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Passivity properties for regulation of DC networks with stochastic load demand ⋆

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Abstract: In this paper we present new (stochastic) passivity properties for Direct Current (DC) power networks, where the unknown and unpredictable load demand is modelled by a stochastic process. More precisely, the considered power network consists of distributed generation units supplying ZIP loads, *i.e.*, nonlinear loads comprised of impedance (Z), current (I) and power (P) components. Differently from the majority of the results in the literature, where each of these components is assumed to be constant, we consider time-varying loads whose dynamics are described by a class of stochastic differential equations. Finally, we prove that an existing distributed control scheme achieving current sharing and (average) voltage regulation ensures the asymptotic stochastic stability of the controlled network.

Keywords: DC power networks, Nonlinear systems, Stochastic modelling, Passivity theory.

1. INTRODUCTION

Power networks are typically classified into Direct Current (DC) and Alternating Current (AC) networks, which are interconnected clusters of Distributed Generation Units (DGUs), loads and energy storage devices. The recent wide spread of renewable energy sources, electronic appliances and batteries (including electric vehicles) motivates the design and operation of DC networks, which are generally more effcient and reliable than AC networks (Justo et al. (2013)), attracting growing interest.

In order to guarantee a proper and safe functioning of the overall network and the appliances connected to it, the main goal in DC networks is voltage stabilization (see for instance Jeltsema and Scherpen (2004); Iovine et al. (2018); Monshizadeh et al. (2019); Machado et al. (2018); Cucuzzella et al. (2019a); Nahata et al. (2020)). Moreover, as different DGUs may generally have different generation (or storage) capacities, an additional goal is to (fairly) share the total demand of the network among its DGUs (see for instance Cucuzzella et al. (2019b); Trip et al. (2019); De Persis et al. (2018); Martinelli et al. (2019)). This goal is usually called power or current sharing and its achievement does not generally permit to regulate the voltage at each node towards the corresponding pre-specified reference value. Consequently, different forms of voltage regulation have been proposed in the literature, where for instance the average value of the voltages of the whole microgrid is controlled towards a desired setpoint (see for instance Cucuzzella et al. (2019b); Trip et al. (2019)).

1.1 Motivation and contributions

In all these works the load components are assumed to be constant. However, it is well known that electric loads are in practice time-varying and, due to the random and unpredictable diversity of usage patterns, it is more realistic to consider unknown time-varying loads described for instance by stochastic processes (see for instance Silani et al. (2019); Verdejo et al. (2016)). In Wilson et al. (2014), a control scheme is proposed to regulate the renewable energy sources and energy storage devices of a DC network with stochastic Z loads. The control strategy requires the design of a filter for estimating the load resistance. A cascade control system for the energy management of DC microgrids with I loads is presented in Pavkovic et al. (2016), where the proposed control scheme includes an adaptive estimation of the quasi-stochastic load current profiles. In Cingoz et al. (2017), a droop control scheme is designed for DC microgrids with stochastic Z loads.

In this paper, (i) each component of the ZIP load is modeled as the sum of an unknown constant and the solution to a stochastic differential equation describing the load dynamics; (ii) sufficient conditions for the stochastic passivity of the open-loop system are presented, facilitating the interconnection with passive control systems; (iii) the asymptotic stochastic stability of the power network

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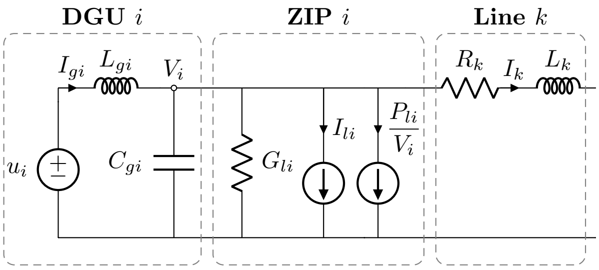

Fig. 1. Electrical scheme of DGU i, ZIP load i and transmission line k, with $i \in \mathcal{V}$ and $k \in \mathcal{E}$.

Table 1. Description of symbols

G_{li}	Unknown conductance load
I_{li}	Unknown current load
P_{li}	Unknown power load
I_{gi}	Generated current
V_i	Load voltage
I_k	Line current
u_i	Control input
L_k	Line inductance
R_k	Line resistance
C_{gi}	Shunt capacitor
L_{gi}	Filter inductance

controlled by the distributed control scheme proposed by Trip et al. (2019) is proved.

