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Event-Triggered Consensus for Euler-Lagrange Systems with Communication Delay

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Abstract: Distributed cooperative control of multi-agent systems, typically requires some form of information exchange in order to achieve coordination between individual agents. Especially in wireless communication systems, communication delays can lead to instability and can not be neglected during the design of the control law. We propose a novel coordination scheme for Euler-Lagrange systems taking into account constant communication delays. Additionally, in order to save limited resources, we propose an event-triggered strategy for the communication between agents, as well as for the local actuator updates of the individual agents. We show that with the proposed algorithms a stable interaction in consensus tasks can be guaranteed. Simulations illustrate the theoretical results.

Keywords: Networked robotic systems, Event-based control, Decentralized and distributed control, Lagrangian and Hamiltonian systems, Delay systems

1. INTRODUCTION

Distributed control of multi-agent systems has become a major research direction, including problems such as agreement/consensus, formation control, and distributed optimization and estimation. Practical applications include, but are not limited to, search and rescue, cooperative manipulation, and surveillance (Budde gen. Dohmann and Hirche, 2020; Olfati-Saber et al., 2007; Cao et al., 2013). For those types of systems exchange of information is required in order to meet the coordination aspect of the task, typically achieved by embedding the agents in a common communication network. As a result, issues arise regarding the networked data transmission, since it is well known that high network traffic increases communication delays and the chance of packet dropouts. This is of particular relevance in the case of limited communication bandwidth, as it is common in wireless networks. In addition, with decreased weight and size of future multi-agent systems, embedded microprocessors become more important for the implementation of the control algorithms. Since many real-world applications require high update frequencies, it becomes unfeasible to implement the underlying control laws in time-driven fashion, while maintaining the often required real-time capabilities of the system. A common plant model for such applications is given by the Euler-Lagrange equations of motion, since they describe mechanical systems. Due to the highly non-linear dynamics of such systems, classical results for first order or linear system can not be directly applied and the dynamics have to be explicitly addressed during the design of the control law.

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A common tool for designing multi-agent systems is given by the passivity framework, since multiple systems can be easily interconnected (Arcak, 2007). Due to the inherent passivity of Euler-Lagrange systems, such types of algorithms are especially useful in this context. An additional important property of passive systems is the possible extension to deal with time-delays via the well-known scattering transformation (Anderson and Spong, 1989). Originally proposed for teleoperator systems, the concept can be extended for the consensus problems of Euler-Lagrange systems (Wang, 2013). Scattering like approaches have also been investigated for event-triggered systems (Yu and Antsaklis, 2014), which has been applied to Euler-Lagrange systems in a teleoperation scenario (Hu et al., 2016). However, in those approaches merely the communication between agents is in even-triggered fashion, while the feedback passivation and with that the controller updates are obtained in time-continuous fashion. In contrast, (Liu et al., 2016) provide a solution for a complete event-triggered framework for Euler-Lagrange systems with parametric uncertainties, including event-triggered communication and actuator updates. However, in this work time delays are neglected and some continuity assumptions have to be made for the underlying dynamics, in order to show Zeno freeness of the triggering scheme. In addition, such model-based approaches which are typically used for feedback passivation are sensible to modeling errors and can be computationally demanding. Consensus for Euler-Lagrange systems with unknown time-delays and event-triggered control and communication remains an open problem.

In this work we present a distributed control approach for cooperating Euler-Lagrange systems. We provide a strategy allowing for event-triggered communication and

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controller updates, while considering constant delays in the communication between agents. This is done by exploiting the properties of first order auxiliary systems, which are coupled to the agent dynamics in an energetically passive fashion. In contrast to previous works, no information about the plant dynamics is required or estimated and as a result, no assumptions on the Euler-Lagrange dynamics are needed.

The remainder of this work is structured as follows. After introducing some preliminaries and the problem statement in Section 2, we present the general idea of how to deal with the communication delays by presenting the basic control law in time-continuous fashion in Section 3.1. In Section 3.2 the event-triggering mechanism for the communication between agents is introduced and extended to the local feedback control of the individual agents in Section 3.3. The proposed algorithm is illustrated in simulation in Section 4.

