

Multi-Objective Following Control for Heavy-Duty Vehicles using Differential Dynamic Programming

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Abstract: The speed with which an ego-vehicle follows a lead vehicle through traffic can significantly affect the former's fuel consumption, safety, average speed, and ride comfort. This paper merges these objectives and constraints into a unified trajectory optimization problem. One of the paper's goals is to provide a unified formulation of problems traditionally tackled independently, e.g., platooning, fuel-minimizing vehicle speed trajectory optimization, etc. Another key goal is to demonstrate the degree to which Differential Dynamic Programming (DDP) provides a conceptually attractive and computationally inexpensive decomposition of the resulting multi-objective problem. In this decomposition, perturbations from an optimal steady-state vehicle trajectory are controlled using a linear quadratic regulation (LQR) law obtained analytically through DDP. We examine the performance of this controller simulating a representative urban drive cycle with a lead vehicle. We also perform a sensitivity study on the parameters in the objective and explore their effect in both fuel economy and deviations from nominal headway distance. Finally, we explore the effect of different levels of collaboration between vehicles by assuming the lead vehicle shares its predicted future average acceleration.

Keywords: Automatic control, optimization, real-time operations in transportation; Trajectory and Path Planning; Trajectory Tracking and Path Following

1. INTRODUCTION

This paper examines the problem of optimizing the speed with which a heavy-duty ego-vehicle follows a lead vehicle through urban/suburban traffic. The paper exploits inter-vehicle connectivity and automation, specifically by assuming that the lead vehicle's velocity and acceleration are known to the ego-vehicle either instantaneously or ahead of time, with some look-ahead horizon. The objective of the paper's speed trajectory optimization problem is threefold, namely, to: (i) minimize fuel consumption, (ii) maximize speed, and (iii) minimize the rate of change of vehicle power demand. The last of these three competing Pareto objectives prioritizes vehicle operator comfort. Constraints on this optimization problem reflect a mix of physical, legal, and safety limitations, namely: engine/braking power constraints, speed limits, and minimum inter-vehicle spacing.

Motivation for the above optimization problem is twofold. First, there is a growing need for fuel economy improvement in the heavy-duty transportation sector. Emissions from the overall transportation sector accounted for 28.5% of greenhouse gases emitted in the United States in 2016, with medium- and heavy-duty trucks and buses accounting for 24% of carbon emissions in that sector (EPA, 2018). To address this issue, Environmental Protection Agency (EPA) mandates will require heavy-duty vehicles to reduce their fuel consumption by 25% by 2027 (EPA, 2016).

Second, emerging vehicle connectivity and automation technologies provide new possibilities for fuel consumption minimization in the transportation sector, as explained below.

One way to significantly improve the fuel economy of heavy-duty-vehicles is by shaping their speed profile so that energy waste is minimized. This opportunity is well-supported by the existing literature. Hellström et al. (2006; 2007), for example, implement a controller that minimizes fuel and trip time given the topography of the route using dynamic programming. They observe fuel savings of up to 8% in on-road experiments. Similarly, Groelke et al. (2018), compare three different model predictive control strategies for fuel economy optimization of a heavy-duty-vehicle. The first strategy directly penalizes fuel consumption, the second one penalizes braking effort, and the third one minimizes a convex objective with a tradeoff between energy expenditure and tracking of a coarse dynamic programming solution. They observe fuel savings in simulation of 4%-7%.

Another way to improve the fuel economy of heavy-duty-vehicles is for them to travel close to each other forming a platoon. This problem is closely related to the problem of adaptive cruise control (ACC). In fact, Li et al. (2011), implemented a model predictive control-based ACC that penalized fuel consumption, and achieved fuel savings of 5.9%. In a platoon of multiple heavy-duty-vehicles, the

following vehicles have the advantage of reducing their aerodynamic drag by traveling in the wake of a leading vehicle. Alam et al. (2010) observe fuel savings of up to 7.7% in experiments with trucks of different masses, platooning at 70 km h^{-1} . Typically, platooning is implemented by algorithms that maintain either a constant headway distance or a constant gap time. It is possible to exploit future predictions/information to further reduce braking and thus fuel consumption: a fact demonstrated by Turri et al.'s work on cooperative look-ahead control (2015). This work's simulation studies suggest fuel savings of up to 12% when compared to conventional platoon controllers. A model predictive controller can also be implemented with non-cooperative lead that communicates its future speed trajectory, (see Turri et al., 2017).

