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Scenario-based stochastic MPC for vehicle speed control considering the interaction with pedestrians

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Abstract: A typical driver spends a lot of the driving time on roads shared with pedestrians and bicyclists. Unlike highway driving, when there are pedestrians and cyclists using the same space as cars, controlling the car is more complicated. This is due to the fact that the behaviors of such agents does not follow strict rules like the cars in a closed highway. Their trajectories can be expressed better with multiple probabilistic functions than deterministic ones. We suggest a scenario-based stochastic model predictive control (MPC) framework to handle this. We consider multiple pedestrian trajectories with their respective probabilities according to an Interacting Multiple-Model Kalman Filter (IMM-KF). The car dynamics and non linear constraints are considered to avoid collision. A sample-based method is used to solve this optimization problem. The control situation was simulated using MATLAB. The proposed controller is observed to give a very natural control behavior for shared road driving compared to a deterministic single scenario MPC.

Keywords: Scenario-based MPC, Stochastic MPC, Vehicle-Pedestrian interaction, Intelligent autonomous vehicle

1. INTRODUCTION

Autonomous driving in closed highways is becoming a well addressed problem as major automotive makers set to launch autonomous driving on highways by 2020. Great progress in sensing, localization, detection, recognition, and control have achieved and realized reliable the autonomous driving in simple driving environments like highways. On the other hand, the autonomous driving in downtown area needs more improvement from the viewpoint of safety and driving comfort. Most of our daily driving are the driving on roads in residential and shopping areas. In these driving environment, road is sometimes shared with pedestrians and bicyclists. We often find it hard to predict the future movement of a pedestrian, especially in the case the person is looking at his mobile phone and walking close to the road. These cases are not as easy as the highway driving scenario because the interactions with pedestrians must be considered. Another fact that needs to be noted while dealing with pedestrians is that vehicle-pedestrian crashes are more likely to result in fatalities and severe injuries compared to vehicle-vehicle crashes. This is evident while looking at the recent statistics on road fatalities as summarized by Onishi et al. (2018). 273,000 pedestrians were killed in the world in 2010 and this makes up to 22% of all the road fatalities. Adding cyclists and motorcyclists, this number increases to 50% of the total fatalities.

In a control perspective, the former case of highway driving can be defined with a small set of strict rules like lane following and obeying traffic rules. It is possible to mathematically express them as deterministic functions or constraints based on sensor data and vehicle state as demonstrated in Niehaus and Stengel (1991); Falcone et al. (2007); Vanholme et al. (2012); Tran et al. (2019). However, the latter case has to deal with the intentions of the pedestrians and cyclists. Since their intentions are not observable, their movement can only be expressed in terms of probabilistic functions. This uncertainty is a big problem on the modeling of the pedestrian behavior and to formulate the planning and control problem with strict safety constraints.

There are various models proposed for pedestrian behaviors in shared roads. It can be noted that the state of the art models use probabilistic functions to represent the possible path of the pedestrian. One popular method to predict pedestrian motion is the Interacting Multiple-Model Kalman Filters (IMM-KF) discussed in Farmer et al. (2002). Schneider and Gavrila (2013) compares recursive Bayesian filters, namely an Extended KF based on a single dynamic model and an IMM-KF that considers multiple future motion models with different probabilities for the pedestrian. This study shows that the usage of an IMM-KF with multiple models provides an improvement of lateral position estimations during maneuvers.

To design controllers for autonomous vehicles while considering pedestrian behaviors, stochastic MPC approach is a promising candidate. This is due to its ability to handle system constraints together with stochastic uncertainties in its framework (Cannon et al. (2007)). While the worst case scenario approach evaluated in conventional MPC can be too conservative, the stochastic MPC is expected to realize the 'non-conservative' but 'enough safety' driving by considering the probabilistic constraints. Even though the pedestrian-vehicle interaction is largely unaddressed as a stochastic MPC control problem, there is some literature on the vehicle-vehicle cases. Carvalho et al. (2014) talks about an ego car trying to negotiate a leading car represented by the IMM-KF model. The framework does not take into account all the possible models of the leading car when predicting its future path, but only consider the most probable model and consider the uncertainty to be propagated within the horizon. While dealing with a pedestrian, the variety of modes that a pedestrian can take is much broader than a leading car.

