Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Cooperative Control of A Flywheel Energy Storage System with Identical Damping ⋆

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Abstract: Motivated by the work of Cai and Hu (2018), this paper considers the dual objective control problem of a flywheel energy storage system targeting simultaneous state-of-energy balancing and reference power tracking. It is first shown that, in the presence of flywheel damping, the steady state solution subject to the dual control objective exists and is defined by a virtual dynamic system. Second, under the identical damping condition, it is proven that the control law proposed in Cai and Hu (2018) can still solve the dual objective control problem.

Keywords: cooperative control, flywheel energy storage system, state-of-energy balancing, power tracking.

1. INTRODUCTION

Energy storage systems play a key role in modern power system providing flexibility when there are imbalances between supply and demand, among which flywheel energy storage system (FESS) is one of the most popular energy storage systems (Arani et al., 2017). FESS has energy fed in the rotational mass of a flywheel, stores it as kinetic energy, and releases out upon demand, which not only has high power and energy density, but also realizes energy conversion with high efficiency yet no environmental pollution (Liu and Jiang, 2007; Molina, 2012; Jin, 2007; Mousavi et al., 2017). FESS also has some other advantages including fast response, low maintenance and geographical free (Amiryar and Pullen, 2017), which makes it a promising solution for energy problems.

So far, there have been extensive results for the control of a single flywheel system. In Chang et al. (2014), active disturbance rejection techniques together with a nonlinear control method developed from the traditional PID control was proposed to improve the performance of the flywheel designed for DC microgrids. In Ghanaatian and Lotfifard (2019), based on model predictive control, an optimal nonlinear controller for a flywheel was synthesized against modeling uncertainties and external disturbances. In Nguyen et al. (2011), a novel control scheme using SPWM as well as a boost converter was studied to protect critical loads on distribution feeders. On the other side, there has been less effort on the control of a FESS, which usually employs two or more flywheels in order to provide sufficient power. The result of Elsayed and Mohammed (2014) shows that three small flywheels perform better than one large flywheel. The control scheme for FESS with multiple flywheels may vary depending on the mechanism of cooperation. The centralized control method, or called master-slave arrangement, includes one master flywheel and some slave flywheels operating under the control of the master flywheel. While, the entire FESS might be inoperable when the master unit fails (Hockney et al., 2003). In Sun et al. (2015), a distributed DC-bus signaling based cooperative control strategy was proposed to realize the power balancing of multi paralleled flywheels coupled in a common DC bus. The FESS can adjust its operation automatically according to variation of DC bus voltage. The strategy is able to maintain a stable DC bus voltage and eliminate some adverse effects.

Motivated by the work of Cai and Hu (2018), this paper considers a dual objective control problem of a FESS targeting simultaneous state-of-energy (SOE) balancing and reference power tracking. In particular, on one hand, the SOEs, defined as the ratio of the stored energy and the energy capacity, of all the flywheels should be balanced so as to keep the maximum power capacity of the FESS; on the other hand, the power output of the FESS should track the given reference which is set up by some upper level control. Note that Cai and Hu (2018) considered the fundamental case of general energy storage units, and in contrast, a specific FESS is considered in this paper. It turns out to be much more difficult to solve the dual objective control problem for FESS dual to the damping of flywheels. First, the steady state solution subject to the dual control objective in Cai and Hu (2018) is static and can be identified straightforwardly. However, for the case of FESS, whether the steady state solution exits or not remains unknown. Second, if the steady state solution exists, how to achieve the dual control objective is still challenging. The contribution of this work are twofold. First, we have shown that, in the presence of flywheel damping, the steady state solution subject to the dual control objective exists and is defined by a virtual dynamic system. Second, under the identical damping condition, we have proven that the control law proposed in Cai and Hu (2018) can still solve the dual objective control problem.

⋆ This work was supported in part by the National Natural Science Foundation of China under Grant 61803160, in part by Science Research Programme (General Programme) of Guangzhou under Grant 201904010242, and in part by Student Research Programme of South China University of Technology.

2. PROBLEM FORMULATION

We consider a FESS consisting of N flywheels. The system dynamics of the ith flywheel are given as (Zhang and Yang, 2017):

$$I_i\dot{\omega}_i = -B_{vi}\omega_i + T_{ei} \quad (1)$$

where I_i, ω_i, B_{vi} and T_{ei} denote moment of inertia, angular velocity, damping coefficient and electrical toque, respectively. In here, T_{ei} is taken as the control input for the ith flywheel.

