Stator flux and load torque observers for PMSM

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Abstract: In this paper, a stator flux observer and constant load torque estimation method are developed for non-salient permanent magnet synchronous motors. The methods described here are based on implementation of LTI filters and linear regression. The motor resistance and inductance are assumed to be known.

Keywords: Permanent magnet synchronous motor, sensorless control, stator flux observer, load torque observer.

1. INTRODUCTION

In the past decades, permanent magnet synchronous motors (PMSM) have become widely used in PMSM control systems, see Nam (2018). In practice the real electrical parameters are not equal to values specified in technical documentation as the parameters usually vary during the system performance. The reason of parameters' variation can be temperature, mechanical wear or external influence, so the quality of sensorless control may become inappropriate. In order to improve it and estimate real values of motor parameters adaptive identification algorithms were developed, see Verrelli et al. (2013), Ichikawa et al. (2006), Pippo et al. (2009). It allows to get values of motor parameters similar to real ones. Verrelli et al. (2013), Kisck et al. (2010), Hinkkanen et al. (2011), Bazylev et al. (2016), Bobtsov et al. (2018) present the algorithms evaluating the stator winding resistance and stator inductance. However in these algorithms position of rotor and rotor speed estimation are not considered.

Position and speed estimation algorithms for high-performance AC motors were introduced in Ortega et al. (2010), Tomei and Verrelli (2011), Bobtsov et al. (2015), Bobtsov et al. (2017a) . The dynamics of such motors is nonlinear and includes uncertain parameters. Angular position of the rotor can be found from trigonometric functions using stator flux. This is a reason of paying special attention to stator flux observers. In particular, lots of papers have been devoted to determining initial conditions of the flux
for this purpose was proposed in Bobtsov et al. (2015). Its implementation is difficult because of the need to
ensure persistent excitation (PE) condition. However it was shown that flux error exponentially approaches to zero. In
Bobtsov et al. (2017a) the dynamic regression extension and mixing (DREM) estimation replaced the gradient
adaptation algorithm. These observers require the knowledge
of the edge of the resistance, magnet flux and inductance. But
in real PMSM driven systems the load torque is usually
unknown and depends on the load shape and dimensions.
Magnetic flux varies with the temperature of the rotor.
It is necessary to mention that flux and load torque are
hard to obtain online. So there are two ways to solve
this problem. The first one is development of observers
which do not rely on the knowledge of those parameters.
The case where the magnet flux is unknown but resistance
and inductance are known and described in Bernard and
of the gradient observer obtained in Lee et al. (2009) was
developed. Also a number of papers is devoted to flux and
load torque estimation. In Lian et al. (2018) a load torque
observer based on sliding mode and two moment of inertia
identification methods is proposed. In Ouvang and Dou
(2018) authors introduce online identification of stator
resistance and permanent magnetic flux linkage on the
base of model reference adaptive system. In this paper,
we consider a new algorithm for estimation of stator flux
and constant load torque of the PMSM with known constant
parameters (winding resistance and inductance).

The remaining of the paper is organized as follows. Section
2 presents the model of the PMSM and the assumptions
needed for the observer design. Section 3 contains main
result. Some representative simulations results can be
found in Section 4.

2. PMSM MODEL AND PROBLEM STATEMENT

The classical fixed-frame ($\alpha$-$\beta$) model of the unsaturated
non-salient PMSM is given by Chiasson (2005), Shah et al.
(2011).

