Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Nonlinear Model Predictive Control Design for BSM-MBR: Benchmark of Membrane Bioreactor ⋆

Xingang Guo * **Peiying Hong** ** **Taous-Meriem Laleg-Kirati** *

* *Computer, Electrical and Mathematical Sciences and Engineering*
** *Biological and Environmental Science and Engineering Division*
King Abdullah University of Science and Technology (KAUST)
Thuwal 23955-6900, Kingdom of Saudi Arabia
e-mail: {xingang.guo, peiying.hong, taousmeriem.laleg}@kaust.edu.sa

Abstract: The optimal control and operation of a Membrane Bioreactor (MBR) process by Nonlinear Model Predictive Control (NMPC) is investigated in this work. First, the Benchmark Simulation Model for MBR (BSM-MBR) provided by Maere et al. (2011) is introduced with a detailed mathematical model. Then, an NMPC is designed by incorporating the nonlinear process model of BSM-MBR to control the dissolved oxygen concentration at a certain level while meeting input and other process constraints. The performance of the NMPC is evaluated under both constant influent scenario and dynamic dry weather influent scenario. The simulation results demonstrate that NMPC works better in the constant influent case compared to the dynamic influent scenario.

Keywords: Nonlinear Model Predictive Control (NMPC), Membrane Bioreactor (MBR), Benchmark Simulation Model for MBR (BSM-MBR).

1. INTRODUCTION

The Membrane Bioreactor (MBR) provides a good alternative technology for wastewater treatment. It combines both the activated sludge process and membrane separation process, which provides better performance compared to the traditional technologies in terms of wastewater quality, plant footprint, and wastewater treatment efficiency. It has been widely used in municipal and industrial wastewater treatment (Judd, 2010).

MBR processes are large complex systems that can be modeled with nonlinear equations. Besides, the processes have to be operated continuously while ensuring that treated effluent meet the regulations imposed for discharge or reuse. It is, therefore, essential to have advanced control strategies for the MBR to ensure continuous optimal performance and high quality of the treated water. Many studies investigated different control strategies for wastewater treatment plants. However, the variable influent and the intricacy of the biological and biochemical phenomena prevent a good performance evaluation of the proposed controllers in both simulation and practice. Nevertheless, a Benchmark Simulation Model No.1 (BSM1) has been proposed in Alex et al. (1999) for conventional wastewater treatment plants, as a tool for the evaluation of controllers. The benchmark is built on an accurate process model. Furthermore, an extension of BSM1 to the Benchmark Simulation Model for Membrane Bioreactor (BSM-MBR) has been provided in Maere et al. (2011), which is a virtual test platform for comparing different control strategies of MBR. A comprehensive overview of the process control of conventional wastewater treatment systems has been proposed in Amand et al. (2013); Olsson (2012); Ferrero et al. (2012); Olsson and Newell (1999).

Dissolved oxygen (DO) control is an important control objective in the wastewater treatment plants (Olsson, 2012). Too little DO in the MBR would favor filamentous bacteria Wilén and Balmér (1999) while supplying too much DO would increase the energy cost. A Proportional Integral (PI) controller has been proposed to control DO concentration for BSM1 Alex et al. (1999) and BSM-MBR Maere et al. (2011). An autotuning controller combined with a Kalman filter that estimates the oxygen transfer rate has been proposed to control the dissolved oxygen level for conventional wastewater treatment plant Carlsson et al. (1994). Moreover, a nonlinear DO controller was developed in Lindberg and Carlsson (1996a) and Lindberg and Carlsson (1996b), which improved the PI performance. A multi-criteria control strategy with Takagi–Sugeno fuzzy supervisor has been used to decrease the global cost of activated sludge processes while keeping good performance in Cadet et al. (2004). Recently, a neural network-based adaptive Proportional-Integral-Derivative (PID) algorithm has been designed for better DO control in Du et al. (2018). Optimal control approaches have also been investigated. For example, in Holenda et al. (2008) where a Model Predictive Control (MPC) strategy has been implemented for DO control in activated sludge systems using a linear black-box state-space model identified from the system input-output data. In this paper, a Nonlinear

⋆ This work has been supported by the King Abdullah University of Science and Technology (KAUST) Base Research Fund (BAS/1/1627-0101) to Taous Meriem Laleg and the center of excellent for NEOM research at KAUST.

MPC (NMPC) of DO control is proposed for BSM-MBR based on the system nonlinear mathematical model.