1.2 Notation

The set of real numbers is denoted by $\mathbb{R}$. The set of positive (nonnegative) real numbers is denoted by $\mathbb{R}_{>0}$ ($\mathbb{R}_{\geq 0}$). Let $\mathbf{0}$ be the vector of all zeros or the null matrix of suitable dimension(s) and let $\mathbb{1}_n \in \mathbb{R}^n$ be the vector containing all ones. The i-th element of vector x is denoted by x_i. A steady-state solution to the system $\dot{x} = \zeta(x)$, is denoted by $\overline{x}$, *i.e.*, $\mathbf{0} = \zeta(\overline{x})$. Moreover, given a signal (or parameter) σ, σ^* denotes that σ is a constant signal (or parameter), while $\hat{\sigma}$ denotes that σ is a stochastic process. Given a vector $x \in \mathbb{R}^n$, $[x] \in \mathbb{R}^{n\times n}$ indicates the diagonal matrix whose diagonal entries are the components of x. Let $A \in \mathbb{R}^{n\times n}$ be a matrix. In case A is a positive definite (positive semi-definite) matrix, we write $A > \mathbf{0}$ ($A \geq \mathbf{0}$). The $n \times n$ identity matrix is denoted by $\mathbb{I}_n$ and the vector consisting of all ones is denoted by $\mathbb{1}_n$.

2. DC NETWORK MODEL

In this section, we introduce the model of the considered DC network comprising of Distributed Generation Units (DGUs), loads and transmission lines (Cucuzzella et al. (2019b); Trip et al. (2019)). Fig. 1 illustrates the structure of the considered DC microgrid and Table 1 reports the description of the used symbols. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a connected and undirected graph that describes the considered DC network. The nodes and the edges are denoted by $\mathcal{V} = \{1, ..., n\}$ and $\mathcal{E} = \{1, ..., m\}$, respectively. Then, the topology of the microgrid is represented by the corresponding incidence matrix $\mathcal{A} \in \mathbb{R}^{n\times m}$. Let the ends of each transmission line k be arbitrarily labeled with a '-', or a '+', then $\mathcal{A}$ can be defined as

$$\mathcal{A}_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

To refer to the impedance, current, and power components of loads, the letters Z, I and P, respectively, are often used in the literature (see for instance De Persis et al. (2018)). Therefore, in presence of the so-called ZIP loads, $I_{li} : \mathbb{R} \to \mathbb{R}$ denotes the current demand of load $i \in \mathcal{V}$ and is given by

$$I_{li}(V_i) = (G_{li}^* + \hat{G}_{li})V_i + I_{li}^* + \hat{I}_{li} + V_i^{-1}(P_{li}^* + \hat{P}_{li}), \tag{2}$$

where $G_{li}^*, I_{li}^*, P_{li}^* \in \mathbb{R}_{>0}$ are *unknown* constants and $\hat{G}_{li}, \hat{I}_{li}, \hat{P}_{li} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ are *stochastic* processes, the dynamics of which will be introduced in Section 4. We also use Z*, I* and P* to denote the unknown constant components of the load [1]. Now, the dynamics of the overall network can be written compactly as

$$\begin{aligned} L_g \dot{I}_g &= -V + u \\ C_g \dot{V} &= I_g + \mathcal{A}I - I_l(V) \\ L\dot{I} &= -\mathcal{A}^\top V - RI, \end{aligned} \tag{3}$$

where $I_g, V, u : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $I : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$, $L_g, C_g \in \mathbb{R}_{>0}^{n\times n}$, $R, L \in \mathbb{R}_{>0}^{m\times m}$ and $I_l : \mathbb{R}^n \to \mathbb{R}^n$ is given by $I_l(V) = [G_l]V + I_l + [V]^{-1}P_l$. Moreover, $G_l = G_l^* + \hat{G}_l$, $I_l = I_l^* + \hat{I}_l$ and $P_l = P_l^* + \hat{P}_l$, where $G_l^*, I_l^*, P_l^* \in \mathbb{R}_{>0}^n$ represent the constant components of the load and $\hat{G}_l, \hat{I}_l, \hat{P}_l : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ the stochastic parts. These dynamics are discussed in Section 4, where we assume that their steady-state solutions are equal to zero.

3. PROBLEM FORMULATION

In this section, we introduce the main control goals of this paper, *i.e.*, current sharing and average voltage regulation. Assuming that the stochastic components of the ZIP loads are zero at the steady-state, we notice that for a constant input $u = u^*$, the steady-state solution $(\overline{I}_g, \overline{V}, \overline{I})$ to (3) satisfies

$$\overline{V} = u^* \tag{4a}$$

$$I_l^* + [G_l^*]\overline{V} + [\overline{V}]^{-1}P_l^* - \overline{I}_g = \mathcal{A}\overline{I} \tag{4b}$$

$$\overline{I} = -R^{-1}\mathcal{A}^\top\overline{V}. \tag{4c}$$

Because of different generation capacities, it is reasonable to require that the total load demand of the microgrid is fairly shared among all the different generation units as $q_i\overline{I}_{gi} = q_j\overline{I}_{gj}$, $\forall i, j \in \mathcal{V}$, where q_i has inverse relationship with the capacity of DGU i. Then, we define the first goal concerning the steady-state value of the generated current as follows:

Goal 1. **(Current sharing).**

$$\lim_{t\to\infty} I_g(t) = \overline{I}_g = Q^{-1}\mathbb{1}_n i_g^*, \tag{5}$$

where $Q = \mathrm{diag}(q_1, \ldots, q_n)$, $q_i \in \mathbb{R}_{>0}$, $\forall i \in \mathcal{V}$ and i_g^* any constant value satisfying at the steady-state $i_g^* = \frac{\mathbb{1}_n^\top([G_l^*]\bar{V} + I_l^* + [\bar{V}]^{-1}P_l^*)}{\mathbb{1}_n^\top Q^{-1}\mathbb{1}_n}$. Indeed, the latter implies that at

[1] For instance, in presence of a Z*IP* load, we have $I_{li}(V_i) = G_{li}^*V_i + I_{li}^* + \hat{I}_{li} + V_i^{-1}P_{li}^*$.

the steady-state the total generated current is equal to the total load demand, *i.e.*, $\mathbb{1}_n^\top \bar{I}_g = \mathbb{1}_n^\top I_l(\bar{V})$.

Now, we observe that achieving Goal 1, does not generally allow to perform also voltage regulation. Indeed, the achievement of current sharing generally implies voltage differences between the nodes of the microgrid. As a consequence, a particular form of voltage regulation has been proposed in the literature, where the (weighted) average of the steady-state voltages is regulated towards the (weighted) average of the voltage references (Cucuzzella et al. (2019b); Trip et al. (2019)). Then, the second goal concerning the steady-state value of the voltages is defined as follows:

Goal 2. **(Average voltage regulation).**

$$\lim_{t\to\infty} \mathbb{1}_n^\top Q^{-1} V(t) = \mathbb{1}_n^\top Q^{-1} \overline{V} = \mathbb{1}_n^\top Q^{-1} V^*, \tag{6}$$

V_i^* being the voltage reference at node $i \in \mathcal{V}$.

Before showing in the next section the stochastic properties of the considered network, we assume that there exists a steady-state solution to (3):

Assumption 1. **(Steady-state solution).** There exists a constant input u^* and a steady-state solution $(\bar{I}_g, \bar{V}, \bar{I})$ to (3) satisfying (4) and achieving Goals 1 and 2.

4. STOCHASTIC PASSIVITY OF DC NETWORKS

In this section, we introduce the dynamics of the stochastic components of the load, *i.e.*, $\hat{I}_l$, $\hat{P}_l$ and $\hat{G}_l$, respectively, and verify the stochastic passivity of the open-loop system. Finally, we prove that the control scheme proposed by Trip et al. (2019) ensures the asymptotic stochastic stability of the controlled network.

First, we notice that the stochastic terms in (3) enter in a multiplicative manner. For this reason, an appropriate mathematical framework such as the Ito calculus framework should be adopted to analyze such a model. Therefore, we recall for the readers' convenience the definitions of stochastic differential equation and stochastic passivity through the Ito calculus framework (Astrom (1970); Khasminskii et al. (2011); Wu et al. (2011); van Handel et al. (2007)).

Definition 1. **(Stochastic differential equation).** We define a stochastic differential equation (SDE) as

$$dx(t) = f(x,u)dt + g(x)dw(t), \tag{7}$$

where $f(x,u) \in \mathbb{R}^N$ and $g(x) \in \mathbb{R}^{N\times M}$ are locally Lipschitz, $x(t) \in \mathbb{R}^N$ is the state vector, $u(t) \in \mathbb{R}^P$ is the input of the system and $w(t) \in \mathbb{R}^M$ is the standard Brownian motion vector. In the following, we define the Ito derivative (Astrom (1970); Khasminskii et al. (2011); Wu et al. (2011); van Handel et al. (2007)).

Definition 2. **(Ito derivative).** Consider a storage function $S(x)$, which is twice continuously differentiable. Then, $\mathcal{L}S(x)$ denotes the Ito derivative of $S(x)$ along the SDE (7), *i.e.*,

$$\mathcal{L}S(x) = \frac{\partial S(x)}{\partial x} f(x,u) + \frac{1}{2}\mathrm{tr}\{g^\top(x)\frac{\partial^2 S(x)}{\partial x^\top \partial x} g(x)\}. \tag{8}$$

Now, for the readers' convenience, we introduce the concept of stochastic passivity (Khasminskii et al. (2011); Wu et al. (2011); van Handel et al. (2007)).