$$\dot{\lambda} = \nu - R_i \lambda,$$
$$\dot{J}\omega = \tau_c - \tau_L,$$  \hspace{1cm} (1)

where $\lambda \in \mathbb{R}^2$ is the stator flux, $i \in \mathbb{R}^2$ are the currents,
$\nu \in \mathbb{R}^2$ the voltages, $R > 0$ is the stator winding resistance,
$J > 0$ is the rotor inertia, $\theta \in S := [0, 2\pi]$ is the rotor
phase, $\omega \in \mathbb{R}$ is the mechanical angular velocity, $\tau_L \in \mathbb{R}$
is load torque, $\tau_c$ is the torque of electrical origin, given by

$$\tau_c = n_p p T J_o \lambda,$$

where $n_p$ is the number of pole pairs and $J_o$ is the skew-

symmetric matrix

$$J_o := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

For surface-mounted PMSMs, the total flux verifies

$$\lambda = L i + \lambda_m C(\theta),$$  \hspace{1cm} (2)

where $L > 0$ is the stator inductance, $\lambda_m$ is the constant
flux generated by permanent magnets. To simplify the
notation, we define

$$C(\theta) := \begin{bmatrix} \cos(n_p \theta) \\ \sin(n_p \theta) \end{bmatrix}. \hspace{1cm} (3)$$

The main contribution of the paper is design and inves-
tigation of the stator flux observer and load torque
estimation for PMSM, assuming that the motor resistance
and inductance are known, load torque is unknown and
constant.

3. MAIN RESULT

3.1 Stator flux observer

Let us derive derivatives of stator flux $\dot{\lambda}_i, i = 1, 2$ as $f_i$, and rewrite (1) and (2) in the following form

$$\dot{\lambda}_1 = -R_i \dot{1} + u_1 =: f_1,$$ \hspace{1cm} (4)
$$\dot{\lambda}_2 = -R_i \dot{2} + u_2 =: f_2,$$ \hspace{1cm} (5)
$$\dot{\lambda}_1 = L_j + \lambda_m \cos(n_p \theta),$$ \hspace{1cm} (6)
$$\dot{\lambda}_2 = L_j + \lambda_m \sin(n_p \theta),$$ \hspace{1cm} (7)

where $R$ and $L$ are known, $1, 1_2, u_1$, and $u_2$ are measured.
The goal is to estimate stator flux $\lambda_1, \lambda_2$. Equations (6)
and (7) can be rewritten as

$$\dot{\lambda}_1 = L \frac{d}{dt} \dot{1} - n_p \dot{\theta} \lambda_m \sin(n_p \theta) =$$
$$= L \frac{d}{dt} \dot{1} - n_p \dot{\theta} (A - L \dot{2})$$ \hspace{1cm} (8)

and

$$\dot{\lambda}_2 = L \frac{d}{dt} \dot{2} + n_p \dot{\theta} \lambda_m \cos(n_p \theta) =$$
$$= L \frac{d}{dt} \dot{2} + n_p \dot{\theta} (A - L \dot{1})$$ \hspace{1cm} (9)

After multiplication of (8) by $(\lambda_1 - L \dot{1})$ and (9) by
$(\lambda_2 - L \dot{2})$ we can consider the sum of these products:

$$\lambda_1 (\lambda_1 - L \dot{1}) + \lambda_2 (\lambda_2 - L \dot{2}) =$$
$$= L \frac{d}{dt} \dot{1} (\lambda_1 - L \dot{1}) + L \frac{d}{dt} \dot{2} (\lambda_2 - L \dot{2}) .$$

Substituting (4) and (5) into previous expression we obtain

$$f_1 (\lambda_1 - L \dot{1}) + f_2 (\lambda_2 - L \dot{2}) =$$
$$= L \frac{d}{dt} \dot{1} (\lambda_1 - L \dot{1}) + L \frac{d}{dt} \dot{2} (\lambda_2 - L \dot{2}) .$$ \hspace{1cm} (10)

Let us now apply the parameter estimation based ob-

servers (PEBO) approach discussed in Bazylev et al.
(2017) and rewrite flux as

$$\lambda_i (t) = z_i (t) + \eta_i, i = 1, 2,$$ \hspace{1cm} (11)

where $\eta_1, \eta_2$ are unknown constants and $z_1, z_2$ are the
solutions of the following equations

$$\dot{z}_i = R \dot{1}_i + u_i$$ \hspace{1cm} (12)

with some initial conditions $z_0 (0)$. Obviously, the problem
of estimation of the signals $\lambda$ becomes equivalent to the
problem of identification of the parameters $\eta$.