Compared to the above literature, our goal in this paper is twofold. First, the paper integrates the vehicle speed trajectory optimization and platooning problems into a single multi-objective trajectory optimization problem statement. This provides a unified framework within which these two problems, traditionally treated in the literature as separate, can be examined. Second, the paper applies differential dynamic programming (DDP) to the resulting unified problem formulation. The use of DDP provides insight into the structure of the solution to this unified problem. Moreover, it leads to a very simple, albeit somewhat approximate, solution structure that naturally lends itself to online real-time implementation.

The remainder of this paper is organized as follows. Sections II and III present the vehicle model and optimization problem statement used in this work, respectively. Sections IV-VI then present the solution framework, focusing, respectively, on: the use of differential dynamic programming to construct a problem solution structure; the underlying development of a simple kinematic representation of the lead vehicle's behavior; and the selection of Pareto optimization weights using Bryson's rule (see Bryson and Ho, 1975). Finally, Section VII presents simulation case studies demonstrating the solution to the DDP problem, and Section VIII summarizes the outcomes of this work.

2. VEHICLE MODEL

We consider a platoon of two heavy-duty vehicles. We can model the dynamics of the following vehicle with a simple longitudinal point mass model. In this model the truck's propulsive force F_{prop} is opposed by the rolling resistance F_{roll} , and the aerodynamic drag F_{air} :

$$\dot{v} = \frac{1}{m}(F_{prop} - F_{air} - F_{roll}) \quad (1)$$

where:

$$F_{air} = \frac{1}{2}\rho C_d A_f v_{air}^2 \quad (2)$$

$$F_{roll} = \mu mg \cos(\theta) + mg \sin(\theta) \quad (3)$$

$$F_{prop} = \frac{P}{v} \quad (4)$$

In this model, ρ is the air density, C_d is the aerodynamic drag coefficient, A_f is the frontal area of the truck, v_{air} is

the wind speed relative to the truck, μ is the rolling friction coefficient, m is the vehicle mass, g the gravitational acceleration, θ the road inclination, v is the truck speed, and P is the propulsion power if positive, or the braking power if negative.

In a platoon, the effective air speed that the trailing vehicle experiences is reduced by the aerodynamic wake of the lead vehicle. It is common to model the effect of the wake of the lead vehicle in terms of an aerodynamic drag coefficient dependent on headway h :

$$F_{air} = \frac{1}{2}\rho C_d(h) A_f v^2 \quad (5)$$

Using the results of wind tunnel experiments in Vohra et al. (2019) we fit a sigmoidal function to the drag coefficient reduction with respect to headway distance and obtain:

$$C_d(h) = C_{d0} \frac{a}{(1 + e^{-bh})} \quad (6)$$

where a and b are constant parameters.

If we assume the vehicles travel on a flat road, the dynamics of our two vehicles platoon can be written in state-space form as follows:

$$\dot{h} = v_{lead} - v \quad (7)$$

$$\dot{v} = \frac{1}{m} \left(\frac{P}{v} - \frac{1}{2}\rho C_d(h) A_f v^2 - \mu mg \right) \quad (8)$$

$$\dot{P} = u \quad (9)$$

In this model we assume that the control input to the system is the derivative of the propulsion power.

3. PROBLEM STATEMENT

Our primary objective is to minimize the fuel consumption of the follower truck. The trivial solution to this problem, is for the truck not to move. This makes it necessary to formulate an optimization objective that performs a Pareto trade-off between fuel consumption and trip time. It is also important to consider both passenger comfort and engine health. Additionally, a truck should not exceed the speed limit of the road, and should maintain a safe following distance vis-a-vis the vehicle ahead. Finally, the engine has a maximum power that it can output, and the brakes have a maximum braking power that they can dissipate. We propose the following optimization problem to capture these goals:

