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Iterative Learning Control for Output Tracking of Systems with Unmeasurable States ⋆

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Abstract: In this work, a new design framework of adaptive iterative learning control (ILC) approach for a class of uncertain nonlinear systems is presented. By making use of the closed-loop reference model which works as an observer, the developed adaptive ILC method is able to be adopted to deal with the output tracking problem of nonlinear systems without requiring the measurability of system states. In the system, the uncertainties are formed by the product of unknown parameters and state functions that are also unknown as the system states are not available. In order to facilitate the controller design and convergence analysis, the composite energy function (CEF) method is employed, and the accurate tracking task can be realized successfully. The proposed approach extends CEF-based ILC approach sucessfully to output tracking control of nonlinear systems without requiring the system states information and complicated observer design. The effectiveness of the proposed ILC scheme is verified through an illustrative numerical example.

Keywords: Iterative learning control (ILC), composite energy function (CEF), output tracking, convergence analysis.

1. INTRODUCTION

As an effective approach for tracking control of repeatable uncertain systems, iterative learning control (ILC) has been intensively investigated in the past three decades, such as Bristow et al. (2006); Moore (1999); Shen and Wang (2014). Compared with other control methods, ILC has many distinct advantages: 1) ILC is designed to deal with repetitive systems and is able to improve the control performance iteratively by its learning ability. While, other control methods generate the same control performance in different iterations as they cannot take the advantages of systems' repetition; 2) Different with other control approaches that aim at asymptotical convergence when the time goes to infinity, the control objective of ILC is to achieve perfect tracking performance on the whole time interval; 3) ILC is easy to be implemented as it can be viewed as a feedforward control method in time domain. Moreover, its control algorithm is simple.

In the past years, many efforts have been made in ILC field to extend its application scope. In literature, there are two main frameworks for ILC. One is the contraction-mapping (CM)-based ILC, and another is the composite energy function (CEF)-based ILC Arimoto et al. (1984); Xu and Tan (2003); Bu and Hou (2016); Devasia (2017); Meng and He (2017); Liu et al. (2017); Sebastian et al. (2019); Oh et al. (2018); de Rozario and Oomen (2019). For CM-based ILC, the structure of the ILC law is very simple where the current control input consist of the previous control inputs and tracking errors as a correction term, which is easy to be implemented in applications. However, due to that the convergence analysis is based on contraction mapping methodology, the global Lipschitz continuous (GLC) condition of control systems is required to avoid finite time escape. In contrast, CEF-based ILC is designed based on the Lyapunov-like theory, and thus is able to deal with control systems with nonlinearities not satisfying the GLC condition. Here, it is however worthwhile to point out that in the framework of CEF-based ILC, it usually requires that the full state information is measurable, which may not be available in practice. Therefore, how to extend CEF-based ILC to output tracking problem for nonlinear system with unmeasurable states is an interesting problem to be investigated.

In literature, many efforts have been devoted to establish adaptive ILC schemes using only system output information to relax/remove the requirement on full state measurement in CEF-based ILC, such as Tayebi and Xu (2003); Xu and Xu (2004); Chien and Yao (2004); Zhu et al. (2017); Wang and Chien (2014); Bouakrif et al. (2013). Most of these works were aiming to design the controllers via developing state observers and then followed the design and analysis procedures for traditional CEF-based ILC. However, most of the observer design in aforementioned literature is complicated. It is well-known that: 1) the design of state observer is highly dependent on the prior knowledge of the system dynamics, which is usually difficult to obtained; 2) Badly controlled transient performance of state observer would affect the resulted control performance of the system; 3) The estimated system states may deviate from the original system states due to the unmodelled noises or disturbances existing in practice, which would also degrades the control performance of the closed-loop system. According to these observations, it is worthy to develop new adaptive ILC schemes with a simple state estimation strategy to improve the

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transient and control performance when the system states are not available or measurable.