The energy stored in the ith flywheel can be calculated by

$$E_i = \frac{1}{2}I_i\omega_i^2. \quad (2)$$

Define $P_i = -\dot{E}_i$. Then

$$\begin{aligned} P_i &= -I_i\omega_i\dot{\omega}_i \\ &= B_{vi}\omega_i^2 - T_{ei}\omega_i \end{aligned} \quad (3)$$

where the first part $B_{vi}\omega_i^2$ denotes the power loss due to damping, and the second part $-T_{ei}\omega_i$ denotes the power injected into the grid. In what follows, define $P_{i,loss} = B_{vi}\omega_i^2$ and $P_{i,out} = -T_{ei}\omega_i$.

Let $\omega_{i,\min}, \omega_{i,\max}$ denote the minimum and maximum admissible angular velocity of the ith flywheel, respectively. Then the energy capacity of the ith flywheel is given by

$$E_{ci} = \frac{1}{2}I_i\omega_{i,\max}^2. \quad (4)$$

Thus, the SOE of the ith flywheel is given by

$$\phi_i = \frac{E_i}{E_{ci}} = \frac{\omega_i^2}{\omega_{i,\max}^2} = \gamma_i\omega_i^2 \quad (5)$$

where $\gamma_i = 1/\omega_{i,\max}^2$.

By (1) and (5),

$$\begin{aligned} \dot{\phi}_i &= \frac{2\gamma_i}{I_i}(-B_{vi}\omega_i^2 + T_{ei}\omega_i) \\ &= -\frac{2\gamma_i}{I_i}(\frac{B_{vi}}{\gamma_i}\phi_i + P_{i,out}) \\ &= -\frac{2B_{vi}}{I_i}\phi_i - \frac{2\gamma_i}{I_i}P_{i,out}. \end{aligned} \quad (6)$$

Let $P_{FESS} = \sum_{i=1}^{N} P_{i,out}$ denote the power output of the FESS, and $P_{REF} \in R$ denote the reference for P_{FESS}. As in Cai and Hu (2018), to achieve reference power tracking, we introduce a command generator (CG) in the following generic form

$$\dot{x}_0 = f_0(t) \quad (7)$$

whose specific dynamics will be detailed later.

The FESS (1) can be viewed as a multi-agent system Σ_N associated with which we can define a graph[1] $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \{\mathcal{V} \times \mathcal{V}\}$. For $i, j = 1, \ldots, N$, $(i, j) \in \mathcal{E}$ if and only if the jth flywheel can receive the information from the ith flywheel. Adding CG to Σ_N defines an extended multi-agent system Σ_{N+1}. Let node 0 represent the CG and an augmented graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ is defined as $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ and $\bar{\mathcal{E}} = \mathcal{E} \cup \{\{0\} \times \mathcal{V}\}$ where for $i = 1, \ldots, N$, $(0, i) \in \bar{\mathcal{E}}$ if and only if the ith flywheel can receive the information from the CG. Let $\bar{\mathcal{A}} = [a_{ij}] \in R^{(N+1)\times(N+1)}$ be the weighted adjacency matrix of $\bar{\mathcal{G}}$, $\mathcal{L} \in R^{N\times N}$ be the Laplacian of $\mathcal{G}$ and $H = \mathcal{L} + \text{diag}\{a_{10}, \ldots, a_{N0}\}$.

[1] See NOTATION of Cai and Hu (2018) for a summary of graph notation.

Now, the problem considered in this paper can be described as follows.

Problem 1. Given systems (1), (7), and the communication network $\bar{\mathcal{G}}$, find the control input T_{ei}, such that the state of the closed-loop system is bounded and satisfies

$$\lim_{t\to\infty}(P_{FESS}(t) - P_{REF}) = 0 \quad (8)$$

and

$$\lim_{t\to\infty}(\phi_i(t) - \phi_j(t)) = 0 \quad (9)$$

for $i, j = 1, \ldots, N$.

Remark 1. Though Problem 1 can be seen as an extension of the result of Cai and Hu (2018) in that it also considers the SOE balancing and reference power tracking problem, when taken into consideration the specific dynamics of flywheel, an additional damping term $-\frac{2B_{vi}}{I_i}\phi_i$ in Eq. (6) greatly increases the difficulties in solving this problem. In particular, as will be seen shortly, the steady state power output of each flywheel will no longer be static, but follow time-varying trajectory generated by a virtual dynamic system.

Some assumptions for solving Problem 1 are listed as follows.

Assumption 1. For $i = 1, \ldots, N$, $\omega_i(0) \in [\omega_{i,\min}, \omega_{i,\max}]$.

Assumption 2. The graph $\bar{\mathcal{G}}$ contains a spanning tree with the node 0 as its root and the graph $\mathcal{G}$ is connected.

Assumption 3. The graph $\bar{\mathcal{G}}$ contains a spanning tree with the node 0 as its root and the graph $\mathcal{G}$ is undirected and connected.