In this paper, we propose a scenario-based stochastic MPC (SSMPC) approach to deal with the pedestrian-vehicle interaction control problem. To capture the highly dynamical behaviors of the pedestrian, we suggest a method based on IMM-KF that considers multiple pedestrian models (9 models in this study) and predicts multiple pedestrian paths along with their probabilities in every step. We also propose a tunable probability of collision parameter to customize the control performance in the SSMPC framework. This parameter can change the risk taking behavior of the controller. The SSMPC problem is formulated and solved using a randomized algorithm (Vidyasagar (2001); Okuda et al. (2018); Tran et al. (2018)). Since driving is a safety critical task, computation in real time is mandatory. One expected bottleneck of the suggested method is the computational time in-order to calculate the cost of samples for all the modes that pedestrian can take. Our previous work Muraleedharan et al. (2019) has proved to extend the real time calculations limit of the randomized algorithm by making use of a GPU instead of CPU. We try to model this MPC problem in such a way that it is compatible with GPU computation.

The organization of the paper is as follows. Section 2 describes the dynamics models of the vehicle and the pedestrian to make the path prediction. Section 3 and 4 detail the safety constraints and the formulation of the scenario-based stochastic MPC problem considering the chance-constraint. The simulation result is presented in Section 5 followed by the conclusion in Section 6.

2. DYNAMICS MODELING AND PREDICTION OF PEDESTRIAN AND VEHICLE

Since pedestrians can change their walking speed and direction abruptly with possibilities of multiple future walking scenarios, it is not possible to represent their dynamics using a single model. Instead, we use a multiple-model approach, which combines several basic models in the framework of IMM-KF. This framework was proven to be beneficial to obtain accurate estimation and prediction of the pedestrian's dynamics as reported by Schneider and Gavrila (2013). In this section, the dynamical model of the pedestrian and path prediction using the IMM-KF will be described in detail.

2.1 Problem setting

In-order to demonstrate the advantage of considering pedestrian behavior as a stochastic model, we model a shared road driving situation of a car driving parallel to a pedestrian in a shared road as seen in Fig. 1. The ego car is controlled to drive straight with a reference velocity v_{ref}. The road is 4 metres wide and pedestrian is 1.5 metres away from the center of the car in y-axis. The pedestrian is moving straight along the road but he has a probability of turning to the side. The ego car has to plan its speed to avoid collision in case the pedestrian is expected to walk into the road. The controller predicts the risk of the pedestrian based on the IMM-KF and reacts accordingly.

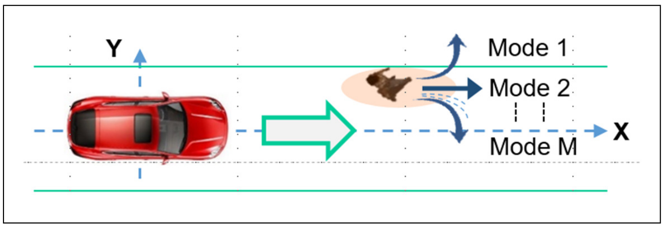

Fig. 1. Target environment which includes a vehicle and a pedestrian.

2.2 Pedestrian model

In this research, the dynamics of the pedestrian are described using a set of Markov jump affine systems as follows

$$\begin{aligned} \xi_{k+1}^p &= F_k^{m_k}\xi_k^p + G_k^{m_k}w_k^{m_k} \\ z_k^p &= H_k^{m_k}\xi_k^p + v_k^{m_k}, \end{aligned} \tag{1}$$

where ξ_k^p, z_k^p denote the pedestrian state and measurement at time k, respectively. The process noise w_k and measurement noise v_k are assumed to be independent and identically distributed (i.i.d.) Gaussian random variable, i.e., $\mathcal{N}(0, Q_k^{m_k})$ and $\mathcal{N}(0, R_k^{m_k})$, respectively. The superscript m_k in (1) indicates the model m in a set of multiple models $\mathbb{M} = \{1, 2, \ldots, M\}$ at time k. As shown in the study of Schneider and Gavrila (2013), the IMM-KF which combines constant velocity model and a "turn expert" one provides the best result for the pedestrian position estimation and the future path prediction among the tested approaches. Therefore, one constant velocity (CV) model and eight constant turn (CT) models with different turn rates are considered in this research ($M = 9$). The details of those models are given below.

