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Robust Global Stabilization of the Third Order Reaction Wheel Pendulum System ⋆

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Abstract: In this paper, the global stabilization of the Reaction Wheel Pendulum, without taking into account the wheel position, is addressed despite the presence of Lipschitz disturbances and/or uncertainties in the model. Using two Second Order Continuous Sliding Modes Algorithms, the control task is performed, reaching finite-time convergence in one part of the dynamics and generating a continuous control signal. Finally, some simulations and experiments in the real systems are presented to test the proposed algorithms and compare them with a linear feedback controller.

Keywords: Sliding mode control, Disturbance rejection, Stability of nonlinear systems.

1. INTRODUCTION

The pendulae systems have been studied vastly in the control theory due to the presence of interesting characteristics, like nonlinear dynamics, underactuation, non-minimum phase and multiple equilibria points. This last property makes the design of a global control for these kind of systems as a really challenging task.

As a result of this, usually in the control of the Reaction Wheel Pendulum (RWP), it is separated the design of such controller in two tasks: the swing-up and the stabilization. Some examples of works that solved the swing-up problem are Spong et al. (2001) and Andrievsky (2011) using the energy based control. Some papers dealing with the stabilization of the RWP are Spong et al. (2001); Moreno-Valenzuela and Aguilar-Avelar (2017) using the feedback linearization control, Andrievsky (2011); Iriarte et al. (2013); Gutiérrez-Oribio et al. (2018) using the sliding mode control, and Ortega et al. (2002); Srinivas and Behera (2008) using the passivity-based control with interconnection and damping assignment.

Nevertheless, there are some results solving both problems, swing-up and stabilization, simultaneously: Olfati-Saber (2001a) and Ye et al. (2008) using backstepping and Ryalat and Laila (2016) using energy shaping. They verify their results by means of some simulations, but the required torque is not available for a real experimental platform, as discussed in (Moreno-Valenzuela and Aguilar-Avelar, 2017, Chapter 8). In any case these papers consider only the nominal system, *i.e.*, free of uncertainties or disturbances, which are unavoidable in a real experimental setup.

In order to deal with such uncertainties and/or disturbances, one can use the Sliding Modes Algorithms. The main disadvantage of this approach is the presence of the so called *chattering effect* due to the discontinuity used in the controller. Continuous Sliding Mode algorithms (Chalanga et al. (2013); Edwards and Shtessel (2016); Laghrouche et al. (2017); Kamal et al. (2018); Moreno (2016, 2018a)), a class of homogeneous sliding mode controllers, are able to compensate Lipschitz uncertainties and/or perturbations theoretically exact, but produce a continuous control signal. When the actuator is fast (see Perez-Ventura and Fridman (2019)), the chattering effect caused by the discontinuity and discretization is strongly attenuated.

These algorithms consist in a static homogeneous finite-time controller for the nominal model of the system and a discontinuous integral action, aimed at estimating and compensating the uncertainties and perturbations. They are an extension of the (classical) Super-Twisting (Levant (1993, 1998); Seeber and Horn (2017, 2018)), and are related to the Continuous Twisting Algorithm (CTA) (Mendoza-Ávila et al. (2017); Torres-González et al. (2017)) and Discontinuous Integral Algorithms (DIA) (Moreno (2016), Moreno (2018a)).

The objective of this work is to design a robust global controller for the third order RWP system using the approach of Olfati-Saber (2001a) and two Second Order Continuous Sliding Modes Algorithms: the CTA and the DIA. Such algorithms ensure global finite-time stability to the zero

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dynamics, which is globally asymptotically stable manifold. This task is performed despite the presence of some Lipschitz matched disturbances/uncertainties and with a continuous control signal. Finally, we provide evidence of the performance of the controllers both in simulations and through experiments on a laboratory setup, and we compare them with the linear controller used in Olfati-Saber (2001a).

The outline of this work is as follows. The problem statement and the main result of the paper are given in Section 2 and Section 3, respectively. The simulation and experimental validation, comparing the proposed algorithms, are shown in Section 4. Finally some concluding remarks are discussed in Section 5.

