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An incremental scenario approach for building energy management with uncertain occupancy ⋆

Arman Karshenas * **Kostas Margellos** * **Simone Garatti** **

* *Department of Engineering Science, University of Oxford, Oxford, UK (e-mail: arman.karshenasnajafabadi@balliol.ox.ac.uk).*
** *Dipartimento di Elettronica e Informazione, Politecnico di Milano, Milano, Italy, (e-mail: simone.garatti@polimi.it)*

Abstract: We deal with the problem of energy management in buildings subject to uncertain occupancy. To this end, we formulate this as a finite horizon optimization program and optimize with respect to the windows' blinds position, radiator and cooling flux. Aiming at a schedule which is robust with respect to uncertain occupancy levels while avoiding imposing arbitrary assumptions on the underlying probability distribution of the uncertainty, we follow a data driven paradigm. In particular, we apply an incremental scenario approach methodology that has been recently proposed in the literature to our energy management formulation. To demonstrate the efficacy of the proposed implementation we provide a detailed numerical analysis on a stylized building and compare it with respect to a deterministic design and the standard scenario approach typically encountered in the literature. We show that our schedule is not agnostic with respect to uncertainty as deterministic approaches, while it requires fewer scenarios with respect to the standard scenario approach, thus resulting in a less conservative performance.

Keywords: Building energy management, Randomized optimization, Robust optimization, Scenario approach.

1. INTRODUCTION

The overall household energy use has increased noticeably over the last decade. As an example, in the United Kingdom, household consumption accounts for 50.2% of the total energy consumed Palmer and Cooper (2013), Waters (2018), while household and domestic heating/cooling is also responsible for over a quarter of the total emission levels Palmer and Cooper (2013). At the same time there have been several advancements in instrumentation, control and efficiency of actuation, which have in turn boosted research towards optimal energy management within buildings.

To this end, optimization based control in building heating and cooling has been extensively studied both from a centralized perspective Oldewurtel et al. (2012), Sturzenegger et al. (2012), Sturzenegger et al. (2014), Sturzenegger et al. (2016), Lehmann et al. (2013), Chen et al. (2013), and by means of a distributed architecture in multi-building settings Causevich et al. (2018), Belluschi et al. (2020). In the aforementioned references, main concern has been the development of energy models for buildings suitable for being integrated within an optimization context, while occupancy has been considered to be deterministic. In the presence of uncertainty the attempts most closely related to our work have been proposed in Oldewurtel et al. (2010), Ioli et al. (2016). In the former a robust approach is proposed (see also Jacomino and Le (2012), Saha et al. (2015) for conceptually similar robust considerations), with uncertainty assumed to be confined in sets with given geometry ignoring the underlying distribution of the uncertainty; the latter, may lead to conservative behaviour and deteriorate performance. In Ioli et al. (2016), a data driven approach is proposed using the so called scenario approach theory for convex optimization Calafiore and Campi (2006), Campi and Garatti (2008), Campi et al. (2009), Campi et al. (2019), thus alleviating the need of imposing certain assumptions on the underlying probability distribution of the uncertainty or the geometry of its support.

In this paper, we follow the data driven route and represent occupancy by means of scenarios. We then determine a sequence of blind positions and heater flux settings, which remains feasible when a new realization of the uncertainty/occupancy is encountered. We extend the developments in Ioli et al. (2016) using an incremental scenario approach algorithm that has been recently proposed in the literature Garatti and Campi (2019), and is motivated by the developments of Campi and Garatti (2018b). It leverages on the fact that the sample size proposed by the standard scenario approach is tight only for a certain class of programs (termed fully supported Campi and Garatti (2008)), while for other problems typically encountered in applications may result in conservative performance. The incremental scheme gradually introduces additional scenarios to a given initial set of scenarios and terminates when the empirical estimate of the support constraints, a

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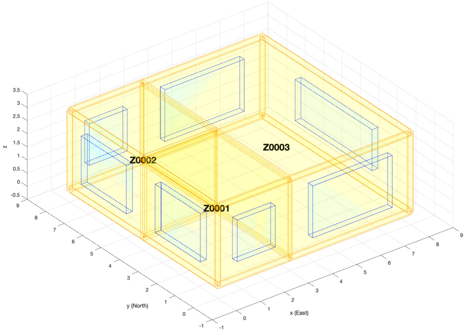

Fig. 1. 3D plot of the considered building model, generated using the MATLAB Toolbox of Sturzenegger et al. (2014).

notion at the core of the scenario approach theory Campi and Garatti (2018a), is no greater than the current algorithm iteration. It is guaranteed that the incremental scenario approach leads to a performance no worse than the standard scenario approach, thus reducing both the level of conservatism of the resulting solution and the sample size required. The latter is of significant importance if sampling is expensive or if only a limited amount of historical data is available. We apply the incremental scenario approach on an energy management problem for a three-zone building, and compare it against a deterministic approach and the standard scenario approach implementation.