- Constant velocity

$$F_k^1 = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G_k^1 = \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix}.$$

- Constant turn

$$F_k^m = \begin{bmatrix} 1 & 0 & \frac{\sin(\omega_m T)}{\omega_m} & -\frac{1-\cos(\omega_m T)}{\omega_m} \\ 0 & 1 & \frac{1-\cos(\omega_m T)}{\omega_m} & \frac{\sin(\omega_m T)}{\omega_m} \\ 0 & 0 & \cos(\omega_m T) & -\sin(\omega_m T) \\ 0 & 0 & \sin(\omega_m T) & \cos(\omega_m T) \end{bmatrix}, \quad G_k^m = \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix},$$

$$m \in \{2, \ldots, 9\}, \quad \omega_m \in \{\pm 20, \pm 50, \pm 80, \pm 110\}(\text{deg/s}).$$

with the pedestrian state $\xi_k^p = \left[x_k^p, y_k^p, v_{x_k}^p, v_{y_k}^p\right]$ containing the pedestrian position in x-axis and y-axis and the pedestrian velocity in x-axis and y-axis at time t, respectively. ω_m is the specified turn rate of the model m, and T is the time step interval. The measurement is chosen as $z_k^p = \left[x_k^p \;\; y_k^p\right]$.

Remark 1. The values of ω_m are chosen heuristically in this paper. In fact, it is not possible to estimate the exact turn rate of pedestrians due to their highly dynamical behaviors. Therefore, we chose a set of turn rates which covers both slow and fast dynamics of pedestrians.

Remark 2. Constant acceleration model is usually used to model the motions of vehicles or airplanes. However, according to Schneider and Gavrila (2013), it does not give better results in estimating and predicting the pedestrian's position compared with just using CV and CT models.

The transition probability matrix (TPM) Π between models is defined as

$$\Pi = [\pi_{ij} := P(m_{k+1} = j | m_k = i)],$$

where m_k is the model at time k. In this research, we fix the values as $\pi_{ij} = 0.95$ for $i = j$ and $\pi_{ij} = \frac{0.05}{M-1}$ for $i \neq j$. Next, the IMM-KF algorithm is implemented. See Li et al. (2000); Blackman and Popoli (1999) for details. By using the IMM-KF, the estimated pedestrian state $\hat{\xi}_t^p$ and model probabilities μ_t^m $(m = 1, \dots, M)$ at the current time t can be estimated.

2.3 Multiple pedestrian path prediction

It is necessary to predict the future path of the pedestrian when designing MPC controllers. In previous studies of stochastic MPC, i.e, Carvalho et al. (2014), the future path of a target vehicle was obtained by propagating the current state over the prediction horizon using the model with the highest probability μ_t^m. Another approach by Schildbach and Borrelli (2015) considered a number of i.i.d. sampled scenarios over the prediction horizon. At each prediction step, scenarios propagate by choosing an arbitrary model from the set of models. We, however, generates M different pedestrian trajectories using all M pedestrian models.

Remark 3. Each trajectory is generated by first fixing a pedestrian model and then propagating the states over the prediction horizon using the same pedestrian model.

According to the pedestrian path prediction procedure described above, M trajectories are not i.i.d but have different probabilities. For simplification, we assume that each trajectory has the same probability μ_t^m according to the models generated by the IMM-KF. Based on these trajectories, the chance of vehicle-pedestrian collision can be estimated. The details will be given in the next section.

2.4 Longitudinal vehicle model

In the shared road scenario, the driver rarely steers the vehicle to avoid the pedestrian due to the limited lane width. Instead, he/she slows down the vehicle when there is an incoming risk or accelerates and passes the pedestrian before being too close. Therefore, only longitudinal motion of the ego car is considered in this research. A point mass model sufficiently express the straight line dynamics of the ego car.

$$\begin{aligned} X_{k+1}^c &= F^c X_k^c + G^c u_k, \\ F^c &= \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G^c = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 0 \end{bmatrix}, \end{aligned} \tag{2}$$

where $X_k^c = [x_k^c \ v_k^c \ y_k^c]^T$ is the vehicle state, x_k^c, v_k^c, u_k^c represent the position, velocity and acceleration of the car in x-axis, respectively, at time k. The position of the car with respect to y-axis, y_k^c, is considered to be constant.