Definition 3. **(Stochastic passivity).** Consider system (7) with output $y = h(x)$. Assume that the deterministic and stochastic terms of the SDE (7) at the equilibrium point are identical to zero, *i.e.*, $f(\bar{x}, u^*) = g(\bar{x}) = \mathbf{0}$. Then, system (7) is said to be stochastically passive with respect to the supply rate $u^\top y$ if there exists a twice continuously differentiable positive semi-definite storage function $S(x)$ satisfying

$$\mathcal{L}S(x) \le u^\top y, \ \forall (x,u) \in \mathbb{R}^N \times \mathbb{R}^P. \tag{9}$$

Now, in the following subsections we introduce the dynamics of the stochastic load components, which we model via an SDE in the Ito calculus framework. More precisely, we adopt the well-known Ornstein-Uhlenbeck process (Karatzas and Shreve (1991)), which is indeed widely used for the description of physical phenomena (see for instance Verdejo et al. (2016)). For the sake of exposition, we first consider Z*IP* loads, *i.e.*, only the load current is described by a stochastic process. Later, we extend the results also to Z*IP and ZIP loads, respectively.

*4.1 Z*IP* loads*

In this subsection, we consider Z*IP* loads type, *i.e.*, equation (2) becomes $I_{li}(V_i) = G_{li}^* V_i + I_{li}^* + \hat{I}_{li} + V_i^{-1} P_{li}^*$, where the SDE describing the dynamics of $\hat{I}_{li}$ is given by

$$d\hat{I}_{li} = -\mu_{I_{li}} \hat{I}_{li} dt + \sigma_{I_{li}} \hat{I}_{li} dw_i, \tag{10}$$

where $\mu_{I_{li}}$ and $\sigma_{I_{li}}$ are positive constants. As mentioned in Section 3, it can be noticed that the deterministic and stochastic parts of (10) at the equilibrium point are identical to zero. Now, in order to verify a passivity property of the considered network, we introduce an assumption on the parameters of the SDE (10) and restrict the trajectories to be inside a subset of the state-space.

Assumption 2. **(Condition on the parameters of (10)).** For all $i \in \mathcal{V}$, the stochastic parameters of (10) satisfy

$$\mu_{I_l} > \frac{1}{2}\sigma_{I_l}^2 - \frac{1}{2}\mathbb{I}_n, \tag{11}$$

where $\mu_{I_l} = \mathrm{diag}\{\mu_{I_{l1}}, \dots, \mu_{I_{ln}}\}$ and $\sigma_{I_l} = \mathrm{diag}\{\sigma_{I_{l1}}, \dots, \sigma_{I_{ln}}\}$.

Let us define the set $\Omega_{\mathrm{Z^*IP^*}} := \{(I_g, V, I, \hat{I}_l) : \frac{1}{2}([G_l^*] - \Pi^{-1}) - [P_l^*][V]^{-1}[\bar{V}]^{-1} > \mathbf{0}\}$. Then, the stochastic passivity property of system (3) with Z*IP* loads is verified via the following lemma:

Lemma 1. **(Stochastic passivity of (3), (10)).** Let Assumption 2 hold and $\Omega_{\mathrm{Z^*IP^*}}$ be nonempty. System (3) with Z*IP* loads and $\hat{I}_l$ given by (10) is stochastically (shifted) passive with respect to the storage function

$$\begin{aligned} S_{\mathrm{Z^*IP^*}}(I_g, V, I, \hat{I}_l) =& \frac{1}{2}(I_g - \bar{I}_g)^\top L_g (I_g - \bar{I}_g) \\ &+ \frac{1}{2}(V - \bar{V})^\top C_g (V - \bar{V}) \\ &+ \frac{1}{2}(I - \bar{I})^\top L (I - \bar{I}) + \frac{1}{2}\hat{I}_l^\top \Pi \hat{I}_l, \end{aligned} \tag{12}$$

and supply rate $(I_g - \bar{I}_g)^\top (u - u^*)$ for all the trajectories $(I_g, V, I, \hat{I}_l) \in \Omega_{\mathrm{Z^*IP^*}}$, where Π is a (suitable) positive definite diagonal constant matrix of appropriate dimensions and $(\bar{I}_g, \bar{V}, \bar{I})$ satisfies (4).

Proof. The proof follows by inspection of the Ito derivative of the storage function (12) and is omitted due to space limitations.

Remark 1. **(Z*I load).** We observe that in presence of only Z*I loads, the result provided in Lemma 1 can be strengthened. Indeed, the absence of P loads implies that the system (3), (10) is stochastically passive for any sufficiently large Π.

