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Fault Detection for Switched Systems based on Pole Assignment and Zonotopic Residual Evaluation

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Abstract: This paper considers fault detection problem for a class of discrete-time switched systems with actuator faults. Pole assignment technique and H_∞ design are used to develop the fault sensitivity and the disturbance attenuation condition of the residual, respectively. The design conditions of the observer are derived in terms of Linear Matrix Inequalities (LMIs). In addition, the disturbances and measurement noises are supposed to be unknown but bounded by zonotopes. A zonotopic method is then presented to evaluate the residual. A numerical example is performed to illustrate the effectiveness of the proposed method through a comparison with the results obtained without considering sensitivity and robustness analysis.

Keywords: Switched systems, fault detection, pole assignment, zonotopic residual evaluation.

1. INTRODUCTION

Due to the rising demand for higher performance and safety, fault diagnosis for dynamic systems is becoming a major technological challenging issue in many engineering fields, such as automotive (Polverino et al. (2017)) and aerospace (Zolghadri (2012), Zolghadri et al. (2016)). Occurrence of faults cannot only make the system unable to reach a number of expected goals but may also lead to severe and irreversible damage to human operator and equipment if they are not detected timely. In the past decades, different approaches have been widely investigated in the field of fault diagnosis such as Zolghadri et al. (2014), Wang et al. (2017b), Zammali et al. (2020) and applied successfully for various systems. In Henry et al. (2015), LPV model-based fault detection schemes are introduced. In Tang et al. (2018), fault detection and isolation are investigated for linear descriptor systems subject to actuator faults. In Buciakowski et al. (2017), the authors propose a new fault estimation approach for uncertain nonlinear systems where both unknown input and actuator faults are considered.

In the literature, a considerable attention has been devoted to fault detection for switched systems. The interest of studying switched systems drives from the facts that they can model and control various physical and engineering systems. This class of systems involves a finite number of subsystems (modes) and a discrete rule which governs the switching among them. Several contributions have been reported to deal with fault detection for a class of switched systems. In Zhong et al. (2015), using the parity space-based approach, the fault detection issue is considered for discrete-time switched linear systems under dwell time constraints. In Rios et al. (2014), a fault detection method based on a high-order sliding-mode observer and a fault identification scheme are proposed. Without the use of decoupling transformations, a residual signal, is designed in Van Gorp et al. (2015) using reduced order sliding mode observers in order to detect faults which occur in the continuous part of the switched system. In the aforementioned references, only actuator faults are considered. In Johnson et al. (2018), a fault estimation scheme is proposed in the presence of simultaneous actuator and sensor faults.

From a practical point of view, the presence of state disturbances and measurement noise can affect the performance of fault detection. Accordingly, robust fault detection approaches are needed in order to deal with false alarm issue. The main idea of robust fault detection approach consists in generating a residual signal as a symptom of faults occurrences. It is required that the generated residual signal should be robust against uncertainties and sensitive to faults. Among several robust fault detection methods, one can find the H_∞ criterion which is considered in Belkhiat et al. (2011) to develop a robust hybrid observer for switched linear systems subject to unknown inputs. The proposed technique allows getting a proper compromise between sensitivity to faults and robustness to the unknown inputs. Based on a mixed H_-/H_∞ formulation, authors in Zhai et al. (2016), Farhat and Koenig (2014) investigate the fault detection issue for switched systems to minimize the effect of disturbance and simultaneously maximize the sensitivity of fault on the residual signal. In the above works, in order to evaluate residuals, a constant, an adaptive threshold or a time-varying one should be computed. It should be mentioned that the design of an appropriate threshold is still a challenge and an arduous task especially for dynamic systems subject to exogenous disturbances. Under the assumption that uncertainties are unknown but bounded, set-membership fault detection approach is used in Ethabet et al. (2019) for continuous-time switched linear systems to handle the effect of uncertainties. The interval observer has additional constraints compared with a conventional one. In fact, the design of interval observers requires not only the stability but also the positivity of the observer errors. Based on coordinates transformation, the cooperativity property is ensured in Ethabet et al. (2019). Although this property is relaxed, a robust observer gain can not be designed simultaneously with the coordinates transformation matrix to obtain a tighter interval width of the residual signal. To remove the above restrictions, a fault detection technique for a class of discrete-time switched systems with actuator faults is considered by combining the benefits of the H_∞ criterion, a pole assignment approach and a zonotope-based residual evaluation.

