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A self-triggered control scheme for Markov jump systems under multiple range performance restrictions

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Abstract: This paper proposes a multi-frequency controller design scheme for Markov jump systems (MJSs) based on the self-triggered strategy in a resource-aware way. Firstly, a derandomization technique is introduced to make sure the transition probability information is included in the finite frequency specification analysis. Then, a self-triggered policy is developed to update the control input of the system via the history measurement. Finally, sufficient conditions are deduced that guarantee the multiple range frequency performances and the reduction of computation and communication occupation for the controlled MJSs, simultanously. The cart- spring system is employed to illustrate the effectiveness of the proposed approach.

Keywords: Self-triggered control, finite frequency performance, Markov jump systems, derandomization, disturbance rejection.

1. INTRODUCTION

Most feedback control approaches are implemented by the traditional time-triggered scheme, which is sampling and updating in a constant interval. This triggering pattern may be conservative that wastes the usage in some cases. Especially for multiple tasks, communication bandwidth, batteries, and computation abilities are limited. To this end, some resource-aware sampling strategies are developed. Event-triggered control is an aperiodic scheme which only updates the control input when the predefined threshold conditions are satisfied (Tabuada (2007)). However, event-triggered scheme needs extra-dedicated hardware to monitor the threshold continuously. On the contrary, the self-triggered policy is another aperiodic scheme. Under this paradigm, the execution intervals are adjusted adaptively to drag the trajectories back to the equilibrium point when far from the equilibrium and relax the update frequency to save the resource usages when close to the equilibrium (Heemels et al. (2012)). Recent years, the self-triggered policy has been widely studied in varies fields such as trajectory tracking of unicycle-type robots (Cao et al. (2019)), the consensus of multiagent systems (Mi et al. (2020)), formation control of nonholonomic robots (Santos et al. (2019)), etc.

As a special type of hybrid systems, Markov jump systems (MJSs) can be described by the continuous state evolution and the discrete mode transition obeying a Markov chain. MJSs have been widely applied for modeling a variety of real systems like economic, power, and failure-prone manufacturing process (Shi et al. (2015), Zhu et al. (2019)). As for the triggering strategy of MJSs, most results are related to event-driven such as Lin et al. (2019) and Cheng et al. (2018)). It is difficult to fuse the self-triggered scheme into MJSs synthesis. A recent attempt of self-triggered control for MJSs was proposed in (Xie et al. (2018)). However, in (Xie et al. (2018)), only constant disturbances have been considered that limits the application of the method.

However, not only the time-varying but also the finite frequency domain characteristics of the external disturbance should be taken into consideration in controller design. In many engineering processes, whether it is system disturbance or reference input signal, its energy often concentrates only on some finite frequency bands. As a result, finite frequency specifications take an important role on MJSs analysis and synthesis, and a considerable amount of related literature has been published including H_∞ control (Luan et al. (2019)), fault detecting (Zhou et al. (2018)), filter design (Wan et al. (2019)), and so on. However, the aforementioned work ignored the effect of transition probabilities when finite frequency analysis which leads to the conservativeness. Moreover, to optimize system performance in frequency domain, different performance indexes are required. Therefore, how to design controllers to ensure the multi-frequency performance for MJSs is of importance in both theory and practice. Furthermore, how to implement the control law in a resource-friendly pattern would be a challenging and interesting issue.

Based on the aforementioned discussion, this paper addresses the issue of multi-frequency controller design for MJSs based on a self-triggered strategy. The main contributions are summarized in the following: (1) a novel self-triggered scheme is proposed to adjust the sampling and updating intervals according to the last measurement; (2) the desired multiple range frequency performances are guaranteed via the designed self-triggered controllers. Consequently, the designed controllers ensure the controlled MJSs satisfying the predefined multiple range frequency performances and reduce the communication and computation usages, simultaneously.

The paper is organized as follows. Section 2 formulates the preliminary introduction. The self-triggered strategy and H_∞ controller restricted to multiple range frequency performances are developed in Section 3. Section 4 presents a two-mode cart-spring system to verify the effectiveness of the proposed method. Finally, Section 5 concludes the paper.