In this paper, a novel adaptive ILC algorithm is developed for output tracking control of a class of nonlinear systems with unmeasurable system state. For the systems considered, the difficulty lies in the input uncertainties formed by the product of unknown parameters and state functions that are also unknown because system state is unavailable. For this kind of uncertainties, we thus cannot treat them as parametric uncertainties as we usually did in traditional CEF-based ILC. To overcome the difficulty, the ideal reference model is modified by adding a feedback term consisting of the tracking error. The closed-loop reference model is thus able to work as an observer, and the proposed adaptive ILC approach can then be applied to the output tracking problem without requiring the system state information. The main contributions of this work can be summarized as follows:

1) We extend CEF-based ILC to output tracking problem of nonlinear systems with unmeasurable system states for the first time;
2) The controller design applies only system output information and a closed-loop reference model without designing complicated state observer as works in literature;
3) The proposed ILC design approach is novel in ILC area and it provides a design framework of adaptive ILC for output tracking of uncertain nonlinear systems;
4) The method proposed in this work makes ILC applicable to more general nonlinear uncertain systems.

This remaining part of this paper is organized as follows. The control problem is formulated in Section 2. Section 3 focuses on the ILC design and convergence analysis for systems with only input uncertainties. Furthermore, an illustrative example is given in Section 4. Section 5 draws a conclusion.

2. PROBLEM FORMULATION

In this section, the control problem will be formulated firstly, and then a kind of closed-loop reference model will be introduced to work as an observer to estimate the system states.

In the present work, we consider the following MIMO nonlinear system

$$\begin{aligned} \dot{x}_k &= Ax_k + B\left[u_k - \Theta^T F(x_k,t)\right], \\ y_k &= C^T x_k, \end{aligned} \tag{1}$$

where the subscript k denotes the iteration index, $x_k \in \mathbf{R}^n$ is the system state that is unmeasurable, $y_k \in \mathbf{R}^m$ is the system output, $u_k \in \mathbf{R}^m$ is the system input, $\Theta \in \mathbf{R}^{n_1\times m}$ represents the parametric uncertainty, which can be either time-invariant or time-varying, and $F(x,t) \in \mathbf{R}^{n_1}$ is a known vector-value function, which is differentiable with respect to x with $\|\frac{\partial F}{\partial x}\| \leq \bar{f}$ and $\bar{f}$ being known. However, due to that the system state x_k is not available, $F(x_k,t)$ is also unknown. A, B, C are matrices with appropriate dimensions.

In this work, to facilitate the controller design, the following assumptions are imposed to the system (1).

Assumption 1. The matrix $C^T B$ is of full rank.

Assumption 2. For system (1), (A, C^T) is observable.

Assumption 3. For system (1), both the parametric uncertainty is bounded

$$\|\Theta\| \leq \bar{\theta}$$

where $\bar{\theta}$ is known.

The objective is to determine a sequence of control inputs u_k, $k \in \mathbf{Z}^+$ such that the system output y_k can track the reference y_d generated by

$$\begin{aligned} \dot{x}_d &= Ax_d + Bu_d, \\ y_d &= C^T x_d, \end{aligned} \tag{2}$$

where u_d is the desired control input. In order to ensure prefect tracking, we assume $x_d(0) = x_k(0) = 0$ which is common in the field of ILC. Due to the fact that the system states are not available, the reference model is modified as follows

$$\begin{aligned} \dot{x}_{d,k} &= Ax_{d,k} + Bu_d - L(y_k - y_{d,k}), \\ y_{d,k} &= C^T x_{d,k}, \end{aligned} \tag{3}$$

to act as an observer to estimate the system states and then facilitate the ILC design, where L is a feedback gain to be designed. Obviously, when the system output y_k converges to the desired output $y_{d,k}$, the modified reference model (3) becomes the same with the ideal reference model (2). Therefore, in the following (3) will be considered as the reference model to be followed.

3. ILC DESIGN

In this section, an adaptive ILC scheme is designed based on the closed-loop reference model, and the convergence analysis is conducted under the framework of CEF method.