In the present work, the FD decision is based on zonotopic residual evaluation methods. It is performed by deciding wether the residual signal is excluded from the residual zonotope when the fault occurs. In recent years, the issue of fault diagnosis using zonotopic techniques has been an active research area. In Wang et al. (2017a), a zonotopic fault detection observer is proposed for discrete-time uncertain system taking into account the H_- fault sensitivity. In Li et al. (2019), a fault detection approach is developed using the zonotopic set-membership theory in the finite-frequency domain. In Tang et al. (2019), a two-step interval estimation approach is introduced for discrete-time linear systems by merging reachability analysis with robust design. In the aforementioned references, the interval estimations are obtained by a zonotope-based method. Instead of improving the accuracy of the residual boundaries, the main contribution of using zonotopic technique consists in providing systematic dynamic thresholds for residual evaluation.

To the best of the authors' knowledge, robust fault detection scheme for switched systems using set-membership approach has not been fully investigated. Inspired by the aforementioned discussions, this paper proposes a new technique to cope with fault detection for discrete-time switched systems subject to disturbances. It is supposed that state disturbances and measurement noises are unknown but bounded by zonotopes. Using a zonotopic approach, non-negativity conditions needed to design classical interval observers (Zammali et al. (2019)) are not considered in this paper. The developed technique is based on H_∞ performance to attenuate the effects of uncertainties and a novel pole assignment approach to increase the sensitivity to faults. LMI conditions are proposed in this paper in order to compute observer gains. Fault detection decision is established through a consistency test of the residual signal.

The paper is structured as follows. Notations and preliminaries about zonotopes are presented in Section 2. In Section 3, the system description and problem statement are provided. The design of a fault detection observer is introduced in Section 4. Then, a zonotopic residual evaluation is described in Section 5. In Section 6, simulation results are shown to illustrate the efficiency of the proposed approach. Finally, the paper is concluded in Section 7.

2. PRELIMINARIES

Notation. In the sequel, the following notations are used. $\mathbb{R}$, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote respectively the sets of real numbers, n dimensional and $m\times n$ dimensional Euclidean space. For a signal $x\in\mathbb{R}^n$, $\|x\|$ represents its $\mathscr{L}_2$-norm. We denote by $\mathscr{I}=\overline{1,N}, N\in\mathbb{Z}_+$ the set of non-negative integers $\{1,...,N\}$. $\mathbb{Z}$ is used to denote the set of all integers. In symmetric block matrices, $(*)$ denotes the terms introduced by symmetry. I_n represents the identity matrix. For a matrix P, $P\succ 0$ is equivalent that P is positive definite, $P^\dagger$ represents its pseudo-inverse. Throughout this paper, the following inequalities $\leq$, $\geq$, $<$ and $>$ should be interpreted elementwise.

The following definitions, lemma and property are introduced in this manuscript.

Definition 1. Consider two sets $\mathcal{X}$ and $\mathcal{Y}$. The Minkowski sum of $\mathcal{X}$ and $\mathcal{Y}$ is defined as:

$$\mathcal{X}\oplus\mathcal{Y}=\{x+y, x\in\mathcal{X}, y\in\mathcal{Y}\}$$

Definition 2.(Li et al. (2019)) The linear product of a matrix $K\in\mathbb{R}^{m\times n}$ and a set $\mathcal{X}\in\mathbb{R}^n$ is denoted as $\odot$ and defined as:

$$K\odot\mathcal{X}=\{Kx, x\in\mathcal{X}\}$$

Definition 3. An s-order zonotope $\mathbf{Z}$ represents the affine image of a hypercube $\mathbb{B}^s=[-1,\,1]^s$ as follows:

$$\mathbf{Z}=\langle p,H\rangle=p+H\mathbb{B}^s=\{p+Hz, z\in\mathbb{B}^s\}$$

$p\in\mathbb{R}^n$ denotes the center of $\mathbf{Z}$ and $H\in\mathbb{R}^{n\times s}$ is the generation matrix of $\mathbf{Z}$, which represents the shape of $\mathbf{Z}$.