Notations: R^n and I_n represent the n-dimensional Euclidean space and unit matrix, respectively. X is a negative define matrix if $X < 0$. A^{T} and $\|A\|$ represent the transpose and the spectral norms of A, respectively. $He\{\cdot\}$ denotes $A + A^{\mathrm{T}}$. The Kronecker product operator and expectation operator are expressed as $\otimes$ and $E\{\cdot\}$, respectively. The nullspace and the range space of A are denoted as $\Im(A)$ and $\Re(A)$, respectively. $A^{\perp}$ denotes a matrix satisfies that $\Im\left(A^{\perp}\right) = \Re(A)$ and $A^{\perp}A^{\perp\mathrm{T}} > 0$.

2. PROBLEM PRELIMINARIES

Consider the following continuous-time Markov jump system (MJS):

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B(r(t))u(t) + B_w(r(t))w(t) \\ z(t) = C(r(t))x(t) + D(r(t))u(t) \end{cases}, \quad (1)$$

where $x(t)$, $u(t)$ and $z(t)$ denote the state vector of the system, the control input, and controlled output, respectively. The external disturbance is denoted by $w(t)$ and satisfies $\int_0^{T_1} w^{\mathrm{T}}(t)w(t)dt \le W$, where W is a positive constant, characterizing the upper energy bound of noises. Furthermore, $A(r(t))$, $B(r(t))$, $B_w(r(t))$, $C(r(t))$ and $D(r(t))$ are real defined mode-dependent matrices with appropriate dimensions. To simplify the presentation, system matrices $A(r(t))$, $B(r(t))$, $B_w(r(t))$, $C(r(t))$ and $D(r(t))$ are shown by A_i, B_i, B_{wi}, C_i and D_i, respectively. The random process $\{r(t),\ t \ge 0\}$, which can take values on a finite set $S = \{1, 2, \ldots, s\}$, is a Markov chain with the following transition probability:

$$P_r\{r(t+\Delta t) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), i \ne j \\ 1 + \pi_{ii}\Delta t + o(\Delta t), i = j \end{cases},$$

where $\Delta t > 0$, $\lim_{\Delta t \to 0} o(\Delta t)/\Delta t \to 0$, $\pi_{ij} \ge 0$ for $i, j \in S$, $i \ne j$ and $\pi_{ii} = -\sum_{j=1, i\ne j}^{s} \pi_{ij}$ for each mode i.

To fully analyse the effects of transition probabilities to the finite frequency specifications, a derandomization technique is introduced as follow.

Define indicator function $\mathbf{1}_{\mathbb{A}}$ by (Benjelloun et al. (1997))

$$\mathbf{1}_{\mathbb{A}}(\omega) = \begin{cases} 1 & \text{if } \omega \in \mathbb{A} \\ 0 & \text{otherwise} \end{cases}.$$

Denote

$$q_i(t) = E\left\{\|x(t)\|\, \mathbf{1}_{\{r(t)=i\}}\right\}, \quad (2)$$

Combining (1) and (2) gives

$$\begin{aligned} dq_j(t) &= E\left\{x(t)d\mathbf{1}_{\{r(t)=j\}} + dx(t)\mathbf{1}_{\{r(t)=j\}}\right\} \\ &= A_i q_j(t)dt + \sum_{i=1}^{s} \pi_{ij} q_j(t)dt + B_i u(t)dt + B_{wi}w(t)dt, \end{aligned} \quad (3)$$