The energy management problem under consideration is introduced in Section 2. Background on the scenario approach and the adopted incremental algorithm is presented in Section 3. Section 4 provides a detailed simulation based study, while Section 5 concludes the paper and provides directions for future work.

2. PROBLEM STATEMENT

2.1 Description and mathematical modelling

We consider the heating and cooling problem for a symmetric building with two bedrooms and a living room labelled by Z0001, Z0002 and Z0003, respectively, in Figure 1. We selected a symmetric case here as it allows verifying the validity of the obtained results, with temperature profiles being identical in the symmetric zones (see upper left panel of Figure 2), however, non-symmetric buildings can also be captured by the proposed methodology.

We consider a finite-horizon decision problem, where we aim at identifying an optimal sequence of actions for the following quantities, that are considered as inputs. These involve

(1) Windows' blinds position (u_1);
(2) Positive (heating) heat flux in W/m^2 by radiators or any external heating sources (u_2);
(3) Negative (cooling) heat flux in W/m^2 by cooling pipes or any external cooling sources (u_3).

For our analysis we consider ambient temperature and solar radiation to be deterministic, while building's occupancy is not fixed and is considered to be uncertain. Under this setting we aim at identifying a sequence of the aforementioned inputs that are (probabilistically) robust with respect to uncertainty in occupancy levels. In particular, we model uncertainty as a heat flux in W/m^2 generated by an uncertain level of occupancy.

To model the heat transfer dynamics we employed the BRCM toolbox developed by Sturzenegger et al. (2014). The internal heat transfer between zones is governed by a linear time-invariant model while the external heat fluxes into the building introduce bilinear terms according to the nature of actuation. The modelling framework of Sturzenegger et al. (2014) considers that i) the air volume of each zone has uniform temperature; ii) temperature within building elements varies only along the direction of the normal surface; iii) there is no conductive heat transfer between different building elements; iv) the temperature within a layer of a building element is constant; v) all model parameters are constant over time; vi) long-wave, thermal radiation is considered in a combined convective heat transfer coefficient. Under these assumptions, the overall dynamical model of heat transfer is then given by the following discrete time, bilinear system; more details about the structure of the model and the energy exchange can be found in Sturzenegger et al. (2012), Sturzenegger et al. (2014).

$$\begin{aligned} x_{k+1} = Ax_k + B_u u_k + B_\delta \delta_k \\ + \sum_{i=1}^{n} \left(B_{u\delta}\delta_k + B_{xu}x_k\right)u_{i,k}, \end{aligned} \tag{1}$$

where for each time instance k, x_k denotes the system state which contains the temperature of each zone and wall layer; u_k contains the actuation inputs, and δ_k the uncertain occupancy level. The total number of inputs is denoted by n. All matrices are of appropriate dimensions.

Our goals is to determine an optimal sequence u by integrating the dynamical system in (1) within a convex optimization context. To this end, the bilinear terms impose a challenge, as they would render the problem non-convex. To alleviate this we impose the following simplifying assumptions.

(1) The ambient air temperature during the summer is assumed to be constant at $35°C$;
(2) The global radiation flux or the solar constant as considered to be constant at 200 W/m^2 (see SoDa (2019) for benchmark values);
(3) No ventilation or airflow between zones is considered.

Under these assumptions, (1) simplifies to a linear, time-invariant dynamical system. The fact that ambient temperature and global radiation are assumed to be deterministic eliminates the input-disturbance bilinear terms. The resulting system is then denoted by

$$x_{k+1} = Ax_k + B_u u_k + B_\delta \delta_k, \tag{2}$$

for appropriately defined matrices.

Out of the imposed assumptions, only the last one appears to be restrictive in practice, as it assumes absence of airflow or ventilation. The main focus of the paper, is to quantify the effect of using the algorithm in Garatti and Campi (2019) as an efficient, data driven manner to deal with uncertain occupancy. This requires the underlying optimization program to be convex, which in turn calls for linear dynamics; the latter is ensured under the imposed assumption. Current work concentrates towards relaxing the convexity assumption, thus allowing airflow or ventilation to be present, using the non-convex developments of Campi and Garatti (2018b).