A lower zonotope can bound a high-dimensional zonotope using the reduction operation. The following property is given to describe the reduction operator.

Property 1. The reduction operator for a zonotope can be described as the following form $\mathbf{Z}=\langle p,H\rangle\subseteq\langle p,\downarrow_l(H)\rangle$ where $\downarrow_l(H)$ represents the complexity reduction operator with $n\leq l\leq s$ denotes the maximum number of columns of generator matrix H. For the detailed algorithm, please refer to Combastel (2003).

Lemma 1. (Wang et al. (2015)) Let $A\in\mathbb{R}^{a\times b}$, $B\in\mathbb{R}^{b\times c}$ and $C\in\mathbb{R}^{a\times c}$, if $\mathrm{rank}(B)=c$, hence the solution of the equation $AB=C$ is given by

$$A=CB^\dagger+S(I-BB^\dagger)$$

where $S\in\mathbb{R}^{a\times b}$ represents an arbitrary matrix.

3. PROBLEM FORMULATION

Consider the following discrete-time switched system:

$$\begin{cases} x_{k+1} &= A_q x_k + B_q u_k + W w_k + F_q f_k \\ y_k &= C x_k + E v_k \end{cases} \tag{1}$$

where $x\in\mathbb{R}^{n_x}$, $y\in\mathbb{R}^{n_y}$, $u\in\mathbb{R}^{n_u}$, $f\in\mathbb{R}^{n_f}$, $w\in\mathbb{R}^{n_w}$ and $v\in\mathbb{R}^{n_v}$ are respectively the state vector, the input, the output, the actuator fault, the state disturbance and the measurement noise. The known matrices A_q, B_q, C, W, E and F_q are given with appropriate dimensions. The index q defines the active subsystem. $q\in\mathscr{I}=\overline{1,N}, N\in\mathbb{Z}_+$, N denotes the number of linear subsystems (modes). The switching signal is assumed to be known in this paper.

The following assumption is introduced.

Assumption 1. The initial state x_0, the measurement noise v_k and the state disturbance w_k are supposed to be unknown but bounded such that:

$$x_0\in\langle p_0,H_0\rangle,\quad w_k\in\langle 0,H_w\rangle,\quad v_k\in\langle 0,H_v\rangle \tag{2}$$

where $H_0=\mathrm{diag}(\overline{x})$, $H_w=\mathrm{diag}(\overline{w})$ and $H_v=\mathrm{diag}(\overline{v})$. The symbol diag(.) is represented by a diagonal matrix where the diagonal terms are given as the arguments of diag.

In the following, the aim is to design a FD approach for the discrete-time switched systems defined by (1). The proposed approach is robust against the state disturbances and the measurement noise, meanwhile sensitive to fault. The fault sensitivity is dealt with a novel pole placement technique. For the proposed fault detection observer (FDO), the residual evaluation is made using zonotopic approaches.

4. FAULT DETECTION OBSERVER DESIGN

In this section, we propose a FDO for the system (1):

$$\begin{cases} \hat{x}_{k+1} = A_q\hat{x}_k + B_q u_k + L_q(y_k - C\hat{x}_k) \\ r_k = y_k - C\hat{x}_k \end{cases} \tag{3}$$

where $\hat{x}_k$ is the estimation of x_k and $L_q \in \mathbb{R}^{n_x \times n_y}$, $q \in \{1,...,N\}$, are the observer gains to be designed. The goal is to compute the FDO gains L_q that improve the sensitivity to fault and minimize the H_∞ performance of the transfer function for each mode from disturbances to the residual vector r_k. By defining the estimation error as $e_k = x_k - \hat{x}_k$ and the residual signal as $r_k = y_k - \hat{y}_k$, the error dynamics are given by:

$$\begin{cases} e_{k+1} &= (A_q - L_qC)e_k + F_q f_k + W w_k - L_q E v_k \\ r_k &= Ce_k + Ev_k \end{cases} \tag{4}$$