2. PRELIMINARIES AND PROBLEM STATEMENT

2.1 Notation

For any given vector $\boldsymbol{x}_i \in \mathbb{R}^n$ and matrix $\boldsymbol{A}_i \in \mathbb{R}^{n\times n}$, where the index refers the quantity to agent i, we denote with $\boldsymbol{x} \in \mathbb{R}^{nN}$ and $\boldsymbol{A} \in \mathbb{R}^{nN\times nN}$ the column vector obtained by stacking the individual vectors for all N agents and $\boldsymbol{A} = \mathrm{blockdiag}(\boldsymbol{A}_1, \ldots, \boldsymbol{A}_N)$, respectively. The smallest eigenvalue of $\boldsymbol{A}$ is denoted with $\lambda_{min}(\boldsymbol{A})$. The $n \times 1$ vector $\mathbf{1}_n$ denotes a column vector full of ones, while $\boldsymbol{I}_n$ is the $n\times n$ identity matrix and $\mathbf{0}_{n\times m}$ is the $n\times m$ zero matrix. The Kronecker product operator is denoted as $\otimes$. The Euclidean norm of a vector and the matrix norm, induced by the Euclidean norm, are denoted by $\|(\cdot)\|_2$. Define $\|\boldsymbol{x}\|_{\boldsymbol{A}} = \boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x}$, for any symmetric positive definite matrix $\boldsymbol{A}$. Given a function $\boldsymbol{f}(t) : \mathbb{R}_0^+ \to \mathbb{R}^n$, we denote with $\boldsymbol{f}(t^-)$ the limit of $\boldsymbol{f}(s)$ when s approaches t from the left. For notational convenience we will omit the dependence of a function $\boldsymbol{f}(t)$ on the time t, if it is clear from the context. A function $\alpha : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said to be of class $\mathcal{K}_\infty$, if it is continuous, strictly increasing, $\alpha(0) = 0$, and $\alpha(s) \to \infty$ if $s \to \infty$.

2.2 Euler-Lagrange Systems

In this work we consider N agents, where the dynamics of the ith agent can be modeled via the Euler-Lagrange formalism as

$$\boldsymbol{M}_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i = \boldsymbol{\tau}_i, \tag{1}$$

where $\boldsymbol{q}_i \in \mathbb{R}^n$ and $\boldsymbol{\tau}_i \in \mathbb{R}^n$ are the vector of generalized coordinates and control inputs of the ith agent and the inertia and Coriolis/centrifugal matrix are denoted with $\boldsymbol{M}_i \in \mathbb{R}^{n\times n}$ and $\boldsymbol{C}_i \in \mathbb{R}^{n\times n}$, respectively. The following basic properties can be found in any textbook about Euler-Lagrange systems.

- P(1) The matrix $\boldsymbol{M}_i(\boldsymbol{q}_i)$ is symmetric positive definite and bounded from above.
- P(2) The matrix $\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$ containing the Coriolis and centrifugal terms is bounded for bounded velocities $\dot{\boldsymbol{q}}_i$.
- P(3) The matrix $\frac{1}{2}\dot{\boldsymbol{M}}_i(\boldsymbol{q}_i) - \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$ is skew symmetric and as a result $\boldsymbol{x}^T\left(\frac{1}{2}\dot{\boldsymbol{M}}_i(\boldsymbol{q}_i) - \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\right)\boldsymbol{x} = \mathbf{0}$ for any $\boldsymbol{x} \in \mathbb{R}^n$.

2.3 Graph Theory

In this work we model the interaction between the N individual agents with a weighted communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \boldsymbol{W})$, with vertex set $\mathcal{V} = \{1, ..., N\}$ representing the N agents and edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where two agents communicate with each other if $(i, j) \in \mathcal{E}$. The neighborhood of agent i is defined as $N_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. The weight matrix $\boldsymbol{W}$ is a positive definite block-diagonal matrix with weight $\boldsymbol{W}_k \in \mathbb{R}^{n\times n}$ on the diagonal for each edge $k \in \mathcal{E}$. For the sake of exposition we assume $\boldsymbol{W}_k = w_k\boldsymbol{I}_n$, where $w_k \in \mathbb{R}^+$, but the results can be easily extended to the more general case. In this work we consider undirected and connected graphs, i.e. if $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$ and there exists a path between any two vertices i and j.

2.4 Problem Statement

The goal of this work is to achieve consensus for a multi-agent system consisting of N agents with dynamics (1) in distributed fashion. This is said to be achieved if

$$\begin{aligned}\|\boldsymbol{q}_i - \boldsymbol{q}_j\|_2 &= 0\\ \|\dot{\boldsymbol{q}}_i\|_2 &= 0,\end{aligned}$$

for all $i, j = 1, ..., N$. In this context distributed means that each agent i has access only to the states of its neighbors in N_i, characterized by the communication graph. We assume constant unknown communication delays, such that at time t agent i can only access the state of agent j delayed by a finite amount of time T_{ji}. It should be noted that the delays $T_{ji}, \forall(i, j) \notin \mathcal{V}$ do not have any physical meaning since the state j is not accessible by agent i and we simply chose $T_{ji} = 0, \forall(i, j) \notin \mathcal{V}$. In addition to solving the problem in time-continuous fashion, we consider the case of event-triggered communication and actuation updates. As a result, signal transmissions between agents and actuation updates only occur during certain discrete time instances. The goal for such schemes is to design a triggering condition, which online determines the event-times t_{k_i} and t_{c_i} for the communication and actuation updates, respectively, as