Notation: Define the function $\lceil \cdot \rfloor^{\gamma} := |\cdot|^{\gamma}\text{sign}(\cdot)$, for any $\gamma \in \mathbb{R}_{\geq 0}$.

2. PROBLEM STATEMENT

Consider the RWP shown in Fig. 1. This figure describes a pendulum rotating in a vertical plane and its pivot pin is mounted in a stationary base. The pivot pin of the wheel is attached to the pendulum. The axes of rotation of the pendulum and the wheel are parallel each other. The wheel is actuated by the torque τ [Nm]. The states vector is $x = [x_1, x_2, x_3, x_4]^T$, where x_1 [rad] is the angle between the upward direction and the pendulum, measured counter-clockwise ($x_1 = 0$ for the upright position of the pendulum), x_2 [rad/sec] is the pendulum angular velocity, x_3 [rad] is the wheel angular position, and x_4 [rad/sec] is the wheel angular velocity.

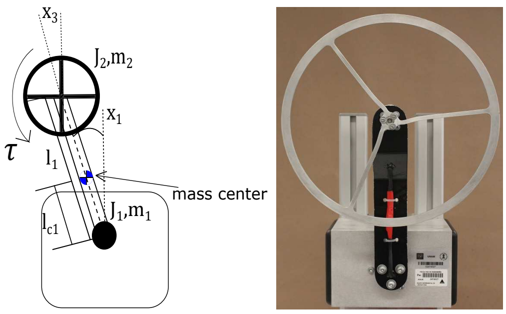

Fig. 1. Reaction wheel pendulum system.

The system dynamics was presented in Spong and Vidyasagar (1989) and can be described by the following equations:

$$\begin{aligned} \dot{x}_1 =& x_2, \quad \dot{x}_2 = \frac{d_{22} W \sin(x_1) + d_{12} b_2 x_4 - d_{12}\tau}{D}, \\ \dot{x}_3 =& x_4, \quad \dot{x}_4 = \frac{-d_{21} W \sin(x_1) - d_{11} b_2 x_4 + d_{11}\tau}{D}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} d_{21} =& d_{12} = d_{22} = J_2, \\ d_{11} :=& m_1 l_{c1}^2 + m_2 l_1^2 + J_1 + J_2, \\ D :=& d_{11} d_{22} - d_{12} d_{21} > 0, \\ \bar{m} :=& m_1 l_{c1} + m_2 l_1, \quad W = \bar{m} g, \end{aligned}$$

and parameters of the system shown in Table 1.

In Olfati-Saber (2001a) and Olfati-Saber (2001b) is proposed a methodology to globally stabilize the origin of the RWP system. First, it is defined the next global change of coordinates

$$z_1 := d_{11} x_2 + d_{12} x_4, \quad z_2 := x_1, \quad z_3 := x_4, \tag{2}$$

rewriting the system as

$$\begin{aligned} \dot{z}_1 =& W \sin(z_2), \quad \dot{z}_2 = \frac{z_1 - d_{12} z_3}{d_{11}}, \\ \dot{z}_3 =& \frac{-d_{21} W \sin(z_2) - d_{11} b_2 z_3 + d_{11}\tau}{D}. \end{aligned} \tag{3}$$

Remark 1. In this paper it is not considered the state x_3 (wheel position) as a control objective, so, only the three states left are going to be controlled. Also, in Olfati-Saber (2001a) and Olfati-Saber (2001b) is not considered the viscous friction of the wheel, but is easy to add it to the system to make it more likely to the reality.