Defining
$q(t) = \left(\, q_1{}^{\mathrm{T}}(t) \cdots q_s{}^{\mathrm{T}}(t)\,\right)^{\mathrm{T}}$, $\tilde{u}(t) = \left(\, u^{\mathrm{T}}(t) \cdots u^{\mathrm{T}}(t)\,\right)^{\mathrm{T}}$, $\tilde{w}(t) = \left(\, w^{\mathrm{T}}(t) \cdots w^{\mathrm{T}}(t)\,\right)^{\mathrm{T}}$, $\tilde{z}(t) = \left(\, z^{\mathrm{T}}(t) \cdots z^{\mathrm{T}}(t)\,\right)^{\mathrm{T}}$, then MJS (1) could be transformed to (Luan et al. (2018))

$$\begin{cases} \dot{q}(t) = \mathcal{A}q(t) + \mathcal{B}\tilde{u}(t) + \mathcal{B}_w\tilde{w}(t) \\ \tilde{z}(t) = \mathcal{C}q(t) + \mathcal{D}\tilde{u}(t) \end{cases}, \quad (4)$$

where
$\mathcal{A} = diag\{A_1, A_2, \ldots, A_s\} + \Pi \otimes I_n$,
$\mathcal{B} = diag\{B_1, B_2, \ldots, B_s\}, \mathcal{B}_w = diag\{B_{w1}, B_{w2}, \ldots, B_{ws}\}$,
$\mathcal{C} = diag\{C_1, C_2, \ldots, C_s\}$, $\mathcal{D} = diag\{D_1, D_2, \ldots, D_s\}$,

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{21} & \cdots & \pi_{s1} \\ \pi_{12} & \pi_{22} & \cdots & \pi_{11} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{1s} & \pi_{2s} & \cdots & \pi_{ss} \end{pmatrix}.$$

Remark 1: The transition probabilities are assumed to be known in this paper, and the accuracy of the proposed algorithm depends on the precision of matrix Π.

According to the self-triggered scheme in (Heemels et al. (2012)), the control input of the extended system (4) is defined as

$$\tilde{u}(t) = \mathcal{K}q(t_k), \qquad t \in [t_k, t_{k+1}), \quad (5)$$

where t_k is the last sampling instant, K_i is the control gain of the subsystem i, and $\mathcal{K} = diag\{K_1, K_2, \ldots, K_s\}$. Then, for $t \in [t_k, t_{k+1})$, the closed-loop self-triggered system based on system (4) could be formulated as:

$$\begin{cases} \dot{q}(t) = (\mathcal{A} + \mathcal{B}\mathcal{K})q(t) + \mathcal{B}\mathcal{K}e_{t_k}(t) + \mathcal{B}_w\tilde{w}(t) \\ \tilde{z}(t) = \mathcal{C}q(t) + \mathcal{D}\tilde{u}(t) \end{cases}, \quad (6)$$

where $e_{t_k}(t) := q(t_k) - q(t)$, $t \in [t_k, t_{k+1})$ represents the measurement errors. The main objective of this paper is designing controllers (5) to ensure the stability of the closed-loop self-triggered system (6) and the multiple range frequency performances as follows:
a) disturbance attenuation ability constraint

$$|G_{\tilde{z}\tilde{w}}(j\omega)| = \sup_{|\tilde{w}(t)|\ne 0} \frac{\|\tilde{z}(t)\|_2}{\|\tilde{w}(t)\|_2} < \gamma, \quad (7)$$

b) control consumption index

$$|G_{\tilde{u}\tilde{w}}(j\omega)| = \sup_{|\tilde{w}(t)|\ne 0} \frac{\|\tilde{u}(t)\|_2}{\|\tilde{w}(t)\|_2} < \rho. \quad (8)$$

3. MAIN RESULTS

In this section, a self-triggered mechanism for the derived extended deterministic system (6) is proposed. Under this paradigm, sufficient conditions that ensure the required multiple range frequency performances are deduced while saving the resources usages.

3.1 Self-triggered implementation for MJS

The key of self-triggered policy is the calculation of the next execution instant only based on the last measurement. The function of the self-triggered execution interval is denoted by $\tau(q(t_k))$. To obtain the aforementioned function, the following event-triggering condition is employed

$$e_{t_k}(t)\Phi e_{t_k}(t) \geq \varepsilon q^{\mathrm{T}}(t)\Phi q(t). \quad (9)$$

The specific algorithm of self-triggered execution intervals function is detailed in Theorem 1.

