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Event-triggered PI control of time-delay systems with parametric uncertainties ⋆

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Abstract: This paper studies sampled-data implementation of event-triggered PI control for time-delay systems with parametric uncertainties. The systems are given by continuous-time linear systems with parameter uncertainty polytopes. We propose an event-triggered PI controller, in which the controller transmits its signal to the actuator when its relative value goes beyond a threshold. A state-space formulation of the Smith predictor is used to compensate the time-delay. An asymptotic stability condition is derived in the form of LMIs using a Lyapunov-Krasovskii functional. Numerical examples illustrate that our proposed controller reduces the communication load without performance degradation and despite plant uncertainties.

Keywords: PI control, event-triggered control, robust control, sampled-data systems, networked control, time-delay systems, linear matrix inequality

1. INTRODUCTION

Control of process plants using wireless sensors and actuators is of growing interest in process automation industries (Isaksson et al. (2017); Park et al. (2018); Ahlén et al. (2019)). Wireless process control offers advantages through massive sensing, flexible deployment, operation, and efficient maintenance. However, there remains an important problem, which is how to limit the amount of information that needs to be exchanged over the network, since the system performance is critically affected by network-induced delay, packet dropout, and sensor energy shortage.

In this context, event-triggered control has received a lot of attention from both academia and industry as a measure to reduce the communication load in networks (Åström and Bernhardsson (1999); Årzén (1999)). Various event-triggered control architectures appeared recently (see the survey in Heemels et al. (2012) and the references therein). Event-triggered PID control for industrial automation systems is considered in some studies. For example, stability conditions of PI control subject to actuator saturation are derived by Kiener et al. (2014); Moreira et al. (2016). Set-point tracking using the PIDPLUS controller (Song et al. (2006)) is discussed in Tiberi et al. (2012). Experimental validation is carried out in Kiener et al. (2014); Lehmann and Lunze (2011). Implementations on a real industrial plant is presented in Norgren et al. (2012); Lindberg and Isaksson (2015); Blevins et al. (2015).

In process control systems, there are several control architectures, such as feedforward control, cascade control and decoupling control (Åström and Hägglund (2006); Seborg et al. (2010)). Event-triggered control for these architectures has also been proposed. In Iwaki et al. (2018), event-triggered feedforward control is discussed. Event-triggered cascade control and feedforward control where the controllers are switched according to the event generation is proposed in Iwaki et al. (2019). Time-delay compensation is important for process control applications. The Smith predictor (Seborg et al. (2010)) is a widely-used technique for this purpose. The Smith predictor compensates large time-delay by predicting the plant output using a simple plant model. Since its development in Smith (1959), modifications were proposed to apply the predictor to integrator systems (Åström et al. (1994)) and unstable systems (Majhi and Atherton (1999); Sanz et al. (2018)). State predictor-based controllers are also studied in more general settings. In Najafi et al. (2013), the authors propose a sequence of subpredictors to stabilize the plant with a long time-delay. Uncertain time-varying delays are investigated in Léchappé et al. (2018).

In this paper, we investigate event-triggered PI control for time-delay systems. We consider continuous-time linear systems with parametric uncertainties. We introduce sample-data PI control with a predictor, as a state-space formulation of the Smith predictor. A stability condition is derived using a Lyapunov-Krasovskii functional via Wirtinger's inequality (Liu and Fridman (2012)) in the form of Linear Matrix Inequalities (LMIs). An event-triggered controller which updates its signal based on a relative threshold (Heemels and Donkers (2013); Selivanov and Fridman (2018)) is considered. The event threshold synthesis is also proposed. Numerical examples show how our proposed controller reduces the communication load without performance degradation compared to conventional sampled-data PI control.

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The remainder of the paper is organized as follows. Section 2 describes the sampled-data PI control system with the predictor. An asymptotic stability conditions are derived. In Section 3, we propose an event-triggered PI controller and derive a stability condition. We provide numerical examples in Section 4. The conclusion is presented in Section 5.

Notation Throughout this paper, $\mathbb{N}$ and $\mathbb{R}$ are the sets of nonnegative integers and real numbers, respectively. The set of n by n positive definite (positive semi-definite) matrices over $\mathbb{R}^{n\times n}$ is denoted as $\mathbb{S}^n_{++}$ ($\mathbb{S}^n_{+}$). For simplicity, we write $X > Y$ ($X \geq Y$), $X, Y \in \mathbb{S}^n_{++}$, if $X - Y \in \mathbb{S}^n_{++}$ ($X - Y \in \mathbb{S}^n_{+}$) and $X > 0$ ($X \geq 0$) if $X \in \mathbb{S}^n_{++}$ ($X \in \mathbb{S}^n_{+}$). Symmetric matrices of the form $\begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}$ are written as $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ with $B^\top$ denoting the transpose of B.