3. EXISTENCE OF THE SOLUTION

In this section, we will answer the fundamental question that whether the steady state of the FESS exists subject to the dual control objectives (8) and (9). In particular, we have the following result.

Lemma 1. The steady state of the FESS satisfying the following constraints

$$P_{REF} = \sum_{i=1}^{N} P_{i,out} \quad (10a)$$

$$\phi_i = \phi_j,\ i, j = 1, \ldots, N \quad (10b)$$

$$\dot{\phi}_i = \dot{\phi}_j,\ i, j = 1, \ldots, N \quad (10c)$$

exists and is defined by $\phi_i(t) \equiv \phi(t)$ for $i = 1, \ldots, N$ and all $t \geq 0$, where

$$\dot{\phi} = -\frac{2}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}P_{REF} - \frac{2\sum_{j=1}^{N}\frac{B_{vj}}{\gamma_j}}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}\phi. \quad (11)$$

Proof: By equation (6) and (10c), for $j = 2, \ldots, N$, we have

$$\frac{2\gamma_1}{I_1}(P_{1,out} + \frac{B_{v1}}{\gamma_1}\phi) = \frac{2\gamma_j}{I_j}(P_{j,out} + \frac{B_{vj}}{\gamma_j}\phi). \quad (12)$$

Thus,

$$\begin{aligned}P_{j,out} &= \frac{\gamma_1 I_j}{\gamma_j I_1}(P_{1,out} + \frac{B_{v1}}{\gamma_1}\phi) - \frac{B_{vj}}{\gamma_j}\phi \\ &= \frac{\gamma_1 I_j}{\gamma_j I_1}P_{1,out} + \left(\frac{B_{v1} I_j}{\gamma_j I_1} - \frac{B_{vj}}{\gamma_j}\right)\phi \\ &= \frac{\gamma_1 I_j}{\gamma_j I_1}P_{1,out} + \frac{B_{v1} I_j - B_{vj} I_1}{\gamma_j I_1}\phi.\end{aligned} \tag{13}$$

Therefore, by (10a), we have

$$\begin{aligned}P_{REF} &= P_{1,out} + \sum_{j=2}^{N} P_{j,out} \\ &= P_{1,out} + \sum_{j=2}^{N}\frac{\gamma_1 I_j}{\gamma_j I_1}P_{1,out} + \sum_{j=2}^{N}\frac{B_{v1} I_j - B_{vj} I_1}{\gamma_j I_1}\phi \\ &= \sum_{j=1}^{N}\frac{\gamma_1 I_j}{\gamma_j I_1}P_{1,out} + \sum_{j=2}^{N}\frac{B_{v1} I_j - B_{vj} I_1}{\gamma_j I_1}\phi.\end{aligned} \tag{14}$$

In the steady state, for $i = 1, \ldots, N$, by equation (6), we have

$$\dot{\phi} = -\frac{2\gamma_i}{I_i}(P_{i,out} + \frac{B_{vi}}{\gamma_i}\phi). \tag{15}$$

Thus, we have

$$\dot{\phi} = -\frac{2\gamma_1}{I_1}(P_{1,out} + \frac{B_{v1}}{\gamma_1}\phi). \tag{16}$$

Substituting (14) into (16) gives

$$\begin{aligned}\dot{\phi} &= -\frac{2\gamma_1}{I_1}\left(\frac{P_{REF} - \sum_{j=2}^{N}\frac{B_{v1} I_j - B_{vj} I_1}{\gamma_j I_1}\phi}{\sum_{j=1}^{N}\frac{\gamma_1 I_j}{\gamma_j I_1}} + \frac{B_{v1}}{\gamma_1}\phi\right) \\ &= -\frac{2}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}P_{REF} + 2\left(\frac{\sum_{j=2}^{N}\frac{B_{v1} I_j - B_{vj} I_1}{\gamma_j I_1}}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}} - \frac{B_{v1}}{I_1}\right)\phi \\ &= -\frac{2}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}P_{REF} \\ &\quad + 2\frac{\sum_{j=2}^{N}\frac{B_{v1} I_j - B_{vj} I_1}{\gamma_j I_1} - \sum_{j=1}^{N}\frac{B_{v1} I_j}{I_1 \gamma_j}}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}\phi \\ &= -\frac{2}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}P_{REF} - \frac{2\sum_{j=1}^{N}\frac{B_{vj}}{\gamma_j}}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}\phi.\end{aligned} \tag{17}$$