Define the tracking error $e_k \triangleq x_k - x_{d,k}$ and $e_{y,k} \triangleq y_k - y_{d,k}$. The following ILC law is proposed

$$u_k = \hat{\Theta}_k^T F(x_{d,k},t) + u_d, \tag{4}$$

$$\hat{\Theta}_k = \hat{\Theta}_{k-1} - \Gamma F(x_{d,k},t) e_{y,k}^T D, \tag{5}$$

where $D \triangleq C^T B$, Γ is the learning gain matrix which are diagonal with positive diagonal elements, $\hat{\Theta}_k$ is the estimation of Θ with $\hat{\Theta}_0 = 0$.

Denote $F_{d,k} \triangleq F(x_{d,k},t)$, $F_k \triangleq F(x_k,t)$ and $\tilde{\Theta}_k \triangleq \hat{\Theta}_k - \Theta$. The error dynamics is derived as follows

$$\begin{aligned} \dot{e}_k &= \dot{x}_k - \dot{x}_d \\ &= (A + LC^T)e_k + B\Pi\left[u_k - \Theta^T F_k\right] - Bu_d \\ &= (A + LC^T)e_k + B\Pi\left[\tilde{\Theta}_k^T F_{d,k} - \Theta^T(F_k - F_{d,k})\right] \end{aligned} \tag{6}$$

where the control law (4) is applied. By the mean value theorem, there exists a x_k^* such that

$$F_k - F_{d,k} = \frac{\partial F(x_k^*,t)}{\partial x}(x_k - x_{d,k}) \triangleq F_k^x(x_k^*,t)e_k \tag{7}$$

which implies that

$$\dot{e}_k = (A + LC^T)e_k + B\Pi\left[\tilde{\Theta}_k^T F_{d,k} - \Theta^T F_k^x e_k\right] \tag{8}$$

with F_k^x denoting $F_k^x(x_k^*,t)$ for simplicity.

Before the convergence analysis of the proposed controller, the design of the feedback gain L is discussed first. According to Assumption 2, namely, (A, C^T) is observable, there exists a feedback gain L_s such that the matrix $A + L_sC^T$ is Hurwitz, from which we have that for any given positive definite matrix Q there exists a positive definite matrix P with $P = P^T$ satisfying

$$(A + L_sC^T)^T P + P(A + L_sC^T) = -Q. \tag{9}$$

Then, we have the following lemma.

Lemma 1. For system (1) and the matrices L_s, P, Q satisfying (9), the matrix $A + LC^T$ is Hurwitz with $L \triangleq L_s - \rho BD^T$ and $\rho > 0$ if $PB = CD$.

Proof. To show the stability of $A+LC^T$, the following calculation is conducted

$$
\begin{aligned}
&(A+LC^T)^T P + P(A+LC^T)\\
&= (A+L_sC^T)^T P + P(A+L_sC^T) - \rho CDB^T P - \rho PBD^T C^T\\
&= -Q - 2\rho CDD^T C^T
\end{aligned} \tag{10}
$$

where the condition $PB = CD$ is used. Due to that Q is positive definite and CDD^TC^T is positive semi-definite, $Q+2\rho CDD^TC^T$ is shown to be positive definite, which thus implies $A+LC^T$ is stable.

The first main result of this work can be summarized in Theorem 1.

Theorem 1. For system (1) satisfying Assumptions 1-3, the controller (4) with the updating law (5) together with the feedback gain chosen as $L = L_s - \rho BD^T$ in (2) ensure both the state and output tracking if

$$\rho > \frac{\bar{\pi}^2 \bar{f}^2}{2\lambda_{min}(Q)} \tag{11}$$

with $\bar{\theta}$ being defined in Assumption 3, $\bar{f}$ is the upper bound of $\frac{\partial F}{\partial x}$ and $\lambda_{min}(Q)$ (>0) being the minimum eigenvalue of Q.

Proof. To show the convergence of the proposed controller, we define a composite energy function (CEF) as follows

$$E_k(t) = e_k^T P e_k + \int_0^t \mathrm{Tr}\left(\tilde{\Theta}_k^T \Gamma^{-1} \tilde{\Theta}_k\right) d\tau. \tag{12}$$

The convergence analysis is presented as follows, which consists of three parts.