2. TIME-TRIGGERED PI CONTROL OF TIME-DELAY SYSTEMS

In this paper, we consider a time-invariant linear plant with a process time-delay. The plant is controlled by a sampled-data PI controller with the Smith predictor (Fig. 1). In this section, we introduce the plant with uncertain parameters, the predictor, and the sampled-data PI controller. A stability condition for the closed-loop is derived.

2.1 System description

Consider a plant with a constant process time-delay given by

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t-\eta), \tag{1}$$
$$y(t) = C_p x_p(t), \tag{2}$$

where $x_p(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$, and $y(t) \in \mathbb{R}$ are the state, input, and output, respectively, and the constant $\eta > 0$, a process time-delay. We assume that the sensor samples and transmits its measurement every h time interval. Let $t_k, k = 0, 1, 2, \ldots$, be the time of transmission k of the sensor, i.e., $t_{k+1} - t_k = h$ for all $t > 0$. A sampled-data implementation of a predictor, which updates its state every h time interval, is given by

$$\dot{\hat{x}}_p(t) = \hat{A}_p \hat{x}_p(t_k) + \hat{B}_p u(t) \quad t \in [t_k, t_{k+1}), \tag{3}$$
$$\hat{y}(t) = C_p \hat{x}_p(t_k), \tag{4}$$

where $\hat{x}_p(t) \in \mathbb{R}^n$ and $\hat{y}(t) \in \mathbb{R}$ are the predictions of the plant state and the output. A PI controller is given by

$$\dot{x}_c(t) = r - e(t_k) - \hat{y}(t_k), \quad t \in [t_k, t_{k+1}), \tag{5}$$
$$u(t) = K_i x_c(t_k) + K_p(r - e(t_k) - \hat{y}(t_k)), \tag{6}$$

where $x_c(t) \in \mathbb{R}$ is the controller state, $r \in \mathbb{R}$ the constant reference signal, $e(t) \triangleq y(t) - \hat{y}(t-\eta)$ the prediction error.

We make the following assumptions on the uncertainty of the plant.

Assumption 1. The system matrix A_p and the vector B_p reside in the uncertain polytopes

$$A_p = \sum_{i=1}^{N} \lambda_i A_p^{(i)}, \quad B_p = \sum_{i=1}^{N} \mu_i B_p^{(i)},$$

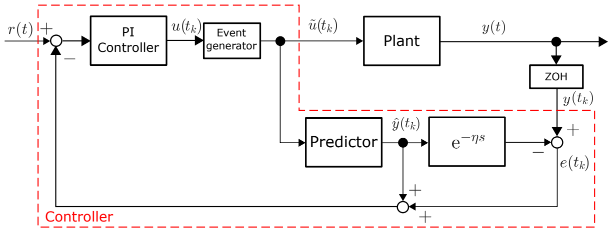

Fig. 1. Event-triggered PI control with the Smith predictor. The event-generator is introduced in Section 3.

where $A^{(i)}$ and $B^{(i)}$, $i \in \mathcal{N}$ are the vertex matrices and vector, respectively, and $\lambda_i, \mu_i \in [0, 1]$, are constants with $\sum_{i=1}^N \lambda_i = 1$ and $\sum_{i=1}^N \mu_i = 1$.

Assumption 2. The system (1)–(2) with the uncertain polytopes is $(A_p^{(i)}, B_p^{(i)})$ controllable and $(C_p, A_p^{(i)})$ observable for all $i = 1, \ldots, N$.

By augmenting the state $x(t) \triangleq [x_p^\top(t+\eta), \hat{x}_p^\top(t), x_c^\top(t)]^\top$, we have the following closed-loop system description

$$\begin{aligned} \dot{x}(t) = {} & Ax(t) + A_1 x(t_k) \\ & + A_2 x(t_k - \eta) + B_R r, \quad t \in [t_k, t_{k+1}) \end{aligned} \tag{7}$$

with

$$A = \begin{bmatrix} A_p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & -B_p K_p C_p & B_p K_i \\ 0 & \hat{A}_p - \hat{B}_p K_p C_p & \hat{B}_p K_i \\ 0 & -C_p & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -B_p K_p C_p & B_p K_p C_p & 0 \\ -\hat{B}_p K_p C_p & \hat{B}_p K_p C_p & 0 \\ -C_p & C_p & 0 \end{bmatrix}, \quad B_R = \begin{bmatrix} B_p K_p \\ \hat{B}_p K_p \\ 1 \end{bmatrix}.$$