Now, if the control τ becomes

$$\tau = \frac{d_{21} W \sin(z_2) + d_{11} b_2 z_3 + D u}{d_{11}}, \tag{4}$$

the final system becomes into

$$\dot{z}_1 = W \sin(z_2), \quad \dot{z}_2 = \frac{z_1 - d_{12} z_3}{d_{11}}, \quad \dot{z}_3 = u. \tag{5}$$

The problem of globally stabilize the pendulae systems, is the presence of infinite equilibria points due to the $\sin(z_2)$ function in the system dynamics. One alternative to deal with this, is to force its argument to only take values around zero, so the only equilibrium point of the system will be the origin and if it is stable, the global stability will be ensured. This is performed saturating the argument of $\sin(z_2)$ with a sigmoidal function like $z_2 = \sigma_1(z_1) := c_0 \tanh(c_1 z_1)$, so the first state of the system (5) becomes

$$\dot{z}_1 = W \sin(z_2) = W \sin(c_0 \tanh(c_1 z_1)), \tag{6}$$

with the origin as globally asymptotically stable if

$$-\pi/2 < c_0 < 0, \quad c_1 > 0, \tag{7}$$

and $V = z_1^2$ as a Lyapunov function.

Now, in order to obtain the desired zero dynamics (6), it is defined the following change of coordinates and control as

$$\mu_1 := z_2 - \sigma_1(z_1), \quad \mu_2 := \dot{\mu}_1, \quad \nu := \dot{\mu}_2, \tag{8}$$

transforming the system (5) as

$$\dot{z}_1 = W \sin(\sigma_1(z_1) + \mu_1), \quad \dot{\mu}_1 = \mu_2, \quad \dot{\mu}_2 = \nu. \tag{9}$$

The linear state feedback used in Olfati-Saber (2001b)

$$\nu = -c_2 \mu_1 - c_3 \mu_2, \quad c_2, c_3 > 0, \tag{10}$$

globally exponentially stabilizes $\mu_1 = \mu_2 = 0$ for the μ-subsystem of (9). Due to the fact that for any exponentially vanishing function $\mu_1(t)$, the solution of the z_1-subsystem in (9) is uniformly bounded and the origin of the system (9) is globally asymptotically and locally exponentially stable due to a theorem by Sontag (1989). The overall nonlinear state feedback u can be explicitly calculated as the following

$$\begin{aligned} u =& \frac{d_{11}}{d_{12}} \left(-\nu + \frac{W}{d_{11}} \sin(z_2) - \ddot{\sigma}_1 \right), \\ \ddot{\sigma}_1 =& c_0 c_1 W (1 - \tanh^2(c_1 z_1)) \times \\ & \left(\cos(z_2) \frac{z_1 - d_{12} z_3}{d_{11}} - 2 c_1 W \tanh(c_1 z_1) \sin^2(z_2) \right). \end{aligned} \tag{11}$$

This is an alternative to globally stabilize the origin of a system that in principle looks like a really difficult result to obtain. The problem is that in the reality, the systems present disturbances/uncertainties $\varphi(t)$ on its dynamics and the system (9) could be presented in the form

$$\begin{aligned} \dot{z}_1 =& W\sin(\sigma_1(z_1)+\mu_1), \\ \dot{\mu}_1 =& \mu_2, \quad \dot{\mu}_2 = \nu + \varphi(t). \end{aligned} \tag{12}$$

In this case, the control (10) can not achieve the task of driving the states to the origin globally. In this paper, the global stability of the origin of RWP system is addressed considering the presence of Lipschitz uncertainties/disturbances, *i.e.*, $|\dot{\varphi}(t)| \leq L$, using sliding modes algorithms that generate continuous control signals, reducing the *chattering* effect.

3. MAIN RESULT

Defining the next global change of coordinates (similar to the one used in Olfati-Saber (2001a))

$$z_1 := d_{11}x_2 + d_{12}x_4, \quad z_2 := x_1, \quad z_3 := x_2, \tag{13}$$

the RWP system can be rewriten as

$$\begin{aligned} \dot{z}_1 =& W\sin(z_2), \quad \dot{z}_2 = z_3, \\ \dot{z}_3 =& \frac{d_{22}W\sin(z_2) + b_2(z_1 - d_{11}z_3) - d_{12}\tau}{D}. \end{aligned} \tag{14}$$