Part I: Difference of CEF. For a time instant $t \in [0,T]$, the difference of CEF at two consecutive iterations is defined as

$$
\begin{aligned}
\Delta E_k(t) &\triangleq E_k(t) - E_{k-1}(t)\\
&= e_k^T P e_k - e_{k-1}^T P e_{k-1}\\
&\quad + \int_0^t \mathrm{Tr}\left[\left(\tilde{\Theta}_k^T \Gamma^{-1}\tilde{\Theta}_k - \tilde{\Theta}_{k-1}^T \Gamma^{-1}\tilde{\Theta}_{k-1}\right)\right] d\tau.
\end{aligned} \tag{13}
$$

For the first term in (13), it can be written as

$$
\begin{aligned}
e_k^T P e_k &= \int_0^t \left(\dot{e}_k^T P e_k + e_k^T P \dot{e}_k\right) d\tau\\
&= \int_0^t e_k^T \left[(A+LC^T)^T P + P(A+LC^T)\right] e_k\\
&\quad + 2e_k^T PB\left[\tilde{\Theta}_k^T F_{d,k} - \Theta^T F_k^x e_k\right] d\tau.
\end{aligned} \tag{14}
$$

According to (9) and Lemma 1, we have

$$(A+LC^T)^T P + P(A+LC^T) = -Q - 2\rho CDD^T C^T. \tag{15}$$

Hence, (14) becomes

$$
\begin{aligned}
e_k^T P e_k = \int_0^t &\left[-e_k^T \left(Q + 2\rho CDD^T C^T\right) e_k\right.\\
&\left. + 2e_k^T PB\left(\tilde{\Theta}_k^T F_{d,k} - \Theta^T F_k^x e_k\right)\right] d\tau.
\end{aligned} \tag{16}
$$

Regarding to the third term of (13), first we can derive that

$$
\begin{aligned}
&\tilde{\Theta}_k^T \Gamma^{-1}\tilde{\Theta}_k - \tilde{\Theta}_{k-1}^T \Gamma^{-1}\tilde{\Theta}_{k-1}\\
&= \left(\hat{\Theta}_{k-1} - \hat{\Theta}_k\right)^T \Gamma^{-1}\left[\left(\hat{\Theta}_k - \hat{\Theta}_{k-1}\right) - 2\tilde{\Theta}_k\right]\\
&= -\left(\hat{\Theta}_k - \hat{\Theta}_{k-1}\right)^T \Gamma^{-1}\left(\hat{\Theta}_k - \hat{\Theta}_{k-1}\right)\\
&\quad - 2\left(\hat{\Theta}_{k-1} - \hat{\Theta}_k\right)^T \Gamma^{-1}\tilde{\Theta}_k\\
&= -D^T e_{y,k} F_{d,k}^T \Gamma F_{d,k} e_{y,k}^T D - 2D^T e_{y,k} F_{d,k}^T \tilde{\Theta}_k
\end{aligned} \tag{17}
$$

with the updating law (5) being used. Therefore, it gives that

$$
\begin{aligned}
&\mathrm{Tr}\left[\left(\tilde{\Theta}_k^T \Gamma^{-1}\tilde{\Theta}_k - \tilde{\Theta}_{k-1}^T \Gamma^{-1}\tilde{\Theta}_{k-1}\right)\right]\\
&= -2\mathrm{Tr}(D^T e_{y,k} F_{d,k}^T \tilde{\Theta}_k) - \mathrm{Tr}\left(D^T e_{y,k} F_d^T \Gamma F_{d,k} e_{y,k}^T D\right)\\
&= -2e_k^T CD\tilde{\Theta}_k^T F_{d,k} - F_{d,k}^T \Gamma F_{d,k} \|D^T e_{y,k}\|^2
\end{aligned} \tag{18}
$$

where the cyclic property of trace is applied. Finally, the third term of (13) becomes