$$\text{Minimize } J = \int_0^T (L(\mathbf{x}, u)) dt \quad (10)$$

Subject to:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u) \quad (11)$$

$$\mathbf{g}(\mathbf{x}) \leq \mathbf{0} \quad (12)$$

where:

$$\mathbf{x} = \begin{bmatrix} h \\ v \\ P \end{bmatrix} \mathbf{f}(\mathbf{x}, u) = \begin{bmatrix} v_{lead} - v \\ \frac{1}{m} \left(\frac{P}{v} - \frac{1}{2} \rho C_d(h) A_f v^2 - \mu m g \right) \\ u \end{bmatrix} \quad (13)$$

$$L(\mathbf{x}, u) = \dot{m}_{idle} + \beta P - cv + \frac{1}{2} R u^2 \quad (14)$$

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} h_{min} - h \\ v - v_{max} \\ v_{min} - v \\ P - P_{max} \\ P_{min} - P \end{bmatrix} \quad (15)$$

Notice that the in first term of (14) we approximate the fuel consumption of the vehicle as having an affine relationship with propulsion power. This is a standard Willans line representation of engine fuel consumption. The second term ensures we maximize speed, and the last term imposes a quadratic penalty on the time derivative of power demand.

We can approximate the inequality constraints by using logarithmic barrier functions in the objective:

$$-\alpha_i (\ln(-g_i(\mathbf{x}))) \quad (16)$$

The resulting optimization problem has the following structure:

$$\text{Minimize } J' = \int_0^T (L'(\mathbf{x}, u)) dt \quad (17)$$

$$\text{Subject to: } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad (18)$$

where:

$$L'(\mathbf{x}, u) = L(\mathbf{x}, t) + \sum_{i=1}^3 C_i \quad (19)$$

$$C_1 = -\alpha_1 (\ln(h - h_{min})) \quad (20)$$

$$C_2 = -\alpha_2 (\ln(v - v_{min}) + \ln(v_{max} - v)) \quad (21)$$

$$C_3 = -\alpha_3 (\ln(P - P_{min}) + \ln(P_{max} - P)) \quad (22)$$

Assuming the lead vehicle eventually reaches a constant speed, this problem has a steady-state optimal solution. At steady-state:

$$\mathbf{f}(\mathbf{x}) = 0 \quad (23)$$

$$h = h_{ss} \quad (24)$$

$$v = v_{ss} = \lim_{t \rightarrow \infty} v_{lead} \quad (25)$$

$$P = P_{ss} = \frac{1}{2} \rho C_d(h_{ss}) A_f v_{ss}^3 + \mu m g v_{ss} \quad (26)$$

We can substitute these values into J' and optimization problem becomes static.

4. DIFFERENTIAL DYNAMIC PROBLEM

Differential dynamic programming (Mayne, 1973) relies on approximating an optimization problem around a nominal

trajectory. We are going to consider a steady-state optimum, and approximate our problem around it.

At steady state, the optimization problem written in (17) becomes:

$$\text{Minimize } L'(\mathbf{x}_0, u_0) \quad (27)$$

$$\text{Subject to: } \mathbf{f}(\mathbf{x}_0, u_0) \quad (28)$$

To solve this problem, we augment our objective with Lagrange multipliers:

$$\text{Minimize } L'(\mathbf{x}_0, u_0) + \boldsymbol{\lambda}_0^T \mathbf{f}(\mathbf{x}_0, u_0) \quad (29)$$

The first optimality condition is:

$$\nabla(L'(\mathbf{x}_0^*, u_0^*) + \boldsymbol{\lambda}_0^{*T} \mathbf{f}(\mathbf{x}_0^*, u_0^*)) = \mathbf{0} \quad (30)$$

where, $\mathbf{x}_0^*, u_0^*, \boldsymbol{\lambda}_0^*$ constitute a solution to this problem.