Now, if the control τ becomes

$$\tau = \frac{d_{22}W\sin(z_2) + b_2(z_1 - d_{11}z_3) - Du}{d_{12}}, \tag{15}$$

the final system becomes into

$$\dot{z}_1 = W\sin(z_2), \quad \dot{z}_2 = z_3, \quad \dot{z}_3 = u. \tag{16}$$

Using the same approach as the section before, the system (12) can be obtained, despite the different change of coordinates used, just with a different state feedback u like

$$\begin{aligned} u =& \nu + \ddot{\sigma}_1, \\ \ddot{\sigma}_1 =& c_0c_1W(1-\tanh^2(c_1z_1))\times \\ & \left(\cos(z_2)z_3 - 2c_1W\tanh(c_1z_1)\sin^2(z_2)\right). \end{aligned} \tag{17}$$

Remark 2. The main advantage of using the transformation (13) instead of (2), is that is less the information you need of the system in order to implement the control (17) in comparison to the control (11) (in concrete the parameters d_{11} and d_{12}).

Now the task is to design a control ν which drives the states μ_1, μ_2 globally to the origin, despite the presence of the Lipschitz function $\varphi(t)$ and then the origin of the system (12) will be globally stable.

Let us introduce two Continuous Higher Order Sliding Modes Algorithm, the second order Continuous Twisting Algorithm (2-CTA) (introduced in Torres-González et al. (2015)):

$$\begin{aligned} \nu =& -k_1\lceil\mu_1\rfloor^{\frac{1}{3}} - k_2\lceil\mu_2\rfloor^{\frac{1}{2}} + \zeta, \\ \dot{\zeta} =& -k_3\lceil\mu_1\rfloor^0 - k_4\lceil\mu_2\rfloor^0, \end{aligned} \tag{18}$$

and the second order Discontinuous Integral Algorithm (2-DIA):

$$\begin{aligned} \nu =& -k_2\left\lceil\lceil\mu_2\rfloor^{\frac{3}{2}} + k_1^{\frac{3}{2}}\mu_1\right\rfloor^{\frac{1}{3}} + \zeta \\ \dot{\zeta} =& -k_{I1}\left\lceil\mu_1 + k_{I2}\lceil\mu_2\rfloor^{\frac{3}{2}}\right\rfloor^0. \end{aligned} \tag{19}$$

which is an integral extension of the static controller presented in Cruz-Zavala and Moreno (2017). In both algorithms, the static part ν drives the states μ_1, μ_2 to the origin in finite-time, while the integral part ζ rejects the disturbance $\varphi(t)$. Note that due to the discontinuous function *sign* is in the integrator, the generated control signal of both algorithms is continuous.

The following theorem is the main result of this paper:

Theorem 1. The origin of μ_1, μ_2-system in (12) is globally stable in finite-time, despite the presence of the Lipschitz disturbances/uncertainties $\varphi(t)$, and z_1 will be driven to the origin globally asymptotically, when the control τ takes the form of (13), (15) and (17), and ν takes the form of (18) or (19).

Remark 3. As a consequence of the Theorem 1, the origin of the RWP system (1), *i.e.*, $x_1, x_2, x_4 = 0$ is globally asymptotically stable. The wheel position x_3 will only be driven to an arbitrary constant, due to the fact that is not a controlled state.

Proof.

The proof can be done by analysing only the system

$$\dot{\mu}_1 = \mu_2, \quad \dot{\mu}_2 = \nu + \varphi(t), \tag{20}$$

because if the states μ_1, μ_2 in (12) are globally taken to zero, it is clear that the state z_1 will be driven to the origin asymptotically as seen in the approach of Olfati-Saber (2001b).