$$t_{k_i+1} = \inf\{t > t_{k_i} | \phi_i(t) \le 0\}, \tag{2}$$

$$t_{c_i+1} = \inf\{t > t_{c_i} | \varphi_i(t) \le 0\}, \tag{3}$$

where ϕ_i and φ_i are functions to be designed. In between events, the respective value is hold constant with a zero-order hold. Important for the design of event-triggered systems is avoidance of the so called Zeno behavior which describes the occurrence of infinite events in a finite time. More formally, a system is said to exhibit Zeno behavior if there exist finite k_i or c_i such that $t_{k_i+1} - t_{k_i} = 0$ or $t_{c_i+1} - t_{c_i} = 0$. It should be noted that the triggering is desired to be asynchronous, since all agents transmitting their data at the same time would defy the purpose of the triggering in the communication. In addition, only locally available information can be used for deciding the event times. Finally, we desire a model-free algorithm in the sense that the system dynamics are assumed to be completely unknown and no estimation/adaptation is required.

3. MAIN RESULTS

In this section, we first introduce the basic ideas with a continuous control law. Then we present a strategy to dynamically obtain the communication instants t_{k_i} and extend the result to event-triggered actuator updates at times t_{c_i}, which occur independently of communication events.

3.1 Time-continuous Communication and Control

Starting with the time-continuous case, we present the following control law

$$\boldsymbol{\tau}_i(t) = -\boldsymbol{D}_i\dot{\boldsymbol{q}}_i(t) - \boldsymbol{K}_i\left(\boldsymbol{q}_i(t) - \boldsymbol{x}_i(t)\right), \tag{4}$$

with symmetric positive definite $\boldsymbol{K}_i, \boldsymbol{D}_i \in \mathbb{R}^{n\times n}$ and where $\boldsymbol{x}_i \in \mathbb{R}^n$ is the state of the dynamical system

$$\dot{\boldsymbol{x}}_i(t) = -\sum_{j\in N_i} w_{ij}\left(\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t-T_{ji})\right) - \boldsymbol{K}_i\dot{\boldsymbol{q}}_i(t). \tag{5}$$

Note that the $\boldsymbol{x}_i$ dynamics consist of two parts. The idea is to exploit the robustness of the first order agreement problem to time delays as reported in (Münz et al., 2010). As such, the first term consists of a standard first order agreement problem with communication delays. The second term is the velocity feedback from the ith agent. Without the first term, $\boldsymbol{x}_i$ would simply integrate the ith agents velocity as commonly done in coordination algorithms for port-Hamiltonian systems (Vos et al., 2014). The dynamics of the ith agent are then coupled to their corresponding controller state with a simple position feedback and a local damping term in (4), ensuring stability of the system as given in the following theorem.

Theorem 1. Consider the multi-agent system consisting of N agents with Euler-Lagrange dynamics (1), driven by the time-continuous control law (4) with controller dynamics (5). If the underlying communication graph is connected, then the agents asymptotically reach consensus with zero velocity, i.e. $\|\dot{\boldsymbol{q}}\|_2 \to \boldsymbol{0}$ and $\|\boldsymbol{q}_i - \boldsymbol{q}_j\|_2 \to \boldsymbol{0}$.

Proof. Define

$$V = \frac{1}{2}\left[\underbrace{\|\dot{\boldsymbol{q}}\|^2_{\boldsymbol{M}} + \|\boldsymbol{q}\|^2_{\boldsymbol{K}}}_{V_a} + \underbrace{\|\boldsymbol{x}\|^2_2}_{V_c}\right]. \tag{6}$$

Now consider the Lyapunov candidate

$$V_1 = V + \underbrace{\sum_{(i,j)\in\mathcal{E}}\int_{t-T_{ij}}^{t} w_{ij}\,\|\boldsymbol{x}_i\|^2_2}_{V_d}, \tag{7}$$

where $w_{ij} > 0$ is the weight of the edge (i,j). The derivative of (7), along the solutions of (1) with (4) and (5), evaluates to

$$\dot{V}_1 = -\dot{\boldsymbol{q}}^T\boldsymbol{D}\dot{\boldsymbol{q}} - \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i} w_{ij}\,\|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t-T_{ji})\|^2_2 \tag{8}$$

and $\|\dot{\boldsymbol{q}}\|_2, \|\boldsymbol{x}_i - \boldsymbol{x}_j\|_2 \to \boldsymbol{0}, \forall i,j$ follows immediately. Combined with the dynamics (1), (4), $\boldsymbol{q}_i \to \boldsymbol{x}_i$ can be concluded. □