Now let us define deviations around this steady-state optimum as: $\delta \mathbf{x}, \delta u, \delta \boldsymbol{\lambda}$, where:

$$\mathbf{x} = \mathbf{x}_0^* + \delta \mathbf{x} \quad (31)$$

$$u = u_0^* + \delta u \quad (32)$$

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}_0^* + \delta \boldsymbol{\lambda} \quad (33)$$

We can write our original problem as:

$$\text{Minimize } J' = \int_0^T (L'(\mathbf{x}_0^* + \delta \mathbf{x}, u_0^* + \delta u)) dt \quad (34)$$

$$\text{Subject to: } \delta \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}_0^* + \delta \mathbf{x}, u_0^* + \delta u) \quad (35)$$

We can approximate the objective as quadratic and the dynamics as linear, around the optimal steady state. The problem becomes:

$$\text{Minimize } J'_{approx} = \int_0^T [L'_{approx}] dt \quad (36)$$

$$\text{Subject to: } \delta \dot{\mathbf{x}} = \mathbf{f}_{\mathbf{x}}|_{\mathbf{x}_0^*, u_0^*} \delta \mathbf{x} + \mathbf{f}_{\mathbf{u}}|_{\mathbf{x}_0^*, u_0^*} \delta u \quad (37)$$

where:

$$L'_{approx} = L'(\mathbf{x}_0^*, u_0^*) + L'_x \delta \mathbf{x} + L'_u \delta u + \frac{1}{2} [\delta \mathbf{x}^T \delta u] \mathbf{H}(L') [\delta \mathbf{x}^T \delta u]^T \quad (38)$$

and $\mathbf{H}(L')$ is the Hessian of L' .

We can remove the constant $L'(\mathbf{x}_0^*, u_0^*)$ in the integrand, since it does not affect the solution of the optimization problem:

$$L'_{approx} := L'_x \delta \mathbf{x} + L'_u \delta u + \frac{1}{2} [\delta \mathbf{x}^T \delta u] \mathbf{H}(L') [\delta \mathbf{x}^T \delta u]^T \quad (39)$$

We can calculate the Hamiltonian of the resulting problem, and get:

$$H = L'_{approx} + (\lambda_0^* + \delta\lambda)^T (f_x \delta x + f_u \delta u) \quad (40)$$

Notice that from (30) we know that terms:

$$L'_x \delta x + \lambda_0^{*T} f_x|_{x_0^*, u_0^*} \delta x = 0 \quad (41)$$

$$L'_u \delta u + \lambda_0^{*T} f_u|_{x_0^*, u_0^*} \delta u = 0 \quad (42)$$

Our Hamiltonian becomes:

$$H = \frac{1}{2} [\delta x^T \delta u] \mathbf{H}(L') [\delta x^T \delta u]^T + \delta\lambda^T (f_x \delta x + f_u \delta u) \quad (43)$$

Equation (43) provides the Hamiltonian of a linear quadratic regulation problem that arises from our use of differential dynamic programming. This LQR problem is stated below:

$$\text{Minimize } J'_{approx} = \int_0^T \left[\frac{1}{2} [\delta x^T \delta u] \mathbf{H}(L') [\delta x^T \delta u]^T \right] dt \quad (44)$$

Subject to:

$$\delta \dot{x} = f_x|_{x_0^*, u_0^*} \delta x + f_u|_{x_0^*, u_0^*} \delta u \quad (45)$$

Notice that the linear fuel consumption and speed terms in the original objective disappears. This suggests that our approximate objective J'_{approx} only captures the penalty in our input and our logarithmic constrains around a static optimum.

5. LEAD VEHICLE MODEL

One of the state variables of the above LQR problem is the perturbation in lead vehicle velocity around a steady-state value. To solve this LQR problem, one needs a dynamic model of this perturbation. To obtain this model, we use an urban/suburban duty cycle for a commercial heavy-duty truck

We fit a second order model to the speed deviations around the average speed of this duty cycle:

$$\delta v(k) = a_1 \delta v(k-1) + a_2 \delta v(k-2) \quad (46)$$

Using least squares estimation, we obtain:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1.85 \\ -0.8543 \end{bmatrix} \quad (47)$$

The discrete transfer function of the system from disturbance to speed deviation is thus:

$$G(z) = \frac{z}{z^2 - 1.85z + 0.8543}, \quad (48)$$

and has poles (0.9572, 0.8925) inside of the unit circle, which means this system is stable.