If $\phi_i(t) \equiv \phi(t)$, $i = 1, \ldots, N$, then (10b) and (10c) are satisfied. Moreover,

$$\begin{aligned}\dot{\phi}_i &= -\frac{2B_{vi}}{I_i}\phi - \frac{2\gamma_i}{I_i}P_{i,out} \\ &= -\frac{2}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}P_{REF} - \frac{2\sum_{j=1}^{N}\frac{B_{vj}}{\gamma_j}}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}\phi\end{aligned} \tag{18}$$

and thus

$$P_{i,out} = \frac{I_i}{\gamma_i}\left(\frac{1}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}P_{REF} + \frac{\sum_{j=1}^{N}\frac{B_{vj}}{\gamma_j}}{\sum_{j=1}^{N}\frac{I_j}{\gamma_j}}\phi - \frac{B_{vi}}{I_i}\phi\right) \tag{19}$$

which concludes that $\sum_{i=1}^{N} P_{i,out} = P_{REF}$. □

Remark 2. It might be interesting to check the equilibrium point of system (17), which is

$$\phi^* = -\frac{P_{REF}}{\sum_{j=1}^{N}\frac{B_{vj}}{\gamma_j}}. \tag{20}$$

Clearly, ϕ^* is meaningful only if $\phi^* \in [0, 1]$, i.e., $-\sum_{j=1}^{N}\frac{B_{vj}}{\gamma_j} \leq P_{REF} \leq 0$. On this special equilibrium point, the power absorbed from the grid happens to meet the power loss on the flywheel due to damping. On the other hand, if $P_{REF} > 0$ or $P_{REF} < -\sum_{j=1}^{N}\frac{B_{vj}}{\gamma_j}$, P_{REF} should be reset from time to time so that the SOEs of the flywheels keep to stay in the admissible range.

4. CONTROL LAW DESIGN

In this section, we will show that under certain condition, the control scheme proposed in Cai and Hu (2018) can still solve Problem 1.

4.1 identical system parameters

First, we consider the situation where the system parameters of all the flywheels are identical. For simplicity, for $i = 1, \ldots, N$, let $I_i = I$, $B_{vi} = B_v$, and $\gamma_i = \gamma$.

The CG is designed to be

$$\dot{\zeta}_0 = 0,\ \zeta_0(0) = P_{REF}/N \tag{21}$$

where $\zeta_0 \in R$.

The control law for the ith flywheel is designed to be

$$\dot{\zeta}_i = \mu_\zeta \sum_{j=0}^{N} a_{ij}\,(\zeta_j - \zeta_i) \tag{22a}$$

$$T_{ei} = \frac{1}{\omega_i}\left(k_\phi \sum_{j=1}^{N} a_{ij}(\phi_j - \phi_i) - \zeta_i\right) \tag{22b}$$

where $\zeta_i \in R$ and $\mu_\zeta, k_\phi > 0$. We have the following result.

Theorem 1. Given systems (1) and (21), under Assumptions 1 and 2, Problem 1 can be solved by the control law (22).

Proof: For $i = 1, \ldots, N$, let $\bar{\zeta}_i = \zeta_i - \zeta_0$ and $\bar{\zeta} = \text{col}(\bar{\zeta}_1, \ldots, \bar{\zeta}_N)$ [2]. Then by (21) and (22a), we have

$$\dot{\bar{\zeta}} = -\mu_\zeta H \bar{\zeta}. \tag{23}$$

Under Assumption 1, by Lemma 1 of Su and Huang (2012), all the eigenvalues of H have positive real parts. Therefore,

$$\lim_{t\to\infty} \bar{\zeta}(t) = 0. \tag{24}$$

Substituting (22b) into (6) gives

$$\dot{\phi}_i = -\frac{2B_v}{I}\phi_i + \frac{2\gamma}{I}\left(k_\phi \sum_{j=1}^{N} a_{ij}(\phi_j - \phi_i) - \zeta_i\right). \tag{25}$$

Let $\phi = \text{col}(\phi_1, \ldots, \phi_N)$, $\zeta = \text{col}(\zeta_1, \ldots, \zeta_N)$. Then we have

$$\begin{aligned}\dot{\phi} &= -\frac{2B_v}{I}\phi - \frac{2\gamma k_\phi}{I}\mathcal{L}\phi - \frac{2\gamma}{I}\zeta \\ &= -\frac{2B_v}{I}\phi - \frac{2\gamma k_\phi}{I}\mathcal{L}\phi - \frac{2\gamma}{I}\bar{\zeta} - \frac{2\gamma\zeta_0}{I}1_N.\end{aligned} \tag{26}$$

[2] For $x_i \in R^{n_i}$, $i = 1, \ldots, m$, $\text{col}(x_1, \ldots, x_m) = [x_1^T, \ldots, x_m^T]^T$. $1_n = \text{col}(1, \ldots, 1) \in R^n$. $0_n = \text{col}(0, \ldots, 0) \in R^n$.