MPC is an algorithm that uses an explicit process model to predict the future evolution of a plant. At each sampling instant, the MPC algorithm solves a constrained optimization problem to optimize the future behavior of the process states by using the plant's initial states and current states. The optimization provides a series of optimal manipulated variables, but only the first control input in this sequence is applied. There are several advantages of MPC compared to other control strategies. First, MPC can incorporate different equality and inequality constraints. Second, compared to conventional Linear Quadratic Regulator (LQR) (Kwakernaak and Sivan, 1972), MPC can work for nonlinear systems without linearization. Third, MPC can be implemented for Multiple Inputs and Multiple Outputs (MIMO) systems easily, which is not the case for some other controllers such as the PID controller. Finally, since the optimization problem is solved online, the MPC can capture system's changes from the measurements at each sampling time, and thus the control input can be adjusted accordingly, which enhances the robustness of the MPC algorithm.

Motivated by the above considerations, in this work, first, we introduce the detailed mathematical model of BSM-MBR. Then, we develop an NMPC control strategy based on the proposed process model to maintain the DO concentration around a certain level, while meeting the input constraints and other process constraints. In particular, the upper limit concentrations of total nitrogen (TN) and ammonia (NH_3) in the effluent water are considered as constraints in the NMPC to enhance the effluent water quality.

The paper organizes as follows. In section 2, a brief review of the BSM-MBR is given, then the bio-kinetic model, detailed process equations of BSM-MBR are described. Section 3 first defines the control problem formally, then develops the NMPC controller for BSM-MBR to maintain the DO concentration around a certain level while meeting the input constraints and other process constraints. In section 4, simulation results are presented with the initialization of the system and influent data description. Finally, section 5 draws some conclusions and some possible future directions.

2. BSM-MBR REVIEW

A dynamic benchmark simulation model for MBR has been proposed in Maere et al. (2011). The model has been used to develop a platform to test different control algorithms and evaluate the effluent quality and operational cost.

Figure 1 gives the layout and flow scheme of the BSM-MBR with all possible control inputs. There are five activated sludge tanks with anoxic tank 1 and tank 2 and aerobic tank 3 to tank 5 as shown in Fig. 1. Wastewater influent enters the system from the first tank and is withdrawn through the membrane filter in tank 5, which is a membrane tank equipped with a membrane area of 71,500 m^2. The air is injected into tank 3 and 4 through fine bubble aeration to supply sufficient oxygen to the process and is injected into tank 5 through coarse bubble aeration for the mitigation of membrane fouling. There are two recycle loops in the process: the sludge is recycled from the second aerobic tank (tank 4) to the first anoxic tank (tank 1) by internal circulation to enhance the denitrification of the nitrate, and then recycled from the tank 5 to the tank 3.

In this section, we will first introduce the bio-kinetic model Activated Sludge Model No.1 (ASM1) (Henze et al., 1987), which is adopted in the BSM-MBR process. The detailed process equations of BSM-MBR will be given in the second subsection.

2.1 Bio-kinetic model of BSM-MBR

Activated Sludge Model No.1 (ASM1) model is considered as the bio-kinetic model of BSM-MBR, which describes the dynamic behavior of heterotrophic and autotrophic biomass, organic substrate, and nitrogen under anoxic and aerobic environments. ASM1 considers 13 different concentrations as system states with the following order:

$$\mathbf{c} = [S_I, S_S, X_I, X_S, X_{BH}, X_{BA}, X_P, S_O, S_{NO}, S_{NH}, S_{ND}, X_{ND}, S_{ALK}]$$

A detailed model description can be found in Henze et al. (1987). The same bio-kinetic parameters are adopted in the BSM-MBR. It is shown that the default parameter values are sufficient (Verrecht et al., 2010).

2.2 System Equations of BSM-MBR

To derive the dynamic equations for BSM-MBR, we first consider a single well-mixed tank reactor as shown in Fig. 2. The mass balance equation can be written as follows:

$$\frac{d}{dt}(V\mathbf{c}) = \sum_{i=1}^{N_{in}} Q_i^{in}\mathbf{c}_i^{in} + \sum_{j=1}^{N_{out}} Q_j^{out}\mathbf{c}_j^{out} + \mathbf{q} + V\mathbf{S}\rho(\mathbf{c}), \quad (1)$$

where V is the reactor's volume, which is considered as constant in this case (i.e. $\sum_{i=1}^{N_{in}} Q_i^{in} = \sum_{j=1}^{N_{out}} Q_j^{out}$), N_{in} and N_{out} are the numbers of flow rate of sludge stream entering and leaving the reactor, respectively, Q_i^{in} $i \in (1, 2, \cdots, N_{in})$, and Q_j^{out} $j \in (1, 2, \cdots, N_{out})$ are the flow rates of sludge entering and leaving the reactor, respectively, $\mathbf{c}_i^{in}$ is the incoming sludge concentrations, $\mathbf{q}$ is the material inputs that are not included in the sludge flow into the reactor (e.g., air or carbon); $\rho(\mathbf{c})$ is the vector of reaction rates, $\mathbf{S}$ is the stoichiometric matrix, and we have $\dim(\mathbf{S}\rho(\mathbf{c}))=\dim(\mathbf{c})$. The stoichiometric matrix $\mathbf{S}$ and the biological process rates $\rho(\mathbf{c})$ are different for different bio-kinetic models.