Remark 3. Suppose that Assumption 1 holds. Then the matrix A, A_1, A_2, and B_R reside in the uncertain polytope

$$A = \sum_{i\in\mathcal{N}} \lambda_i A^{(i)}, \quad 0 \leq \lambda_i \leq 1, \quad \sum_{i\in\mathcal{N}} \lambda_i = 1,$$
$$A_1 = \sum_{i\in\mathcal{N}} \mu_i A_1^{(i)}, \quad 0 \leq \mu_i \leq 1, \quad \sum_{i\in\mathcal{N}} \mu_i = 1,$$
$$A_2 = \sum_{i\in\mathcal{N}} \mu_i A_2^{(i)}, \quad 0 \leq \mu_i \leq 1, \quad \sum_{i\in\mathcal{N}} \mu_i = 1,$$
$$B_R = \sum_{i\in\mathcal{N}} \mu_i B_r^{(i)}, \quad 0 \leq \mu_i \leq 1, \quad \sum_{i\in\mathcal{N}} \mu_i = 1,$$

where

$$A^{(i)} = \begin{bmatrix} A_p^{(i)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1^{(i)} = \begin{bmatrix} 0 & -B_p^{(i)} K_p C_p & B_p^{(i)} K_i \\ 0 & \hat{A}_p - \hat{B}_p K_p C_p & \hat{B}_p K_i \\ 0 & -C_p & 0 \end{bmatrix},$$

$$A_2^{(i)} = \begin{bmatrix} -B_p^{(i)} K_p C_p & B_p^{(i)} K_p C_p & 0 \\ -\hat{B}_p K_p C_p & \hat{B}_p K_p C_p & 0 \\ -C_p & C_p & 0 \end{bmatrix}, \quad B_R^{(i)} = \begin{bmatrix} B_p^{(i)} K_p \\ \hat{B}_p K_p \\ 1 \end{bmatrix}.$$

2.2 Stability condition of time-triggered PI control

Now, we derive the stability condition of the system (7).

Theorem 4. Consider the system (7). Suppose that Assumption 1 holds. Given K_p, $K_i \in \mathbb{R}$, and decay rate $\alpha > 0$, assume that there exist $P, R_1, R_2, W_1, W_2 \in \mathbb{S}^{2n+1}_{++}$, such that

$$\Phi^{(i)} = \Phi^{(i)\top} = \{\Phi^{(i)}_{\ell m}\} < 0, \quad \ell, m = 1, \ldots, 5, \tag{8}$$

where

$$\Phi_{11}^{(i)} = P(A^{(i)} + A_1^{(i)}) + (A^{(i)} + A_1^{(i)})^\top P + 2\alpha P + R_1 - \mathrm{e}^{-2\alpha\eta} R_2,$$
$$\Phi_{12}^{(i)} = PA_2^{(i)} + \mathrm{e}^{-2\alpha\eta} R_2, \quad \Phi_{13}^{(i)} = PA_1^{(i)}, \quad \Phi_{14}^{(i)} = PA_2^{(i)}$$
$$\Phi_{15}^{(i)} = (A^{(i)} + A_1^{(i)})^\top Q, \quad \Phi_{22}^{(i)} = -\mathrm{e}^{-2\alpha\eta}(R_1 + R_2),$$
$$\Phi_{23}^{(i)} = 0, \quad \Phi_{24}^{(i)} = 0, \quad \Phi_{25}^{(i)} = A_2^{(i)\top} Q,$$
$$\Phi_{33}^{(i)} = -\frac{\pi^2}{4} W_1, \quad \Phi_{34}^{(i)} = 0, \quad \Phi_{35}^{(i)} = A_1^{(i)\top} Q,$$
$$\Phi_{44}^{(i)} = -\frac{\pi^2}{4}\mathrm{e}^{-2\alpha\eta} W_2, \quad \Phi_{45}^{(i)} = A_2^{(i)\top} Q, \quad \Phi_{55}^{(i)} = -Q.$$

and $Q \triangleq \eta^2 R_2 + h^2 \mathrm{e}^{2\alpha h}(W_1 + \mathrm{e}^{-2\alpha\eta} W_2)$ for all $i = 1, \ldots, N$. Then the system (7) with $r = 0$ is exponentially stable with decay rate α.

Proof. See Appendix A.

3. EVENT-TRIGGERED CONTROL OF TIME-DELAY SYSTEMS

In this section, we discuss the event-triggered control introduced in Heemels and Donkers (2013); Selivanov and Fridman (2018) for a time-delay system with parametric uncertainties. We derive a stability condition and propose how to design the event-triggering condition with given control parameters.

