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Describing government formation processes through collective multiagent dynamics on signed networks ⋆

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Abstract: The formation of a government in parliamentary democracies can be seen as a collective decision-making process where the members of the parliament (the "agents") cast a vote of confidence (the "decision") to the candidate cabinet coalition. If no party or alliance has managed to attain the majority of the votes at the election, this coalition will be the outcome of government negotiation talks between the political parties, talks which often require a long period of time as the parties need to overcome their ideological differences.
To explain this process, we propose a dynamical model of collective decision-making over the signed networks representing the parliament, where the signs describe the competitive and cooperative interactions among its members and the crossing of a bifurcation represents the completion of the negotiation process. The general philosophy is that the bifurcation parameter represents the social effort, identifiable with the duration of the cabinet negotiations, required from the political parties to reach a nontrivial decision (i.e., the confidence vote). In our model the social effort grows with the frustration of the parliamentary networks, that is, the amount of "disorder" encoded in the network. We show that indeed the frustration is a good indicator of the complexity of the government formation process by analyzing the legislative elections in 29 European countries with a parliamentary system in the last 40 years.

Keywords: Multi-agent systems, signed graph, frustration, government formation, multiparty parliamentary system, complex networks.

1. INTRODUCTION

In multiparty parliamentary democracies the confidence of the parliament is necessary for a government to rule, which implies that countries with a parliamentary system are often characterized by post-elections protracted and complex periods of government formation bargaining, in which the political parties have to negotiate to achieve a majority cabinet coalition. Statistical or game-theoretical models are proposed in the literature to model the process of government formation, see for instance Debus (2009); Diermeier and van Roozendaal (1998); Golder (2006, 2010); Ecker and Meyer (2015). These papers investigate such process in terms of ideological distance between parties, number of parties and number of parliamentary members. This work instead wants to bring a new perspective to this field by utilizing a dynamical model for decision-making to describe the process of government formation and the knowledge of the signed parliamentary networks to predict both the duration of the government negotiation process and the likely cabinet that will form after the elections.

We represent the Members of the Parliament (MPs hereafter) and their interactions as the nodes and edges of a signed complete graph, denoted *parliamentary network*: a positive edge indicates cooperation between the corresponding MPs (that is, the MPs belong to the same party or to the same pre-electoral coalition), while a negative edge indicates competition. The network we obtain is structurally balanced (see Altafini (2013)) if it is possible to divide the parliament into two subgroups such that there are only positive interactions (cooperation) between the MPs inside each subgroup, and negative interactions (antagonism) between MPs from different subgroups. In our context, this notion translates into a parliamentary network representing a two-party parliament (e.g., the Maltese parliament), or a parliament split into two coalitions. Equivalently, we say that the network is structurally balanced if the smallest eigenvalue of its normalized signed Laplacian is zero. Instead, if a network is not structurally balanced, the least eigenvalue of the normalized signed Laplacian is strictly positive and can be used to approximate the frustration of the network, that is, the distance of the network from a structurally balanced state (see Kunegis et al. (2010); Fontan and Altafini (2018a)). The notion of frustration comes from the statistical physics literature on Ising spin glasses (see Facchetti et al. (2011)): if we assign a binary variable (± 1, or "spin up/spin down") to each node of the network and compute an energy-like function for each spin configuration, the frustration corresponds to the energy in the ground state. In this work we claim that the frustration of the parliamentary networks (seen as the amount of "disorder" introduced by

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the antagonism between the MPs) affects the government formation process: when a parliamentary network has high frustration, then the period of government negotiation talks is long. Moreover, the successful coalition outcome of the negotiations is given by the group of parties achieving a majority in the ground state configuration. This intuition comes from our choice to model the process of government formation through an interconnected and nonlinear system for collective decision-making, used for instance in Gray et al. (2018); Fontan and Altafini (2018b,a), characterized by sigmoidal and saturated nonlinearities (describing how each agent expresses its opinion to its neighbors), a Laplacian-like structure at the origin and a "social effort" parameter, added to the model to represent the strength of the commitment between the agents, here interpreted as the duration of the cabinet negotiation phase. The equilibria of this system represent the decision states for the community and it is shown (see Fontan and Altafini (2018b,a)) that in order to achieve a decision (i.e., a nontrivial equilibrium point) the social effort parameter, which plays the role of bifurcation parameter in the analysis, needs to be higher than a certain threshold proportional to the smallest eigenvalue of the normalized signed Laplacian of the signed network representing the community.