Based on the analysis above, Table 1 shows the nonlinear dynamic equations for BSM-MBR derived from the mass balance equations among five reactors. Nonlinearity comes from the reaction rates $\rho(\mathbf{c})$ of ASM1 and the material input vector $\mathbf{q}$. V_1 to V_5 are the volume of reactor 1 to reactor 5, $\mathbf{c}_1$ to $\mathbf{c}_5$ are the system states for reactor 1 to reactor 5, respectively. Q_{feed} and $\mathbf{c}_{feed}$ are the flow rate and concentrations of influent, respectively. Q_{int} and Q_r are the flow rate of two recycle streams, Q_w is the flow rate of waste stream. These model parameters can be found in the first block of Table 2. The vector of input flow rates

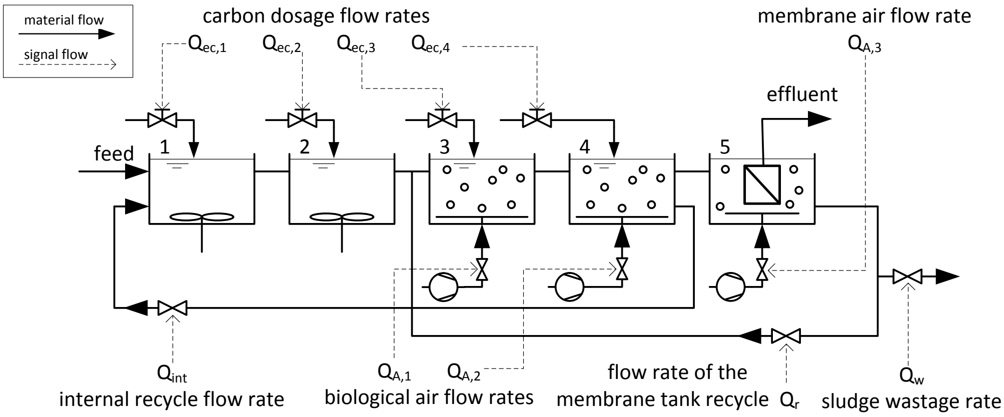

Fig. 1. BSM-MBR (Maere et al., 2011) layout and flow scheme, with all possible control inputs (Elixmann and Nopens, 2016).

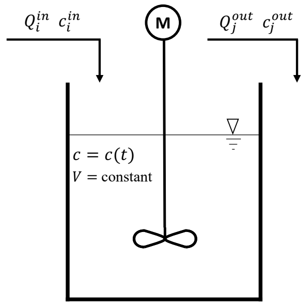

Fig. 2. Graphical representation of a well-mixed tank reactor with constant active volume Janus (2013).

$\mathbf{q}_i$ contains the carbon dosage rates $q_{EC,i}$ and the oxygen transfer rates $q_{O_2,i}$:

$$(\mathbf{q}_i)_j = \begin{cases} c_{EC}q_{EC,i}, & j = 2 \\ 0, & \text{otherwise} \end{cases} \quad i \in \{1, 2\}, \tag{2}$$

$$(\mathbf{q}_i)_j = \begin{cases} c_{EC}q_{EC,i}, & j = 2 \\ q_{O_2,i}, & j = 8 \\ 0, & \text{otherwise} \end{cases} \quad i \in \{3, 4, 5\}. \tag{3}$$

The oxygen transfer rates $q_{O_2,i}$ are associated to the air flow rates $Q_{A,i}$ through the aeration model (Maere et al., 2011):

$$q_{O_2,i} = Q_{A,i} \cdot \rho_A \cdot O_{Am,i} \cdot \text{AOTE}_i, \tag{9}$$

$$\text{AOTE}_i = y_i \cdot \text{SOTE}_i \frac{\beta S_{O,i}^{sat,av,cw}(T) - (\mathbf{c}_i)_8}{S_O^{sat,cw}(20\,°\text{C})} \phi^{T-20\,°\text{C}} \alpha_i F_i, \tag{10}$$