2.2 Energy Management Optimization

We consider a finite-horizon energy management problem, where M denotes the number of time steps. Let $X = [x_1^\top \dots x_M^\top]^\top$ denotes a stacked vector including the states of all time instances, and define U and δ similarly. Propagating the linear, time-invariant dynamics in (2) by M steps, we obtain the following compact representation of the systems temperature evolutions.

$$X = Fx_0 + GU + H\delta, \tag{3}$$

where matrices F, G and H are of appropriate dimension; e.g., the reader is referred to Campi et al. (2019) for dimensions. Note that X depends on the history of inputs and disturbances that are included in U and δ, respectively, as well as on the initial state x_0, which is given.

We aim at minimizing the following objective function.

$$J(U) = \mathbb{E}[\, X^\top QX + U^\top RU \,], \tag{4}$$

where we denote the objective function as being a function of U only, as X could be substituted by (3) which is also a function of U and the uncertainty vector δ. We assume that δ is distributed according to some probability distribution $\mathbb{P}$ with support Δ (possibly unknown if only data are available), and denote by $\mathbb{E}$ the corresponding expectation operator. The objective function in (4) involves the expected value of the sum of two components: one term penalizing the state (temperature) and one the input, where $Q \succeq 0$ and $R \succ 0$ are given matrices. In the numerical case study we only consider the second term, seeking a minimum effort sequence of actuation commands.

The minimization of $J(U)$ is subject to the following constraints.

Input constraints: The following constraints on the elements of the input vector U are considered. For all k,

(1) $u_{1,k} \in (0, 90)\%$: effective reduction of solar gain in the form of percentage for window blinds;
(2) $u_{2,k} \le 1kW/m^2$: upper limit on the heating system;
(3) $u_{3,k} \le 1kW/m^2$: upper limit on the cooling system.

State constraints: We differentiate between different seasons, namely, summer and winter, in line with the developments in Hussain et al. (2017). This distinction allows us to model state constraints by means of single-sided inequality constraints; in the opposite case we would have double-sided inequalities; the latter may lead to feasibility issues, but this is outside the scope of this paper. We then have the following constraints on the temperature profiles in X that encode comfort, which by means of (3), result in constraints on U:

Summer day:

$$\mathbb{P}\{\delta \in \Delta : \; Fx_0 + GU + H\delta \le T_{\max}\} \ge 1 - \epsilon, \tag{5}$$

Winter day:

$$\mathbb{P}\{\delta \in \Delta : \; Fx_0 + GU + H\delta \ge T_{\min}\} \ge 1 - \epsilon, \tag{6}$$

where $\epsilon \in (0, 1)$. Constraints (5), (6), are chance constraints, i.e., the guarantee that an upper ($T_{\max}$) and a lower ($T_{\min}$) temperature limit, respectively, is achieved, with probability at least $1 - \epsilon$.

Note that we use an upper limit during summer because there exists a continuous heat flux from the surroundings, and a lower limit for winter time as there exist a continuous heat flux to surroundings. For the considered case study, in winter time occupancy level provides enough heating for the temperatures to stay consistently above the lower limit; to highlight all features of the proposed algorithm we select a summer day for our numerical investigations.

By means of (3), we can collectively represent all input and state constraints outlines above as constraints involving only U and δ. To facilitate the algorithmic developments of the following section, the energy management problem under consideration is of the form

$$\min_{U \in \mathbb{R}^{nM}} \; J(U) \tag{7}$$

$$\text{subject to } \mathbb{P}\{\delta \in \Delta : \; g(U, \delta) \le 0\} \ge 1 - \epsilon, \tag{8}$$

where g is a scalar valued function (corresponding to the maximum among all constraints imposed on U), convex with respect to U for any fixed δ; the dependency on δ could be arbitrary. Note that in our formulation we have a total of $nM = 3M$ decision variables, i.e., three scalar inputs times the number of time-steps M.