Let $\bar{\phi} = \mathcal{L}\phi$. Then we have

$$\begin{aligned}\dot{\bar{\phi}} &= -\frac{2B_v}{I}\mathcal{L}\phi - \frac{2\gamma k_\phi}{I}\mathcal{L}^2\phi - \frac{2\gamma}{I}\mathcal{L}\bar{\zeta} - \frac{2\gamma\zeta_0}{I}\mathcal{L}1_N \\ &= -\frac{2B_v}{I}\bar{\phi} - \frac{2\gamma k_\phi}{I}\mathcal{L}\bar{\phi} - \frac{2\gamma}{I}\mathcal{L}\bar{\zeta} \\ &\triangleq -A_\phi\bar{\phi} - \frac{2\gamma}{I}\mathcal{L}\bar{\zeta}\end{aligned} \tag{27}$$

where $A_\phi = \frac{2B_v}{I}I_N + \frac{2\gamma k_\phi}{I}\mathcal{L}$. Then by Lemma 1.18 of Ren and Cao (2011), all the eigenvalues of A_ϕ have positive real parts and thus A_ϕ is Hurwitz. Since $\lim_{t\to\infty}\bar{\zeta}(t) = 0$, it follows that $\lim_{t\to\infty}\bar{\phi}(t) = 0$. Note that by Lemma 1.3 of Ren and Cao (2011), under Assumption 2, $\bar{\phi} = 0$ if and only if $\phi_i - \phi_j = 0$ for $i, j = 1, \ldots, N$. Therefore,

$$\lim_{t\to\infty}(\phi_i(t) - \phi_j(t)) = 0. \tag{28}$$

Then, by (22b) and (28), noticing that

$$\begin{aligned}P_{FESS} - P_{REF} &= \sum_{i=1}^{N} P_{i,out} - N\zeta_0 \\ &= -k_\phi\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(\phi_j - \phi_i) + \sum_{i=1}^{N}\zeta_i - N\zeta_0 \\ &= -k_\phi\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(\phi_j - \phi_i) + \sum_{i=1}^{N}\bar{\zeta}_i\end{aligned} \tag{29}$$

gives $\lim_{t\to\infty}(P_{FESS}(t) - P_{REF}) = 0$.

□

4.2 non-identical system parameters

Next, we consider the general situation where the system parameters are non-identical. To solve Problem 1, the following assumption is further needed.

Assumption 4. For $i = 1, \ldots, N$, $B_{vi}/I_i = \rho$ where ρ is some constant.

The CG is designed to be

$$\dot{\eta}_0 = \alpha(P_{REF} - P_{FESS}) \tag{30}$$

where $\eta_0 \in R$, $\alpha > 0$.

The control law for the ith flywheel is designed to be

$$\dot{\xi}_i = \sum_{j=1}^{N}a_{ij}(\phi_j - \phi_i) \tag{31a}$$

$$\dot{\eta}_i = \mu_\eta\sum_{j=0}^{N}a_{ij}(\eta_j - \eta_i) \tag{31b}$$

$$T_{ei} = \frac{1}{\omega_i}\left(k_\phi\sum_{j=1}^{N}a_{ij}(\phi_j - \phi_i) + k_\xi\xi_i - k_\eta\eta_i\right) \tag{31c}$$

where $\xi_i, \eta_i \in R$, $\mu_\eta, k_\phi, k_\xi, k_\eta > 0$. We have the following result.

Theorem 2. Given systems (1) and (30), under Assumptions 1, 3 and 4, Problem 1 can be solved by the control law (31).

Proof: Let $\xi_{sum} = \sum_{i=1}^{N}\xi_i$, $\bar{P}_{REF} = P_{REF} + \kappa_\xi\xi_{sum}$, $\tilde{\eta}_i = \eta_i - \bar{P}_{REF}/(\kappa_\eta N)$, $i = 0, 1, \ldots, N$, $\tilde{\eta}_a = \text{col}(\tilde{\eta}_0, \tilde{\eta}_1, \ldots, \tilde{\eta}_N)$. Then by the proof of Theorem 2 of Cai and Hu (2018), the origin of the following system is exponentially stable

$$\dot{\tilde{\eta}}_a = \begin{pmatrix} 0 & -\alpha k_\eta & \cdots & -\alpha k_\eta \\ \mu_\eta a_{10} & & & \\ \vdots & & -\mu_\eta H & \\ \mu_\eta a_{N0} & & & \end{pmatrix}\tilde{\eta}_a \triangleq \bar{H}\tilde{\eta}_a \tag{32}$$

and thus $\bar{H}$ is Hurwitz. Moreover, $\lim_{t\to\infty}(P_{FESS}(t) - P_{REF}) = 0$ exponentially.