3.1 System model of event-triggered PI control

Consider the system

$$\dot{x}_p(t) = A_p x_p(t) + B_p \tilde{u}(t - \eta)$$

where $\tilde{u}(t)$ is the event-triggered control signal. We assume that $\tilde{u}(t)$ is updated by checking the event condition

$$(u(t_k) - \tilde{u}(t_{k-1}))^2 > \sigma u^2(t_k) \tag{9}$$

at every sampling time t_k, $k = 0, 1, \ldots$, where $\sigma \in [0, 1)$ is a relative threshold. Thus, the event-triggered control signal is given by

$$\tilde{u}(t) = \begin{cases} u(t_k), & t \in [t_k, t_{k+1}), \quad \text{if (9) is true,} \\ \tilde{u}(t_{k-1}), & t \in [t_k, t_{k+1}) \quad \text{if (9) is false,} \end{cases}$$

with $\tilde{u}_0 = u(t_0)$. Define the control signal error as

$$\begin{aligned} v_3(t) &\triangleq \tilde{u}(t) - u(t) \\ &= \tilde{u}(t_k) - u(t_k), \quad t \in [t_k, t_{k+1}). \end{aligned}$$

Then the closed-loop system is given by

$$\dot{x}(t) = Ax(t) + A_1 x(t_k) + A_2 x(t_k - \eta) + Bv_3(t) + B_R r \tag{10}$$

where

$$B = \begin{bmatrix} B_p \\ \hat{B}_p \\ 0 \end{bmatrix}.$$

Remark 5. Suppose that Assumption 1 holds. Then B resides in the uncertain polytope

$$B = \sum_{i=1}^{N} \mu_i B^{(i)}, \quad 0 \le \mu_i \le 1, \quad \sum_{i=1}^{N} \mu_i = 1,$$

where

$$B^{(i)} = \begin{bmatrix} B_p^{(i)} \\ \hat{B}_p \\ 0 \end{bmatrix}.$$

3.2 Stability condition of event-triggered PI control

We have the following stability condition.

Theorem 6. Consider the system (10). Suppose that Assumption 1 holds. Given K_p, $K_i \in \mathbb{R}$, and decay rate $\alpha > 0$, assume that there exist $P, R_1, R_2, W_1, W_2 \in \mathbb{S}_{++}^{2n+1}$, $w > 0$, and $\sigma > 0$, such that

$$\Psi^{(i)} = \left[\begin{array}{ccccc|cc} & & & & & PB^{(i)} & w\sigma K_1^\top \\ & & & & & 0 & w\sigma K_2^\top \\ & & \Phi^{(i)} & & & 0 & w\sigma K_1^\top \\ & & & & & 0 & w\sigma K_2^\top \\ & & & & & QB^{(i)} & 0 \\ \hline * & * & * & * & * & -w & 0 \\ * & * & * & * & * & 0 & -w\sigma \end{array}\right] < 0, \tag{11}$$

for all $i = 1, \ldots, N$, where

$$K_1 = [0 \ -K_p C_p \ K_i], \quad K_2 = [-K_p C_p \ K_p C_p \ 0].$$

Then the system (10) with $r = 0$ is exponentially stable with decay rate α.

Proof. See Appendix B.

Remark 7. For $r \neq 0$, we need to apply a coordinate transformation $\tilde{x}(t) = x(t) - x_e$ where $x_e = -(A + A_1 + A_2)^{-1} B_R r$ is the equilibrium point. Note that $A + A_1 + A_2$ is invertible when the continuous controller (i.e., $h = 0$) stabilizes the system and therefore $A + A_1 + A_2$ is Hurwitz. The event condition (9) is replaced by

$$(u(t_k) - \tilde{u}(t_{k-1}))^2 > \sigma(u(t_k) - u_e)^2$$

where $u_e = (K_1 + K_2)x_e$ is the steady-state input signal. Thus, Theorem 6 can be applied if we know the exact model $\hat{A}_p = A_p$ and $\hat{B}_p = B_p$. Otherwise, we use the prediction of A, A_1, A_2, B_R denoted as $\hat{A}, \hat{A}_1, \hat{A}_2, \hat{B}_R$. The prediction matrices are given by replacing A_p, B_p of A, A_1, A_2, B_R by $\hat{A}_p, \hat{B}_p$. The event condition (9) is replaced by

$$(u(t_k) - \tilde{u}(t_{k-1}))^2 > \sigma(u(t_k) - \hat{u}_e)^2$$

where $\hat{u}_e = -(K_1 + K_2)(\hat{A} + \hat{A}_1 + \hat{A}_2)^{-1}\hat{B}_R r$ is the prediction of steady-state input. In this case, the prediction error $e_u \triangleq \hat{u}_e - u_e$ leads to oscillation of $y(t)$ around r.