Remark 2. Note that the Lyapunov candidate (7) can be divided into V_a and $V_c + V_d$, corresponding to the agent and controller dynamics, respectively. From a passivity point of view, it can be shown that with the proposed control law the agent and controller dynamics are passive with input-output pairs $(\boldsymbol{x}_i, \boldsymbol{K}_i\dot{\boldsymbol{q}}_i)$ and $(-\boldsymbol{K}_i\dot{\boldsymbol{q}}_i, \boldsymbol{x}_i)$, respectively, since $\dot{V}_a \le \boldsymbol{x}_i^T\boldsymbol{K}_i\dot{\boldsymbol{q}}_i$ and $\dot{V}_c + \dot{V}_d \le -\boldsymbol{x}_i^T\boldsymbol{K}_i\dot{\boldsymbol{q}}_i$. This is achieved by introducing the term $\boldsymbol{K}_i\dot{\boldsymbol{q}}_i$ in (5). As a result, the cross term in the Lyapunov function vanishes and the feedback interconnection of the two systems is stable. In addition, with output $\boldsymbol{x}_i$, the synchronization of the controller states can be interpreted from a scattering transformation point of view. In this work however, since the synchronization is achieved for the first order system (5) and the agents dynamics (1) are merely coupled to those, no knowledge of the agents dynamics is required.

3.2 Event-Triggered Communication

With the control law presented in the last section, the controller states $\boldsymbol{x}_i$ are send over the communication network in time-continuous fashion, leading to an inefficient use of the communication bandwidth. Here we present a modified version, where transmissions between agents are limited to occur only at certain discrete time-instances t_{k_i}. For this consider the modified controller dynamics with event-triggered communication

$$\dot{\boldsymbol{x}}_i(t) = -\sum_{j\in N_i} w_{ij}\left(\bar{\boldsymbol{x}}_i(t) - \bar{\boldsymbol{x}}_j(t-T_{ji})\right) - \boldsymbol{K}_i\dot{\boldsymbol{q}}_i(t), \tag{9}$$

where

$$\bar{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t_{k_i}). \tag{10}$$

Following (Girard, 2015), we design a dynamic triggering law based on the auxiliary variable η_i with dynamics

$$\dot{\eta}_i(t) = -\alpha(\eta_i(t)) + f_i(t), \tag{11}$$

with initial condition $\eta_i(0) > 0$, class $\mathcal{K}_\infty$ function α, and

$$\begin{aligned} f_i(t) =& \frac{1}{2}\sum_{j\in N_i}\sigma_i w_{ij}\,\|\bar{\boldsymbol{x}}_i(t) - \bar{\boldsymbol{x}}_j(t-T_{ji})\|^2_2 \\ &- \left\|\sum_{j\in N_i} w_{ij}(\bar{\boldsymbol{x}}_i(t) - \bar{\boldsymbol{x}}_j(t-T_{ji}))\right\|_2 \|\hat{\boldsymbol{x}}_i(t)\|_2, \end{aligned} \tag{12}$$

with $0 < \sigma_i < 1$, and the trigger induced error is given by

$$\hat{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t) - \boldsymbol{x}_i(t_{k_i}). \tag{13}$$

With the triggering law

$$t_{k_i+1} = \inf\left\{t > t_{k_i} | \eta_i(t) + \theta_i f_i \le 0\right\}, \tag{14}$$

with $\theta_i > 0$, we can establish following lemma, which will prove useful for the analysis to follow.

Lemma 3. If the event-times t_{k_i+1} are chosen according to (14), then $\eta_i(t) > 0, \forall 0 \le t < \infty$.

The proof follows (Girard, 2015) and is omitted here. The triggering instances in (14) are determined by the state of the auxiliary system (11) and the triggering function (12). The latter is commonly used to maintain stability of the triggered system (Tabuada, 2007), while the former can increase the triggering performance and allows us to show that no Zeno behavior occurs. The control scheme with event-triggered communication is depicted in Fig. 1. With the proposed triggering strategy, we obtain the following stability result.

Theorem 4. Consider the multi-agent system consisting of N agents with Euler-Lagrange dynamics (1), driven by the time-continuous control law (4) with event-triggered