The closed-loop system of (20), with the controller (18) and the new variable $\mu_3 = \zeta + \varphi(t)$ is

$$\begin{aligned} \dot{\mu}_1 =& \mu_2, \\ \dot{\mu}_2 =& -k_1\lceil\mu_1\rfloor^{\frac{1}{3}} - k_2\lceil\mu_2\rfloor^{\frac{1}{2}} + \mu_3 \\ \dot{\mu}_3 =& -k_3\lceil\mu_1\rfloor^0 - k_4\lceil\mu_2\rfloor^0 + \dot{\varphi}(t), \end{aligned} \tag{21}$$

whose origin is proven to be finite-time stable in Torres-González et al. (2017) designing properly the gains k_1, k_2, k_3, k_4 and using the following Lyapunov function

$$\begin{aligned} V(\mu) =& \alpha_1|\mu_1|^{\frac{5}{3}} + \alpha_2\mu_1\mu_2 + \alpha_3|\mu_2|^{\frac{5}{2}} + \\ & \alpha_4\mu_1\lceil\mu_3\rfloor^2 - \alpha_5\mu_2\mu_3^3 + \alpha_6|\mu_3|^5. \end{aligned}$$

On the other hand, the closed-loop system of (20), with the controller (19) and the new variable $\mu_3 = \zeta + \varphi(t)$ is

$$\begin{aligned} \dot{\mu}_1 =& \mu_2, \\ \dot{\mu}_2 =& -k_2\left\lceil\lceil\mu_2\rfloor^{\frac{3}{2}} + k_1^{\frac{3}{2}}\mu_1\right\rfloor^{\frac{1}{3}} + \mu_3 \\ \dot{\mu}_3 =& -k_{I1}\left\lceil\mu_1 + k_{I2}\lceil\mu_2\rfloor^{\frac{3}{2}}\right\rfloor^0 + \dot{\varphi}(t). \end{aligned} \tag{22}$$

whose origin is proven to be finite-time stable in Moreno (2018b) designing properly the gains k_1, k_2, k_{I1}, k_{I2} and using the following Lyapunov function

$$V(\mu) = \frac{3}{5}\gamma_1|\xi_1|^{\frac{5}{3}} + \xi_1\xi_2 + \frac{2}{5}k_1^{-\frac{3}{2}}|\xi_2|^{\frac{5}{2}} + \frac{1}{5}|\xi_3|^5,$$

where

$$\xi_1 = \mu_1 - \lceil\xi_3\rfloor^3, \quad \xi_2 = \mu_2, \quad \xi_3 = k_1^{-\frac{1}{2}}k_2^1\mu_3.$$

Therefore, it is not difficult to prove that the origin of the system (12) in closed loop with controllers (18) or (19) is asymptotically and globally stable. Furthermore, by the change of coordinates (8) and (13), the original states of the RWP system x_1, x_2, x_4 will be driven to the origin asymptotically and globally.

■

4. SIMULATION AND EXPERIMENTAL VALIDATION

In order to illustrate the performance of the presented algorithms, simulations and experiments have been done over the RWP system described in (1). The used system parameters were obtained with an off-line identification algorithm, given in Table 1.

Table 1. Parameters of the Reaction Wheel Pendulum system

Name	Description	Value
$\bar{m}$	Equivalent mass of the system	0.191[kgm]
d_{11}	Equivalent moment of inertia of the system	0.0543[kgm^2]
J_2	Moment of inertia of the wheel	0.0027[kgm^2]
b_2	Friction coefficient of the wheel	0.01[Ns/m^2]
g	Acceleration of the gravity	9.81[m/s^2]

The experiments were made in the real RWP system depicted in Fig. 1. This system was developed in the Institut für Regelungs- und Automatisierungstechnik of TU Graz, Austria. The experiments are performed in Matlab Simulink over a data-acquisition board connected to the RWP system.

The real RWP system only counts with the information of the positions, so it was implemented a second order robust exact differentiator Levant (2008) in order to obtain an estimate of the velocities x_2, x_4 for the control.

Also, it is important to say, that the control signal τ from the algorithm has to be converted to a voltage signal using the next expression $V = -0.2778\tau + 3.6 \times 10^{-5} x_4$. This signal is saturated between $[-0.9, 0.9]$[V] and, after an amplification stage, is connected to a 12[V] DC motor.