*4.2 Z*IP loads*

In this subsection, we consider Z*IP loads type, *i.e.*, equation (2) becomes $I_{li}(V_i) = G^*_{li}V_i + I^*_{li} + \hat{I}_{li} + V_i^{-1}(P^*_{li} + \hat{P}_{li})$, where the SDE describing the dynamics of $\hat{P}_{li}$ is given by

$$d\hat{P}_{li} = -\mu_{P_{li}}\hat{P}_{li}dt + \sigma_{P_{li}}\hat{P}_{li}dw_i, \tag{13}$$

where $\mu_{P_{li}}$ and $\sigma_{P_{li}}$ are positive constants. Also in this case, we notice that the equilibrium points of the deterministic and stochastic parts of (13) are zero. Now, in order to verify a passivity property of the considered network, we introduce an assumption on the parameters of the SDE (13) and restrict the trajectories to be inside a subset of the state-space.

Assumption 3. **(Condition on the parameters of** (13)**).** For all $i \in \mathcal{V}$, the stochastic parameters of (13) satisfy

$$\mu_{P_l} > \sigma^2_{P_l}, \tag{14}$$

where $\mu_{P_l} = \mathrm{diag}\{\mu_{P_{l1}}, \ldots, \mu_{P_{ln}}\}$ and $\sigma_{P_l} = \mathrm{diag}\{\sigma_{P_{l1}}, \ldots, \sigma_{P_{ln}}\}$.

Let us define the set $\Omega_{\mathrm{Z^*IP}} := \{(I_g, V, I, \hat{I}_l,\ \hat{P}_l) : \frac{1}{2}([G^*_l] - \Pi^{-1}) - [P^*_l][V]^{-1}\ [\bar{V}]^{-1} - \frac{1}{2}\Sigma\mu_{P_l}[V]^{-2} > \mathbf{0}\}$. Then, the stochastic passivity property of system (3) with Z*IP loads is verified via the following lemma:

Lemma 2. **(Stochastic passivity of** (3)**,** (10)**,** (13)**).** Let Assumptions 2, 3 hold and $\Omega_{\mathrm{Z^*IP}}$ be nonempty. System (3) with Z*IP loads and $\hat{I}_l, \hat{P}_l$ given by (10), (13) is stochastically (shifted) passive with respect to the storage function

$$S_{\mathrm{Z^*IP}}(I_g, V, I, \hat{I}_l, \hat{P}_l) = S_{\mathrm{Z^*IP^*}}(I_g, V, I, \hat{I}_l) + \frac{1}{2}\hat{P}_l^\top \Sigma \hat{P}_l, \tag{15}$$

and supply rate $(I_g - \bar{I}_g)^\top(u - u^*)$ for all the trajectories $(I_g, V, I, \hat{I}_l, \hat{P}_l) \in \Omega_{\mathrm{Z^*IP}}$, where Σ is a (suitable) positive-definite diagonal constant matrix of appropriate dimensions.

Proof. The proof follows by inspection of the Ito derivative of the storage function (15) and is omitted due to space limitations.

4.3 ZIP loads

In this subsection, we finally consider ZIP loads, where the SDE describing the dynamics of $\hat{G}_{li}$ is given by

$$d\hat{G}_{li} = -\mu_{G_{li}}\hat{G}_{li}dt + \sigma_{G_{li}}\hat{G}_{li}dw_i, \tag{16}$$

where $\mu_{G_{li}}$ and $\sigma_{G_{li}}$ are positive constants. Also in this case, the deterministic and stochastic parts of (16) at the equilibrium point are identical to zero. Now, in order to verify a passivity property of the considered network, we introduce an assumption on the parameters of the SDE (16) and restrict the trajectories to be inside a subset of the state-space.

Assumption 4. **(Condition on the parameters of** (16)**).** For all $i \in \mathcal{V}$, the stochastic parameters of (16) satisfy

$$\mu_{G_l} > \sigma^2_{G_l}, \tag{17}$$

where $\mu_{G_l} = \mathrm{diag}\{\mu_{G_{l1}}, \ldots, \mu_{G_{ln}}\}$ and $\sigma_{G_l} = \mathrm{diag}\{\sigma_{G_{l1}}, \ldots, \sigma_{G_{ln}}\}$.

Let us define the set $\Omega_{\mathrm{ZIP}} := \{(I_g, V, I, \hat{I}_l, \hat{P}_l,\ \hat{G}_l) : \frac{1}{2}([G^*_l] - \Pi^{-1}) - [P^*_l][V]^{-1}[\bar{V}]^{-1} - \frac{1}{2}\Sigma\mu_{P_l}[V]^{-2} - [V - \bar{V}]^2\Lambda\mu_{G_l} > \mathbf{0}\}$. Then, the stochastic passivity property of system (3) with ZIP loads is verified via the following lemma.