We can find a state space realization of this system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1.85 & -0.8543 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (49)$$

$$y(k) = x_1(k) \quad (50)$$

where x_1 corresponds to the deviation from the steady-state optimal speed, and x_2 corresponds to a linear combination of both the speed and acceleration deviations. We can write this in continuous time as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = A_{lead} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Bw \quad (51)$$

$$y = x_1 \quad (52)$$

where:

$$A_{lead} = \begin{bmatrix} 0.9217 & -0.9241 \\ 1.082 & -1.079 \end{bmatrix} \quad (53)$$

We can augment our linear model in (45) and include the dynamics of the lead vehicle.

$$\begin{bmatrix} \delta \dot{h} \\ \delta \dot{v} \\ \delta \dot{P} \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{ego} & 0_{3 \times 1} \\ 0_{1 \times 3} & A_{lead} \end{bmatrix} \begin{bmatrix} \delta h \\ \delta v \\ \delta P \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \delta u \quad (54)$$

where:

$$A_{ego} = f_x|_{x_0^*, u_0^*} \quad (55)$$

Notice that the resulting system is stabilizable, meaning that the LQR problem is solvable.

6. SELECTION OF PARETO OBJECTIVE WEIGHTS

Using ideas from differential dynamic programming, we have constructed a LQR problem around an optimal steady state. This LQR emerges from the approximation (17) of our original optimization problem in (10). To capture the relative importance of different terms in this objective, we introduce the weights: $(c, R, \alpha_1, \alpha_2, \alpha_3)$. In this section, we focus on how we choose values for these weights.

In section 4 we defined our nominal trajectory as a steady state optimum of the optimization problem in (17), and assumed we could find a solution to that problem. In reality, the optimum steady-state trajectory depends on the objective weights.

To select these weights, we can impose a steady state optimum speed and headway, and use first order optimality conditions, in combination with Bryson's rule, to obtain a system of equations that we can solve for the objective weights.

$$\frac{\partial L'}{\partial v_{ss}} = 0 \quad (56)$$

$$\frac{\partial L'}{\partial h_{ss}} = 0 \quad (57)$$

$$\delta h_{bad}^2 Q_{11} = \delta P_{bad}^2 Q_{33} \quad (58)$$

$$\delta P_{bad}^2 Q_{33} = \delta v_{bad}^2 Q_{22} \quad (59)$$

$$\delta u_{bad}^2 Q_{66} = \delta P_{bad}^2 Q_{33} \quad (60)$$

where $Q = \mathbf{H}(L')$, and the variable with subscript *bad* represent equivalent unacceptable deviations from the steady state. With this approach the problem of selecting the optimization weights is replaced to the much intuitive problem of selecting the relative importance of the parameters in the Bryson's equations (58),(59),(60). We will explore the effect varying these parameters has on the optimization result in the following section.

7. SIMULATION & RESULTS

7.1 Sensitivity Analysis of Bryson's Weights

While it is possible to use engineering intuition to pick the values of equivalent unacceptable deviations in Bryson's rule, in this section we explore the effect of varying these in simulation.

We consider a platoon of two identical trucks, where the lead vehicle is following a representative urban cycle with an average speed of 9.1m s^{-1} . The ego vehicle follows the lead using the feedback law obtained from solving the LQR in previous sections.

We perform a sensitivity study on the equivalent unacceptable deviations of speed, headway, power, and input. We determined that changing the deviations in power had an effect in fuel consumption more significant than changing the other parameters. In Figure 1 we show the fuel saving of our vehicle compared to the lead vehicle, for different values of the deviations in power.

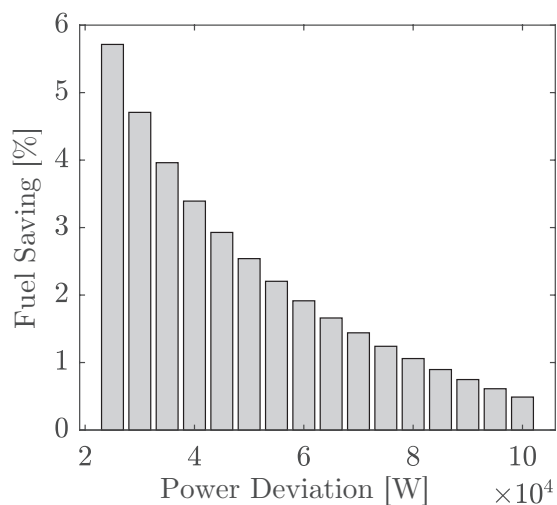


Fig. 1. Fuel savings for different values of the unacceptable deviation in Power used in Bryson's rule.