4.1 Simulation results

The control (15) and (17), the transformation (13), the 2-DIA described by (19) with $k_1 = 4.16$, $k_2 = 28.5$, $k_{I1} = 6.3$, $k_{I2} = 0$, and the 2-CTA described by (18) with $k_1 = 47.57$, $k_2 = 19.84$, $k_3 = 16.1$, $k_4 = 7.7$ were implemented in Matlab Simulink with the Runge-Kutta's integration method of fixed step and a sampling time equal to $1 \times 10^{-4}[s]$. On the other hand, to make a comparison with the algorith used in Olfati-Saber (2001a), the control (4) and (11), the transformation (2) and the linear feedback described by (10) with $c_2 = c_3 = 30$ was implemented as well. The simulations were made with the same unbounded perturbation $\varphi(t) = 2\sin(2t) + 2t + 2$, the same initial condition $x_0 = [100\pi, -50, 0, 50]^T$ and the same gains $c_0 = -1.5$ and $c_1 = 5$.

The results are shown in Figs. 2-4. The sliding modes algorithm drive the μ_1, μ_2 states to the origin in finite-time making the original states of the RWP system to be attracted to the origin asymptotically and maintain there despite the presence of the unbounded disturbance (due to the fact that they are able to identified the disturbance exactly as shown in Fig. 4). The linear algorithm, at the beginning of the simulation, achieve the stabilization of the states but when the disturbance is big enough, the controller is not able to maintain the states in the origin and this is more evident in the wheel velocity. With respect to the control signal, the three algorithms present a continuous one, but the linear algorithm shows a large overshoot.

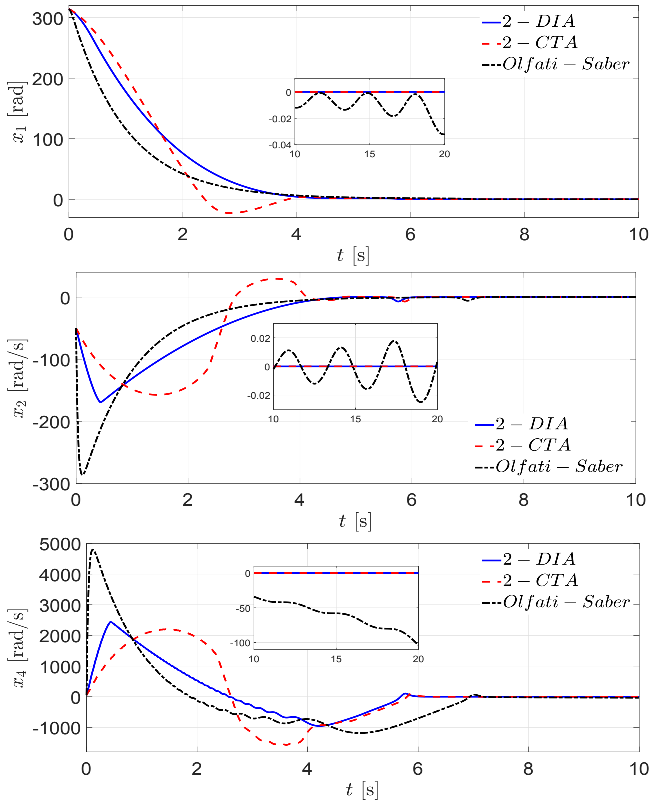

Fig. 2. State trajectories in the simulations.

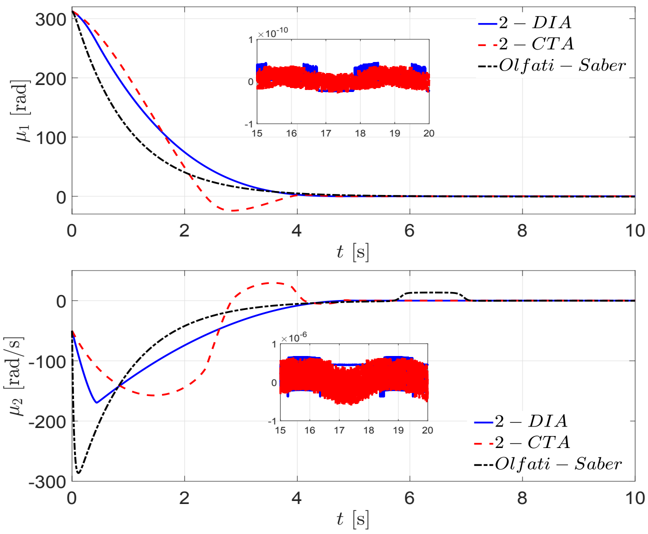

Fig. 3. μ_1-μ_2 trajectories in the simulations.