3. SCENARIO APPROACH IMPLEMENTATION

3.1 Standard scenario approach

The scenario approach provides a way to obtain a solution that is feasible for the chance constraint in (8) with certain confidence, without requiring knowledge of $\mathbb{P}$ and/or Δ. To this end, associate with (7)-(8) a scenario program, which involves replacing the chance constraint with N constraints, each of them corresponding to realization/scenario of the uncertain occupancy level δ, extracted in an i.i.d. (independent and identically distributed) fashion. This is given by

$$\min_{U \in \mathbb{R}^{nM}} \; J(U) \tag{9}$$

$$\text{subject to } g(U, \delta^i) \le 0, \text{ for all } i = 1, \dots, N. \tag{10}$$

We assume that (9)-(10) is feasible for any multi-scenario extraction, and admits a unique solution, denoted by U^*. In case of multiple solutions, a convex tie-break rule could be adopted to single out a particular minimizer. The feasibility assumption can be relaxed as shown in Calafiore and Campi (2006).

Under the feasibility and uniqueness requirements, the standard scenario approach Campi and Garatti (2008), shows that for a given $\beta, \epsilon \in (0, 1)$, if N is chosen so that it satisfies

$$\sum_{j=0}^{nM-1} \binom{N}{j} \epsilon^j (1 - \epsilon)^{N-j} \le \beta, \tag{11}$$

then

$$\mathbb{P}^N\{(\delta^1, \dots, \delta^N) \in \Delta^N : \\ \mathbb{P}\{\delta \in \Delta : \; g(U^*, \delta) \le 0\} \ge 1 - \epsilon\} \ge 1 - \beta, \tag{12}$$

where $\mathbb{P}^N$ denotes the product probability measure, as U^* depends on all extracted scenarios used to solve (9)-(10), hence it is a random variable in Δ^N. In other words, with confidence at least $1-\beta$, the optimal solution U^* of (9)-(10) is feasible for the chance constraint in (8). Note that the only structural characteristic appearing in the confidence is the number of decision variables $3M$.

We can obtain a sufficient, albeit explicit, condition for the number of scenarios N that need to be extracted so that (11) is satisfied. This is given by Campi and Garatti (2018a), Campi et al. (2019),

$$N \geq \frac{2}{\epsilon}\big(\ln\frac{1}{\beta} + nM\big). \tag{13}$$

3.2 Incremental scenario approach

The incremental approach Garatti and Campi (2019) leverages on the fact that the sample size proposed by the standard scenario approach is tight only for a certain class of programs (termed fully supported in Campi and Garatti (2008)), while for other problems typically encountered in applications may result in conservative performance. The incremental scheme gradually introduces additional scenarios to a given initial set of scenarios and is guaranteed to result in a performance no worse than the standard scenario approach, thus reducing both the level of conservatism of the resulting solution and the sample size required. The general structure of the incremental scenario approach scheme is presented in Algorithm 1.

Algorithm 1 Incremental scenario approach

1: $j = 0$, $N_{-1} = 0$,
2: **while** $S_j^* > j$ **do**
3: Add $(N_j - N_{j-1})$ additional scenarios
4: Compute U_j^* by solving (9)-(10) with $N = N_j$
5: Determine the number of support constraints S_j^*
6: **if** $S_j^* < j$ **then**
7: Break and return $U^* = U_j^*$
8: **else**
9: $j \leftarrow j+1$
10: **end if**
11: **end while**

For each j let U_j^* denote the optimal solution of (9)-(10) with $N = N_j$ (step 4). Variable S_j^* denotes the number of support constraints (a constraint is of support if its removal results in a different optimal solution Campi and Garatti (2018a)) of U_j^*. It can be determined by means of an iterative process, where constraints are removed one by one, checking whether their removal resulted in a change in the optimal solution. For non-degenerate convex programs (see Campi and Garatti (2008) for a formal definition of non-degeneracy), this is equivalent to determining the number of constraints active at the optimal solution; one way of achieving this is by considering the constraints corresponding to non-zero dual variables.

If at a given iteration j, $S_j^* < j$, then the algorithm terminates returning U_j^* (step 7). This termination condition involves comparing the estimate of the support constraints with the iteration counter; this relationship between iterations and support constraints is due to the fact that the algorithm is guaranteed to terminate after at most nM iterations, as the number of support constraints for convex programs can be no greater than the number of decision variables Campi and Garatti (2018a). Due to this feature, the incremental scenario approach is guaranteed to require a number of scenarios no worse than the standard scenario approach.