These fuel savings mostly come from a reduction in energy dissipated by braking. In Figure 2 we show this reduction for for different values of the deviations in power.

We can take a closer look to one of these cases. In Figure 3 we show a snippet of the speed trajectories of both the lead and ego vehicles. For this case we can also look at the changes in energy losses in Figure 4. While there is a reduction in aerodynamic losses from the reduction in aerodynamic drag in the platoon. Its effect is not as significant as a the reduction in energy dissipated while braking.

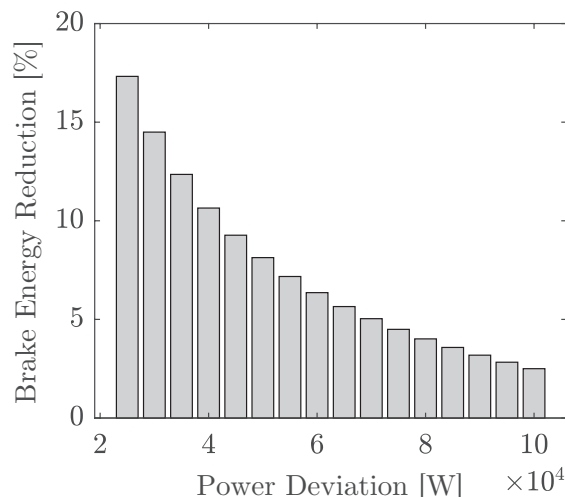


Fig. 2. Brake energy reduction for different values of the unacceptable deviation in Power used in Bryson's rule.

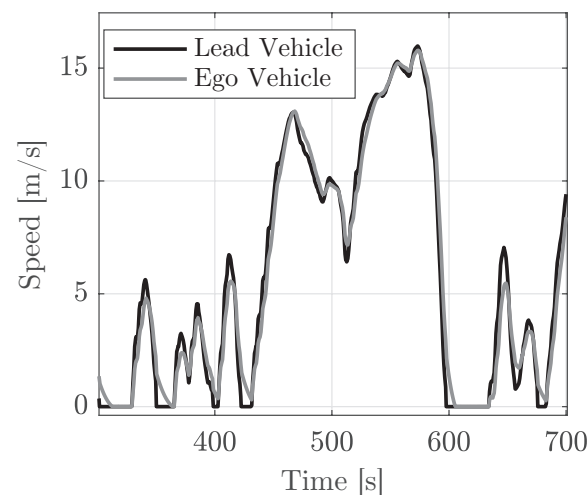


Fig. 3. Snippet of speed trajectory of both lead and ego vehicles

7.2 Effect of prediction in fuel savings

So far we have assumed that we have knowledge of the lead vehicle speed and acceleration in real time. However, as connectivity and automation becomes more prevalent, we could assume that the lead vehicle communicates its future plans. We can explore the effect this have in our controller.

We assume that instead of using the current speed and acceleration we take the average future speed and acceleration for a given preview horizon. We then feed this signals back to our controller. We can then explore the effect of changing the preview horizon. In Figure 5 we show the effect of varying the both the preview horizon and the power deviation in fuel savings and headway root mean square error.

We can identify a fundamental trade-of between saving fuel and maintaining a constant headway.