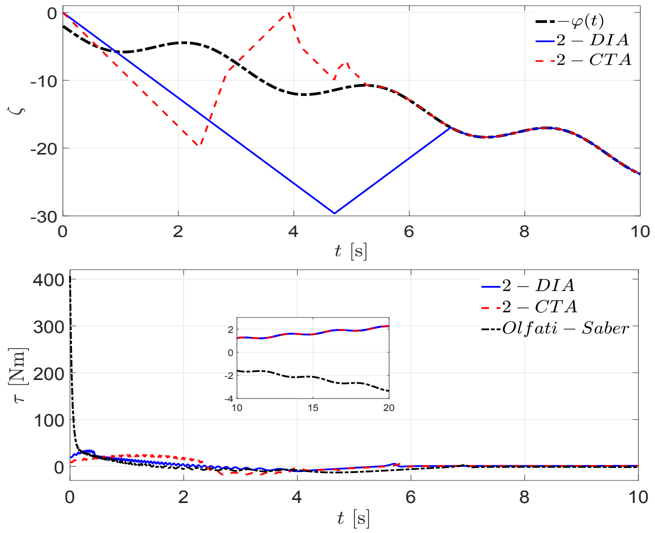

Fig. 4. Perturbation identification and control signal in the simulations.

4.2 Experimental validation

The control (15) and (17), the transformation (13), the 2-DIA described by (19) with $k_1 = 2.43$, $k_2 = 14.62$, $k_{I1} = 1.26$, $k_{I2} = 0$, $c_0 = -1.5$ $c_1 = 6.5$ and the 2-CTA described by (18) with $k_1 = 36.3$, $k_2 = 19.84$, $k_3 = 4.02$, $k_4 = 1.92$, $c_0 = -1.5$ $c_1 = 7$ were implemented in the real RWP system with the Runge-Kutta's integration method of fixed step and a sampling time equal to $1 \times 10^{-4}[s]$. As was done in the simulations, the control (4) and (11), the transformation (2) and the linear feedback described by (10) with $c_0 = -1.5$ $c_1 = 4$, $c_2 = c_3 = 30$, was implemented as well. The experiments were made with the same initial condition $x_0 = [\pi, 0, 0, 0]^T$.

The results are shown in Figs. 5-7. The three algorithms drive the states of the pendulum from the downward position close to the origin and stay around it, but the linear controller performs this in a large time than the others (around 15[s]). The wheel velocity is again a robust indicator, because only the sliding modes algorithm maintain it close to the origin and the linear one does not. This is because the linear algorithm can not deal with the uncertainties present in the real system and can not drive the μ_1 state to the origin. Again, the chattering effect is diminished as seen in the continuous control signal of the sliding modes algorithms.

In theory, the three algorithms can achieve global stabilization, but in practice, with the saturation of the real control signal (voltage) is no longer possible. Despite this, the three algorithms achieve the swing-up and stabilization in one step.

5. CONCLUSION

The global stabilization of the third order RWP system origin was obtained, robustly due to the rejection of Lipschitz disturbances/uncertainties in the system. Using two continuous second order sliding modes algorithm, the finite-time convergence of a part of the dynamics is ensured with a continuous control signal. Simulations and experimental validation were performed to test the proposed algorithm