To verify our model we collect elections data from 29 European countries with a parliamentary system, in a time interval that goes from 1980 to 2020. We propose three scenarios (denoted **I**, **II**, **III**) corresponding to different ways to determine the edge weights of the parliamentary networks, which consider different party grouping criteria (based on a priori information over pre-electoral coalitions between the political parties) and weight selection methods (based on the ideological distance between the parties). To confirm our hypotheses we calculate the correlation between the duration of the government negotiation talks and the frustration of the parliamentary networks, and introduce two indexes to measure the accuracy of our prediction for the cabinet composition.

2. SIGNED PARLIAMENTARY NETWORKS

A parliamentary network is a complete and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V}$ is the vertex set, $\mathcal{E}$ is the edge set and A is the adjacency matrix. Each node in $\mathcal{V}$ represents an elected Member of the Parliament (MP) (and $\text{card}(\mathcal{V}) = n$, where n is the total number of seats in the parliament), each edge (i, j) in $\mathcal{E}$ represents the relationship between MPs i and j and each element a_{ij} of $A \in \mathbb{R}^{n\times n}$ is the weight of the edge $(j, i) \in \mathcal{E}$. A positive weight $a_{ij} > 0$ indicates that MPs i and j cooperate, while a negative weight $a_{ij} < 0$ that the MPs compete.

Each MP belongs to a political party p_i, $i = 1, \ldots, n_\text{p}$ where n_p is the total number of parties obtaining seats in the parliament after the election. Since the aim of this work is to model the government formation process immediately following the elections, we will assume that each MP will vote along its party's line.

Assumption 1. Parties behave as homogeneous entities, i.e., as fully connected signed subgraphs with all identical positive edge weights.

Under Assumption 1, MPs from the same party can be gathered in a single "party node" rendering the parliamentary network a clustered network with n_p clusters and the adjacency matrix a $n_\text{p} \times n_\text{p}$ block matrix:

$$A = \begin{bmatrix} w_{11}(E_{c_1} - I_{c_1}) & \ldots & w_{1n_\text{p}} E_{c_1 c_{n_\text{p}}} \\ & \ddots & \\ w_{n_\text{p}1} E_{c_{n_\text{p}} c_1} & \ldots & w_{n_\text{p} n_\text{p}}(E_{c_{n_\text{p}}} - I_{c_{n_\text{p}}}) \end{bmatrix} \quad (1)$$

where c_i is the number of seats gained by the i-th party, $E_{c_i c_j} = \mathbb{1}_{c_i} \mathbb{1}_{c_j}^T$ (simplified to E_{c_i} if $j = i$) is the matrix of all 1s, and $W = [w_{ij}] \in \mathbb{R}^{n_\text{p}\times n_\text{p}}$ is the matrix of party-party weights. Its signed entries w_{ij} describe the interaction between MPs of the party p_i and party p_j, in terms of political affinity. We assume that each element of the matrix W belongs to the interval $[-1, +1]$, and that each diagonal element satisfies $w_{ii} = +1$ for all $i = 1, \ldots, n_\text{p}$. To determine the remaining off-diagonal (party-party) weights we propose three scenarios (denoted **I** to **III**), based on different party grouping criteria and weight selection methods: each off-diagonal weight w_{ij} represents an alliance (if $w_{ij} > 0$) or rivalry (if $w_{ij} < 0$) between the parties p_i and p_j. If the network is unweighted, $w_{ij} \in \{-1, 1\}$ for all $i, j = 1, \ldots, n_\text{p}$. If instead the network is weighted, the party-party weights are determined by the ideological distance between the parties, defined as the distance between the "political position" of each party on a left-right grid describing the political spectrum.