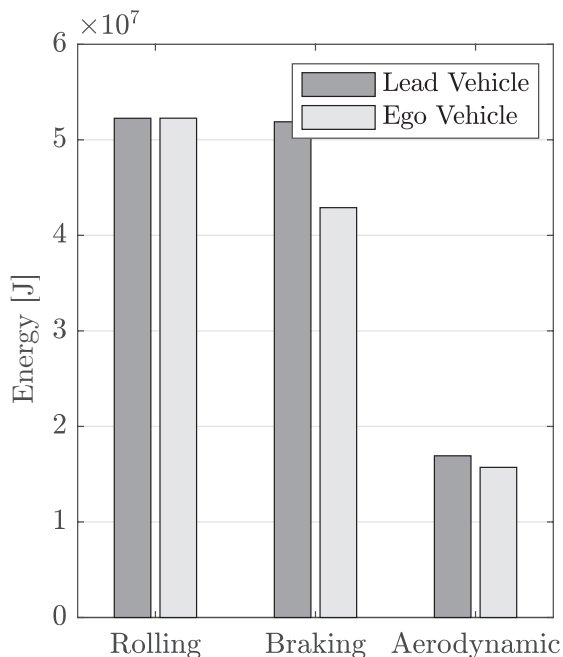


Fig. 4. Energy losses by lead and ego vehicles

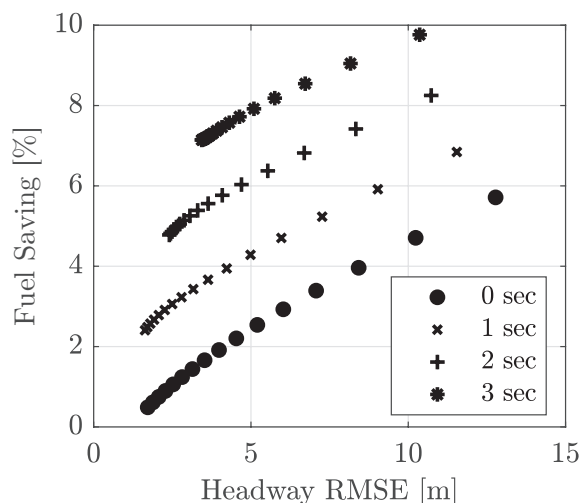


Fig. 5. Effect of changing the unacceptable deviation of power and preview horizon on fuel savings and headway root mean square error.

We can also look at one of these cases and dig a bit deeper into what is causing the savings. In Figure 6 we show a snippet of the speed trajectories of both lead and ego vehicle, and in Figure 7 we break down the energy losses.

8. CONCLUSION

In this paper we used differential dynamic programming approximate the speed trajectory optimization a platooning vehicle as a linear quadratic regulator problem that tracks an optimal steady-state trajectory. We used experimental data to fit a linear model for the speed of an uncontrollable lead vehicle. We used Bryson's rule and first optimality conditions to obtain the objective weights

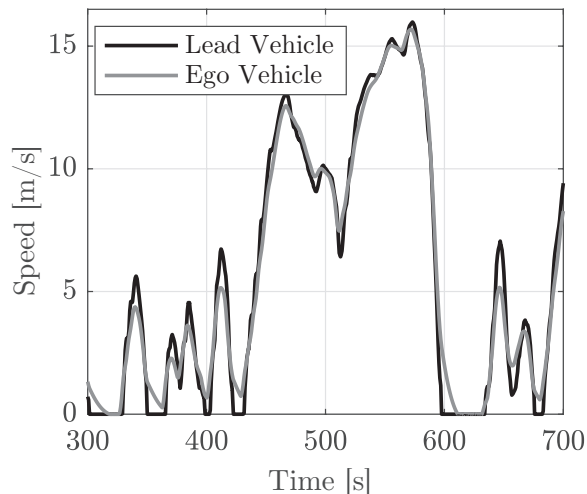


Fig. 6. Snippet of speed trajectory of both lead and ego vehicles for a preview horizon of 3s.