$$\alpha_i = e^{-\omega_i \cdot X_{TSS,i}}, \tag{11}$$

$$X_{TSS,i} = i_{BM,TSS} \sum_{k=3}^{7} (\mathbf{c}_i)_k, \tag{12}$$

$$S_{O,i}^{sat,av,cw}(T) = \frac{1}{2} S_O^{sat,cw}(T)\left(\frac{P_d}{P_{atm}} + \frac{O_{out,i}}{O_{A,v}}\right), \tag{13}$$

$$P_d = P_{atm} + \rho_{sludge} \cdot g \cdot h, \tag{14}$$

$$O_{out,i} = \frac{O_{A,v}(1 - \text{AOTE}_i)}{1 - O_{A,v} \cdot \text{AOTE}_i}, \tag{15}$$

with $i \in \{3, 4, 5\}$. Second block of Table 2 gives the values of all parameters used in Eq. 9 to Eq. 15.

The effluent concentrations $\mathbf{c}_e$ can be calculated as follows:

$$(\mathbf{c}_e)_j = \gamma(\mathbf{c}_5)_j, \quad j \in \{3, 4, 5, 6, 7, 12\},$$
$$(\mathbf{c}_e)_j = (\mathbf{c}_5)_j, \quad j \in \{1, 2, 8, 9, 10, 11, 13\}. \tag{16}$$

where γ is a modeling parameters related to the membrane property. In this work, membrane is considered as an ideal separator by taking $\gamma = 0$, which means only soluble component can be went through the membrane.

In the membrane tank, the following mass balance equations must hold:

$$(Q_{feed} + Q_r)\mathbf{c}_5 = (Q_{feed} - Q_w)\mathbf{c}_e + (Q_r + Q_w)\mathbf{c}_r. \tag{17}$$

Therefore, combining Eq. 16 and Eq. 17, the recycle concentrations $\mathbf{c}_r$ can be expressed as follow:

$$(\mathbf{c}_r)_j = \frac{(1-\gamma)Q_{feed} + Q_r - \gamma Q_w}{Q_r + Q_w}(\mathbf{c}_5)_j, \tag{18}$$
$$j \in \{3, 4, 5, 6, 7, 12\},$$
$$(\mathbf{c}_r)_j = (\mathbf{c}_5)_j, \quad j \in \{1, 2, 8, 9, 10, 11, 13\}. \tag{19}$$

3. CONTROLLER DESIGN OF BSM-MBR

In this section, we will first give the control problem formulation in the first subsection, in which the general system model, control objective will be discussed. Then the NMPC design for BSM-MBR will be introduced in the section subsection.

3.1 Control problem formulation

As mentioned above, the objective of the NMPC design is to maintain the DO concentration around a certain level. In Maere et al. (2011), the DO control problem is defined to keep the DO concentration in the second aerobic reactor at 1.5 g.m^{-3} by using a PI controller. In this work, the NMPC will be designed to achieve such control objective by using the airflow rate of $Q_{A,2}$ as the manipulated variable. Hence, in this scenario, we can treat the DO concentration at the second aerobic reactor as the output $y(t)$ of the whole process, while the airflow rate $Q_{A,2}$ as the input of the process $u(t)$.

The compact states space representation of the considered system can be written as follows:

Table 1. Nonlinear dynamic equations for BSM-MBR

$$V_1\dot{\mathbf{c}}_1 = Q_{feed}\mathbf{c}_{feed} + Q_{int}\mathbf{c}_4 - (Q_{feed} + Q_{int})\mathbf{c}_1 + V_1 S_{ASM1}\rho_{ASM1}(\mathbf{c}_1) + \mathbf{q}_1 \quad (4)$$

$$V_2\dot{\mathbf{c}}_2 = (Q_{feed} + Q_{int})(\mathbf{c}_1 - \mathbf{c}_2) + V_2 S_{ASM1}\rho_{ASM1}(\mathbf{c}_2) + \mathbf{q}_2 \quad (5)$$

$$V_3\dot{\mathbf{c}}_3 = (Q_{feed} + Q_{int})\mathbf{c}_2 + Q_r\mathbf{c}_r - (Q_{feed} + Q_{int} + Q_r)\mathbf{c}_3 + V_3 S_{ASM1}\rho_{ASM1}(\mathbf{c}_3) + \mathbf{q}_3 \quad (6)$$

$$V_4\dot{\mathbf{c}}_4 = (Q_{feed} + Q_{int} + Q_r)(\mathbf{c}_3 - \mathbf{c}_4) + V_4 S_{ASM1}\rho_{ASM1}(\mathbf{c}_4) + \mathbf{q}_4 \quad (7)$$

$$V_5\dot{\mathbf{c}}_5 = (Q_{feed} + Q_r)\mathbf{c}_4 - (Q_{feed} - Q_w)\mathbf{c}_e - (Q_r + Q_w)\mathbf{c}_r + V_5 S_{ASM1}\rho_{ASM1}(\mathbf{c}_5) + \mathbf{q}_5 \quad (8)$$