3. SAFETY CONSTRAINTS

3.1 Collision avoidance

We directly restrict the distance of the vehicle and pedestrians to be above a certain safety value $d_{\min}$ to avoid the collision. Without the loss of generality, we consider one pedestrian on the road. As described in Section 2.3, M pedestrian trajectories s^m $(m = 1, \dots, M)$ are generated by using all M pedestrian models. Therefore, the collision between the vehicle and the pedestrian considering that the pedestrian is moving according to model m can be represented by the constraint

$$d_k^m - d_{\min} \leq 0, \quad k = 0, \dots, N-1, \tag{3}$$

where d_k^m is the distance between the vehicle and the pedestrian at time k in the prediction horizon, N is the number of predicted steps.

Remark 4. Even if the constraint (3) is violated for some model m, it does not mean that the vehicle is going to collide with the pedestrian. The model probabilities should also be considered in such a judgement.

Note that each model of the pedestrian has its own probability, which is calculated from the IMM-KF. Therefore, we take this into account by formulating (3) as a chance-constraint to be satisfied to avoid the collision. That is,

$$\sum_{m=1}^{M} \mu_t^m C_{t,N}^m \leq p, \tag{4}$$

where

$$C_{t,N}^m = \begin{cases} 1, & \text{if } \exists k \in \{0, \dots, N-1\},\ d_k^m - d_{\min} \leq 0 \\ 0, & \text{if } \forall k \in \{0, \dots, N-1\},\ d_k^m - d_{\min} \geq 0, \end{cases}$$

and p is a specified probability representing a risk level. Naturally, a high value of p might lead to a higher collision risk. On the other hand, a low level of p might make the behavior of the vehicle too conservative. We study the effect of the parameter p on the driving behavior in Section 5.

3.2 Acceleration and velocity constraints

The constraints on the velocity and acceleration of the vehicle are considered as below

$$v_{\min} \leq v_k^c \leq v_{\max}, \quad a_{\min} \leq u_k \leq a_{\max} \tag{5}$$

4. SCENARIO-BASED STOCHASTIC MODEL PREDICTIVE CONTROL FOR COLLISION AVOIDANCE

In this section, an MPC speed controller for the vehicle is designed to operate in a shared road environment with a pedestrian. The designed controller takes into account the uncertainties of the pedestrian behavior in the prediction horizon, evaluates the chance of collision and calculates the optimal control input that keeps that chance of collision below a specified probability threshold p.

4.1 Problem formulation

The receding horizon control problem is formulated using the models of the vehicle and the pedestrian and the safety constraints. At each time step, a finite time horizon optimal control problem is solved to obtain a sequence of control input to minimizes a predefined cost function. The first element of the control input sequence is implemented to the system and the process is repeated at the next time step with new measurement data. The optimization problem at each time step is formulated as follows.

given: $X_t^c,\ \xi_t^p,\ v_{\mathrm{ref}},\ \mu_t^m$
find: $u_{k|t},\quad k=0,\ldots,N-1$
which minimizes:

$$J_{\mathrm{cost}} = \Phi(X^c_{N|t}, v_{\mathrm{ref}}) + \sum_{k=0}^{N-1}\sum_{m=1}^{M} \mu^m_{k|t}\mathscr{L}^m(X^c_{k|t}, u_{k|t}) \tag{6a}$$

$$\Phi(X^c_{N|t}, v_{\mathrm{ref}}) = S_c(v^c_{\mathrm{ref}} - v^c_{N|t})^2 \tag{6b}$$

$$\begin{aligned}\mathscr{L}^m(X^c_{k|t}, u_{k|t}, \xi^p_{k|t}) &= Q_c(v^c_{\mathrm{ref}} - v^c_{k|t})^2 \\ &\quad + R_c(u_{k+1|t} - u_{k|t})^2 + \mathscr{B}^m(X^c_{k|t}, \xi^p_{k|t})\end{aligned} \tag{6c}$$

$$\mathscr{B}^m(X^c_{k|t}, \xi^p_{k|t}) = P_c \exp\left(-\alpha\left(d^m_{k|t} - d_{\min}\right)\right) \tag{6d}$$

subject to:

$$X^c(k+1|t) = F^c X^c(k|t) + G^c u(k|t),\ \ k=0,\ldots,N-1 \tag{6e}$$

$$\sum_{m=1}^{M} \mu_t^m C^m_{t,N} \le p \tag{6f}$$

$$v_{\min} \le v_k^c \le v_{\max},\quad a_{\min} \le u_k \le a_{\max} \tag{6g}$$

$$X^c_{0|t} = X^c_t,\quad \xi^p_{0|t} = \xi^p_t, \tag{6h}$$

where $N = 20$ is the number of steps in the prediction horizon, $T = 0.1$ (s) is the time step interval, $d_{\min}$ is the safety distance. $\alpha = 10$ is a constant, S_c, Q_c, R_c and P_c are weight parameters, and d_k^m is the distance between the vehicle and the pedestrian at time k considering that the pedestrian is following the model m. Note that each pedestrian's predicted trajectory has its own probability μ_t^m obtained from the IMM-KF. Thus, if the distance constraint between vehicle-pedestrian is violated according to a model m whose probability μ_t^m is very small, then the chance of collision between vehicle and pedestrian is also negligible.

4.2 Randomized approach for semi-global optimal solution

The optimization problem in hand need to consider multiple modes for the pedestrian. Having done some previous works on developing fast sample-based optimization with GPU, we choose the sample-based optimization approach for this problem. In the previous work Okuda et al. (2018) have demonstrated that the sample-based method can give accurate solutions without the need of linearization or approximations. It is also proved by Vidyasagar (2001) that having more than a certain number of samples guarantees the solution to be close enough to the global optimum. The steps involved in this process is as follows.

The first step is to generate N_s number of control input series u^i_{IDCT} according to (7b). We also make sure that the series does not exceed the maximum acceleration limits of the car. Then we compute the state function values for each series following the vehicle dynamic model (2) and predict the pedestrian path using a pedestrian model. This step is repeated for the M number of modes that the pedestrian may follow within the horizon. Then, we calculate cost for each control input series in each given models (6a). In order to find the most optimum sample, we filter out the samples that have a higher probability of collision than p. First element of the series with minimum cost is our semi optimal solution. This goes as input to the ego car. These steps are repeated in every control cycle (Algorithm 1).

Algorithm 1 Sample-based optimization

1: Generate N_s number of of control input series.
2: **for** $Series = 1,2,\ldots,N_s$ **do**
3: **for** $Models = 1,2,\ldots,M$ **do**
4: Calculate the predicted car position based on the control input series and M possible pedestrian paths.
5: Calculate the cost (6a) for each series and the chance-constraint (6f) .
6: **end for**
7: **end for**
8: Filter out the samples that have a higher probability of collision than p.
9: Find the control input series corresponding to the minimum cost. The first element is implemented to the system.

4.3 Random input sampling in frequency domain

Generating samples by random numbers cannot guarantee samples that are smooth enough for cases like driving a car. The samples that we use here are instead generated from the frequency domain and then converted to the time domain. This is done using Inverse Discrete Cosine Transform (IDCT). This method of generation leads to smoother samples and hence smoother driving performance. The advantages in terms of control performance can be found in the author's previous work Okuda et al. (2018). The details of generating input series u^i_{IDCT} by using IDCT transform are explained below:

$$u^i_{IDCT}(0|t) = u(t-1), \tag{7a}$$

$$\begin{aligned}u^i_{IDCT}(k|t) &= u^i_{IDCT}(k-1|t) + \Delta u^i_{IDCT}(k|t) \\ &\qquad \forall k \in \{1,\ldots,N\},\end{aligned} \tag{7b}$$

$$\Delta u^i_{IDCT}(t)^T = \gamma D U^i_{IDCT}(t)^T, \tag{7c}$$

$$U^i_{IDCT}(l|t) = \begin{cases} \sim \mathscr{U}(0,1) & \text{if } l \le F_{c/o} \\ = 0 & \text{other} \end{cases}, \tag{7d}$$

where the coefficients matrix of IDCT transform is $D \in R^{N\times N}$ and l is the frequency component index. One element of D is defined as:

$$D_{ij} = \sqrt{\frac{2}{N}} k_i \cos\left(\frac{(i-1)(j-1/2)\pi}{N}\right),$$
$$i \in \{1,2,\ldots,N\},\ j \in \{1,2,\ldots,N\}, \tag{8}$$

where N is the length of sampled control input series. Normalizing factor k_i $(i = 1,2,...,N)$ has two values:

$$k_i = \begin{cases} \dfrac{1}{\sqrt{2}}, & \text{if } i = 1 \\ 1, & \text{if } i \ne 1. \end{cases} \tag{9}$$