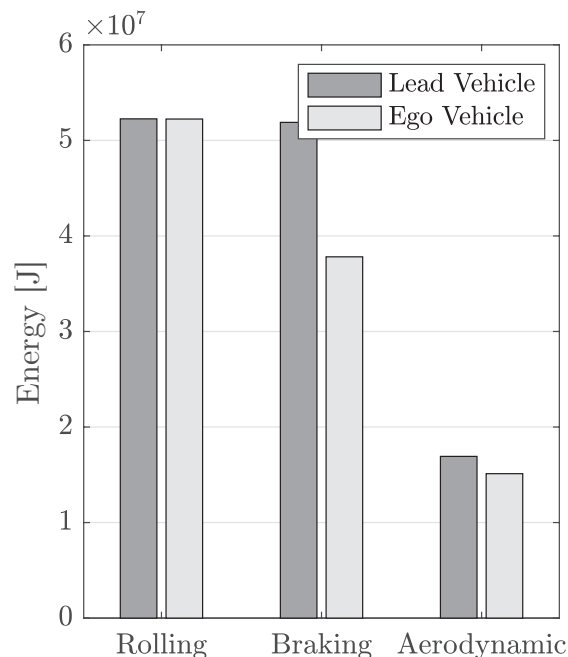


Fig. 7. Energy losses by lead and ego vehicles for a preview horizon of 3s.

that produce a desired steady-state optimum trajectory. We explore the effect of changing the Bryson's rule unacceptable deviations from the optimal steady state. An unacceptable deviation of $25 \times 10^3 \text{W}$ resulted in the highest fuel savings of 5.7% compared to the lead vehicle. We also identified a fundamental trade-off between fuel savings and maintaining a constant headway. Finally we explored the effect of leveraging knowledge of the future speed and acceleration of the lead vehicle. For a prediction horizon of 3s we obtained fuel savings of 9.7%.

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REFERENCES

- Alam, A.A., Gattami, A., and Johansson, K.H. (2010). An experimental study on the fuel reduction potential of heavy duty vehicle platooning. *Intelligent Transportation Systems (ITSC), 2010 13th International IEEE Conference on*, 306–311.
- Bryson, A.E. and Ho, Y.C. (1975). Applied optimal control, revised printing. *Hemisphere, New York*.
- EPA (2016). Rules and Regulations: Greenhouse Gas Emissions and Fuel Efficiency Standards for Medium- and Heavy-Duty Engines and Vehicles-Phase 2. Technical Report 206, Environmental Protection Agency(EPA) National Highway Traffic Safety Administration (NHTSA), Department of Transportation (DOT).
- EPA (2018). U.S. Greenhouse Gas Emissions and Sinks, 1990-2016. Technical report, U.S. Environmental Protection Agency (EPA). doi:EPA 430-R-12-001.
- Groelke, B., Borek, J., Earnhardt, C., Li, J., Geyer, S., and Vermillion, C. (2018). A comparative assessment of economic model predictive control strategies for fuel economy optimization of heavy-duty trucks. In *2018 Annual American Control Conference (ACC)*, 834–839. doi:10.23919/ACC.2018.8431050.
- Hellström, E., Fröberg, A., and Nielsen, L. (2006). A real-time fuel-optimal cruise controller for heavy trucks using road topography information. In *SAE World Congress*, 724, 1–10. doi:10.4271/2006-01-0008.
- Hellström, E., Ivarsson, M., Aslund, J., and Nielsen, L. (2007). Look-ahead control for heavy trucks to minimize trip time and fuel consumption. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 5(PART 1), 439–446. doi:10.1016/j.conengprac.2008.07.005.
- Li, S., Li, K., Rajamani, R., and Wang, J. (2011). Model predictive multi-objective vehicular adaptive cruise control. *IEEE Transactions on Control Systems Technology*, 19(3), 556–566. doi:10.1109/TCST.2010.2049203.
- Mayne, D.Q. (1973). Differential dynamic programming—a unified approach to the optimization of dynamic systems. In *Control and Dynamic Systems*, volume 10, 179–254. Elsevier.
- Turri, V., Besselink, B., and Johansson, K.H. (2015). Cooperative look-ahead control for fuel-efficient and safe heavy-duty vehicle platooning. *arXiv:1505.00447 [cs.SY]*, Submitted(1), 1–16. doi:10.1109/TCST.2016.2542044.
- Turri, V., Kim, Y., Guanetti, J., Johansson, K.H., and Borrelli, F. (2017). A model predictive controller for non-cooperative eco-platooning. *Proceedings of the American Control Conference*, 2309–2314. doi:10.23919/ACC.2017.7963297.
- Vohra, V., Wahba, M., Akarslan, G., Ni, R., and Brennan, S. (2019). An examination of vehicle spacing to reduce aerodynamic drag in truck platoons. In *2018 IEEE*