Table 2. BSM-MBR model and the membrane fouling model parameters a: coarse bubble, b: fine bubble.

parameter	unit	value
$V_1 \cdots V_5$	m^3	1500
c_{EC}	mg COD l^{-1}	400
Q_{int}	$m^3\ d^{-1}$	2000
Q_r	$m^3\ d^{-1}$	2000
Q_w	$m^3\ d^{-1}$	200
β	-	0.95
$F_1 \cdots F_5$	-	$0.9^a - 0.7^b$
g	$m\ s^{-2}$	9.81
$O_{A,m}$	-	0.232
$O_{A,v}$	-	0.21
P_{atm}	Pa	101325
ρ_A	$g\ m^{-3}$	1200
ρ_{sludge}	$kg\ m^{-3}$	1000
$SOTE_{1\cdots5}$	m^{-1}	$0.02^a - 0.06^b$
T	℃	15
$y_1 \cdots y_5$	m	$3.5^a - 5^b$
h	m	5
ϕ	-	1.024
$\omega_1 \cdots \omega_5$	m	$0.05^a - 0.083^b$

$$\dot{\mathbf{z}} = \Gamma(\mathbf{z}, Q_{feed}, \mathbf{c}_{feed}, Q_{int}, Q_r, Q_w, u)$$
$$y = H\mathbf{z} \quad (20)$$

where $\mathbf{z} = [\mathbf{c}_1; \mathbf{c}_2; \mathbf{c}_3; \mathbf{c}_4; \mathbf{c}_5] \in \Omega \in R^{65}$ consists of all state variables of BSM-MBR; $\Gamma(\cdot)$ presents the nonlinear behavior of the process; Q_{feed} and $\mathbf{c}_{feed}$ are the influent flow rate and concentrations, respectively; Q_{int}, Q_r, and Q_w are the recycle flow rates and wastage flow rate, respectively; $u(t)$ is the manipulate variable of the process, which refers to the air flow rate $Q_{A,2}$ of the second aerobic reactor; y is the output of the process, which refers to the dissolved oxygen concentration of reactor 4; H is the corresponding output matrix.

3.2 NMPC design for BSM-MBR

The control objective of the NMPC designed for BSM-MBR is to steer the DO level at the second aerobic zone (tank 4) to the desired value while meeting input and other process constraints. This is done by minimizing a quadratic function that penalizes the deviation of the process output. Specifically, the following optimization problem is solved iteratively to drive the process output to the desired value:

$$\min_{u(t)\in S(\Delta)} \int_{t_k}^{t_{k+N}} (\tilde{y}(\tau) - y_r)^T Q_c (\tilde{y}(\tau) - y_r) \quad (21a)$$

$$\text{s.t. } \dot{\tilde{\mathbf{z}}} = \Gamma(\tilde{\mathbf{z}}, Q_{feed}, \mathbf{c}_{feed}, Q_{int}, Q_r, Q_w, u) \quad (21b)$$

$$\tilde{y} = H\tilde{\mathbf{z}} \quad (21c)$$

$$u \in U,\ \forall\ t \in [t_k, t_{k+N}) \quad (21d)$$

$$\tilde{\mathbf{z}} \in Z,\ \forall\ t \in [t_k, t_{k+N}) \quad (21e)$$

$$\tilde{\mathbf{z}}(t_k) = \mathbf{z}(t_k) \quad (21f)$$

where $S(\Delta)$ is the family of piece-wise constant functions with sampling period Δ, N is the prediction horizon of NMPC, and the weighting matrix Q_c is a positive constant since we only have one output in this case. In the NMPC optimization problem (21), (21a) gives the quadratic cost function, which minimizes to drive the system output to the desired value; constraint (21b) and (21c) refer to the predicted model that used to predict the behavior of the system within the predicted horizon, where $\tilde{\mathbf{z}}$ is the predicted state trajectory of the process under the input trajectory $u(t)$ calculated by the NMPC optimization problem; due to the physical constraints on the actuators, the manipulated input $u(t)$ is restricted to be in a convex set over $N\Delta$ constraint (21d); (21e) is the constraint on the process states; (21f) denotes the initial condition at time instant t_k. In NMPC, only the first control action $u(t_k)$ of the NMPC (21) is implemented, and then the NMPC horizon is rolled again over the next time step. Specifically, if we denote the optimal solution to (21) as $u^*_{\text{NMPC}}(t)$, throughout the sampling period (i.e., $t_k \to t_{k+1}$), the first control action is applied in a sampled-and-hold fashion:

$$u(t) = u^*_{\text{NMPC}}(t|t_k), \quad \forall t \in [t_k, t_{k+1}). \quad (22)$$