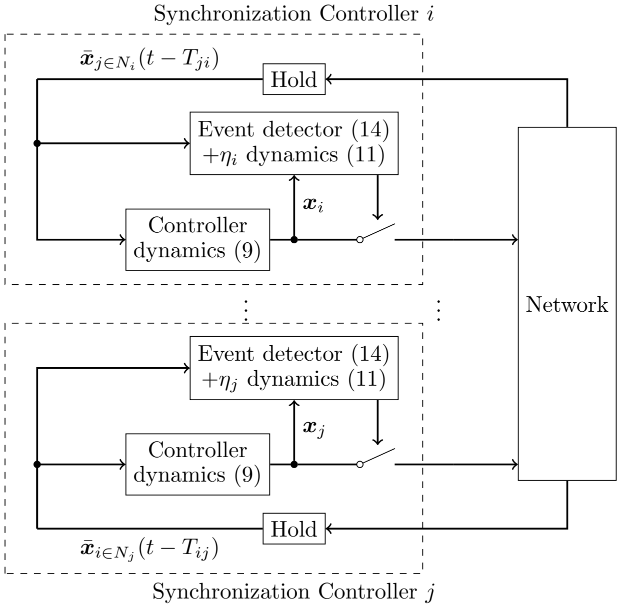

Fig. 1. Block diagram of the event-triggered communication scheme. For clarity the local dynamics of the plant are omitted.

controller dynamics (9). If the underlying communication graph is connected and the triggering instances t_{k_i} are locally chosen by each agent i as (14) then the agents asymptotically reach consensus with zero velocity, i.e. $\|\dot{\boldsymbol{q}}\|_2 \to 0$ and $\|\boldsymbol{q}_i - \boldsymbol{q}_j\|_2 \to 0$ and no Zeno behavior occurs.

Proof. Consider the Lyapunov candidate

$$V_2 = V + \sum_{(i,j)\in\mathcal{E}} \int_{t-T_{ij}}^{t} w_{ij} \|\bar{\boldsymbol{x}}_i\|_2^2 + \sum_{i=1}^{N} \eta_i, \tag{15}$$

the derivative of which, along the solutions of (1) with (4) and (9), evaluates to

$$\begin{aligned}
\dot{V}_2 =& -\dot{\boldsymbol{q}}^T \boldsymbol{D}\dot{\boldsymbol{q}} - \sum_{i=1}^{N}\sum_{j\in N_i} w_{ij}(\hat{\boldsymbol{x}}_i + \bar{\boldsymbol{x}}_i)\,(\bar{\boldsymbol{x}}_i - \bar{\boldsymbol{x}}_j(t-T_{ji}))\\
&- \sum_{i=1}^{N}\sum_{j\in N_i} \left[\frac{w_{ij}}{2}\bar{\boldsymbol{x}}_j(t-T_{ji})^T\bar{\boldsymbol{x}}_j(t-T_{ji}) - \frac{w_{ij}}{2}\bar{\boldsymbol{x}}_i^T\bar{\boldsymbol{x}}_i\right]\\
&+ \sum_{i=1}^{N} [f_i - \alpha(\eta_i)]\\
\leq& -\sum_{i=1}^{N}\left[\lambda_{min}(\boldsymbol{D}_i)\|\dot{\boldsymbol{q}}_i\|_2^2 + \alpha(\eta_i)\right]\\
&- \sum_{i=1}^{N}\frac{1-\sigma_i}{2}\sum_{j\in N_i} w_{ij}\|\bar{\boldsymbol{x}}_i - \bar{\boldsymbol{x}}_j(t-T_{ji})\|_2^2
\end{aligned}$$

and as a result $\|\dot{\boldsymbol{q}}_i\|_2, \|\bar{\boldsymbol{x}}_i - \bar{\boldsymbol{x}}_j(t-T_{ji})\|_2, \eta_i \to 0$. Note that due to (14) and Lemma 3, $f_i \geq 0$ and thus

$$\|\hat{\boldsymbol{x}}_i(t)\|_2 \leq \frac{\sum_{j\in N_i}\sigma_i w_{ij}\|\bar{\boldsymbol{x}}_i(t) - \bar{\boldsymbol{x}}_j(t-T_{ji})\|_2^2}{2\left\|\sum_{j\in N_i} w_{ij}(\bar{\boldsymbol{x}}_i(t) - \bar{\boldsymbol{x}}_j(t-T_{ji}))\right\|_2}. \tag{16}$$

Since $\|\bar{\boldsymbol{x}}_i - \bar{\boldsymbol{x}}_j(t-T_{ji})\|_2 \to 0$, it follows that $\hat{\boldsymbol{x}}_i(t) \to 0$ and thus $\|\boldsymbol{x}_i - \boldsymbol{x}_j(t-T_{ji})\|_2 \to 0$ can be concluded. Finally, from $\|\dot{\boldsymbol{q}}_i\|_2 \to 0$ and the dynamics (1), (4) it can be concluded that $\boldsymbol{q}_i \to \boldsymbol{x}_i$ and thus $\|\boldsymbol{q}_i - \boldsymbol{q}_j(t-T_{ji})\|_2 \to 0$. Note that $\|\hat{\boldsymbol{x}}(t_{k_i})\|_2 = 0$ at the triggering instances t_{k_i} and with above stability results it can be verified that