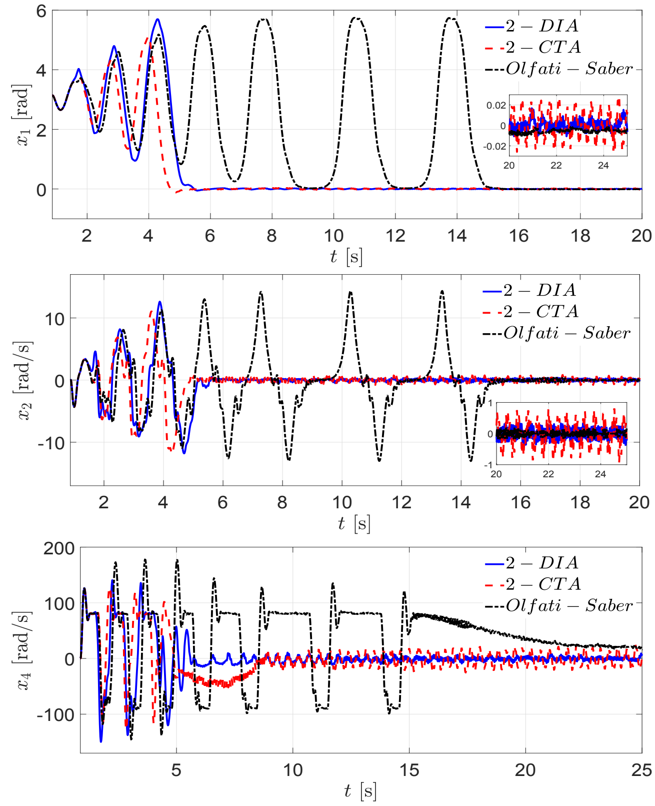

Fig. 5. State trajectories in the experiment.

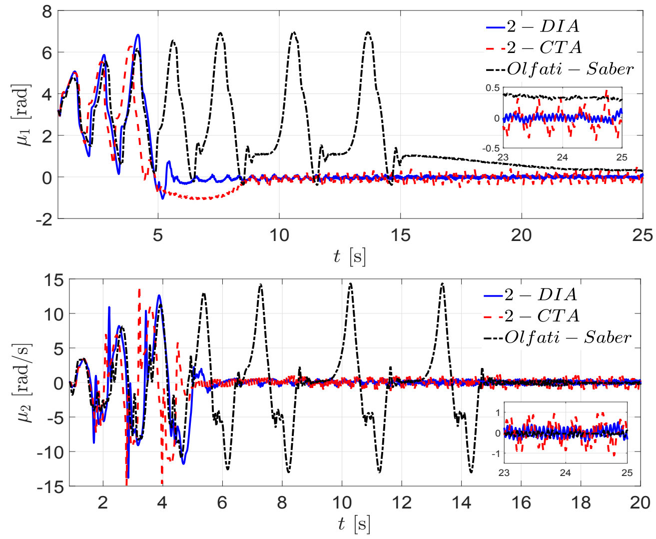

Fig. 6. μ_1-μ_2 trajectories in the experiment.

and compare them with a linear controller, obtaining better results due to the disturbance rejection.

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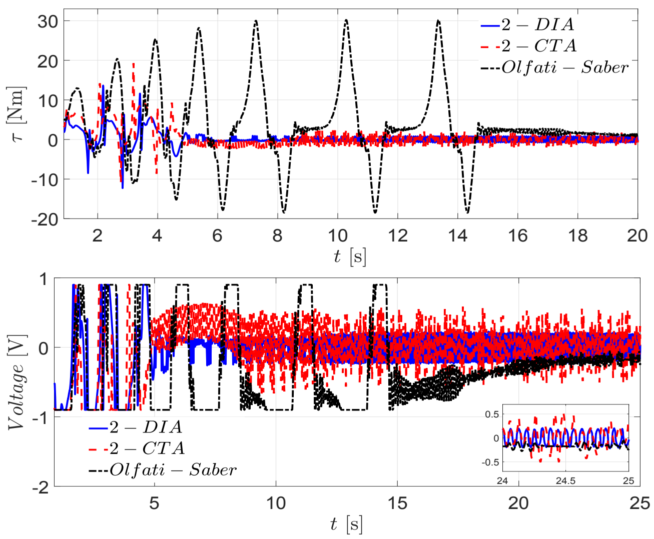

Fig. 7. Torque control signal and voltage control signal in the experiment.

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