$U^i_{IDCT}(t)$ is sampled from a uniform distribution $\mathscr{U}(0,1)$ in this case. This indicates the magnitude of each frequency component. γ can be used to adjust the resulting input and $F_{c/o}$ is a threshold for cut-off frequency, it checks the higher-frequency components. Smaller $F_{c/o}$ leads to a smoother input series, the rest of the paper assumes $F_{c/o}$ to be 20.

5. SIMULATION RESULTS

Simulations are conducted using MATLAB to evaluate the effectiveness of the method. The number of samples N_s in Section 4.2 is chosen to be 500 samples. In these simulations, the pedestrian is assumed to be walking ahead of the vehicle. He/she has a chance to cross the road afterward. The measurement of the pedestrian position is corrupted by Gaussian disturbance. Note that, we are considering the shared road driving scenario in a narrow space. Thus, it is limited to the longitudinal motion control problem for the car.

5.1 Test scenario 1: The pedestrian only walks on one side of the road

In this test scenario, the vehicle is running at a reference velocity $v_{\text{ref}} = 5$ (m/s) and there is a pedestrian walking on the left-hand side of the road ahead of the vehicle at the speed of 1.2 (m/s). The lateral distance between vehicle and the pedestrian is 1.5 (m) while the safety distance $d_{\min}$ is set as 1 (m).

In a deterministic case where there is no uncertainty, the vehicle will just keep the same speed without risking of colliding with the pedestrian. However, because of the highly dynamical behavior of the pedestrian, the vehicle should anticipate a sudden change in the walking direction of the vehicle and slow down properly to keep the risk of collision below a certain level. Fig. 2 displays 9 different predicted trajectories corresponding to the chosen dynamic models of the pedestrian, and Fig. 3 shows the mode probabilities. It can be seen that the pedestrian is predicted to be walking straight in the CV mode with a very high probability.

Based on the above prediction, the vehicle anticipates the risk of collision, even though it is small due to the small probability of turning right of the pedestrian, and slow down accordingly. Fig. 4 illustrates the speed profile of the vehicle at different chance-constraint probability p. It can be seen that by setting the chance-constraint probability to a lower value, the behavior of the vehicle is more conservative, and the vehicle slows down more when approaching the pedestrian. At higher chance-constraint probability p, the vehicle only slightly reduce the speed; therefore, the risk of collision will be high if the pedestrian suddenly changes his/her walking direction.

5.2 Test scenario 2: The pedestrian crosses the road at a given time with different crossing angles

In this scenario, the pedestrian who initially walks along the road, suddenly crosses the road at a given time and with a crossing angle β with respect to the y-axis. A record of one test scenario, which includes the true pedestrian position, the measurement noise and the estimated position of the pedestrian is given in Fig. 5.

Before the pedestrian crosses the road, the prediction of pedestrian path is very similar to the Test scenario 1. However, when