We consider the following three scenarios:

I the networks are unweighted and all the parties compete against each other. The party-party weights are chosen through the following rule:

$$w_{ij} = \begin{cases} +1 & \text{if } p_i \text{ and } p_j \text{ are the same party} \\ -1 & \text{if } p_i \text{ and } p_j \text{ are different parties.} \end{cases}$$

II the networks are weighted according to the left-right position of each party in the political spectrum given by their electoral manifestos (the so-called "rile" index, see Laver and Budge (1992)), and pre-electoral coalitions are taken into account.

III the networks are weighted according to a randomized and optimized left-right position, and pre-electoral coalitions are taken into account.

3. STRUCTURAL BALANCE AND FRUSTRATION

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a signed connected network with $\text{card}(\mathcal{V}) = n$ and normalized signed Laplacian $\mathcal{L}$ defined as

$$[\mathcal{L}]_{ij} = \begin{cases} 1 & j = i \\ -\frac{a_{ij}}{\sum_{k\neq i} |a_{ik}|} & j \neq i. \end{cases} \quad (2)$$

The signed (connected) network $\mathcal{G}$ is *structurally balanced* if any of the following condition holds (see Altafini (2013)): *(i)* there exists a partition of the node set, $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, such that every edge between $\mathcal{V}_1$ and $\mathcal{V}_2$ is negative while every edge within $\mathcal{V}_1$ or $\mathcal{V}_2$ is positive; *(ii)* there exists a signature matrix $S = \text{diag}\{s_1, \ldots, s_n\}$, with $s_i = \pm 1$ for all i, such that $S\mathcal{L}S$ has all nonpositive off-diagonal entries; *(iii)* $\lambda_1(\mathcal{L}) = 0$. The implication is that $\mathcal{G}$ is structurally unbalanced iff the smallest eigenvalue of its normalized signed Laplacian is strictly positive, $\lambda_1(\mathcal{L}) > 0$.

The signed parliamentary networks $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, as described in Section 2, are in general not structurally balanced, hence they have nonzero frustration, which is a standard measure introduced in the literature to characterize the distance of a signed network to a structurally balanced state, see Facchetti et al. (2011). Under Assumption 1, to calculate the frustration of each network $\mathcal{G}$ we consider block signature matrices (with each block corresponding to a political party): $S = \text{diag}\{s_1 I_{c_i}, \ldots, s_{n_\text{p}} I_{c_{n_\text{p}}}\}$, $s_i = \pm 1$, where c_i and n_p are the size of the political party p_i and the total number of parties in the parliament, respectively. We define the energy functional of a configuration S as follows:

$$e(S) = \frac{1}{2} \sum_{i,j \neq i} [\,|\mathcal{L}| + S\mathcal{L}S\,]_{ij}. \quad (3)$$

The *party-wise frustration* of a parliamentary network $\mathcal{G}$ is then the minimum of the values of energy functionals (3) over all 2^{n_p} possible (block) signature matrices S:

$$\zeta(\mathcal{G}) = \min_{\substack{S=\text{diag}\{s_1 I_{c_1}, \ldots, s_{n_\text{p}} I_{c_{n_\text{p}}}\} \\ s_i = \pm 1}} \frac{1}{2} \sum_{i,j \neq i} [\,|\mathcal{L}| + S\mathcal{L}S\,]_{ij}. \quad (4)$$

The configuration S yielding this minimum is denoted *ground state*. The frustration (4) can be computed exactly.

4. A DYNAMICAL MODEL FOR COLLECTIVE DECISION-MAKING OVER SIGNED NETWORKS

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be the signed parliamentary network with adjacency matrix A describing the interactions between the n MPs, as defined in Section 2. The process of government formation is modeled as a nonlinear multiagent system for collective decision-making:

$$\dot{x} = -\Delta x + \pi A \psi(x), \quad (5)$$

where $x = [x_1 \ldots x_n]^T \in \mathbb{R}^n$ is the vector of the opinions of the MPs, $\Delta = \text{diag}\{\delta_1, \ldots, \delta_n\}$ is a diagonal matrix where each element is defined as $\delta_i = \sum_{j \neq i} |a_{ij}|$, $\psi(x) = [\psi_1(x_1) \ldots \psi_n(x_n)]^T$ is a vector of nonlinearities and $\pi > 0$ is a scalar parameter. Each nonlinear function $\psi_i(x_i)$ is assumed sigmoidal and saturated and indicates how an agent i expresses its opinion to its neighbors in the network: opinions which are too "extreme" are disregarded. When an agent i communicates its opinion to a neighbor j in the network, the term $\psi_i(x_i)$ is weighted first by the corresponding element a_{ji} of the adjacency matrix of the network, which will be positive if the agents cooperate or negative if they compete, and then by the positive scalar parameter π, denoted *social effort* and representing the strength of the commitment between the agents. Each equilibrium point of the system (5) represents a decision for the community of agents: in this case, the vote of confidence cast by the parliament to a candidate cabinet coalition.