Substituting (31c) into (6) gives

$$\begin{aligned}\dot{\phi}_i &= \frac{2\gamma_i}{I_i}(T_{ei}\omega_i - \frac{B_{vi}}{\gamma_i}\phi_i) \\ &= \frac{2\gamma_i}{I_i}\left(k_\phi\sum_{j=1}^{N}a_{ij}(\phi_j - \phi_i) + k_\xi\xi_i - k_\eta\eta_i - \frac{B_{vi}}{\gamma_i}\phi_i\right) \\ &= \frac{2\gamma_i k_\phi}{I_i}\sum_{j=1}^{N}a_{ij}(\phi_j - \phi_i) + \frac{2\gamma_i k_\xi}{I_i}\xi_i - \frac{2\gamma_i k_\eta}{I_i}\eta_i - 2\rho\phi_i.\end{aligned} \tag{33}$$

Let $\Gamma_I = \text{diag}\{\gamma_1/I_1, \ldots, \gamma_N/I_N\}$, $\phi = \text{col}(\phi_1, \ldots, \phi_N)$, $\xi = \text{col}(\xi_1, \ldots, \xi_N)$, $\eta = \text{col}(\eta_1, \ldots, \eta_N)$, $\bar{\xi} = \xi - \frac{\bar{P}_{REF}}{\kappa_\xi N}1_N$, $\tilde{\eta} = \text{col}(\tilde{\eta}_1, \ldots, \tilde{\eta}_N)$. Then

$$\begin{aligned}\dot{\phi} &= -2\rho\phi - 2k_\phi\Gamma_I\mathcal{L}\phi + 2k_\xi\Gamma_I\xi - 2k_\eta\Gamma_I\eta \\ &= -2\rho\phi - 2k_\phi\Gamma_I\mathcal{L}\phi + 2k_\xi\Gamma_I\left(\bar{\xi} + \frac{\bar{P}_{REF}}{k_\xi N}1_N\right) - 2\kappa_\eta\Gamma_I\eta \\ &= -2\rho\phi - 2k_\phi\Gamma_I\mathcal{L}\phi + 2k_\xi\Gamma_I\bar{\xi} - 2k_\eta\Gamma_I\left(\eta - \frac{\bar{P}_{REF}}{k_\eta N}1_N\right) \\ &= -2\rho\phi - 2k_\phi\Gamma_I\mathcal{L}\phi + 2k_\xi\Gamma_I\bar{\xi} - 2k_\eta\Gamma_I\tilde{\eta} \\ &= -2\rho\phi - 2k_\phi\Gamma_I\mathcal{L}\phi + 2k_\xi\Gamma_I\bar{\xi} - 2k_\eta\bar{\Gamma}_I\tilde{\eta}_a\end{aligned} \tag{34}$$

where $\bar{\Gamma}_I = (\, 0_N \;\; \Gamma_I \,)$.

Let $\bar{\phi} = \mathcal{L}\phi$. Then we have

$$\begin{aligned}\dot{\bar{\phi}} &= -2\rho\mathcal{L}\phi - 2k_\phi\mathcal{L}\Gamma_I\mathcal{L}\phi + 2k_\xi\mathcal{L}\Gamma_I\bar{\xi} - 2k_\eta\mathcal{L}\bar{\Gamma}_I\tilde{\eta} \\ &= -2\rho\bar{\phi} - 2k_\phi\mathcal{L}\Gamma_I\bar{\phi} + 2k_\xi\mathcal{L}\Gamma_I\bar{\xi} - 2k_\eta\mathcal{L}\bar{\Gamma}_I\tilde{\eta}\end{aligned} \tag{35}$$

and

$$\dot{\bar{\xi}} = \dot{\xi} = -\mathcal{L}\phi = -\bar{\phi}. \tag{36}$$

Since $\mathcal{L}$ is positive semidefinite under Assumption 3, $\Psi \triangleq \rho\Gamma_I + k_\phi\Gamma_I\mathcal{L}\Gamma_I$ is positive definite. Let $\psi_{\min}$ denote the minimum eigenvalue of Ψ. Since $\bar{H}$ is Hurwitz, let $\bar{\Pi} > 0$ be such that

$$\bar{\Pi}\bar{H} + \bar{H}^T\bar{\Pi} = -\alpha I_N$$

where

$$\alpha = 1 + \frac{||k_\eta\Gamma_I\mathcal{L}\bar{\Gamma}_I||^2}{2\psi_{\min}}.$$

Let

$$V = \frac{1}{4}\bar{\phi}^T\Gamma_I\bar{\phi} + \frac{k_\xi}{2}\bar{\xi}^T\Gamma_I\mathcal{L}\Gamma_I\bar{\xi} + \tilde{\eta}_a^T\bar{\Pi}\tilde{\eta}_a. \tag{37}$$