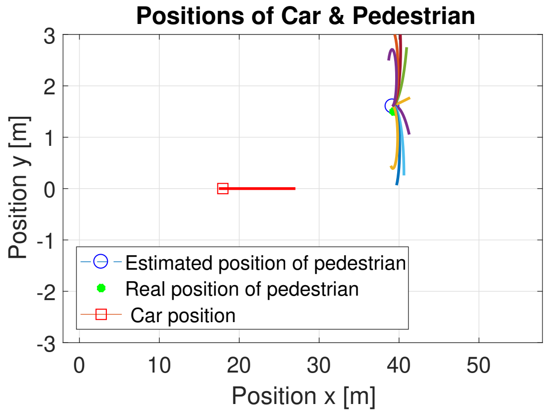

Fig. 2. Pedestrian is walking parallel to the vehicle without crossing the road

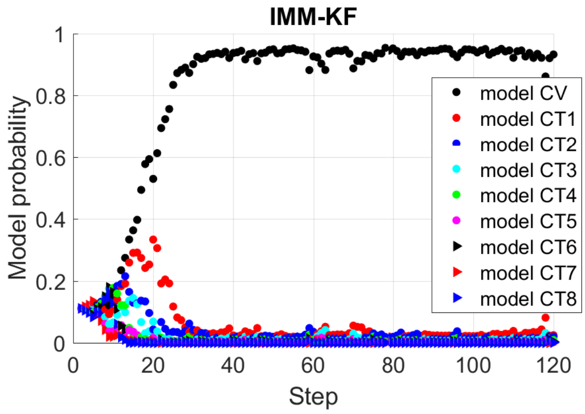

Fig. 3. Model probability of the pedestrian

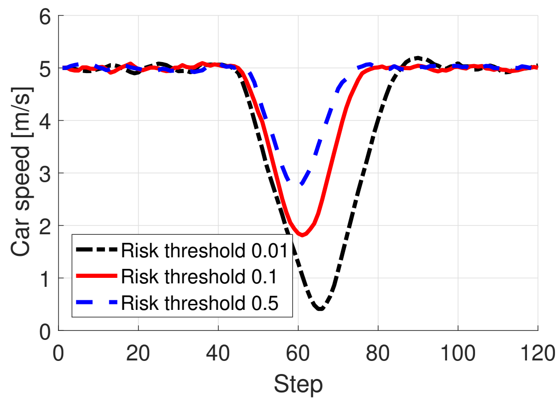

Fig. 4. Speed profile of the vehicle at different risk threshold p

the pedestrian starts crossing, the prediction of pedestrian path changes accordingly. Fig. 6 displays the predicted trajectories of the vehicle and the pedestrian when the pedestrian is crossing in front of the vehicle. The probability of each trajectory is illustrated in Fig. 7. It can be seen that the pedestrian is predicted to walk in some trajectories with fair possibilities. By taking this into account, the vehicle can slow down the speed appropriately to deal with the anticipated risk of collision.

Fig. 8 shows the probability of collision of the SSMPC controller in the prediction horizon which is calculated from (4) with a probability threshold $p = 0.001$. It can be seen that

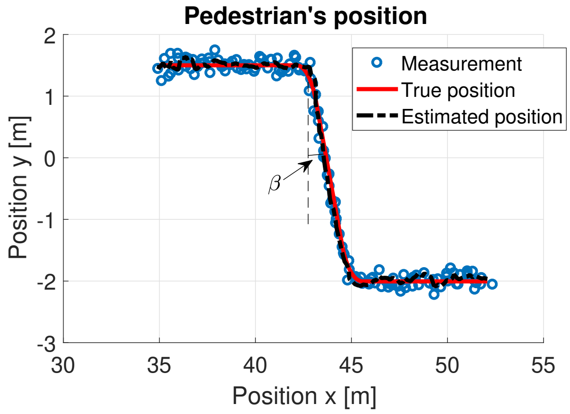

Fig. 5. Scenario of pedestrian crossing the road. Includes the true position, measurement noise and estimated position of the pedestrian.

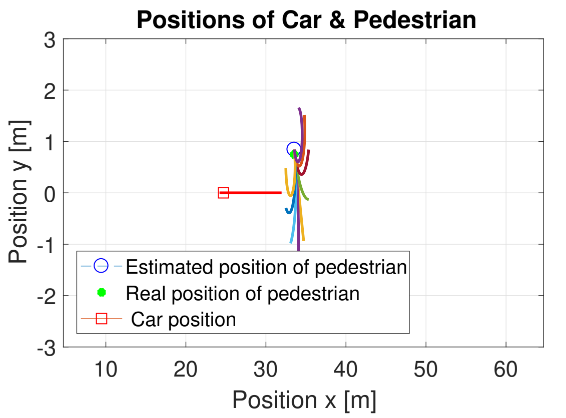

Fig. 6. Predicted trajectories of the pedestrian when crossing the road.