Lemma 3. **(Stochastic passivity of** (3)**,** (10)**,** (13)**,** (16)**).** Let Assumptions 2–4 hold and Ω_{ZIP} be nonempty. System (3), with $\hat{I}_l$, $\hat{P}_l$, $\hat{G}_l$ given by (10), (13), (16) is stochastically (shifted) passive with respect to the storage function

$$S_{\mathrm{ZIP}}(I_g, V, I, \hat{I}_l, \hat{P}_l, \hat{G}_l) = S_{\mathrm{Z^*IP}}(I_g, V, I, \hat{I}_l, \hat{P}_l) + \frac{1}{2}\hat{G}_l^\top \Lambda \hat{G}_l \tag{18}$$

and supply rate $(I_g - \bar{I}_g)^\top(u - u^*)$ for all the trajectories $(I_g, V, I, \hat{I}_l, \hat{P}_l, \hat{G}_l) \in \Omega_{\mathrm{ZIP}}$, where Λ is a (suitable) positive-definite diagonal constant matrix of appropriate dimensions.

Proof. The proof follows by inspection of the Ito derivative of the storage function (18) and is omitted due to space limitations.

Remark 2. **(Sufficient conditions).** We observe that for large Π and small Σ, Λ, the sufficient conditions for Ω_{ZIP} to be nonempty are similar to the (well-known) sufficient conditions provided in the literature for DC networks with (non-stochastic) Z*I*P* loads, *i.e.*, high voltage and large values of the load conductance (De Persis et al. (2018); Monshizadeh et al. (2019); Cucuzzella et al. (2019a); Nahata et al. (2020)).

4.4 Closed-loop analysis

In this subsection, we consider the distributed controllers proposed by Trip et al. (2019) and show that the closed-loop system is asymptotically stochastically stable, achieving at the steady state Goals 1 and 2. Before introducing the controller proposed by Trip et al. (2019), we recall for the readers' convenience the definition of (asymptotic) stochastic stability (Astrom (1970); Khasminskii et al. (2011); Wu et al. (2011); van Handel et al. (2007)).

Definition 4. **((Asymptotic) stochastic stability).** System (7) is (asymptotically) stochastically stable if a twice continuously differentiable positive definite Lyapunov function $S : \mathbb{R}^N \longrightarrow \mathbb{R}_{>0}$ exists such that $\mathcal{L}S$ is (negative definite) negative semi-definite.

Now, for the sake of completeness, we report below the controller proposed by Trip et al. (2019) for $i \in \mathcal{V}$, *i.e.*,

$$\begin{aligned}\tau_{\xi i}\dot{\xi}_i =& -\sum_{j\in\mathcal{N}_i^{com}} \rho_{ij}(q_i I_{gi} - q_j I_{gj})\\ \tau_{\eta i}\dot{\eta}_i =& -\eta_i + I_{gi} \qquad (19)\\ u_i =& -K_i(I_{gi}-\eta_i) + q_i \sum_{j\in\mathcal{N}_i^{com}} \rho_{ij}(\xi_i - \xi_j) + V_i^*.\end{aligned}$$

where $\tau_{\xi i}, \tau_{\eta i}, K_i \in \mathbb{R}_{>0}$ are design parameters and $\mathcal{N}_i^{com}$ is the set of DGUs communicating with DGU i via a communication network (possibly different from the electric network), where $\rho_{ij} \in \mathbb{R}_{>0}$ are the edge weights. The distributed controller (19) can be written compactly for all $i \in \mathcal{V}$ as

$$\tau_\xi \dot{\xi} = -\mathcal{L}^{com} Q I_g \qquad (20a)$$
$$\tau_\eta \dot{\eta} = -\eta + I_g \qquad (20b)$$
$$u = -K(I_g - \eta) + Q\mathcal{L}^{com}\xi + V^*, \qquad (20c)$$

where τ_ξ, K, τ_η are positive definite diagonal matrices of appropriate dimensions and $\mathcal{L}^{com}$ is the weighted Laplacian matrix associated with the communication network.

In the following theorem, we show that the closed-loop system is asymptotically stochastically stable and the average voltage regulation and current sharing goals are attained.

Theorem 1. **(Closed-loop analysis).** Let Assumptions 1–4 hold and Ω_{ZIP} be non-empty. System (3) with $\hat{I}_l$, $\hat{P}_l$, $\hat{G}_l$ given by (10), (13), (16) and controlled by (20) is asymptotically stochastically stable and achieves Goals 1 and 2 for all the trajectories $(I_g, V, I, \hat{I}_l, \hat{P}_l, \hat{G}_l) \in \Omega_{\text{ZIP}}$.