$$\frac{d}{dt}\|\hat{\boldsymbol{x}}(t_{k_i})\|_2 \leq \gamma, \tag{17}$$

in the interval $t \in [t_{k_i}, t_{k_i+1})$, where $\gamma > 0$. As a result we obtain by integration over $[t_{k_i}, t]$,

$$\|\hat{\boldsymbol{x}}(t)\|_2 \leq (t - t_{k_i})\gamma. \tag{18}$$

In addition, note that due to the triggering condition (14) events can only occur in the case $\left\|\hat{\boldsymbol{x}}_i(t_{k_i+1}^-)\right\|_2 \geq \epsilon > 0$ and thus we obtain

$$(t_{k_i+1}^- - t_{k_i}) \geq \frac{\epsilon}{\gamma} > 0. \tag{19}$$

□

3.3 Event-Triggered Control and Communication

In the previous section, we introduced event-triggered communication between agents in order to achieve a more efficient use of the communication bandwidth. However, some applications with limited computational resources might also not be able to cope with the high sampling frequencies typical used to mimic time-continuous behavior. In order to overcome this limitation, we extend the proposed control law with an event-triggered controller update strategy, as

$$\boldsymbol{\tau}_i(t) = -\boldsymbol{D}_i\dot{\boldsymbol{q}}_i(t_{c_i}) - \boldsymbol{K}_i(\boldsymbol{q}_i(t_{c_i}) - \boldsymbol{x}_i(t_{c_i})), \tag{20}$$

where the dynamics of $\boldsymbol{x}_i$ are given as in (9). Similar to the previous section, we design a dynamic triggering law based on the auxiliary variable μ_i with dynamics

$$\dot{\mu}_i(t) = -\beta(\mu_i(t)) + h_i(t), \tag{21}$$

with initial condition $\mu_i(0) > 0$, class $\mathcal{K}_\infty$ function β, and

$$h_i(t) = -\|\tilde{\boldsymbol{\tau}}_i\|_2\|\dot{\boldsymbol{q}}_i\|_2 + \delta_i\lambda_{min}(\boldsymbol{D})\|\dot{\boldsymbol{q}}_i\|_2^2, \tag{22}$$

with $0 < \delta_i < 1$ and the trigger-induced error defined as

$$\tilde{\boldsymbol{\tau}}_i(t) = \boldsymbol{\tau}_i^c(t) - \boldsymbol{\tau}_i(t_{c_i}), \tag{23}$$

where $\boldsymbol{\tau}_i^c$ refers to the time-continuous control law (4). Note the difference between $\boldsymbol{x}_i(t) - \boldsymbol{x}_i(t_{c_i})$ in $\tilde{\boldsymbol{\tau}}_i(t)$ and $\hat{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t) - \boldsymbol{x}_i(t_{k_i})$. The error $\boldsymbol{x}_i - \boldsymbol{x}_i(t_{c_i})$ is induced by the triggering in the control-law (20), while $\hat{\boldsymbol{x}}_i$ is related to the event-triggered communication in the controller dynamics (9). This also means that there is asynchronous triggering for the communication and control. This allows for a certain degree of flexibility, since event-triggering in the communication and control are motivated by different factors and can thus be designed independently with the concrete task characteristics in mind. Similar to Lemma 3, if the event-times t_{c_i+1} are chosen according to

$$t_{c_i+1} = \inf\{t > t_{c_i} | \mu_i(t) + \vartheta_i h_i \leq 0\}, \tag{24}$$

with $\vartheta_i > 0$, the following lemma can be easily verified.

Lemma 5. If the event-times t_{c_i+1} are chosen according to (24), then $\mu_i(t) > 0, \forall 0 < t < \infty$.

The complete control and communication architecture can be found in Fig. 2. With the proposed triggering strategy, we can state the following stability result for the asynchronous event-triggered communication and control law.

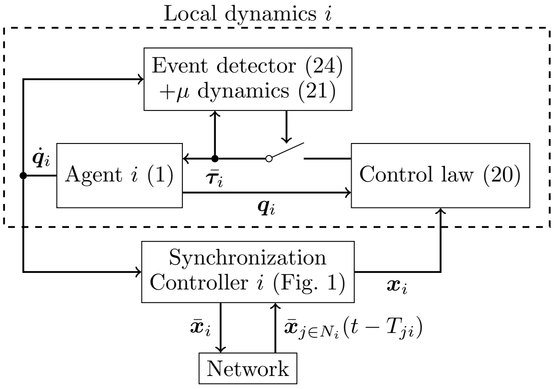

Fig. 2. Block diagram of the control scheme with event-triggered control and communication.