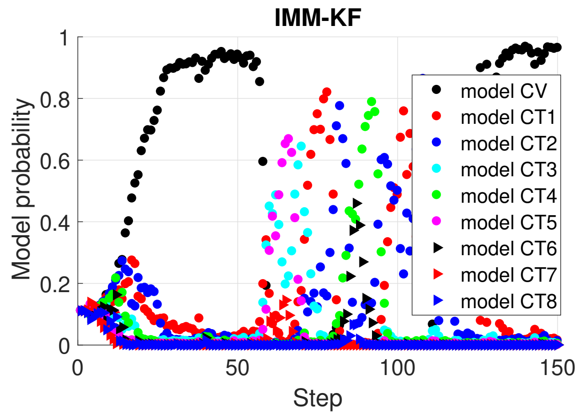

Fig. 7. Model probabilities of the pedestrian

the probability of collision is well kept below the specified threshold. A deterministic MPC controller that acts as a benchmark is designed using the above method. This controller being standard MPC has a single scenario. Here that scenario is the CV model of the pedestrian, i.e., the model probability of the CV model is 1 in the controller design process. However this is tested for its probability of collision by taking into account all M modes in the same scenario as the SSMPC (4). It can be seen in Fig. 8 that the probability of collision of the deterministic controller is very high at certain steps, which are the time when

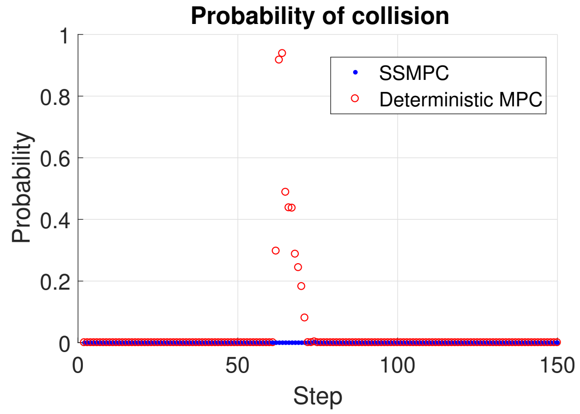

Fig. 8. Predicted probability of collision in the prediction horizon calculated from (4). The probability threshold of the SSMPC controller is $p = 0.001$.

the pedestrian crosses the road. That is because the deterministic MPC controller does not consider the uncertainty of the pedestrian walking behavior. Therefore, it cannot anticipate and react quickly when the pedestrian changes the walking direction and crosses the road.

We used Monte Carlo method to conduct 100 simulations at different walking conditions of the pedestrian to test the performance of 3 controllers: a SSMPC controller with the probability threshold of 0.001, a SSMPC controller with the probability threshold of 0.1, and a deterministic MPC which only considers the CV model of the pedestrian. In these simulations, the pedestrian starts crossing the road in front of the vehicle at different time and different crossing angle β with respect to the y-axis ($\beta \in [-25, 25]$ in degree). The collision is evaluated by comparing the actual vehicle-pedestrian distance to the safety distance $d_{\min}$. The result is shown in Table 1. It can be seen that the chance of collision of the SSMPC controller is below the specified probability threshold. Restricting collision probability with a soft constraint is not ideal considering the risk associated in case of a constraint violation. Since all predicted state are known in RMPC algorithm, a deterministic hard constraint can be considered strictly and naturally in this framework to guarantee safety.

The calculation time of the SSMPC controller is about 61 (ms) using MATLAB. Note that the structure of the proposed SSMPC controller is very suitable for parallel computing. Therefore, a significant improvement in the calculation time of the proposed controller can be obtained by implementing it in GPU as discussed by Muraleedharan et al. (2019).

Table 1. Constraint violation in 100 simulations at different conditions

Controller	Constraint violation	CPU time (ms)
Deterministic MPC	12%	19
SSMPC (p = 0.1)	5%	61
SSMPC (p = 0.001)	0%	61

6. CONCLUSIONS

The proposed SSMPC framework has guaranteed safe navigation with a proactive collision avoidance while driving around a pedestrian in a shared road situation. The controller can consider a number of predicted trajectories of different probabilities corresponding to different dynamic behaviors of the pedestrian at a same time. This enables it to anticipate the potential collision risk and react properly. The proposed framework also allows a tunable risk acceptance that lets the car behave less conservative and avoid deadlock situations compared to a conventional deterministic MPC. Numerous simulation results confirm that the automated vehicle can successfully avoid a moving pedestrian with probabilistic behaviors in a narrow shared road environment. An ongoing work is to implement the SSMPC controller on a real vehicle using GPU for faster execution. This work will be extended to consider both lateral and longitudinal motion control of ego-car. We also plan to consider multiple agents like pedestrians and cyclists in the framework along with their interaction with each other.

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