$$
\begin{aligned}
&\int_0^t \mathrm{Tr}\left[\left(\tilde{\Theta}_k^T \Gamma^{-1}\tilde{\Theta}_k - \tilde{\Theta}_{k-1}^T \Gamma^{-1}\tilde{\Theta}_{k-1}\right)\right] d\tau\\
&= -\int_0^t \left\{2e_k^T CD\tilde{\Theta}_k^T F_{d,k} + F_{d,k}^T \Gamma F_{d,k}\|D^T e_{y,k}\|^2\right\} d\tau\\
&\le -\int_0^t 2e_k^T CD\tilde{\Theta}_k^T F_{d,k} d\tau
\end{aligned} \tag{19}
$$

since $F_{d,k}^T \Gamma F_{d,k}\|D^T e_{y,k}\|^2$ is positive.

By combining (16) and (19) with (13), it yields

$$
\begin{aligned}
\Delta E_k(t) &\le -\int_0^t \left[e_k^T \left(Q + 2\rho CDD^T C^T\right) e_k + 2e_k^T PB\Theta^T F_k^x e_k\right] d\tau\\
&\quad - e_{k-1}^T P e_{k-1}\\
&= -\int_0^t \xi_k^T \bar{Q}(\rho)\xi_k d\tau - e_{k-1}^T P e_{k-1}
\end{aligned} \tag{20}
$$

where $\xi_k \triangleq [e_{y,k}, e_k]^T$ and

$$\bar{Q}(\rho) \triangleq \begin{bmatrix} 2\rho DD^T & D\Theta^T (F_k^x)^T \\ F_k^x \Theta D^T & Q \end{bmatrix}. \tag{21}$$

Let Q_S be the Schur complement of Q in the matrix $\bar{Q}(\rho)$, it gives $Q_S = 2\rho DD^T - D\Theta^T \left(F_k^x\right)^T Q^{-1} F_k^x \Theta D^T$. It is not difficult to show that Q_S is positive definite if

$$\rho > \frac{\bar{\theta}^2 \bar{f}^2}{2\lambda_{min}(Q)}. \tag{22}$$

By the Schur complement, the matrix $\bar{Q}(\rho)$ is positive definite. Therefore, we have

$$
\begin{aligned}
\Delta E_k(t) &\le -\int_0^t \xi_k^T \bar{Q}(\rho)\xi_k d\tau - e_{k-1}^T P e_{k-1}\\
&\le -\lambda_{min}^{\rho}(\bar{Q}) \int_0^t \xi_k^T \xi_k d\tau - \lambda_{min}(P) e_{k-1}^T e_{k-1}
\end{aligned} \tag{23}
$$

with $\lambda_{min}^{\rho}(\bar{Q}) (>0)$ and $\lambda_{min}(P) (>0)$ being the minimum eigenvalues of $\bar{Q}(\rho)$ and P, respectively.

Remark 1. Due to the existence of $2e_k^T PB\Theta^T F_k^x e_k$ on the right hand side of (20), the sign of $\Delta E_k(t)$ is not determined if the feedback gain in (2) is chosen to be L_s. Therefore, the feedback gain $L = L_s - \rho BD^T$ designed in Lemma 1 is used to contend with the term $2e_k^T PB\Theta^T F_k^x e_k$.

Part II: Convergence of the Tracking errors. For a given time instant $t \in [0,T]$, consider a finite sum of $\Delta E_k(t)$,

$$\sum_{i=2}^{k} \Delta E_i(t) = \sum_{i=2}^{k} \left[E_i(t) - E_{i-1}(t)\right] = E_k(t) - E_1(t) \tag{24}$$

and apply the inequality (23), it gives

$$
\begin{aligned}
E_k(t) &= E_1(t) - \sum_{i=2}^{k} \Delta E_i(t)\\
&\le E_1(t) - \lambda_{min}^{\rho}(\bar{Q}) \sum_{i=2}^{k} \int_0^t \|\xi_k\|^2 d\tau - \lambda_{min}(P) \sum_{i=2}^{k} \|e_{i-1}\|^2.
\end{aligned} \tag{25}
$$

Because of the positiveness of $E_k(t)$, $e_k(t)$ converges to zero in a pointwise fashion and ξ_k converges to zero in the sense of L^2-norm as k goes to infinity if E_1 is bounded, namely,

$\lim_{k\to\infty} e_k(t) = 0$ and $\lim_{k\to\infty}\int_0^t \|\xi_k\|^2 d\tau = 0$, which thus implies that $\lim_{k\to\infty}\int_0^t \|e_{y,k}\|^2 d\tau = 0$.