Thus, we obtain

$$f_1 z_1 + f_1 \eta_1 - L \dot{1}_1 f_1 + f_2 z_2 + f_2 \eta_2 - L \dot{2} f_2 =$$
$$= L \frac{d}{dt} \dot{1} z_1 + L \eta_1 \frac{d}{dt} \dot{1} - L^2 \frac{d}{dt} \dot{1} +$$
$$+ L \eta_2 \frac{d}{dt} \dot{2} - L^2 \dot{2} .$$ \hspace{1cm} (13)
Let us apply LTI filter $\frac{k}{p+k}$, where $p = \frac{d}{dt}$ and $k$ is gain.

Then recalling the Swapping Lemma, see Morse (1980), Sastry and Bodson (1989) we have

$$\frac{k}{p+k} \left[ \frac{d}{dt} \right] \tilde{i}_1 = \frac{pk}{p+k} \left[ \frac{d}{dt} \right] i_1, \quad (14)$$

$$\frac{k}{p+k} \left[ \frac{d}{dt} \right] \tilde{i}_1 = z_i \frac{kp}{p+k} \left[ i_1 \right] = \frac{1}{p+k} \left( \left( \frac{pk}{p+k} \left[ i_1 \right] \right) \left( R_i + u_i \right) \right), \quad (15)$$

$$\frac{k}{p+k} \left[ \frac{d}{dt} \right] i_1 = \frac{1}{2} \frac{pk}{p+k} \left[ i_2 \right]. \quad (16)$$

Let us denote

$$z_i \frac{kp}{p+k} \left[ i_1 \right] = \frac{1}{p+k} \left( \left( \frac{pk}{p+k} \left[ i_1 \right] \right) \left( R_i + u_i \right) \right) =: \psi_i. \quad (17)$$

After applying the LTI filter $\frac{k}{p+k}$ to the both sides of (13) and using (17) we obtain

$$\frac{k}{p+k} \left[ f_z z_1 - L_i f_1 + f_2 z_2 - L_i f_2 \right] - L \psi_1 + \frac{L^2}{2} \frac{pk}{p+k} \left[ i_2 \right] - L \psi_2 + \frac{L^2}{2} \frac{pk}{p+k} \left[ i_2 \right] = \left( L \frac{pk}{p+k} \left[ i_1 \right] - f_1 \right) \eta_1 + \left( L \frac{pk}{p+k} \left[ i_2 \right] - f_2 \right) \eta_2. \quad (18)$$

Previous expression can be considered as a linear regression of the form

$$y = \phi^T \eta, \quad (19)$$

where

$$y := \frac{k}{p+k} \left[ f_z z_1 - L_i f_1 + f_2 z_2 - L_i f_2 \right] - L \psi_1 + \frac{L^2}{2} \frac{pk}{p+k} \left[ i_2 \right] - L \psi_2 + \frac{L^2}{2} \frac{pk}{p+k} \left[ i_2 \right], \quad (20)$$

$$\phi := \left[ L \frac{pk}{p+k} \left[ i_1 \right] - f_1 \right] \left[ L \frac{pk}{p+k} \left[ i_2 \right] - f_2 \right]^T. \quad (21)$$

Now, the parameters $\eta$ can be estimated using any proper technique, for instance

$$\hat{\eta} = \gamma \phi \left( y - \phi^T \hat{\eta} \right), \quad (22)$$

where $\gamma > 0$ is adaptation coefficient.

From (22) we have $\hat{\eta}$ and then from (11) we can find stator flux estimate $\lambda$.