Using (11), we can tune the event threshold σ to give a minimum communication load satisfying a given stability margin α.

Corollary 8. Suppose that Assumption 1 holds. Given $K_p, K_i \in \mathbb{R}$, $w > 0$, and $\alpha > 0$, if the semi-definite programming problem (SDP):

$$\sigma^* \triangleq \max \quad \sigma \tag{12a}$$
$$\text{s.t.} \quad \Psi^{(i)} < 0, \quad i = 1, \ldots, N, \tag{12b}$$

is feasible, then the closed-loop system (10) under the event condition (9) with σ^* is exponentially stable with decay rate α.

4. NUMERICAL EXAMPLE

In this section, we provide numerical examples to illustrate our theoretical results. Consider a first-order linear system

$$\dot{x}_p(t) = a x_p(t) + b\tilde{u}(t - \eta), \tag{13}$$
$$y(t) = x_p(t), \tag{14}$$

	Comm. until $t = 50$	Comm. Reduction	IAE
EBS ($\eta = 1$)	285	43.0%	3.39
SDS ($\eta = 1$)	500	0%	3.52
SPI ($\eta = 1$)	500	0%	4.52
EBS ($\eta = 3$)	316	36.8%	5.37
SDS ($\eta = 3$)	500	0%	5.43
SPI ($\eta = 3$)	500	0%	$+\infty$
Open-loop	–	–	18.94

Table 1. Number of communications, their reductions, and the IAE for each strategy with $\eta = 1$ and $\eta = 3$.

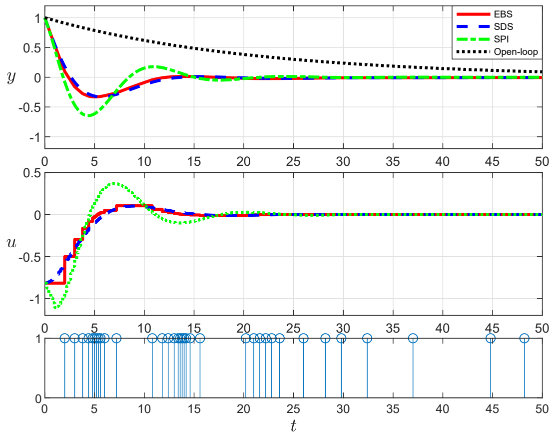

Fig. 2. Responses to the initial state $x(t) = [1, 0, 0]$, $t \in [-1, 0]$ (top: $y(t)$, middle: $u(t)$) of the four cases with time-delay $\eta = 1$: Sampled-data event-triggered PI control with Smith predictor (EBS: red solid line), Sampled-data PI control with Smith predictor (SDS: blue dashed line), Sampled-data PI control without Smith predictor (SPI: green dash-dot line), and open loop system (black dot line). The below plot shows the event generation at the controller.

where the system parameters take their values in $a \in [-0.055, -0.045]$ and $b \in [0.45, 0.55]$. We use a plant model

$$\dot{\hat{x}}_p(t) = -0.05\hat{x}_p(t) + 0.5\tilde{u}(t - \eta).$$

The LMI 8 guarantees the asymptotic stability of the system (13)–(14) with the time-triggered PI control with $K_p = 0.816$, $K_i = 0.293$, the sampling interval $h = 0.2$, the decay rate $\alpha = 0.04$ for the time-delay $\eta \leq 3.4$.

Initial response. We first see the responses of two different time delays $\eta = 1$ and $\eta = 3$ with reference $r = 0$ and the initial state $x(t) = [1, 0, 0]$, $t \in [-3, 0]$. By solving SDP (12), we obtain the event thresholds $\sigma^* = 0.245$ and $\sigma^* = 0.014$ for $\eta = 1$ and $\eta = 3$, respectively. The SDP can be solved effectively by YALMIP toolbox (Löfberg (2004)). To evaluate the system performance, we use the Integral of the Absolute Error (IAE) which is calculated as

$$\text{IAE} = \int_0^{+\infty} |r - y(t)| \text{d}t.$$

The results for three strategies: the sampled-data event-triggered PI control with Smith predictor (EBS), the sampled-data PI control with Smith predictor (SDS), and the conventional sampled-data PI control (SPI) are summarized in Table 1. The responses for $\eta = 1$ and $\eta = 3$

