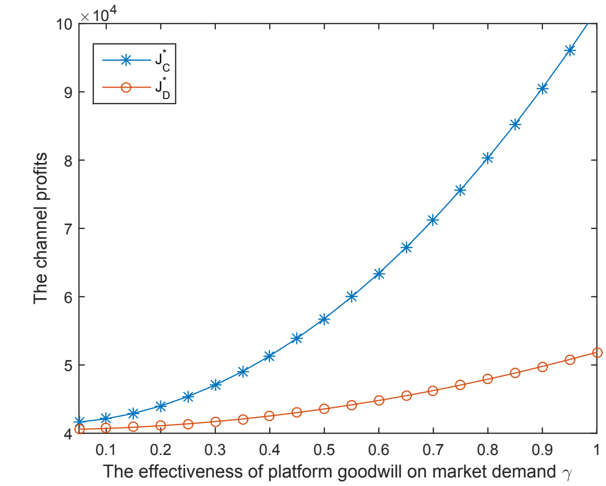

Fig. 5. Impact of γ on total profits.

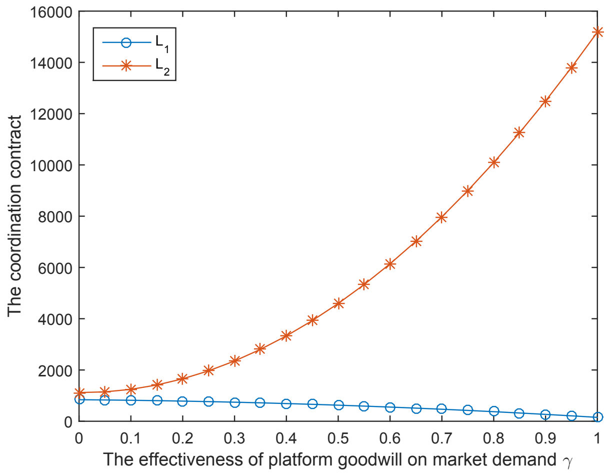

Fig. 6. Impact of γ on coordination contract.

7. CONCLUSIONS

In this paper, we have discussed the channel coordination problem for dynamic supply chain, where the manufacturer's product demand is dependent on the own advertising effort and the platform reputation. The equilibrium strategies under the different decision structures in the context of consignment have been proposed and an appropriate contract has been designed to achieve the profit Pareto improvements for the channel players. The following results have been obtained: 1) if the retailer sets a high share of the revenue (i.e., ϕ), then the manufacturer would always invest more in advertising and choose a low retail price in the centralized channel; 2) the consignment contract with the platform use fees and the cooperative advertising can perfectly coordinate the decentralized channel. Finally, some simulations have been utilized to demonstrate the coordination capability of the proposed contract.

REFERENCES

Adida, E. and Ratisoontorn, N. (2011). Consignment contracts with retail competition. *European Journal of Operational Research*, 215 (1):136–148.

Burattoa, A., Cesarettoa, R., and Giovann, P.De. (2019). Consignment contracts with cooperative programs and price discount mechanisms in a dynamic supply chain. *International Journal of Production Economics*, 48(2): 72–82.

Chen, L.T. (2013). Dynamic supply chain coordination under consignment and vendor-managed inventory in retailer-centric B2B electronic markets. *Industrial Marketing Management*, 114:518-531.

Chen, J., Lin, I., and Cheng, H. (2010). Channel coordination under consignment and vendor-managed inventory in a distribution system. *Transportation Research*, 30(1):831–843.

Chen, J., Cheng, H., and Chien, M.C. (2011). On channel coordination through revenue-sharing contracts with price and shelf-space dependent demand. *Applied Mathematical Modelling*, 35(10):4886-4901.

Giovann, P.De., Karray, S., and Martín-Herrán, G. (2019). Vendor Management Inventory with consignment contracts and the benefits of cooperative advertising. *European Journal of Operational Research*, 272(2):465–480.

Hu, W., Li, J., and Wang, W. (2017). Benefit and risk analysis of consignment contracts. *Annals of Operations Research*, 257 (1-2):641–659.

Hu, W., Li, Y., and Goinda, K. (2014). The impact of consumer returns policies on consignment contracts with inventory control. *European Journal of Operations Research*, 233 (2):398–407.

Jin, Y., Wang, S., and Hu, Q. (2015) Contract type and decision right of sales promotion in supply chain management with a capital constrained retailer. *European Journal of Operational Research*, 240(2):415–424.

Li, S., Zhu, Z., and Huang., L. (2009). Supply chain coordination and decision making under consignment contract with revenue sharing. *International Journal of Production Economics*, 120(1):88–99.

Lu, F., Zhang, J., and Tang, W. (2019). Wholesale price contract versus consignment contract in a supply chain considering dynamic advertising. *International Transactions in Operational Research*, 26(5):1977–2003.

Wang, R., Guo, Q., Choi, T.M., and Liang, L. (2016). Advertising strategies for mobile platforms with "Apps". *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 48(5):767–778.

Wu, Z., Chen, D., and Yu, H. (2016). Coordination of a supply chain with consumer return under vendor-managed consignment inventory and stochastic demand. *International Journal of General Systems*, 45(5):502–516.

Wu, Z., Feng, L., and Chen, D. (2018). Coordinating Pricing and Advertising Decisions for Supply Chain under Consignment Contract in the Dynamic Setting. *Complexity*, 2018, Article ID 7697180, 11 pages.

Wu, Z., Liu, G.-P., and Chen, D. (2020) Advertising strategies and coordination for supply chain based on consignment platform with delayed effect. *Systems Science & Control Engineering*, 8(1):162–172.

Xie, J. and Neyret, A. (2009). Co-op advertising and pricing models in manufacturer-retailer supply chains. *Computers and Industrial Engineering*, 56 (4):1375-1385.