Then we have

$$\begin{aligned}\dot{V} &= -\bar{\phi}^T(\rho\Gamma_I + k_\phi\Gamma_I\mathcal{L}\Gamma_I)\bar{\phi} + k_\xi\bar{\phi}^T\Gamma_I\mathcal{L}\Gamma_I\bar{\xi} \\ &\quad - k_\eta\bar{\phi}^T\Gamma_I\mathcal{L}\bar{\Gamma}_I\tilde{\eta}_a - k_\xi\bar{\xi}^T\Gamma_I\mathcal{L}\Gamma_I\bar{\phi} - \alpha\|\tilde{\eta}_a\|^2 \\ &\le -\psi_{\min}\|\bar{\phi}\|^2 + \frac{\psi_{\min}}{2}\|\bar{\phi}\|^2 \\ &\quad + (\frac{\|k_\eta\Gamma_I\mathcal{L}\bar{\Gamma}_I\|^2}{2\psi_{\min}} - \alpha)\|\tilde{\eta}_a\|^2 \\ &= -\frac{\psi_{\min}}{2}\|\bar{\phi}\|^2 - \|\tilde{\eta}_a\|^2 \le 0\end{aligned} \quad (38)$$

which in turn implies that $\bar{\phi}$ and $\mathcal{L}^{1/2}\Gamma_I\bar{\xi}$, and therefore $\mathcal{L}\Gamma_I\bar{\xi}$ is bounded, and so is $\ddot{V}$. Then, by Barbalats' Lemma,

$$\lim_{t\to\infty} \dot{V}(t) = 0,$$

and thus $\lim_{t\to\infty} \bar{\phi}(t) = 0$. Again, by by Lemma 1.3 of Ren and Cao (2011), under Assumption 3, $\bar{\phi} = 0$ if and only if $\phi_i - \phi_j = 0$ for $i, j = 1, \ldots, N$. Therefore,

$$\lim_{t\to\infty} (\phi_i(t) - \phi_j(t)) = 0. \quad (39)$$

□

Remark 3. The key to the success of the control laws (22) and (31) lies in that the damping of the flywheels are identical, which in essence eliminates the effect of damping on the SOE balancing objective.

Remark 4. In both control laws (22b) and (31c), it is required that $\omega_i(t) \neq 0$ for all $t \geq 0$, which can be easily guaranteed in practice. On one hand, for flywheels with large inertial, the change of the angular velocities of flywheels is rather slow. On the other hand, the reference angular velocity, as indicated by (17), can be adjusted by resetting P_{REF} from time to time, so that the angular velocities of all the flywheels will remain in the admissible range. Such results can be seen in the numerical simulations presented in the next section.

5. SIMULATION

In this section, we consider an FESS consisting of fifteen flywheels. The communication network associated with the FESS is shown in Fig. 1. The initial SOE for the ith flywheel is given by $\phi_i(0) = 0.9 - 0.005 * (i - 1)$, $i = 1, 2, \ldots, 15$ and let $P_{REF} = 30$kw.

For the case of identical system parameters, we set $B_v = 10^{-4}$, $I = 300$kg·m^2, $\omega_{i,\max} = 10^3$rad/s and the control gains are selected to be $k_\phi = 2 * 10^5$, $\mu_\zeta = 10$. The initial state for the controller is $\zeta_i(0) = 0, i = 1, 2, \ldots, 15$.

For the case of non-identical system parameters, the directed edges of $\mathcal{G}$ are reset to be undirected, and we set $I_i = 300 - 0.5 * (i - 1)$kg·m^2, $I_i/B_{vi} = 3 * 10^6$, $\omega_{i,\max} = 1000 * \sqrt{1 - 0.01 * i}$rad/s, $i = 1, 2, \ldots, 15$. The control gains are selected to be $\alpha = 1$, $\mu_\eta = 1000$, $k_\phi = 2 * 10^5$, $k_\xi = 1$, $k_\eta = 100$. The initial state for the controller is $\eta_0(0) = 0$, $\xi_i(0) = 0$ and $\eta_i(0) = 0$, $i = 1, 2, \ldots, 15$.

The profiles of SOE, power outputs of each flywheel, and the power output error $P_e = P_{REF} - P_{ESS}$ of the entire FESS under control law (22) and (31) are shown in Fig. 2 and Fig. 3, respectively. From the figures we learn, in both cases, reference power tracking and SOE balancing have been achieved simultaneously.

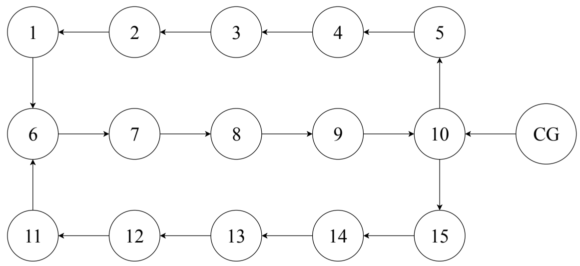

Fig. 1. Communication network.