The error dynamics (4) can be split into two subsystems where the subsystem (5) is decoupled from the effects of f_k and the subsystem (6) is only affected by the actuator fault.

$$\begin{cases} e^d_{k+1} &= (A_q - L_qC)e^d_k + W w_k - L_q E v_k \\ r^d_k &= Ce^d_k + Ev_k \end{cases} \tag{5}$$

$$\begin{cases} e^f_{k+1} &= (A_q - L_qC)e^f_k + F_q f_k \\ r^f_k &= Ce^f_k \end{cases} \tag{6}$$

where $e_k = e^f_k + e^d_k$, $e^f_0 = 0$ and $e^d_0 = 0$.
The reason behind splitting the error dynamics given in (4) is to handle the nonzero condition in fault sensitivity analysis.
For efficient FD, it is required that the sensitivity to fault on the residual vector should be improved. Herein, since the error dynamics (4) can be split and only the subsystem (6) is affected by the additive fault, a pole placement technique is considered for (6). The FDO gains L_q are designed to improve fault sensitivity on residual signal.
In the FD scheme, we design the observer gain matrices L_q such that:

$$(A_q - L_qC)F_q = \lambda F_q \tag{7}$$

where λ is a scalar satisfying $0 < \lambda < 1$.
In fact, if the condition (7) holds, it can be easily shown from (6) that:

$$e^f_k = \lambda^{k-1}F_q f_0 + ... + \lambda F_q f_{k-2} + F_q f_{k-1}$$

It follows from $r^f_k = Ce^f_k$ that:

$$r^f_k = \lambda^{k-1}CF_q f_0 + ... + \lambda CF_q f_{k-2} + CF_q f_{k-1} \tag{8}$$

From (8), it is worth noting that r_k depends on a weighting scalar λ. Thus, in order to improve fault sensitivity, it is required to increase the value of λ as much as possible and to let it closer to 1. To that end, a pole assignment method is considered. Based on Lemma 1, the FDO gains L_q can be obtained by solving (7). They are given by:

$$L_q = (A_qF_q - \lambda F_q)(CF_q)^\dagger + S(I - CF_q(CF_q)^\dagger) \tag{9}$$

where $S \in \mathbb{R}^{n_x \times n_y}$ is a matrix to be designed.
Then, from (9), an H_∞ criterion is used to attenuate state disturbances and measurement noises. Based on the bounded real lemma (Boyd et al. (1994)), the performance

$$||r^d|| < \gamma\sqrt{(||w||^2 + ||v||^2)} \tag{10}$$

is considered. Sufficient conditions are given in terms of LMIs in the following theorem.

Theorem 1. For a given scalar $\gamma > 0$, the error system (5) is stable and satisfies the H_∞ performance, if there exist a positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$ and $Q_q \in \mathbb{R}^{n_x \times n_y}$ such that:

$$\begin{bmatrix} -P + C^TC & * & * & * \\ 0 & -\gamma^2 I_n & 0 & * \\ E^TC & 0 & E^TE - \gamma^2 I_n & * \\ PA_q - Q_qC & PW & -Q_qE & -P \end{bmatrix} \prec 0 \tag{11}$$

Moreover, the FDO gains L_q are given by:

$$L_q = P^{-1}Q_q \tag{12}$$

Proof. To prove the stability of (5) and the H_∞ performance (10), sufficient conditions are given in terms of LMIs using the bounded real lemma and based on a common quadratic Lyapunov function given by $V(e^d_k) = e^{d\,T}_k P e^d_k$, $P^T = P \succ 0$, such that:

$$\Delta V(e^d_k) + r^{d\,T}_k r^d_k - \gamma^2 w_k^T w_k - \gamma^2 v_k^T v_k < 0 \tag{13}$$

One can obtain from (13)

$$\begin{aligned} V(e^d_{k+1}) - V(e^d_k) + r^{d\,T}_k r^d_k - \gamma^2 w_k^T w_k - \gamma^2 v_k^T v_k &< 0 \\ e^{d\,T}_{k+1} P e^d_{k+1} - e^{d\,T}_k P e^d_k + r^{d\,T}_k r^d_k - \gamma^2 w_k^T w_k - \gamma^2 v_k^T v_k &< 0 \end{aligned}$$