Previous works such as Gray et al. (2018); Fontan and Altafini (2018b,a) have investigated how the social effort parameter π (a bifurcation parameter in the analysis) affects the presence (and stability) of the equilibria of the system (5). It is shown that there exists a threshold value π_1 such that for $\pi < \pi_1$ the origin is the only equilibrium point of (5), while at $\pi = \pi_1$ the system (5) undergoes a pitchfork bifurcation and two (opposite) nontrivial (locally asymptotically stable) equilibria appear. The threshold value π_1 depends on the smallest eigenvalue of $\mathcal{L}$: $\pi_1 = \frac{1}{1-\lambda_1(\mathcal{L})}$. If the network is structurally balanced $\lambda_1(\mathcal{L}) = 0$ which implies that π_1 is fixed: $\pi_1 = 1$. When instead the network is not structurally balanced, the smallest eigenvalue of $\mathcal{L}$ not only is strictly positive but also constitutes a measure of the frustration of the network (see Fontan and Altafini (2018a)), which means that π_1 is not fixed but depends on the frustration. Hence, the social effort π needed to achieve a nontrivial equilibrium point will grow with the frustration of the signed networks.

5. APPLICATION TO THE GOVERNMENT FORMATION PROCESS

We model the government formation process through the nonlinear system for collective decision-making (5) over signed (parliamentary) networks. The social effort (π) required from the MPs can be interpreted as the duration of the government negotiation process: if the parliamentary network has high frustration, the political parties will need a long period to propose a candidate government enjoying the confidence of the parliament. Our aim is then to show that the frustration of the parliamentary networks can be used to predict the duration of the government negotiation periods. To verify this hypothesis, we calculate the Pearson's correlation index between the duration of the negotiation phase (defined as the number of days between the election date and the date the government is sworn in) and the frustration of the parliamentary network (4), for each country and scenario. In addition, we want to show that the notion of frustration can be used to infer the composition of the successful cabinet coalition (enjoying the confidence of the parliament). The estimate we propose corresponds to the group of parties achieving a majority in the ground state. Let S_best be defined as follows:

$$S_\text{best} = \arg\min_{S} \; e(S)$$
$$\text{s.t. } (i)\; S = \text{diag}\{s_1 I_{c_1}, \ldots, s_{n_\text{p}} I_{c_{n_\text{p}}}\},\; s_i = \pm 1;$$
$$(ii) \sum_{i:\, s_i = +1} c_i \neq \sum_{i:\, s_i = -1} c_i. \quad (6)$$

The second condition is set to avoid the situation (common in scenario **I**) where S_best splits perfectly the parliament into two factions. The first condition in (6) instead chooses S_best as the signature matrix yielding the minimum value of energy: without condition *(ii)*, $\zeta(\mathcal{G}) = e(S_\text{best})$. Our government estimate $\mathcal{P}_\text{best,maj}$ then is given by the party set holding majority in the configuration S_best. To evaluate the accuracy of our estimate, we propose two indexes:

$$\rho_\text{gov} = \frac{\text{card}\,(\mathcal{P}_\text{best,maj} \cap \mathcal{P}_\text{gov})}{\text{card}\,(\mathcal{P}_\text{gov})} \quad (7)$$

$$\eta_\text{gov} = 1 - \frac{e(S_\text{gov}) - \zeta(\mathcal{G})}{\max_S e(S) - \zeta(\mathcal{G})}, \quad (8)$$

where $\mathcal{P}_\text{gov}$ is the set of parties representing the government formed after the election and S_gov is the corresponding signature matrix. The index ρ_gov measures how accurate is our government prediction ($\mathcal{P}_\text{best,maj}$) in terms of number of correctly guessed parties. The index η_gov measures how energetically close is our prediction to the true government: $e(S)$ is the energy functional (3) corresponding to the configuration S, $\zeta(\mathcal{G})$ is the frustration of the signed parliamentary network (4) and $e(S_\text{gov})$ is the energy functional corresponding to the government configuration S_gov.