In Theorem 1 in Garatti and Campi (2019), it is shown that for non-degenerate problem instances, if the number of scenarios $N_0, N_1, \ldots, N_{nM}$ (recall that at most nM algorithm iterations are required) is selected as shown in the sequel, then for a given $\beta, \epsilon \in (0,1)$,

$$\mathbb{P}^{nM}\{(\delta^1,\ldots,\delta^N) \in \Delta^N : \\ \mathbb{P}\{\delta \in \Delta : \ g(U^*,\delta) \leq 0\} \geq 1-\epsilon\} \quad \geq 1-\beta, \tag{14}$$

where U^* denotes the solution returned upon termination of Algorithm 1. The number of scenarios required at each iteration $j = 0, 1, \ldots, nM$ is given by

$$N_j \geq \min\Bigg\{N : N \geq M_j \text{ and} \tag{15}$$

$$\binom{N}{j}(1-\epsilon)^{N-j} \leq \frac{\beta}{(d+1)(M_j+1)}\sum_{m=j}^{M_j}\binom{m}{j}(1-\epsilon)^{m-j}\Bigg\},$$

where M_j is given by

$$M_j \geq \min\big\{N : \sum_{\ell=0}^{j-1}\binom{N}{\ell}\epsilon^\ell(1-\epsilon)^{N-\ell} \leq \beta\big\}, \tag{16}$$

We now provide sufficient conditions for the satisfaction of the conditions in (16) and (15), to determine the number of scenarios M_j and N_j, respectively. Due to the equivalence between (16) and (11), similarly to Section 3.1, by Campi and Garatti (2018a), Campi et al. (2019), one can take

$$M_j = \frac{2}{\epsilon}\big(\ln\frac{1}{\beta} + j - 1\big). \tag{17}$$

To obtain an explicit relation for N_j, set

$$\beta_j = \frac{\beta}{(d+1)(M_j+1)}\sum_{m=j}^{M_j}\binom{m}{j}(1-\epsilon)^{m-j}. \tag{18}$$

We then seek a value for N_j that satisfies $\binom{N}{j}(1-\epsilon)^{N-j} \leq \beta_j$. By Corollary 1 in Calafiore and Campi (2006), we obtain (the maximum ensures $N_j \geq M_j$)

$$N_j = \max\{M_j, (\frac{2}{\epsilon}\ln\frac{1}{\beta_j} + 2j + \frac{2j}{\epsilon}\ln\frac{2}{\epsilon})\}. \tag{19}$$

4. CASE STUDY

4.1 Simulation set-up

To quantify the efficacy of the proposed incremental scenario approach of Section 3.2, we compare it against a deterministic variant, where a nominal/forecasted occupancy level is considered, and against the standard scenario approach implementation of Section 3.1.

To this end, we revisit the energy management problem of Section 2, considering (4) with $Q = 0$ and $R = I$, thus resulting in minimizing $J(U) = U^TRU = U^TU$. Assuming a summer day, this is subject to (5), where we consider $T_{\max} = 24°C$. Moreover, the constraints on U defined in

Table 1. Comparison between the deterministic, the standard, and the incremental scenario approach.

Algorithm	Number of Scenarios	Cost ($\frac{W^2}{m^4}$)	Theoretical Risk	Empirical Risk
Deterministic	1	2.55×10^4	N/A	85.27%
Standard Scenario Approach	3065	2.36×10^5	10%	0%
Incremental Scenario Approach	376	1.94×10^5	10%	5.1%

Section 2 are also imposed. A time horizon of 12 hours with granularity of 15 minutes is considered, resulting in $M = 48$ time-steps.

We considered that the occupancy level follows a discrete Poisson distribution with a mean equal to 3. All scenarios, for the standard and the incremental scenario approach, as well as the ones used for validation purposes, were generated in an i.i.d. fashion from the same distribution. For the deterministic analysis we considered occupancy to be the empirical expectation from a certain number of scenarios extracted from the aforementioned distribution.

4.2 Simulation results

We consider three algorithmic alternatives, namely, the deterministic approach, the standard scenario approach and the incremental one. We compare them in terms of the number of scenarios required by each alternative, resulting cost of the optimization program, theoretical risk (constraint violation level ϵ) level, and empirical risk. The latter is determined by calculating the number of violations encountered by the solution returned by each approach over 3000 validation scenarios.

The comparison outcomes are summarised in Table 1. The following observations are in order.
1) The cost incurred in the deterministic approach is significantly lower, however, this comes at the expense of not being robust with respect to uncertainty in the occupancy level. This can be witnessed by the high empirical risk, i.e., the empirical frequency of constraint violation against the 3000 validation scenarios. 2) The standard scenario approach requires 3065, as opposed to 376 scenarios required by the incremental algorithm. Within a data driven context this is a desirable feature, in particular if data resources are limited.
3) As a result of the lower number of scenarios required for the same theoretical risk level $\epsilon = 10\%$, the incremental algorithm leads to a less conservative behaviour. This is witnessed by the lower cost, as well as by the higher empirical risk (5% as opposed to 0%), which is closer to the theoretical value.