Then, according to (5), the following inequality can be obtained:

$$[e^{d\,T}_k \quad w^T_k \quad v^T_k]^T \Sigma_q [e^{d\,T}_k \quad w^T_k \quad v^T_k] < 0 \tag{14}$$

where $\Sigma_q =$

$$\begin{bmatrix} \Theta_{11} & * & * \\ \Theta_{21} & \Theta_{22} & * \\ \Theta_{31} & \Theta_{32} & \Theta_{33} \end{bmatrix}$$

and

$$\begin{aligned} \Theta_{11} &= (A_q - L_qC)^T P (A_q - L_qC) + C^TC - P \\ \Theta_{21} &= W^T P (A_q - L_qC) \\ \Theta_{22} &= W^T P W - \gamma^2 I_n \\ \Theta_{31} &= -(L_qE)^T P (A_q - L_qC) + E^TC \\ \Theta_{32} &= -(L_qE)^T P W \\ \Theta_{33} &= (L_qE)^T P (L_qE) + E^TE - \gamma^2 I_n \end{aligned}$$

The inequality (14) can be rewritten as:

$$\begin{bmatrix} -P + C^TC & * & * \\ 0 & -\gamma^2 I_n & * \\ E^TC & 0 & E^TE - \gamma^2 I_n \end{bmatrix} - \Omega^T P \Omega \prec 0 \tag{15}$$

with $\Omega = [(A_q - L_qC) \quad W \quad (L_qE)]$.
From (15) and based on the Schur complement, it is easy to show that (11) implies (14). The observer gains are deduced with $Q_q = PL_q$. The stability and the H_∞ are satisfied.

In the following, robustness against disturbances and sensitivity to fault are investigated. Convenient LMI conditions are derived and given in the following theorem by means of the H_∞ performance criterion and a novel pole placement method.

Theorem 2. For a given value of λ with $0 < \lambda < 1$, if there exist a symmetric positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$ and $Y \in \mathbb{R}^{n_x \times n_y}$ such that:

$$\begin{bmatrix} \Psi_{11} & * & * & * \\ 0 & \Psi_{22} & * & * \\ \Psi_{31} & 0 & \Psi_{33} & * \\ \Psi_{41} & \Psi_{42} & \Psi_{43} & \Psi_{44} \end{bmatrix} \prec 0 \tag{16}$$

where

$$\begin{aligned}
\Psi_{11} &= -P + C^T C \\
\Psi_{22} &= -\gamma^2 I_n \\
\Psi_{31} &= E^T C \\
\Psi_{33} &= E^T E - \gamma^2 I_n \\
\Psi_{41} &= PA_q - P(A_q F_q - \lambda F_q)(CF_q)^\dagger C - Y(CF_q)^\perp C \\
\Psi_{42} &= PW \\
\Psi_{43} &= -(P(A_q F_q - \lambda F_q)(CF_q)^\dagger C - Y(CF_q)^\perp)E \\
\Psi_{44} &= -P
\end{aligned}$$

$Y = PS$, $(CF_q)^\perp = (I - CF_q(CF_q)^\dagger)$, $L_q = (A_qF_q - \lambda F_q)(CF_q)^\dagger + S(CF_q)^\perp$ and $S \in \mathbb{R}^{n_x \times n_y}$ is an arbitrary matrix, the following statements hold true:

(1) The stability of the observer (3) and the H_∞ performance (10) are satisfied.
(2) The sensitivity of the residual r_k to the fault is improved.