Theorem 1. For given scalars λ, ε and the appropriate dimensions matrices $\mathcal{K}$, and $\Phi = diag\{\Phi_1, \Phi_2, \ldots, \Phi_s\}$, the self-triggered policy is constructed if the following execution intervals function is existed

$$\tau(q(t_k)) = t_{k+1} - t_k = \frac{1}{M_1}\ln\left[\frac{M_1 N}{M_2} + 1\right], \quad (10)$$

where $M_1 = \left\|\sqrt{\Phi}\mathcal{A}\sqrt{\Phi}^{-1}\right\|$, $N = \sqrt{\frac{\varepsilon(1-\lambda)}{1-\varepsilon(1-\lambda^{-1})} q^{\mathrm{T}}(t_k)\Phi q(t_k)}$, $M_2 = \left[\left\|\sqrt{\Phi}\mathcal{A}\right\| + \left\|\sqrt{\Phi}\mathcal{B}\mathcal{K}\right\|\right]\|q(t_k)\| + W\left\|\sqrt{\Phi}\mathcal{B}_w\right\|$.

Proof. According to (6), we obtain the following property for $\forall\ t \in\ [t_k,\ t_{k+1})$

$$\begin{aligned}&\frac{d\left\|\sqrt{\Phi}e_{t_k}(t)\right\|}{dt} \\ &\leq \left\|\sqrt{\Phi}\mathcal{A}\sqrt{\Phi}^{-1}\right\|\left\|\sqrt{\Phi}e_{t_k}(t)\right\| + W\left\|\sqrt{\Phi}\mathcal{B}_w\right\| \\ &\quad + \left[\left\|\sqrt{\Phi}\mathcal{A}\right\| + \left\|\sqrt{\Phi}\mathcal{B}\mathcal{K}\right\|\right]\|q(t_k)\|.\end{aligned} \quad (11)$$

Let $y(t) = \left\|\sqrt{\Phi}e_{t_k}(t)\right\|$, (11) could be formulated as

$$\frac{dy(t)}{dt} \leq M_1 y(t) + M_2, \quad (12)$$

Considering $\frac{dh(t)}{dt} = M_1 h(t) + M_2$, and applying Lemma 1 (see the Appendix), one gets

$$\left\|\sqrt{\Phi}e_{t_k}(t)\right\| \leq \frac{M_2}{M_1}\left[e^{M_1(t-t_k)} - 1\right]. \quad (13)$$

Considering $1 - \varepsilon(1-\lambda^{-1}) > 0$ and $t \in [t_k,\ t_{k+1})$, a combination of condition (9) and Lemma 2 (see the Appendix) gives

$$\left\|\sqrt{\Phi}e_{t_k}(t)\right\| < \sqrt{\frac{\varepsilon(1-\lambda)}{1-\varepsilon(1-\lambda^{-1})} q^{\mathrm{T}}(t_k)\Phi q(t_k)}. \quad (14)$$

Then, the system (6) is triggered if the above inequality is violated. Combining (13) and the triggering threshold leads to

$$e^{M_1(t-t_k)} - 1 \geq \frac{M_1 N}{M_2}. \quad (15)$$

By taking natural logarithm, condition (10) could be easily derived from (15), which completes the proof.

Remark 2 When the execution interval function $\tau(\bullet)$ related to the current measurement $q(t)$, the proposed self-triggered strategy degenerates into the event-triggered schemes. In comparison, continuously monitoring of the system states with extra hardware is not necessary for the developed self-triggered policy.

3.2 Controller design restricted to multiple range frequency performances

Based on the self-triggered scheme derived in Theorem 1, the following Theorem provides an algorithm to design controller meeting the desired multiple range frequency specifications.