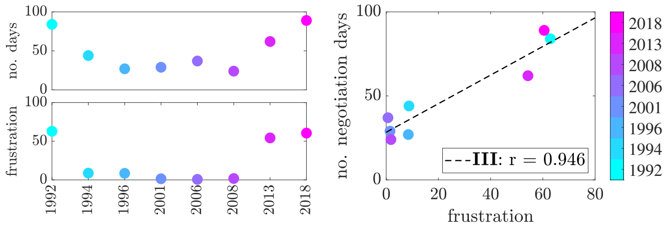
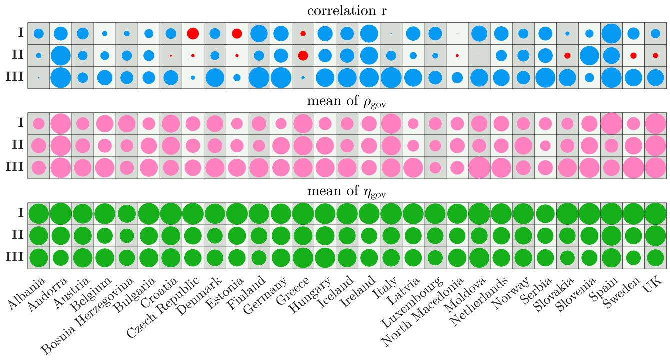


Fig. 1. Scenario **III**. (left): Duration of the government negotiation talks (no. negotiation days) and frustration of the Italian parliamentary networks for each election. (right): Linear regression (days vs frustration). The value of correlation is reported in the plot legend.



scenario	correlation r	mean of ρ_{gov}	mean of η_{gov}
I	0.437	0.678	0.948
II	0.363	0.710	0.793
III	0.714	0.788	0.799

Fig. 2. Results for the 29 European countries for the correlation and the accuracy of our predictions on the cabinet composition. The mean values for each scenario are reported in the table. (top): Correlation between duration of the government negotiation period and frustration of the parliamentary network for each country and scenario. The color of the circle indicates positive (blue) or negative correlation (red), while the size the absolute value. (middle): Index ρ_{gov} (7). (bottom): Index η_{gov} (8).

5.1 *Results*

For the 29 parliamentary democracies we consider, we gather information on the duration of the government negotiation periods, the number of political parties and their position in the left-right political spectrum and "rile" index, the pre-electoral alliances and the composition of the government formed after the elections. Some of the sources used to collect the data are the Manifesto Project Database (Volkens et al. (2019)), the Parliaments and Governments Database (Holger and Manow (2018)), the new Parline (Inter-Parliamentary Union (2019)) and Wikipedia. For each scenario we construct the parliamentary networks, compute the frustration (4), and the correlation between the duration of the government negotiations and the frustration (see this procedure for Italy and scenario **III** in Fig. 1). We show that the duration of the cabinet negotiation process correlates well with the frustration of the parliamentary networks, with average values of correlation equal to 0.44 in scenario **I**, 0.36 in scenario **II** and 0.71 in scenario **III** (see Fig. 2). Interestingly, for Estonia and Greece where our hypothesis fails (i.e., where we obtain negative values of correlation), the constitution defines and limits the period of time the parties are given to propose a cabinet coalition (see Article 89 of the Estonian constitution and Article 37 of the Greek constitution). Such a time constraint likely explains the low values we obtain for the correlation. Finally, we show in Fig. 2 that the accuracy of our predictions on the composition of the post-election cabinet in terms of ρ_{gov} (i.e., percentage of parties in the cabinet which are "correctly guessed") ranges between 67.8% (in scenario **I**) and 78.8% (in scenario **III**), while it is around (or greater) 80% in each scenario when we consider the index η_{gov} (i.e., how "energetically close" our prediction is to the government).

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