Moreover, if (16) is solvable, the optimal FDO gain matrices can be determined by:

$$L_q = (A_q F_q - \lambda F_q)(CF_q)^\dagger + S(CF_q)^\perp \tag{17}$$

and computed by resolving the following minimization problem:

$$\begin{aligned} &\min_{L_q} \gamma^2, \quad q = 1, ..., N \\ &s.t. \quad (16) \end{aligned} \tag{18}$$

Proof. Consider the common Lyapunov function defined by $V(e_k^d)$ where $V(e_k^d) = e_k^{d^T} P e_k^d$, $P^T = P \succ 0$. The common Lyapunov function $V(e_k^d)$ satisfies the following relation $||r^d|| < \gamma\sqrt{(||w||^2 + ||v||^2)}$ where γ is a positive scalar. The H_∞ index γ is minimized to attenuate the effect of disturbance by minimizing the cost function (18). Besides, the result can be easily achieved by allowing $L_q = (A_qF_q - \lambda F_q)(CF_q)^\dagger + S(CF_q)^\perp$ and using Theorem 1. In fact, by substituting the expression of L_q in (11), one can obtain (16). Note that λ is chosen such that $0 < \lambda < 1$. Robustness against disturbances and sensitivity to fault are then improved.

5. ZONOTOPIC RESIDUAL EVALUATION

The FD decision is evaluated based on zonotopic technique. It is based on determining whether the residual signal r_k, defined in (4), is excluded from the residual zonotope $\mathbf{R_k}$ or not. The FD decision scheme is obtained as follows:

$$\begin{cases} r_k \in \mathbf{R_k} & \text{Fault-free} \\ r_k \notin \mathbf{R_k} & \text{Faulty} \end{cases} \tag{19}$$

The fault detection decision can be checked by solving the following constraint problem:

$$\begin{aligned} &R_k z_i = r_k \\ &s.t. \qquad -1 \le z_i \le 1 \quad \forall i = 1, \ldots, l. \end{aligned} \tag{20}$$

where R_k denotes the generation matrix of the residual zonotope $\mathbf{R_k}$ and z_i denotes the ith element of uncertain vector z bounded by hypercube $\mathbb{B}^l$ (the residual zonotope $\mathbf{R_k}$ is reduced to dimension l).

Theorem 3. The residual signal r_k, defined in (4), is bounded by the zonotope $\mathbf{R_k} = \langle 0, R_k \rangle$. R_k satisfies the following iteration equation:

$$\begin{cases} R_k &= [CH_k \quad EH_v] \\ H_{k+1} &= [(A_q - L_qC)\downarrow_l(H_k) \quad WH_w \quad -L_qEH_v] \end{cases} \tag{21}$$

Proof. Based on Assumption 1, we have $x_0 \in \langle p_0, H_0 \rangle$. We assume that $\hat{x}_0 = p_0$ then according to Definition 1, $e_0^d \in \langle 0, H_0 \rangle$. From (5) and based on Definition 2, we can obtain the error zonotope iteratively such that:

$$\begin{cases} e_k^d &\in \langle 0, H_0 \rangle \\ H_{k+1} &= [(A_q - L_qC)\downarrow_l(H_k) \quad WH_w \quad -L_qEH_v] \end{cases} \tag{22}$$

As mentioned in the preliminaries, a high-dimensional zonotope can be bounded by a lower dimensional one using the reduction operator. Thus, (22), is determined taking into account this property $H_k \subset \downarrow_l(H_k)$. It follows from system (1) and (22) that $r_k \in \langle 0, R_k \rangle$ such that:

$$\begin{cases} R_k &= [CH_k \quad EH_v] \\ H_{k+1} &= [(A_q - L_qC)\downarrow_l(H_k) \quad WH_w \quad -L_qEH_v] \end{cases}$$

The proposed FD decision provides better fault sensitivity performance and accurate fault detection results and avoids also the design of residual evaluation functions and thresholds.