Part III: Finiteness of $E_1(t)$. When $k=1$, $\hat{\Theta}_1 = -\Gamma F_{d,k} e_{y,1}^T D$. The composite energy function E_1 is

$$E_1(t) = e_1^T P e_1 + \int_0^t \mathrm{Tr}\left(\tilde{\Theta}_1^T \Gamma^{-1} \tilde{\Theta}_1\right) d\tau. \tag{26}$$

Taking the derivative of $E_1(t)$

$$\begin{aligned}\dot{E}_1(t) = &-e_1^T \tilde{Q} e_1 + 2e_1^T PB\tilde{\Theta}_1^T F_{d,1} - 2e_1^T PB\Theta^T F_1^x e_1 \\ &+ \mathrm{Tr}\left(\tilde{\Theta}_1^T \Gamma^{-1}\tilde{\Theta}_1\right)\end{aligned} \tag{27}$$

with $\tilde{Q} \triangleq Q + 2\rho CDD^T C^T$. Firstly, let's expend the term

$$\begin{aligned}\tilde{\Theta}_1^T\Gamma^{-1}\tilde{\Theta}_1 &= \tilde{\Theta}_1^T\Gamma^{-1}\hat{\Theta}_1 - \tilde{\Theta}_1^T\Gamma^{-1}\Theta \\ &= -\tilde{\Theta}_1^T F_{d,1} e_1 CD + D^T C^T e_1 F_{d,1}^T \Theta + \Theta^T \Gamma^{-1}\Theta.\end{aligned} \tag{28}$$

Hence, we have

$$\begin{aligned}\mathrm{Tr}\left(\tilde{\Theta}_1^T\Gamma_1^{-1}\tilde{\Theta}_1\right) &= -e_1^T CD\tilde{\Theta}_1^T F_{d,1} + e_1^T CD\Theta^T F_{d,1} \\ &\quad + \mathrm{Tr}\left(\Theta^T\Gamma^{-1}\Theta\right) \\ &= -2e_1^T PB\tilde{\Theta}_1^T F_{d,1} - e_1^T CDD^T C^T e_1 F_{d,1}^T \Gamma F_{d,1} \\ &\quad + \mathrm{Tr}\left(\Theta^T\Gamma^{-1}\Theta\right) \\ &\le -2e_1^T PB\tilde{\Theta}_1^T F_{d,1} + \mathrm{Tr}\left(\Theta^T\Gamma^{-1}\Theta\right)\end{aligned} \tag{29}$$

due to the positiveness of $e_1^T CDD^T C^T e_1 F_{d,1}^T \Gamma F_{d,1}$. Substitute (29) into (27), it gives

$$\begin{aligned}\dot{E}_1(t) &\le -e_1^T \tilde{Q} e_1 - 2e_1^T PB\Theta^T F_1^x e_1 + \mathrm{Tr}\left(\Theta^T\Gamma^{-1}\Theta\right) \\ &= -\xi_1^T \bar{Q}(\rho)\xi_1 + \mathrm{Tr}\left(\Theta^T\Gamma^{-1}\Theta\right) \\ &\le -\lambda_{min}^{\rho}(\bar{Q})\|\xi_1\|^2 + \mathrm{Tr}\left(\Theta^T\Gamma^{-1}\Theta\right).\end{aligned} \tag{30}$$

Consequently, once the inequality

$$\lambda_{min}^{\rho}(\bar{Q})\|\xi_1\|^2 \ge \mathrm{Tr}\left(\Theta^T\Gamma^{-1}\Theta\right)$$

holds, $\dot{E}_1(t)$ is negative. Therefore, the boundedness of $E_1(t)$ over $[0,T]$ is obtained.