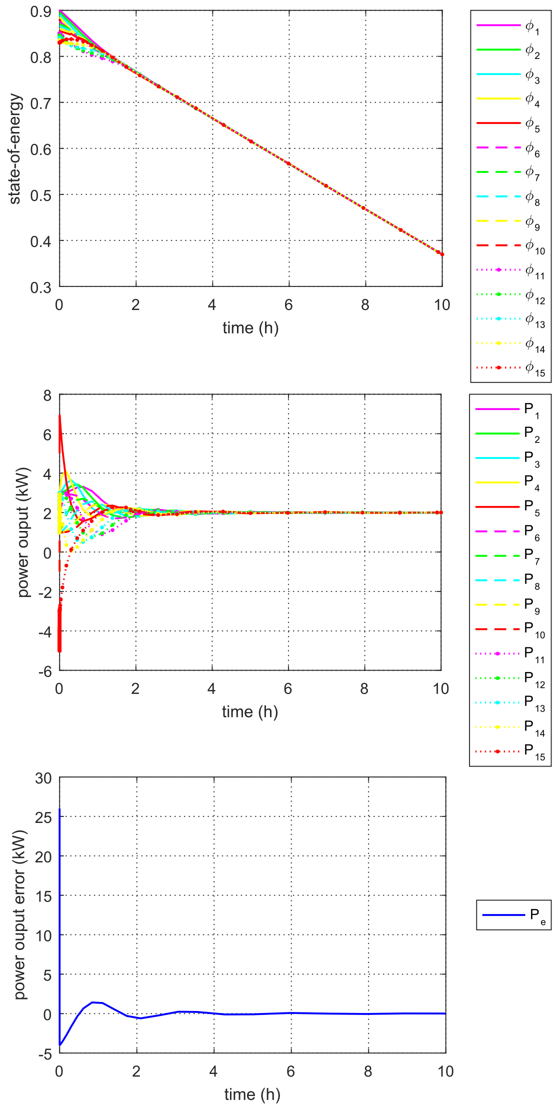

Fig. 2. The case of identical system parameters.

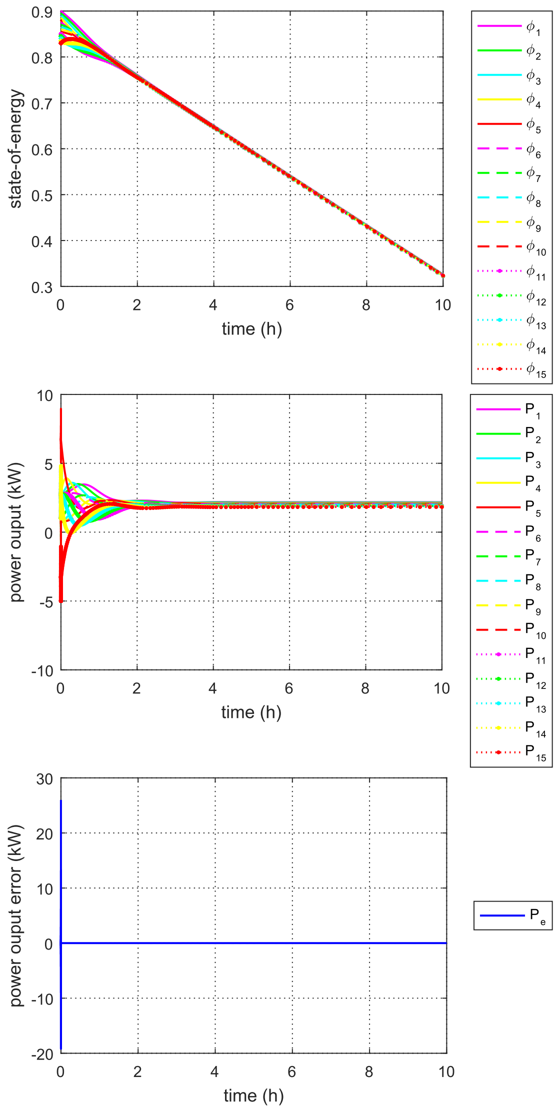

Fig. 3. The case of non-identical system parameters.

6. CONCLUSION

This paper has considered the dual objective control problem of a FESS targeting simultaneous state-of-energy balancing and reference power tracking. It is first shown that, in the presence of flywheel damping, the steady state solution subject to the dual control objective exists and is defined by a virtual dynamic system. Second, under the identical damping condition, it is proven that the control law proposed in Cai and Hu (2018) can still solve the dual objective control problem. In the future, we might further consider the same dual objective control problem without the identical damping assumption.

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