Theorem 6. Consider the multi-agent system consisting of N agents with Euler-Lagrange dynamics (1), driven by the event-triggered control law (20), with event-triggered communication in the controller dynamics (9). If the underlying communication graph is connected and the triggering instances t_{k_i} and t_{c_i} are locally chosen by each agent i according to (14) and (24), respectively, then the agents asymptotically reach consensus with zero velocity, i.e. $\|\dot{\boldsymbol{q}}\|_2 \to \mathbf{0}$ and $\|\boldsymbol{q}_i - \boldsymbol{q}_j\|_2 \to \mathbf{0}$ and no Zeno behavior occurs.

Proof. Consider the Lyapunov candidate

$$V_3 = V_2 + \sum_{i=1}^{N} \mu_i. \tag{25}$$

the derivative of which, along the solutions of (1) with (20) and (9), evaluates to

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{N} \left[\dot{\boldsymbol{q}}_i^T \tilde{\boldsymbol{\tau}}_i + f_i - \alpha(\eta_i) + h_i - \beta(\mu_i)\right] \\ &+ \sum_{i=1}^{N} \hat{\boldsymbol{x}}_i^T \sum_{j\in N_i} w_{ij}(\bar{\boldsymbol{x}}_i - \bar{\boldsymbol{x}}_j(t-T_{ji})) \\ &- \dot{\boldsymbol{q}}^T \boldsymbol{D} \dot{\boldsymbol{q}} - \sum_{i=1}^{N} \sum_{j\in N_i} \frac{w_{ij}}{2} \|\bar{\boldsymbol{x}}_i - \bar{\boldsymbol{x}}_j(t-T_{ji})\|_2^2 \\ \leq& -\sum_{i=1}^{N} \Big[(1-\delta_i)\lambda_{min}(\boldsymbol{D}) \|\dot{\boldsymbol{q}}_i\|_2^2 + \alpha(\eta_i) + \beta(\eta_i) \\ &+ \frac{1-\sigma_i}{2} \sum_{j\in N_i} w_{ij} \|\bar{\boldsymbol{x}}_i - \bar{\boldsymbol{x}}_j(t-T_{ji})\|_2^2 \Big].\end{aligned}$$

The rest of the proof follows the same argumentation as the proof of Theorem 4 and is omitted here. □

4. SIMULATIONS

In this section we present simulation results in order to illustrate the presented results. Due to space limitations we only provide results for the more general case with event-triggered control and communication as proposed in Section 3.3.

4.1 Setup

The dynamics (1) and the initial positions of the agents, moving in two dimensional space, are chosen as in (Liu et al., 2016) and the communication graph corresponds to a circular graph, where each agent has two neighbors with equal edge weights $w_{ij} = w, \forall(i,j) \in \mathcal{E}$. The controller parameters can be found in Table 1 and we chose $\alpha_i(\eta_i) = \eta_i$ and $\beta_i(\mu_i) = \mu_i$. The simulation frequency is chosen as $\frac{1}{t_s} = 10\text{kHz}$ and lasts for 10s. The results are obtained with a communication delay of $T_{ij} = 100\text{ms}, \forall(i,j) \in \mathcal{E}$.

Table 1. Simulation parameters

$\boldsymbol{D}_i$	$\boldsymbol{K}_i$	w	δ_i	σ_i	$\eta_i(0)$	$\mu_i(0)$	θ_i, ϑ_i
$15\boldsymbol{I}_n$	$10\boldsymbol{I}_n$	10	0.9	0.45	5	2.5	0.1

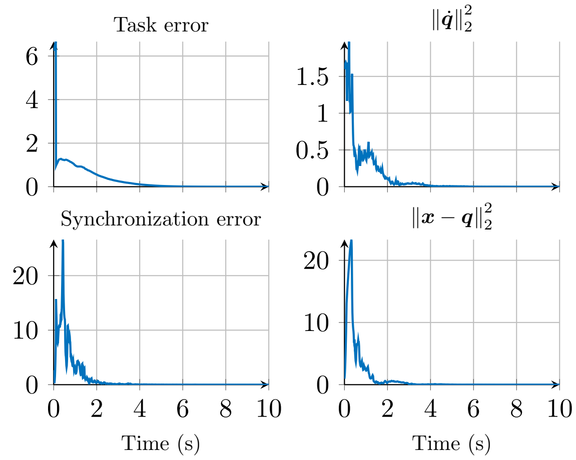

Fig. 3. Clockwise: Task Error (26), velocity error $\|\dot{\boldsymbol{q}}_i\|_2^2$, synchronization error (27) and tracking error $\|\boldsymbol{x}-\boldsymbol{q}\|_2^2$ of the proposed event-triggered communication and control scheme.