Figure 2 shows the temperature profiles of zone 3 (Z0003) generated by the three algorithms. The grey shadow corresponds to the span of the system trajectories when the optimal solution of each algorithmic alternative is monitored for the 3000 validation scenarios. The dashed red line is the upper bound for temperature during the summer. The red trajectory corresponds to the response of the system in the deterministic case, when occupancy is considered to be equal to the forecasted value. For completeness, the temperature profile of zones 1 (Z0001) and 2 (Z0002) is also illustrated for the incremental algorithm.

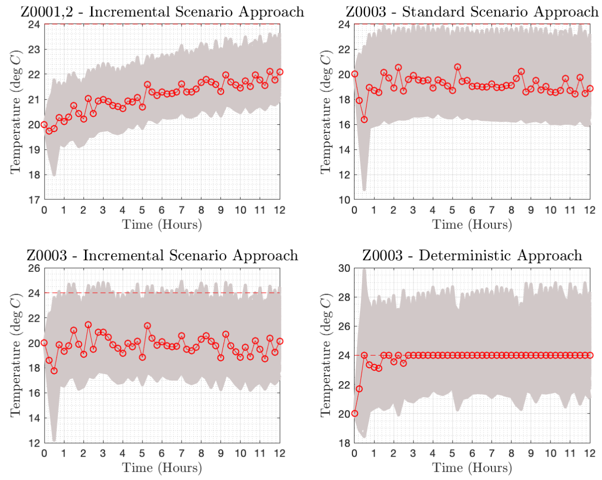

Fig. 2. Temperature profile for zones Z0001 and Z0002 for the incremental scenario approach (they coincide as a result of the symmetry of the zones). Temperature profile for zone Z0003 using the incremental (lower left panel), the standard scenario approach (upper right panel), and the deterministic approach (lower right panel).

Note that in the deterministic case (lower right panel of Figure 2), the vast majority of the validation scenarios leads to violation of the temperature limit. At the other extreme, no violation is encountered for the standard scenario approach (upper right panel of Figure 2). The incremental scenario approach results in a moderate number of violations, close to the theoretical acceptable risk level (lower left panel of Figure 2).

The probability of violation is itself a random variable, hence different sets of validation scenarios will result in a different empirical risks. We thus generated 100 sets, each with 3000 validation scenarios, and for each of them calculated the empirical risk. The resulting distribution is shown in Figure 3. It must be noted that in all 100 runs of the validation process, not more than 160 scenarios were violated within each set of 3000 scenarios which is below the 10% theoretical risk level.

5. CONCLUDING REMARKS

We applied an incremental scenario approach algorithm recently introduced in Garatti and Campi (2019) to the problem of building energy management. To quantify the trade-off between performance and robustness we provide a detailed numerical analysis on a stylized building and compare it with respect to a deterministic design and the standard scenario approach implementation.

Current work concentrates towards relaxing the convexity assumption, thus allowing airflow or ventilation to be present. To this end, we aim at employing the non-convex scenario approach developments of Campi and Garatti

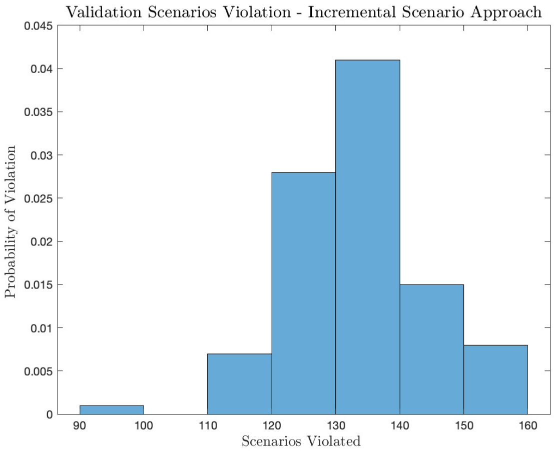

Fig. 3. Histogram showing the empirical probability of constraint violation for 100 sets, each with 3000 scenarios. The empirical risk was calculated as the fraction out of 3000 validation scenarios that were violated by the solution returned by the incremental scenario approach algorithm.

(2018b). Moreover, we aim at replacing the open-loop sequence of decisions with affine feedback policies with respect to uncertainty Goulart et al. (2006), and quantify the potential improvement in terms of performance.

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