Proof. Let $x := (I_g, V, I, \hat{I}_l, \hat{P}_l, \hat{G}_l, \xi, \eta)$ and consider the following storage function

$$S(x) = S_{\text{ZIP}}(I_g, V, I, \hat{I}_l, \hat{P}_l, \hat{G}_l) + \frac{1}{2}(\xi - \bar{\xi})^\top \tau_\xi (\xi - \bar{\xi}) + \frac{1}{2}(\eta - \bar{\eta})^\top \tau_\eta (\eta - \bar{\eta}), \qquad (21)$$

The Ito derivative of the storage function (21) satisfies

$$\begin{aligned}\mathcal{L}S =& (I_g - \eta)^\top K (I_g - \eta) - (I - \bar{I})^\top R (I - \bar{I})\\ &- \hat{I}_l^\top (\mu_{I_l} - \frac{1}{2}\sigma_{I_l}^2 - \frac{1}{2}\mathbb{I}_n)\Pi \hat{I}_l\\ &- \frac{1}{2}(\Pi^{-\frac{1}{2}}(V-\bar{V}) + \Pi^{\frac{1}{2}}\hat{I}_l)^\top(\Pi^{-\frac{1}{2}}(V-\bar{V})\\ &+ \Pi^{\frac{1}{2}}\hat{I}_l) - \frac{1}{2}\hat{P}_l^\top(\mu_{P_l} - \sigma_{P_l}^2)\Sigma\hat{P}_l\\ &- \frac{1}{2}\left(\Sigma^{\frac{1}{2}}\mu_{P_l}^{\frac{1}{2}}\hat{P}_l - \Sigma^{-\frac{1}{2}}\mu_{P_l}^{-\frac{1}{2}}(-\mathbb{1}_n^\top + \bar{V}^\top[V]^{-1})^\top\right)^\top\\ &\left(\Sigma^{\frac{1}{2}}\mu_{P_l}^{\frac{1}{2}}\hat{P}_l - \Sigma^{-\frac{1}{2}}\mu_{P_l}^{-\frac{1}{2}}(-\mathbb{1}_n^\top + \bar{V}^\top[V]^{-1})^\top\right)\\ &- \frac{1}{2}\hat{G}_l^\top(\mu_{G_l} - \sigma_{G_l}^2)\Lambda\hat{G}_l - \frac{1}{2}\left(\Lambda^{\frac{1}{2}}\mu_{G_l}^{\frac{1}{2}}\hat{G}_l\right.\\ &\left.+ \Lambda^{-\frac{1}{2}}\mu_{G_l}^{-\frac{1}{2}}(V-\bar{V})(V-\bar{V})^\top\mathbb{1}_n\right)^\top\left(\Lambda^{\frac{1}{2}}\mu_{G_l}^{\frac{1}{2}}\hat{G}_l\right.\\ &\left.+ \Lambda^{-\frac{1}{2}}\mu_{G_l}^{-\frac{1}{2}}(V-\bar{V})(V-\bar{V})^\top\mathbb{1}_n\right) - (V-\bar{V})^\top\\ &\left(\frac{1}{2}([G_l^*] - \Pi^{-1}) - [P_l^*][V]^{-1}[\bar{V}]^{-1} - \frac{1}{2}\Sigma\mu_{P_l}[V]^{-2}\right.\\ &\left.- [V-\bar{V}]^2\Lambda\mu_{G_l}\right)(V-\bar{V}),\end{aligned} \qquad (22)$$

along the solutions to the closed-loop system (3), (10), (13), (16), (20). Then, it follows that $\mathcal{L}S \le 0$, for all the trajectories $(I_g, V, I, \hat{I}_l, \hat{P}_l, \hat{G}_l) \in \Omega_{\text{ZIP}}$. Then, as a preliminary result we can conclude that the solutions to the closed-loop system (3), (10), (13), (16), (20) are bounded. Moreover, according to LaSalle's invariance principle, these solutions converge to the largest invariant set contained in $\Psi := \{I_g, I, V, \hat{I}_l, \hat{P}_l, \hat{G}_l, \xi, \eta : I_g = \eta, I = \bar{I}, V = \bar{V}, \hat{I}_l = \hat{P}_l = \hat{G}_l = \mathbf{0}\}$. Hence, the behavior of the closed-loop system (3), (10), (13), (16), (20) on the set Ψ can be described by

$$L_g \dot{I}_g = -\bar{V} + Q\mathcal{L}^{com}\xi + V^* \qquad (23a)$$
$$\mathbf{0} = I_g + \mathcal{A}\bar{I} - I_l^* - [G_l^*]\bar{V} - [\bar{V}]^{-1}P_l^* \qquad (23b)$$
$$\mathbf{0} = -\mathcal{A}^\top\bar{V} - R\bar{I} \qquad (23c)$$
$$d\hat{I}_l = \mathbf{0} \qquad (23d)$$
$$d\hat{P}_l = \mathbf{0} \qquad (23e)$$
$$d\hat{G}_l = \mathbf{0} \qquad (23f)$$
$$\tau_\xi\dot{\xi} = -\mathcal{L}^{com}QI_g \qquad (23g)$$
$$\tau_\eta\dot{\eta} = \mathbf{0}. \qquad (23h)$$