For convergence of errors $\hat{\eta} = \eta - \hat{\eta}$ to zero PE condition on the regressor must hold (see for instance Aranovskiy et al. (2016)).

3.2 Estimation of load torque ($\tau_L = \text{const}$)

Consider model of the unsaturated nonsalient PMSM

$$\begin{cases} 
\lambda_1 - L \dot{I}_1 = \lambda_m \cos (n \theta), \\
\lambda_2 - L \ddot{I}_2 = \lambda_m \sin (n \theta), 
\end{cases} \quad (23)$$

$$\ddot{\tau} = \frac{1}{J} (-\tau_e + n_p i_p^T i_{a,b}) = -\frac{1}{J} \tau_L + \frac{1}{J} \tau_e. \quad (24)$$

The goal is to estimate load torque $\tau_L$, under assumption that $\tau_e = n_p i_p^T i_{a,b} \lambda$ and $\lambda$ are known.

Let us denote

$$\begin{cases} 
r_1 = \lambda_1 - L \dot{I}_1 = \lambda_m \cos (n \theta), \\
r_2 = \lambda_2 - L \ddot{I}_2 = \lambda_m \sin (n \theta). 
\end{cases} \quad (25)$$

Then after applying the LTI filter $\frac{1}{p+1}$ to (25) we have the following chain of expressions

$$\dot{r}_1 = \frac{1}{p+1} r_1 = \frac{1}{p+1} \lambda_m \cos (n \theta) = \lambda_m \cos (n \theta) + \frac{n_p}{p+1} (\dot{\theta} \lambda_m \sin (n \theta)) = r_1 + \frac{n_p}{p+1} (\ddot{\tau}_2) = r_1 + \frac{\dot{\theta}}{p+1} \tau_2 - \frac{n_p}{p+1} (\ddot{\theta} \frac{1}{p+1} \tau_2) = r_1 + \frac{n_p}{p+1} \dot{\tau}_2 - \frac{n_p}{p+1} (-\frac{1}{J} \tau_L \dot{\tau}_2 + \frac{1}{J} \tau_e \dot{\tau}_2). \quad (26)$$

$$\dot{r}_2 = \frac{1}{p+1} r_2 = \frac{1}{p+1} \lambda_m \sin (n \theta) = \lambda_m \sin (n \theta) - \frac{n_p}{p+1} (\dot{\theta} \lambda_m \cos (n \theta)) = r_2 - \frac{n_p}{p+1} (\dot{\tau}_1) = r_2 - \frac{\dot{\theta}}{p+1} r_1 - \frac{n_p}{p+1} (\ddot{\theta} \frac{1}{p+1} r_1) = r_2 - \frac{n_p}{p+1} \dot{\tau}_1 + \frac{n_p}{p+1} (-\frac{1}{J} \tau_L \dot{\tau}_1 + \frac{1}{J} \tau_e \dot{\tau}_1). \quad (27)$$

Let us rewrite (26) and (27). Then we obtain

$$\begin{cases} 
\dot{r}_1 = r_1 + n_p \dot{\tau}_2 - n_p \frac{1}{p+1} \tau_L \frac{1}{p+1} \ddot{\tau}_2 - n_p \frac{1}{p+1} (\tau_e \ddot{\tau}_2), \\
\dot{r}_2 = r_2 - n_p \dot{\tau}_1 + n_p \frac{1}{p+1} \tau_L \frac{1}{p+1} \ddot{\tau}_1 + n_p \frac{1}{p+1} (\tau_e \ddot{\tau}_1), 
\end{cases} \quad (28)$$

$$\begin{cases} 
\xi_1 = n_p \dot{\tau}_2 + \tau_L \omega_1 = n_p \omega \dot{\tau}_2 + \tau_L \omega_1, \\
\xi_2 = -n_p \dot{\tau}_1 - \tau_L \omega_2 = -n_p \omega \dot{\tau}_1 - \tau_L \omega_2, 
\end{cases} \quad (29)$$

where $\omega = \frac{n_p}{J} \frac{1}{p+1} \ddot{\tau}_2$ and $\omega = \frac{n_p}{J} \frac{1}{p+1} \ddot{\tau}_1$.