4.2 Results

The convergence of the relevant quantities is depicted in Fig. 3. In the top left plot the task error is defined as

$$\sum_{i=1}^{N} \sum_{j\in N_i} \|\boldsymbol{q}_i(t) - \boldsymbol{q}_j(t-T_{ji})\|_2^2, \tag{26}$$

and describes the achievement of the consensus task. This is a result of the convergence of the synchronization error defined as

$$\sum_{i=1}^{N} \sum_{j\in N_i} \|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t-T_{ji})\|_2^2 \tag{27}$$

to zero, as shown in the bottom left, and the fact the vector of generalized coordinates converges to the state of the controller, i.e. $\|\boldsymbol{x}-\boldsymbol{q}\|_2 \to 0$ as depicted in the bottom right plot. Finally, it can be seen in the top right plot that the velocity $\|\dot{\boldsymbol{q}}\|_2 \to 0$ and the system remains in the equilibrium. Additionally, the control scheme is illustrated by providing the first dimension of the individual state trajectories for the agents and controller in Fig. 4. Regarding the triggering, it becomes apparent from Table 2 that the events for all agents can be drastically reduced to less than 100 and 150 events in the $10s$ of simulation for the communication and controller, respectively. This is further supported by the minimum inter-event time, which is bigger than $0.015 = 150t_s$ for all agents. In addition, this

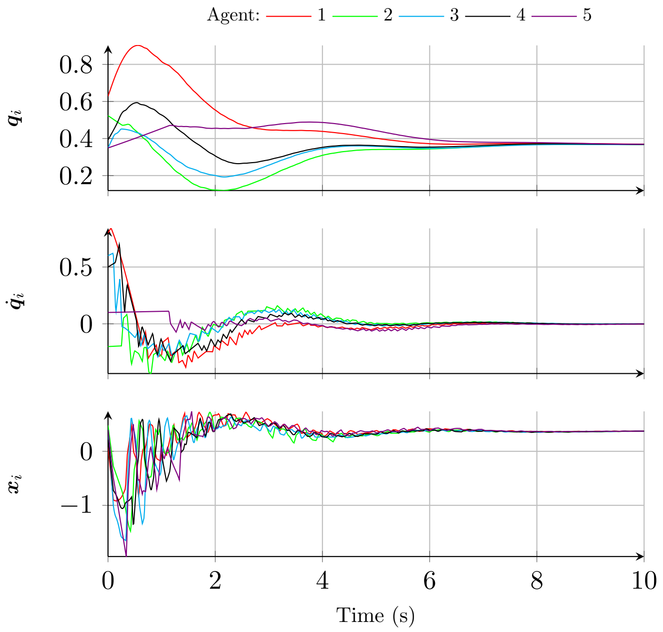

Fig. 4. Trajectories of the systems states for each agent i. From top to bottom: vector of generalized coordinates $\boldsymbol{q}_i$, velocity thereof $\dot{\boldsymbol{q}}_i$, and controller state $\boldsymbol{x}_i$. For clarity, the results are presented only for the first dimension of each state.

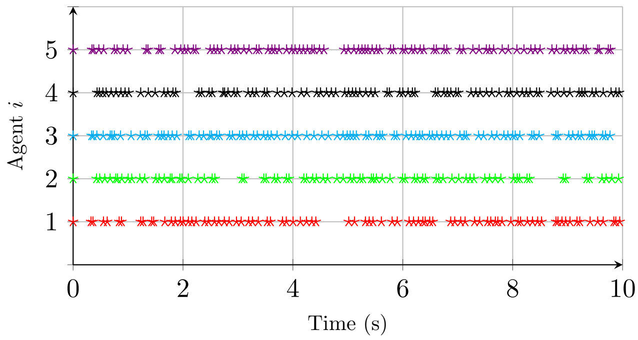

Fig. 5. Event-times t_{k_i} for the communication for all agents using the proposed event-triggered communication and control scheme.

Table 2. Number of event and minimum inter-event times for the event-trigger in the communication (14) and control (24).

i	Events (14)	Events (24)	t_{min} (14)	t_{min} (24)
1	85	133	0.0353	0.0202
2	80	136	0.0303	0.0185
3	96	143	0.0299	0.0165
4	90	144	0.0293	0.0198
5	92	149	0.0303	0.0158

result confirms that no Zeno behavior occurred during the simulation, which can also be observed from Fig. 5, depicting the event-times t_{k_i} for the communication instances of all agents. Similar figures can be obtained for controller updates, but are omitted here due to space limitations.

5. CONCLUSION

In this work we present a control law for the consensus problem for distributed Euler-Lagrange systems. We provide an even-triggering strategy in order to reduce the number of transmission between the agents, as well as the actuator updates of the controller, in order to safe valuable resources. In order to cope with communication delays we use results on first order agreement problems and introduce an auxiliary system, which is then coupled to the agent in an energetically passive fashion. As a result no knowledge and with that no assumptions on the Euler-Lagrange dynamics are required.

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