It follows from (23h) that η is constant on the largest invariant set. Then, I_g is also constant. Since I_g is constant, it follows from (23g) that $\mathcal{L}^{com}QI_g$ is also constant, implying that ξ would increase unbounded if $\mathcal{L}^{com}QI_g \neq \mathbf{0}$, contradicting the preliminary result on the boundedness of the solutions. Then, $\mathcal{L}^{com}Q\bar{I}_g$ must necessarily be equal to zero, implying that the goal of current sharing is achieved (see Goal 1). Since I_g is constant, we can pre-multiply (23a) by $\mathbb{1}_n^\top Q^{-1}$ and obtain $\mathbb{1}_n^\top Q^{-1}\bar{V} = \mathbb{1}_n^\top Q^{-1}V^*$, *i.e.*, average voltage regulation (see Goal 2). ■

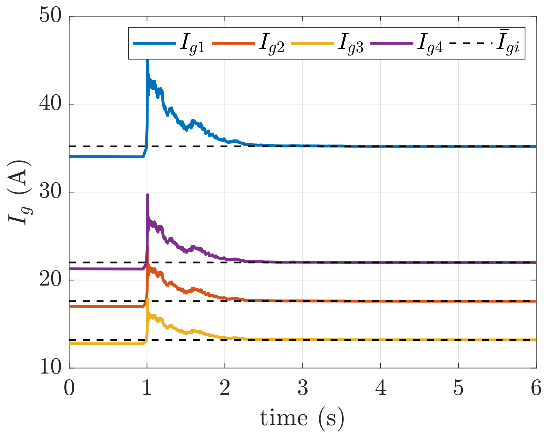

Fig. 2. The generated current at different nodes.

Note that, in analogy with Theorem 1 and by virtue of Lemmas 1 and 2, similar results can be proved for the cases of Z*IP* and Z*IP loads, respectively.

5. SIMULATION RESULTS

In this section, the performance of the proposed method is evaluated via a case study example. We consider the closed-loop system (3), (10), (13), (16), (20) describing a DC network composed of 4 nodes (*i.e.*, DGUs). The parameters of the DC network are taken from Cucuzzella et al. (2019b). The parameters of the controllers are chosen as $K_i = 0.4, \tau_{\eta i} = 0.005, \tau_{\xi i} = 1, i = 1, ..., 4$. The

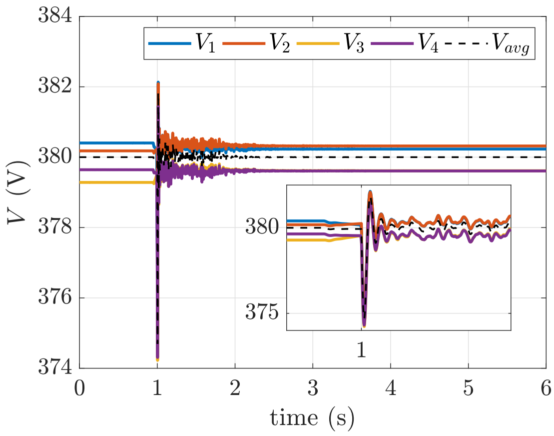

Fig. 3. Voltage of PCC at different nodes.

initial values of P_l^* are $[25, 10, 25, 20]$. Their variations at the time instant $t = 1s$ are $\Delta P_l^* = [8, 5, -8, 3]$, while the initial values of I_l^*, and G_l^* are $[0.07, 0.045, 0.06, 0.08]$ and $[8, 4, 5, 12]$. The stochastic parameters related to the load current, power and conductance are selected as $\mu_{I_{li}} = 2.5, \sigma_{I_{li}} = 1, \mu_{P_{li}} = 2, \sigma_{P_{li}} = 0.7, \mu_{G_{li}} = 1.3, \sigma_{G_{li}} = 0.2, i = 1, ..., 4$, respectively. Fig. 2 illustrates the currents generated by each DGU. We can see that the generated currents converge to the values that ensure fair current sharing (dashed lines). Moreover, Fig. 3 shows the voltage at the PCC of each node. It can be seen that the weighted average voltage (dashed line) converges to the weighted average of the desired voltages, *i.e.*, 380 V.

6. CONCLUSIONS

In this paper, we have considered stochastic dynamics for the impedance (Z), current (I) and power (P) components of loads in DC networks. Then, we have verified the stochastic passivity of the considered system. In order to achieve average voltage regulation and current sharing, we have used an existing distributed control scheme, proving the asymptotic stochastic stability of overall system.

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