Theorem 2. For predefined finite frequency indices γ, ρ, and given frequency ω_l and ω_h, if symmetric matrices $\bar{W}$, P_l, P_h, $Q_l > 0$, $Q_h > 0$, and matrices $\bar{\mathcal{K}}$, $\bar{\Phi}$, Y_l, and Y_h are existed, such that

$$\begin{pmatrix} -Q_l & 0 & P_l & 0 \\ 0 & I & 0 & 0 \\ P_l & 0 & \omega_l^2 Q_l & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{pmatrix} < He\begin{pmatrix} -\bar{W}R_l \\ Y_l \\ V_l \\ \mathcal{C}_2\bar{W}R_l \end{pmatrix}, \quad (16)$$

$$\begin{pmatrix} Q_h & 0 & P_h & 0 \\ 0 & I & 0 & 0 \\ P_h & 0 & -\varpi_h^2 Q_h & 0 \\ 0 & 0 & 0 & -\rho^2 I \end{pmatrix} < He\begin{pmatrix} -\bar{W}R_h \\ -Y_h \\ V_h \\ \bar{\mathcal{K}}R_h \end{pmatrix}, \quad (17)$$

$$\begin{pmatrix} \Lambda & \mathcal{B}\bar{\mathcal{K}} \\ * & \Phi \end{pmatrix} < 0, \quad (18)$$

where
$V_l = \mathcal{A}\bar{W}R_l + \mathcal{B}_w Y_l + \mathcal{B}\bar{\mathcal{K}}R_l$, $V_h = \mathcal{A}\bar{W}R_h + \mathcal{B}_w Y_h + \mathcal{B}\bar{\mathcal{K}}R_h$, $\bar{\Phi} = \bar{W}^{-1}\Phi\bar{W}^{-1}$, $\Lambda = \bar{W}\mathcal{A}^{\mathrm{T}} + \mathcal{A}\bar{W} + \left(\mathcal{B}\bar{\mathcal{K}}\right)^{\mathrm{T}} + \mathcal{B}\bar{\mathcal{K}} + \varepsilon\bar{\Phi}$.

Then, the self-triggered system (6) is said to be stochastically stable and meets the desired multiple range frequency performances. The controller gains are given by $\mathcal{K} = \bar{\mathcal{K}}\bar{W}^{-1}$.

Proof. From GKYP Lemma, low-frequency constraint (7) is equivalent to

$$\begin{pmatrix} \bar{\mathcal{A}} & \mathcal{B}_w \\ I & 0 \end{pmatrix}^{\mathrm{T}} \Xi \begin{pmatrix} \bar{\mathcal{A}} & \mathcal{B}_w \\ I & 0 \end{pmatrix} + \begin{pmatrix} \mathcal{C} & \mathcal{D} \\ 0 & I \end{pmatrix}^{\mathrm{T}} \Omega \begin{pmatrix} \mathcal{C} & \mathcal{D} \\ 0 & I \end{pmatrix} < 0, \quad (19)$$

where $\bar{\mathcal{A}} = \mathcal{A} + \mathcal{B}\mathcal{K}$.

By Schur complement Lemma and reconstruction, condition (16) can be transformed to

$$\Gamma W\Lambda + (\Gamma W\Lambda)^{\mathrm{T}} + (J\Xi J^{\mathrm{T}} + H\Omega H^{\mathrm{T}}) < 0, \quad (20)$$

where
$\Gamma = \begin{bmatrix} -I & \bar{\mathcal{A}} & \mathcal{B}_w \end{bmatrix}^{\mathrm{T}}$, $\Lambda = \begin{bmatrix} 0 & I & 0 \end{bmatrix}$, $J = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}^{\mathrm{T}}$, $H = \begin{bmatrix} 0 & \mathcal{C} & 0 \\ 0 & 0 & I \end{bmatrix}^{\mathrm{T}}$, $\Xi = \begin{bmatrix} -Q_l & P_l \\ P_l & \omega^2 Q_l \end{bmatrix}$, $\Omega = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$.

According to Lemma 3 (see the Appendix), equation (20) is equivalent to

$$\Gamma^{\perp}(J\Xi J^{\mathrm{T}} + H\Omega H^{\mathrm{T}})\Gamma^{\perp\mathrm{T}} < 0, \quad (21)$$

$$\Lambda^{\mathrm{T}\perp}(J\Xi J^{\mathrm{T}} + H\Omega H^{\mathrm{T}})\Lambda^{\mathrm{T}\perp\mathrm{T}} < 0. \quad (22)$$

Combining equations (21) and (22) results in equation (19), which implies the finite frequency index (7) can be ensured by condition (16). That means the self-triggered system (6) meets the required disturbance attenuation level in the low-frequency bands. Similarly, the desired control consumption index (8) could be guaranteed by condition (17).