4. ILLUSTRATIVE EXAMPLES

Consider a circuit model Xu and Xu (2004)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} -\dfrac{R_1L_2}{L_1L_2-M^2} & \dfrac{R_2M}{L_1L_2-M^2} \\ \dfrac{R_1M}{L_1L_2-M^2} & -\dfrac{R_2L_1}{L_1L_2-M^2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\dfrac{L_2-M}{L_1L_2-M^2} \\ \dfrac{L_1-M}{L_1L_2-M^2}\end{bmatrix}(u-\theta^T\xi) \tag{31}$$

where x_1, x_2 represent the loop currents, u is the input voltage, Π represents the input distribution uncertainty, $\theta^T\xi$ denotes the input perturbation with $\theta = -[\sin^3(t), 0.8\sin^2(t)]^T$, $t\in[0,2\pi]$, $\xi = [3x_2, 3\sin(x_1)]^T$, and the system parameters are presented in Table 1 with R_1, R_2 being the resistors and L_1, L_2, M being inductors. The system output is $y = x_1$.

With simple calculations, we can obtain that

$$A = \begin{bmatrix}-3.1746 & 0.9524\\ 0.9524 & -2.2857\end{bmatrix},\ B = \begin{bmatrix}2.2222\\ 1.3333\end{bmatrix},\ C = \begin{bmatrix}1\\ 0\end{bmatrix}. \tag{32}$$

It is easy to verify that Assumptions 1,2 and 3 hold. The desired trajectory y_d is generated by system (3) with matrices presented in (32) and $L = L_s - \rho BD^T$ with $L_s = [0.5397, 0.4381]^T$ and $\rho = 3$.

Table 1. System Parameters

Symbol	Value
R_1	1Ω
R_2	1Ω
L_1	0.36H
L_2	0.5H
M	0.15H

In order to show the effectiveness of the proposed control algorithm, simulations will be conducted by applying the controller (4) with the updating law (5) to the system (31).

Set the learning gain $\Gamma = \mathrm{diag}(0.1, 0.05)$. By applying the proposed controller (4) with the updating law (5), Figs. 1 and 2 present the control input and reference profiles at different iterations. Clearly, Fig. 2 shows that the modified target trajectory varies from iteration to iteration, which converges to the ideal reference gradually as the iteration number increases. The tracking performance of the system outputs at the 1st and 60th iterations are shown in Fig. 3, from which we can see that the system output at the 1st iteration deviates from the reference, while the difference between the output profile and reference becomes invisible at the 60th iteration. This can be further verified by the convergence of the maximum output tracking error, $|e_{y,k}|_s \triangleq \sup_{t\in[0,2*\pi]} |e_{y,k}(t)|$, presented in Fig. 4. The maximum tracking error $|e_{y,k}|_s$ is reduced by 96.22% within 40 iterations. Therefore, the efficacy of the proposed ILC approach has been demonstrated.

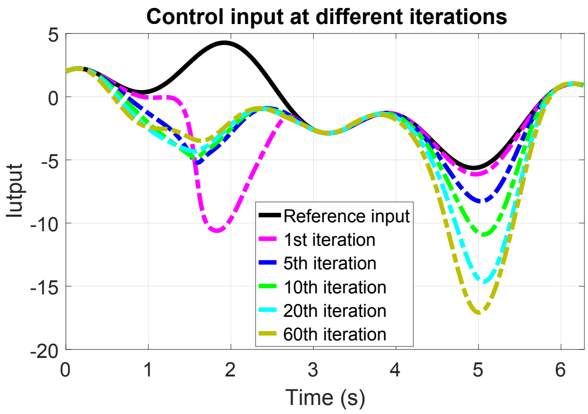

Fig. 1. Variation of the control input profiles in the iteration domain for Case 1.