In this work, the input of the system is constrained as follows: $U = \{u(t) \subset S(\Delta) | 0\ \text{Nm}^3\text{h}^{-1} \le u \le 7000\text{Nm}^3\text{h}^{-1}\}$ Maere et al. (2011). Throughout the sampling period, the following system process constraints are incorporated:

$$\text{TN}_e \le 18 \text{ g N m}^{-3}, \quad (23)$$

$$\text{NH}_{3e} \le 4 \text{ g N m}^{-3}. \quad (24)$$

which is the effluent upper limit proposed for the BSM-MBR (Maere et al., 2011; Elixmann and Nopens, 2016), TN_e and NH_{3e} are calculated as follows:

$$\text{TN}_e = (\mathbf{c}_e)_{10},$$

$$\text{NH}_{3e} = \sum_{j=9}^{12} (\mathbf{c}_e)_j + i_{XB}(\sum_{k=5}^{6} (\mathbf{c}_e)_k) + i_{XP}((\mathbf{c}_e)_3 + (\mathbf{c}_e)_7).$$

where $\mathbf{c}_e$ is defined in Eq. 16, i_{XB} and i_{XP} are the stoichiometric parameters of ASM1.

4. SIMULATION RESULTS AND DISCUSSION

In this section, the dynamic closed-loop simulation results of BSM-MBR will be given. In the first two subsections, the system initialization and influent data used in the simulation will be introduced briefly. Then we provide the simulation results of BSM-MBR under the proposed NMPC scheme.

4.1 System Initialization

In Maere et al. (2011), the simulation procedure is provided, in which the initial condition for closed-loop simulation is obtained from open-loop steady-state simulation, which is shown in Table 3.

4.2 Influent Data

As mentioned above, we will simulate the BSM-MBR process under both constant influent and dynamic influent. For the dynamic influent, simulated influent data are available online (http://iwa-mia.org/benchmarking/#BSM1) in three two-week files derived from real operating data corresponding to three different weather situations: dry weather, rainy weather and stormy weather, respectively (Vanhooren and Nguyen, 1996; Copp, 1999). Here we choose the dry weather scenario as the dynamic influent. In the data-set, the time is given in days, the flow rate is given in (m^3 d^{-1}) and the unit of concentrations are given in (g m^{-3}) with the following order:

$$[\text{ Time}, S_I, S_S, X_I, X_S, X_{BH}, X_{BA}, X_P, S_O, S_{NO}, S_{NH}, S_{ND}, X_{ND}, S_{ALK}, Q_{feed}],$$

in which:

$$S_O = 0 \text{ g COD m}^{-3},$$
$$X_{BA} = 0 \text{ g COD m}^{-3},$$
$$S_{NO} = 0 \text{ g N m}^{-3},$$
$$X_P = 0 \text{ g COD m}^{-3},$$
$$S_{ALK} = 7 \text{ mol m}^{-3}.$$

Table 4 shows the flow-weighted average influent compositions for BSM-MBR in dry weather condition, which are calculated as follows:

$$Q_{av} = \frac{\int_{t_0}^{t_f} Q(t)dt}{t_f - t_0}, \quad (25)$$

$$c_{av,k} = \frac{\int_{t_0}^{t_f} Q(t)c(t)_k dt}{\int_{t_0}^{t_f} Q(t)dt}, \quad k \in \{1, 2, \cdots, 13\}. \quad (26)$$

The constant influent was chosen as the flow-weighted average influent composition under dry weather conditions.

Fig. 3 to Fig. 5 show the trajectories of the influent flow rate, total nitrogen, and ammonia in dry weather condition, respectively. It is obvious that wastewater influent does not satisfy the effluent water quality.

4.3 Simulation Results

In this section, we apply the NMPC scheme proposed for the BSM-MBR process and compare the closed-loop performance under both constant influent and dry weather influent.

Table 3. Initial condition for closed-loop simulation (steady-state open loop BSM-MBR results for reactor zones 1 to 5.)