In the sequel, the stochastic stability of the self-triggered system is proved. Consider Lyapunov candidate function

$$V(q(t)) = q^{\mathrm{T}}(t)Pq(t).$$

For $\tilde{w}(t) = 0$, we have

$$\begin{aligned}\dot{V}(x(t)) &= \dot{q}^{\mathrm{T}}(t)Pq(t) + q^{\mathrm{T}}(t)P\dot{q}(t) \\ &= \eta^{\mathrm{T}}(t)\begin{bmatrix} \bar{\mathcal{A}}^{\mathrm{T}}P + P\bar{\mathcal{A}} & P\mathcal{BK} \\ * & 0 \end{bmatrix}\eta(t),\end{aligned} \tag{23}$$

where $\bar{\mathcal{A}} = \mathcal{A} + \mathcal{BK}$, $\eta^{\mathrm{T}}(t) = \left(q^{\mathrm{T}}(t) \;\, e^{\mathrm{T}}(t_k) \right)$.

According to the self-triggered policy (10), one has the following inequality for $t \in [t_k, t_{k+1})$

$$\dot{V}(q(t)) \leq \eta^{\mathrm{T}}(t)\Theta\eta(t), \tag{24}$$

where $\Theta = \begin{bmatrix} \bar{\mathcal{A}}^{\mathrm{T}}P + P\bar{\mathcal{A}} + \varepsilon\Phi & P\mathcal{BK} \\ * & -\Phi \end{bmatrix}$.

Pre-multiply and post-multiply inequality (18) by $diag\left\{\bar{W}^{-1}, I\right\}$ implies $\Theta < 0$, where $\bar{W}^{-1}{=}P$. Accordingly, $\dot{V}(x(t)) \leq 0$. Thus, it can be deduced that

$$\lim_{t\to\infty} q(t) = 0,$$

which ensures the stability of the closed-loop system (6). This means the original MJS (1) is stochastically stable. This completes the proof.

Remark 3 In Theorem 2, the transition probabilities of original MJSs are fused with extended system matrix $\mathcal{A}$. In this way, the effect of randomness on the finite frequency performance is analyzed. Furthermore, the low and high-frequency performances both included in the proposed algorithm.

4. SIMULATION

To verify the effectiveness and applicability of the proposed method, a two-mode cart-spring system is employed and the system parameters and initial conditions are given in Table 1.

Table 1. System parameters

	Mode 1	Mode 2
A_i	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 2 & 0 & 0 \\ 2 & -2 & 0 & 0 \end{pmatrix}$
B_i	$\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}$	$\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}$
B_{wi}	$\begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}^{\mathrm{T}}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}^{\mathrm{T}}$
C_i	$\begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$
	$\Pi = \begin{pmatrix} -0.8 & 0.8 \\ 0.2 & -0.2 \end{pmatrix}$,	$\omega_l = 2, \omega_h = 10, \varepsilon = 0.2$

By solving LMIs (16)-(18), the controller gains and self-triggering parameters are obtained:

$$K_1 = \left(4.8210 \;\, -4.8932 \;\, -1.8415 \;\, 6.2955 \right),$$

$$K_2 = \left(0.2901 \;\, -0.2462 \;\, 0.3160 \;\, -1.2891 \right),$$

$$\Phi_1 = \begin{pmatrix} 0.4029 & 0.4212 & -0.0428 & 0.1826 \\ 0.4212 & 0.5538 & 0.1318 & -0.1608 \\ -0.0428 & 0.1318 & -0.0206 & -0.2051 \\ 0.1826 & -0.1608 & -0.2051 & 0.3944 \end{pmatrix},$$

$$\Phi_2 = \begin{pmatrix} -0.0510 & 0.0533 & 0.0534 & -0.0588 \\ 0.0533 & -0.0590 & 0.0063 & 0.0141 \\ 0.0534 & 0.0063 & 0.2202 & -0.3008 \\ -0.0588 & 0.0141 & -0.3008 & 0.2316 \end{pmatrix}.$$