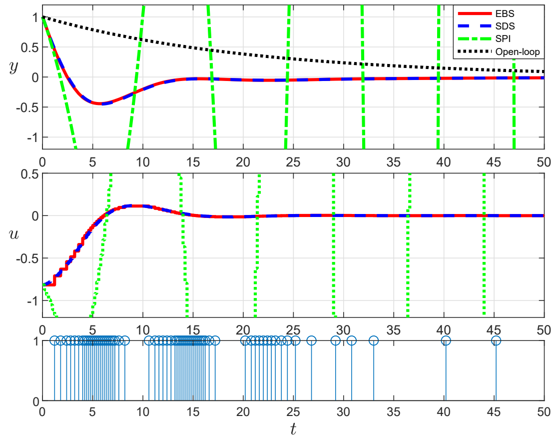

Fig. 3. Responses to the initial state $x(t) = [1, 0, 0]$, $t \in [-3, 0]$ (top: $y(t)$, middle: $u(t)$) of the four cases with time-delay $\eta = 3$. The below plot shows the event generation at the controller.

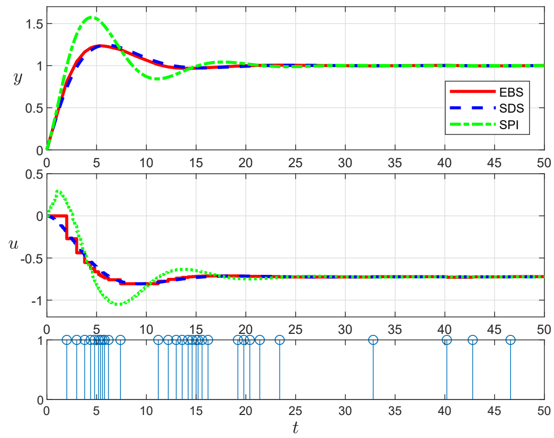

Fig. 4. Responses to the setpoint tracking $r(t) = 1$, $\forall t \geq 0$ of the three cases with time-delay $\eta = 1$ of two The below plot shows the event generation at the controller.

are shown in Fig. 2 and Fig. 3, respectively, where we assume that the unknown system parameters are given by $a = -0.048$ and $b = 0.52$. It can be found that the EBS and the SDS well compensate the time-delay and the outputs converge to the origin. However, the SPI is more oscillative in Fig. 2 ($\eta = 1$) and does not stabilize the system in Fig. 3 ($\eta = 3$). In fact, the IAE for the EBS and the SDS are close as in Table 1, while those for the SPI is larger or diverges. The third plot in Fig. 2 and Fig. 3 show the time instances of the control signal updates. We can see, as well as Table 1, that the communications between the controller and the actuator are performed only 35 times and 66 times until $t = 50$. Including the communications between the sensor and the controller, the EBS reduces the communications by 43.0% and 36.8% compared to the SDS.

Setpoint tracking. Next, we show the responses with $r = 1$. The results are shown in Fig. 4. In setpoint tracking, we need to apply a coordinate transformation $\tilde{x}(t) = x(t) - x_e$ as in Remark 7. In Fig. 4, the EBS has similar response as the SDS with only 30 samplings until $t = 50$, even though there remains very small oscillation due to inexact $\hat{u}_e$.

5. CONCLUSION

In this paper, we investigated the sampled-data implementation of PI control with the Smith predictor. The Smith predictor was formulated as a form of a state predictor. We derived the stability condition under the assumption that the system parameters resided in uncertain polytopes. Furthermore, event-triggered control was introduced and the stability condition was derived. Numerical examples showed that our proposed controller reduces the communication load with slight performance degradation. Future work will consider uncertain time-delays and other types of predictors.

Appendix A. PROOF OF THEOREM 4

Before presenting the proof, we introduce the following lemma.

Lemma 9. (Selivanov and Fridman (2016)) Let $z : [a, b] \to \mathbb{R}^n$ be an absolutely continuous function with a square integrable first order derivative such that $z(a) = 0$ or $z(b) = 0$. Then for any $\alpha > 0$ and $W \in \mathbb{S}^n_{++}$, the following inequality holds:

$$\int_a^b \mathrm{e}^{2\alpha\xi} z^\top(\xi) W z(\xi) \mathrm{d}\xi \leq \mathrm{e}^{2|\alpha|(b-a)} \frac{4(b-a)^2}{\pi^2} \int_a^b \mathrm{e}^{2\alpha\xi} \dot{z}^\top(\xi) W \dot{z}(\xi) \mathrm{d}\xi.$$