Reactors	1	2	3	4	5
S_I	30.00	30.00	30.00	30.00	30.00
S_S	2.25	1.31	0.85	0.77	0.67
X_I	2678.62	2678.62	3554.43	3554.43	4722.18
X_S	82.52	76.19	65.13	59.35	67.25
X_{BH}	2699.15	2697.86	3573.19	3572.44	4739.59
X_{BA}	233.30	233.07	311.13	311.33	413.41
X_P	1781.17	1782.50	2372.10	2373.11	3155.87
S_O	0.01	0.00	2.46	2.19	8.11
S_{NO}	4.09	1.48	10.08	11.54	12.57
S_{NH}	8.57	9.22	1.58	0.33	0.07
S_{ND}	1.08	0.68	0.65	0.63	0.58
X_{ND}	5.38	5.16	4.73	4.40	5.14
S_{ALK}	5.07	5.30	4.14	3.95	3.85

Table 4. Flow-weighted average influent for BSM-MBR in dry weather condition

Component	Value	Component	Value
S_I	30.00	S_S	69.50
X_I	51.20	X_{BH}	28.17
S_{NH}	31.56	S_{ND}	6.95
X_{ND}	10.59	S_{ALK}	7
Q_{av}	18446.33		

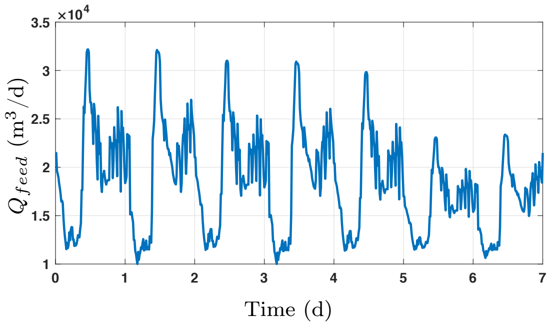

Fig. 3. Trajectory of influent flow rate under dry weather condition.

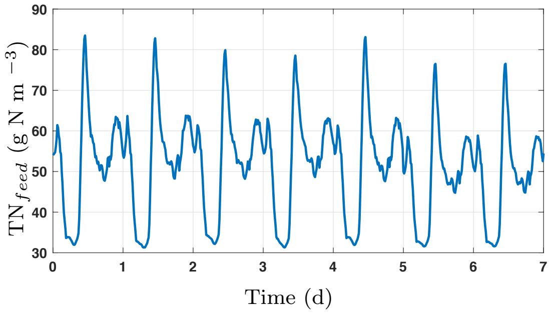

Fig. 4. Trajectory of TN at influent under dry weather condition.

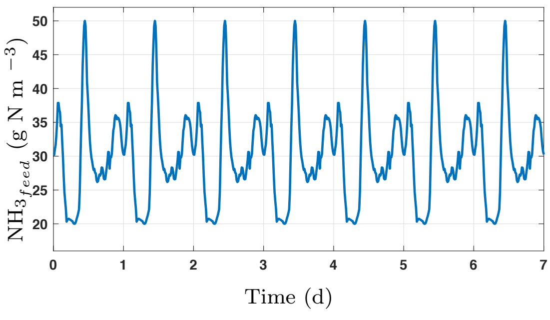

Fig. 5. Trajectory of NH_3 at influent under dry weather condition.

Throughout the simulation, the MATLAB built-in function called *ode15s* with an integration step size of $h = 0.01$ was used to simulate the BSM-MBR process. This numerical integration method is developed to solve stiff differential equations (Shampine et al., 1999), which is the case for the BSM-MBR process model (Janus, 2013). The optimization problem of Eq. 21 was solved by the MATLAB built-in function *fmincon*. The parameters of

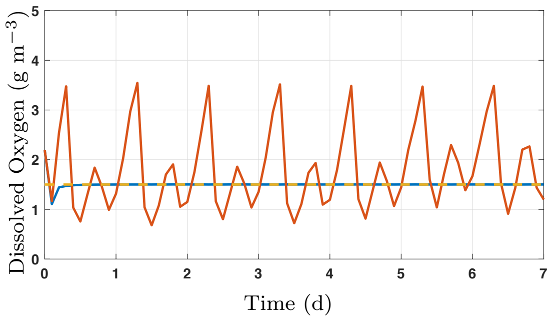

Fig. 6. Trajectory of the dissolved oxygen at second aerobic reactor under the design of NMPC (solid blue line is under constant influent condition, solid red line is under dry weather condition), and the reference value (dashed yellow line).