The predefined indices are $\gamma = 0.5$, $\rho = 1.45$. The initial condition and extra noise are considered as $x_0 = [0.5 \; 0.2 \; 1.5 \; 1.4]^{\mathrm{T}}$ and $w(t) = 0.1e^{1-t}$, respectively. Putting controller gains into MJS (1) and implementing it by the proposed self-triggered policy with the calculated triggering matrices, Fig. 2 to Fig. 7 show the simulation results of the considered example.

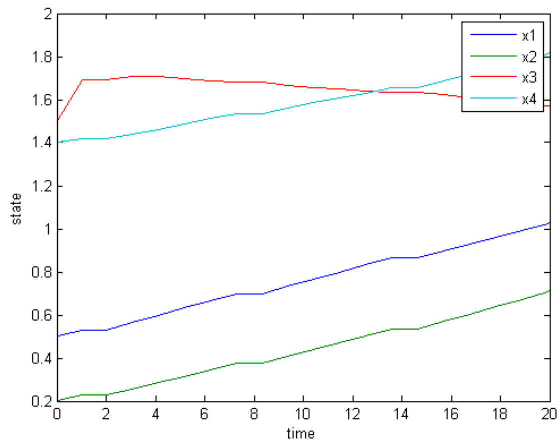

Fig. 1. State responses of the open-loop MJS.

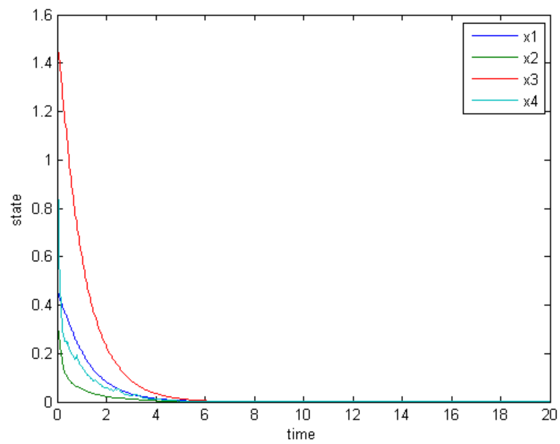

Fig. 2. State responses of the self-triggered MJS.

In particular, the state responses of the open-looped and the controlled system are shown in Fig. 1 and Fig. 2, respectively. It could be seen that the closed-loop system is stable under the designed controller while the free system is unstable. Fig. 3 presents the execution intervals of the self-triggered MJS. We could see that the intervals increase as the state of the controlled system close to the

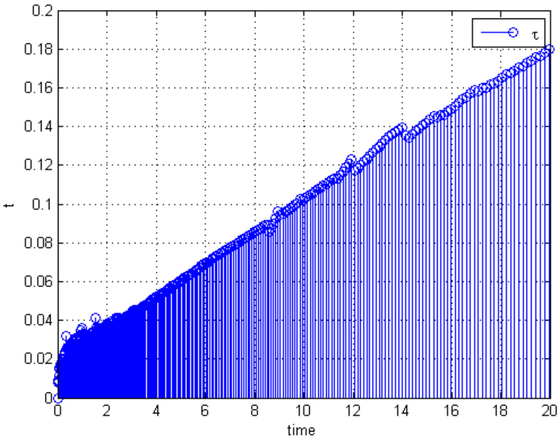

Fig. 3. Execution intervals of the self-triggered policy.

equilibrium point, which decreases the sampling time to save resources.

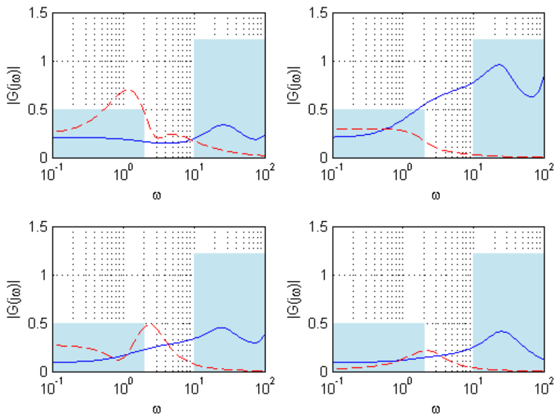

Fig. 4. Frequency responses of the self-triggered MJS.