6. SIMULATION RESULTS

To demonstrate the effectiveness and the advantages of the proposed FD method, a numerical example is considered for a discrete-time switched system defined with three subsystems, $N = 3$, with:

$$A_1 = \begin{bmatrix} 0.78 & 0 & 0.05 \\ 0 & 0.45 & -0.015 \\ 0.06 & 0.012 & 0.01 \end{bmatrix}, A_2 = \begin{bmatrix} -0.02 & -0.2 & 0.4 \\ 0.08 & 1 & -0.2 \\ 0.09 & 0.8 & -0.1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.55 & 0 & 0.25 \\ 0 & 0.45 & 0 \\ 0 & 0.075 & -0.03 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0.17 \\ -0.12 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 \\ 0 \\ 0.1 \end{bmatrix}, B_3 = \begin{bmatrix} 0.1 \\ -0.1 \\ 0 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.005 & -0.01 & -0.002 \\ 0.01 & 0 & 0.005 \\ 0.001 & 0.001 & 0.008 \end{bmatrix}, E = \begin{bmatrix} 0.01 & -0.002 & 0.01 \\ 0.001 & 0.001 & 0.01 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, F_2 = \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}, F_3 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} C = \begin{bmatrix} 1 & -0.5 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

Remark 1. It is pointed out that the state disturbance and the measurement noise matrices W and E are supposed to be common for all modes in this paper. However, there is no theoretical difficulty with allowing them to be switched.

In the simulation study, the initial state x_0 is bounded by the zonotope $\mathbf{X}_0 = \langle p_0, H_0 \rangle$ with:

$$p_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, H_0 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

The disturbances w_k and measurement noise v_k are considered with the random signals that are bounded by $[-0.1, 0.1]$. In order to reduce the column number of zonotope generator matrix, the reduction order of the matrix $\downarrow_l(H)$ is chosen as $l = 5$. By setting $\lambda = 0.7$ and solving the LMIs in (16) we obtain the H_∞ index value $\gamma = 6.56$ and the following gain matrices L_1, L_2 and L_3:

$$L_1 = \begin{bmatrix} -0.038 & -0.112 \\ -0.219 & -0.391 \\ -0.304 & -0.059 \end{bmatrix}, \ L_2 = \begin{bmatrix} -0.099 & -0.064 \\ -0.378 & -0.264 \\ -0.340 & -0.030 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 0.136 & -0.236 \\ 0.143 & -0.356 \\ -0.821 & 0.058 \end{bmatrix}$$

In the simulation study, S is obtained as follows:

$$S = \begin{bmatrix} 0.349 & -0.423 \\ 1.767 & -1.980 \\ 2.306 & 1.542 \end{bmatrix}$$

Under the switching sequence exposed in Fig. 1, simulation results are provided. Figure 1 indicates the active mode of the discrete-time switched system. To show the effectiveness of the designed fault detection approach, a comparison has been made with the fault detection approach without considering sensitivity and robustness analysis. We call in the sequel, method without optimization, a method where only the observer's stability is considered using a common Lyapunov function. An abrupt actuator fault f_k is carried out and represented as follows:

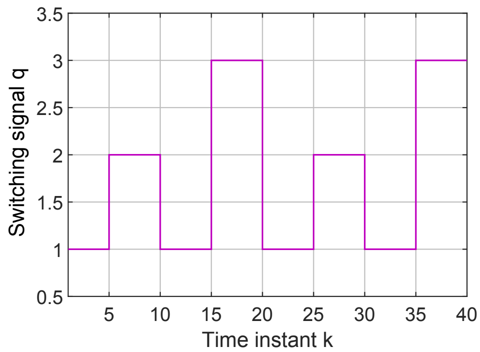
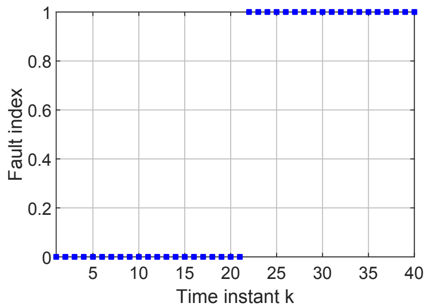


Fig. 1. Evolution of the switching signal

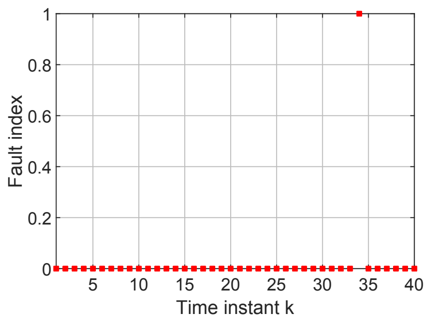
$$f_k = \begin{cases} 0.002 & k \geq 20 \\ 0 & otherwise \end{cases}$$

The comparison results of the detection moments are depicted in Fig. 2.The fault f_k can be detected at the time instant $k = 22$ using the designed approach where both sensitivity to faults and robustness against uncertainties are considered, however the same fault f_k can hardly be detected using method without optimization. Thus, it should be pointed out that the designed approach is able to ensure better fault detection performances. The fault detection logic is developed based on residual zonotopes. The comparison results are shown in Fig. 3 and Fig. 4.