Now, we derive the stability condition of the system (7). Consider the functional

$$V = V_0 + V_R + V_{W_1} + V_{W_2} \tag{A.1}$$

where

$$\begin{aligned}
V_0 &\triangleq x^\top(t) P x(t) \\
V_R &\triangleq \int_{t-\eta}^t \mathrm{e}^{2\alpha(s-t)} x^\top(s) R_1 x(s) \mathrm{d}s \\
&\quad + \eta \int_{-\eta}^0 \int_{t+\theta}^t \mathrm{e}^{2\alpha(s-t)} \dot{x}^\top(s) R_2 \dot{x}(s) \mathrm{d}s\mathrm{d}\theta, \\
V_{W_1} &\triangleq h^2 \mathrm{e}^{2\alpha h} \int_{t_k}^t \dot{x}^\top(s) W_1 \dot{x}(s) \mathrm{d}s \\
&\quad - \frac{\pi^2}{4} \int_{t_k}^t \mathrm{e}^{2\alpha(s-t)} [x(s) - x(t_k)]^\top W_1 [x(s) - x(t_k)] \mathrm{d}s, \\
V_{W_2} &\triangleq h^2 \mathrm{e}^{2\alpha h} \int_{t_k-\eta}^t \dot{x}^\top(s) W_2 \dot{x}(s) \mathrm{d}s - \frac{\pi^2}{4} \int_{t_k-\eta}^{t-\eta} \mathrm{e}^{2\alpha(s-t)} \\
&\quad \times [x(s) - x(t_k - \eta)]^\top W_2 [x(s) - x(t_k - \eta)] \mathrm{d}s.
\end{aligned}$$

Using Lemma 9 and $t - t_k \leq h$, we have $V_{W_1} \geq 0$ and $V_{W_2} \geq 0$. We take the derivatives of each term:

$$\begin{aligned}
\dot{V}_0 + 2\alpha V_0 &= x^\top(t) P \dot{x}(t) + \dot{x}^\top(t) P x(t) + 2\alpha x^\top(t) P x(t), \\
&= x^\top(t) \big(P(A + A_1) + P(A + A_1)^\top + 2\alpha P\big) x(t) \\
&\quad + x^\top(t) P A_2 x(t - \eta) + x^\top(t) P A_1 v_1(t) \\
&\quad + x^\top(t) P A_2 v_2(t) + x^\top(t - \eta) A_2^\top P x(t) \\
&\quad + v_1^\top(t) A_1^\top P x(t) + v_2^\top(t) A_2^\top P x(t),
\end{aligned}$$

and

$$\begin{aligned}
&\dot{V}_{W_1} + 2\alpha V_{W_1} = h^2 \mathrm{e}^{2\alpha h} \dot{x}^\top(t) W_1 \dot{x}(t) - \frac{\pi^2}{4} v_1^\top(t) W_1 v_1(t), \\
&\dot{V}_{W_2} + 2\alpha V_{W_2} \\
&\quad = h^2 \mathrm{e}^{2\alpha h} \dot{x}^\top(t) W_2 \dot{x}(t) - \frac{\pi^2}{4} \mathrm{e}^{-2\alpha\eta} v_2^\top(t) W_2 v_2(t),
\end{aligned}$$

where $v_1(t) \triangleq x(t_k) - x(t)$ and $v_2(t) \triangleq x(t_k - \eta) - x(t - \eta)$. For V_R, by Jensen's inequality (Fridman (2014)), we have

$$\begin{aligned}
&\dot{V}_R + 2\alpha V_R \\
&= x^\top(t) R_1 x(t) - \mathrm{e}^{-2\alpha\eta} x^\top(t - \eta) R_1 x(t - \eta) \\
&\quad + \eta^2 \dot{x}^\top(t) R_2 \dot{x}(t) - \eta \int_{t-\eta}^t \mathrm{e}^{-2\alpha(t-s)} \dot{x}^\top(s) R_2 \dot{x}(s) \mathrm{d}s \\
&\leq x^\top(t) R_1 x(t) - \mathrm{e}^{-2\alpha\eta} x^\top(t - \eta) R_1 x(t - \eta) \\
&\quad + \eta^2 \dot{x}^\top(t) R_2 \dot{x}(t) - \mathrm{e}^{-2\alpha\eta} \int_{t-\eta}^t \dot{x}^\top(s) \mathrm{d}s R_2 \int_{t-\eta}^t \dot{x}(s) \mathrm{d}s \\
&= x^\top(t)(R_1 - \mathrm{e}^{-2\alpha\eta} R_2) x(t) + \mathrm{e}^{-2\alpha\eta} x^\top(t) R_2 x(t - \eta) \\
&\quad + \mathrm{e}^{-2\alpha\eta} x^\top(t - \eta) R_2 x(t) + \eta^2 \dot{x}^\top(t) R_2 \dot{x}(t) \\
&\quad - \mathrm{e}^{-2\alpha\eta} x^\top(t - \eta)(R_1 + R_2) x(t - \eta).
\end{aligned}$$