The frequency responses of the controlled system are shown in Fig. 4. The shadow area represents the controlled energy-consuming constraint and disturbance suppression index in high- and low-frequency bands, respectively. The red dashed line represents the low-frequency performance $|G_{\tilde{z}\tilde{w}}(j\omega)|$ while the blue solid line denotes the high-frequency performance $|G_{\tilde{u}\tilde{w}}(j\omega)|$. It is obvious from Fig. 4 that both the expected low- and high-frequency specifications are guaranteed, which proves the effectiveness of the proposed approach. Moreover, the curve $|G_{\tilde{z}\tilde{w}}(j\omega)|$ does not meet the constraint γ on the high frequency in Fig. 4. This displays the merit of the proposed method that there is no need to satisfy the constraints in full frequency bands to leave some margin for other performance of the controlled system.

5. CONCLUSIONS.

The self-triggered control issue of MJSs under multiple range frequency restrictions is addressed in this paper. The self-triggered strategy is developed to implement the extended system in a resources-saving pattern. Under this scheme, the sampling and updating are self-adjusting according to the degree of the system stability. Then, a self-triggered controller is designed to guarantee the expected multiple range frequency specifications while reduces the resources occupation. Finally, an example is applied to verify the validity of the proposed algorithm.

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Appendix

Lemma 1 (Khalil 2002) (Comparison Lemma) For all $t \geq 0$, consider continuous functions $u(t)$, $v(t)$ and $f(t, u(t))$, if

$$\begin{aligned} \dot{u}(t) &= f(t, u(t)), \quad u(t_0) = u_0, \\ \dot{v}(t) &\leq f(t, v(t)), \quad v(t_0) = v_0, \end{aligned}$$

then, $v(t) \leq u(t)$ for all $t \geq 0$, $v_0 \leq u_0$.

Lemma 2 (Petersen et al. 1986): The following inequality holds for arbitrary real matrices X, Y of appropriate dimensions and any positive scalar $\lambda > 0$,

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leq \lambda X^{\mathrm{T}}X + \lambda^{-1}Y^{\mathrm{T}}Y.$$

Lemma 3 (Iwasaki et al. (1994)) Given matrices Γ, Λ and Ψ with appropriate dimensions and with Ψ symmetrical, there exists a matric F such that

$$\Gamma F \Lambda + (\Gamma F \Lambda)^{\mathrm{T}} + \Psi < 0,$$

if and only if the following holds

$$\Gamma^{\perp}\Psi\Gamma^{\perp\mathrm{T}} < 0, \ \Lambda^{\mathrm{T}\perp}\Psi\Lambda^{\mathrm{T}\perp\mathrm{T}} < 0.$$

Lemma 4 (Iwasaki et al. (2005)): Considering the self-triggered system (6), the following two expressions are equivalent for a symmetric matrix Ω:

1) The inequality of finite frequency meets

$$\begin{pmatrix} G(e^{j\omega}) \\ I \end{pmatrix}^{\mathrm{T}} \Omega \begin{pmatrix} G(e^{j\omega}) \\ I \end{pmatrix} < 0 \quad \forall \omega \in \Theta.$$

2) There exist matrices satisfying

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ I & 0 \end{pmatrix}^{\mathrm{T}} \Xi \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ I & 0 \end{pmatrix} + \begin{pmatrix} \mathcal{C} & \mathcal{D} \\ 0 & I \end{pmatrix}^{\mathrm{T}} \Omega \begin{pmatrix} \mathcal{C} & \mathcal{D} \\ 0 & I \end{pmatrix} < 0,$$

where ω denotes the frequency range, Ξ and Ω takes different values for different frequency ranges and performances.