These figures concern the residuals and the residual zonotopes of the FD technique designed without considering fault sensitivity and robustness performances and with the proposed method. In the fault free case, one can remark from Fig. 3 and Fig. 4 that $r(k) \in R(k)$ at the time instant $k = 19$. When a fault occurs ($k = 20$), the additive actuator fault is detected at the time instant $k = 22$ under the proposed FD approach $r(k) \notin R(k)$. However, under the approach without optimization, it cannot be detected at this instant of time ($k = 22$) and the zonotope



(a) Fault detection with optimisation



(b) Fault detection without optimisation

Fig. 2. Comparison between the results of fault detection instant obtained with and without optimisation

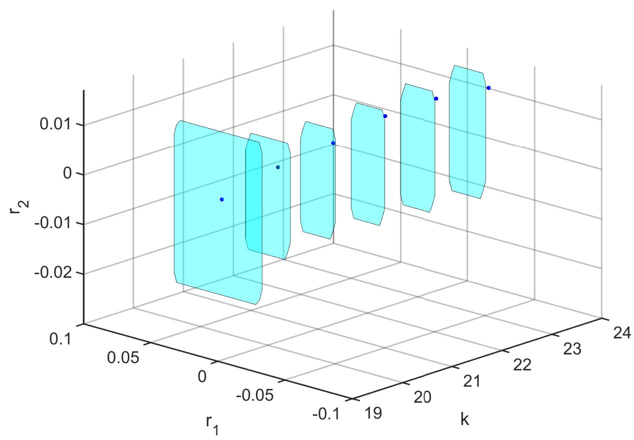


Fig. 3. Residual and residual zonotope of the proposed FD observer

$\mathbf{R}(k)$ still contains the residual signal $r(k)$. This is reasonable since the proposed approach has larger fault sensitivity performances and the impact of uncertainties is attenuated using H_∞ performances.

Therefore, these results press the effectiveness of the designed approach in actuator fault detection.

7. CONCLUSION

This paper presents new LMIs formulation to design a robust observer-based fault detection scheme for a discrete-time switched system. The proposed technique takes into account the robustness of the FDO against disturbances and sensitivity to fault. H_∞ performance is investigated to minimize the effect

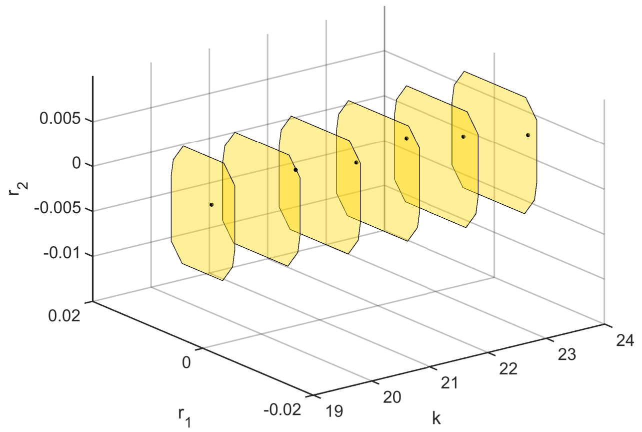


Fig. 4. Residual and residual zonotope of method without optimization

of disturbance. The impact of fault on the residual signal is maximised by a novel pole placement method. Sufficient Conditions for the design of such a FDO are derived in terms of LMI. The fault detection decision is evaluated based on zonotopic techniques. Simulation results are provided to illustrate the efficiency of the approach. For future works, based on L_∞ performances, extensions of these results will be considered in the case of an unknown switching signal and an application on a robotics systems will be considered.

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