From previous equations we can find

$$\xi_1 \dot{r}_1 + \xi_2 \dot{r}_2 = \tau_L (\omega_1 \dot{r}_1 - \omega_2 \dot{r}_2). \quad (30)$$

Now we have linear regression model

$$y_r = \phi^T \tau_L, \quad (31)$$

where $y_r = \xi_1 \dot{r}_1 + \xi_2 \dot{r}_2$ and $\phi^T = \omega_1 \dot{r}_1 - \omega_2 \dot{r}_2$.

Now we can estimate $\tau_L$ using any proper technique, for instance

$$\dot{\tau}_L = \gamma (\omega_1 \dot{r}_1 - \omega_2 \dot{r}_2) (\xi_1 \dot{r}_1 + \xi_2 \dot{r}_2 - \tau_L (\omega_1 \dot{r}_1 - \omega_2 \dot{r}_2)), \quad (32)$$

where $\gamma > 0$ is adaptation coefficient.

For convergence of load torque estimation error $\dot{\tau}_L = \tau_L - \dot{\tau}_L$ to zero PE condition on the regressor must hold (see for instance Aranovskiy et al. (2016))

4. SIMULATIONS

For simulation we use parameters of the motor BMP0701F as in Bobtsov et al. (2017a), Bobtsov et al. (2017b) listed in Table 1.

Simulation results are shown in Fig. 1-Fig. 11.
Table 1. Parameters of the PMSM BMP07F1

<table>
<thead>
<tr>
<th>Parameter (units)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductance $L$ (mH)</td>
<td>40.03</td>
</tr>
<tr>
<td>Resistance (Ω)</td>
<td>8.875</td>
</tr>
<tr>
<td>Drive inertia $J$ ($kg\cdot m^2$)</td>
<td>$60 \times 10^{-6}$</td>
</tr>
<tr>
<td>Pairs of poles $n_p$ (−)</td>
<td>5</td>
</tr>
<tr>
<td>Magnetic flux (Wb)</td>
<td>0.2086</td>
</tr>
</tbody>
</table>

Fig. 1. Stator flux $\lambda_1$ and stator flux estimate $\hat{\lambda}_1$

Fig. 2. Stator flux $\lambda_2$ and stator flux estimate $\hat{\lambda}_2$

Fig. 3. Convergence of estimation error $e_{\lambda_1} = \lambda_1 - \hat{\lambda}_1$ for $\gamma = 25$

Fig. 4. Convergence of estimation error $e_{\lambda_2} = \lambda_2 - \hat{\lambda}_2$ for $\gamma = 25$

Fig. 5. Convergence of estimation error $e_{\lambda_1} = \lambda_1 - \hat{\lambda}_1$ for $\gamma = 100$

Fig. 6. Convergence of estimation error $e_{\lambda_2} = \lambda_2 - \hat{\lambda}_2$ for $\gamma = 100$

5. CONCLUSION

In this paper we propose a stator flux observer. The problem of flux estimation is transformed into the identification problem via implementation of LTI filters and linear regression. A similar method is used for load torque estimation.
Fig. 7. Convergence of estimation error $e_{\lambda_2} = \lambda_2 - \hat{\lambda}_2$ for $\gamma = 8$ and $q = 10$

Fig. 8. Convergence of estimation error $e_{\lambda_2} = \lambda_2 - \hat{\lambda}_2$ for $\gamma = 80$ and $q = 100$

Fig. 9. Load torque estimate $\hat{\tau}_L$ for $\tau_L = 0, 1$

Fig. 10. Load torque estimate $\hat{\tau}_L$ for $\tau_L = 0, 5$

REFERENCES