Thus,

$$\dot{V} + 2\alpha V \leq \phi^\top \left(\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & \bar{\Phi}_{13} & \bar{\Phi}_{14} \\ * & \bar{\Phi}_{22} & \bar{\Phi}_{23} & \bar{\Phi}_{24} \\ * & * & \bar{\Phi}_{33} & \bar{\Phi}_{34} \\ * & * & * & \bar{\Phi}_{44} \end{bmatrix} + \begin{bmatrix} \bar{\Phi}_{15} \\ \bar{\Phi}_{25} \\ \bar{\Phi}_{35} \\ \bar{\Phi}_{45} \end{bmatrix} \bar{\Phi}_{55}^{-1} \begin{bmatrix} \bar{\Phi}_{51} & \bar{\Phi}_{52} & \bar{\Phi}_{53} & \bar{\Phi}_{54} \end{bmatrix} \right) \phi < 0$$

where $\phi \triangleq [x^\top(t), x^\top(t - \eta), v_1^\top(t), v_2^\top(t)]^\top$ and

$$\begin{aligned}
&\bar{\Phi}_{11} = P(A + A_1) + (A + A_1)^\top P \\
&\qquad\qquad + 2\alpha P + R_1 - \mathrm{e}^{-2\alpha\eta} R_2, \\
&\bar{\Phi}_{12} = P A_2 + \mathrm{e}^{-2\alpha\eta} R_2, \quad \bar{\Phi}_{13} = P A_1, \quad \bar{\Phi}_{14} = P A_2, \\
&\bar{\Phi}_{15} = (A + A_1)^\top Q, \quad \bar{\Phi}_{22} = -\mathrm{e}^{-2\alpha\eta}(R_1 + R_2), \\
&\bar{\Phi}_{23} = 0, \quad \bar{\Phi}_{24} = 0, \quad \bar{\Phi}_{25} = A_2^\top Q, \\
&\bar{\Phi}_{33} = -\frac{\pi^2}{4} W_1, \quad \bar{\Phi}_{34} = 0, \quad \bar{\Phi}_{35} = A_1^\top Q, \\
&\bar{\Phi}_{44} = -\frac{\pi^2}{4} \mathrm{e}^{-2\alpha\eta} W_2, \quad \bar{\Phi}_{45} = A_2^\top Q, \quad \bar{\Phi}_{55} = -Q.
\end{aligned}$$

The proof completes by Schur complements and since $\bar{\Phi} = \{\bar{\Phi}_{\ell m}\}$, $\ell, m = 1, \ldots, 5$, is affine in A, A_1, and A_2.

Appendix B. PROOF OF THEOREM 6

First, note that by the event condition (9), for some $w \geq 0$, we have $w\sigma u^2(t_k) - w v_3^2(t) \geq 0$. Introducing the functional (A.1) gives

$$\begin{aligned}
\dot{V} + 2\alpha V &\leq \phi^\top \bar{\Phi}_{1:4} \phi + x^\top(t) P B v_3(t) + v_3^\top(t) B^\top P x(t) \\
&\quad + \dot{x}^\top(t) Q \dot{x}(t) + w\sigma u^2(t_k) - w v_3^2(t) \\
&= \psi^\top \left[\begin{array}{c|c} \bar{\Phi}_{1:4} & \begin{matrix} PB \\ 0 \\ 0 \\ 0 \end{matrix} \\ \hline \begin{matrix} B^\top P & 0 & 0 & 0 \end{matrix} & -w \end{array} \right] \psi + \dot{x}^\top(t) Q \dot{x}(t) + w\sigma u^2(t_k),
\end{aligned}$$

where $\psi = [x^\top(t), x^\top(t - \eta), v_1^\top(t), v_2^\top(t), v_3^\top(t)]^\top$ and $\bar{\Phi}_{1:4}$ is the submatrix obtained by omitting the 5-th row and column vectors from $\bar{\Phi}$. Since $u(t_k) = K_1 x(t_k) + K_2 x(t_k - \eta)$ and by Schur complements, we have that $\dot{V} + 2\alpha V < 0$ if

$$\bar{\Psi} \triangleq \left[\begin{array}{c:cc} & PB & w\sigma K_1^\top \\ & 0 & w\sigma K_2^\top \\ \bar{\Phi} & 0 & w\sigma K_1^\top \\ & 0 & w\sigma K_2^\top \\ & QB & 0 \\ \hdashline * * * * * & -w & 0 \\ * * * * * & 0 & -w\sigma \end{array}\right] < 0.$$

The proof completes since $\bar{\Psi}$ is affine in A, A_1, A_2, and B.

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