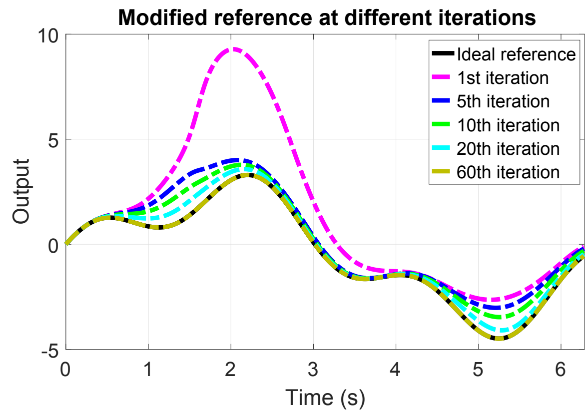

Fig. 2. Variation of the modified reference profiles in the iteration domain for Case 1.

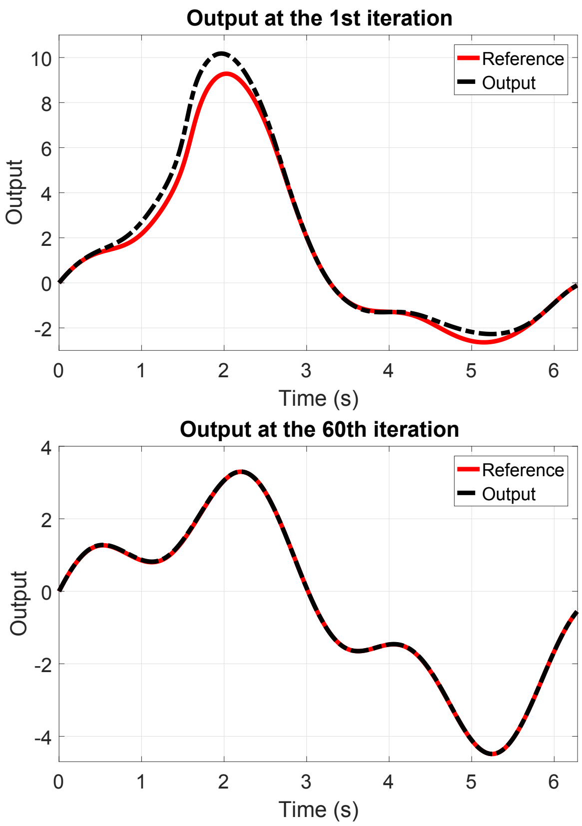

Fig. 3. Tracking performance at the 1st and 60th iterations for Case 1.

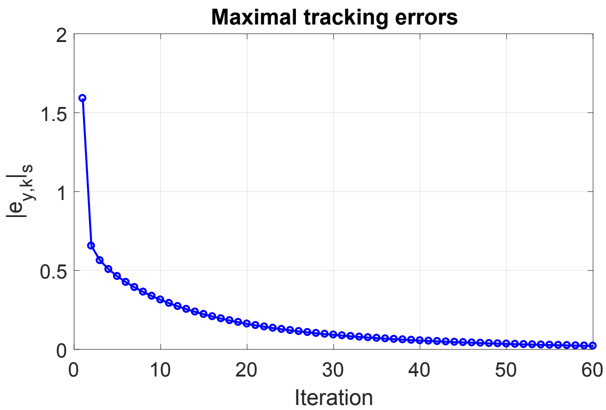

Fig. 4. The maximum tracking error profile along the iteration axis for Case 1.

5. CONCLUSION

In this paper, a novel iterative learning control (ILC) approach is developed for a class of uncertain nonlinear systems with unmeasurable system states. By utilizing the closed-loop reference model as an observer, the proposed adaptive ILC approach can be adopted to deal with the output tracking problem of nonlinear systems without requiring the measurability of system states. In the systems,two kinds of uncertainties, namely, parametric input disturbances and uncertainties caused by the unavailable states, are taken into account. To facilitate the controller design and convergence analysis, the composite energy function (CEF) is employed. This approach extends the CEF-based ILC approach to output tracking control of nonlinear systems with unmeasurable states. The numerical example illustrated has shown the effectiveness of the proposed ILC scheme.

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