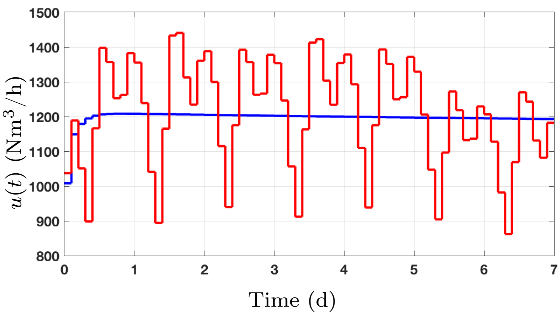

Fig. 7. The air flow rate $u(t)$ profile under the design of NMPC (blue line is under constant influent condition, red line is under dry weather condition).

the NMPC used in the simulations are presented in Table 5.

Table 5. Parameters of NMPC

Parameters	Value
Prediction Horizon N	5
Δ	0.1 d
h	0.01 d
t_f	7 d
Q_c	10

Fig. 6 to Fig. 9 gives the simulation results under the design of the NMPC of Eq. 21 with both constant and dynamic influents. Specifically, Fig. 6 shows the DO concentration of the second aerobic reactor, from which one can observe that after around six sampling periods (i.e., t = 0.6 d), the DO concentration achieves the desired value and stay there for the rest simulation period due to the constant influent. However, for the dynamic influent scenario, due to the periodic behavior of the influent, the DO concentration shows a similar periodic behavior around the reference value. A similar conclusion can be obtained from Fig. 7, in which the air flow rate $Q_{A,2}$ increases first, then stays around some steady-state for the rest simulation period under constant influent condition, while the process input $Q_{A,2}$ has the periodic profile within the input constraints under dynamic weather condition. TN and NH_3 concentrations in the effluent are shown in Fig. 8, and Fig. 9, respectively. It is clear that constraints Eq. 23 to 24 are satisfied under both influent scenarios.

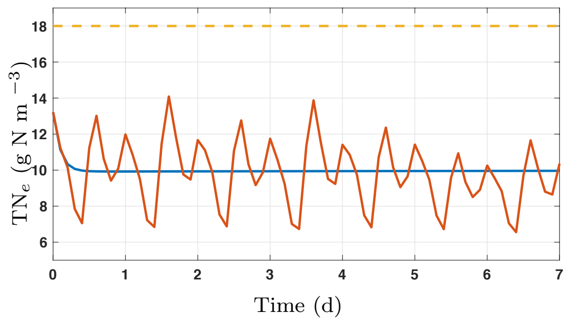

Fig. 8. Trajectory of TN at effluent under the design of NMPC (solid blue line is under constant influent condition, solid red line is under dry weather condition), and its upper limit (dashed yellow line).

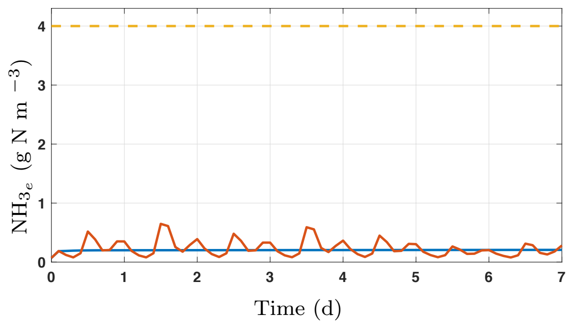

Fig. 9. Trajectory of NH_3 at effluent under the design of NMPC (solid blue line is under constant influent condition, solid red line is under dry weather condition), and its upper limit (dashed yellow line).

Table 6 gives the control performance Integral of Absolute Error (IAE), Integral of Squared Error (ISE), and variance of error of NMPC under two different influent scenarios, respectively. Due to variant influent load in the dry weather scenario, the closed-loop control performance is worse than the case in the constant influent scenario as expected.

Table 6. Control performance of NMPC under different influent scenarios

Index	Constant influent	Dry weather
IAE	0.0903	4.4161
ISE	0.0399	5.0484
Variance of error	0.0091	0.6433

5. CONCLUSION

This paper proposed an NMPC design for MBR processes based on the nonlinear dynamic model of BSM-MBR. The NMPC aims to keep the dissolved oxygen S_{O_4} at the desired value by minimizing a quadratic cost function while meeting the input constraints and other process constraints. Two influent scenarios are investigated in this work: constant influent and dry weather influent. As expected, the constant influent is easier for NMPC to control due to its invariant influent nature.

As for future investigations, one can consider solving different control problems by using MPC algorithms, such as minimize the operation cost while ensuring the effluent water quality. In addition, states estimation is critical for the MBR process and MPC algorithm, since not all the system states are measurable. This technique may also help to detect fouling events, which is a critical issue in the MBR process.

ACKNOWLEDGEMENTS

This work has been supported by the King Abdullah University of Science and Technology (KAUST) Base Research Fund (BAS/1/1627-0101) to Taous Meriem Laleg and the center of excellent for NEOM research at KAUST.

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