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An optimization methodology for self-consumption of residential photovoltaic energy

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Abstract: This paper considers the optimization of the self-consumed power locally produced by residential photovoltaic panels. We focus on a simplified set-up where only the Electric Water Heater (EWH) is regulated, while the rest of the appliances represents an uncontrollable load. We formulate an unconstrained optimization problem by inverting the EWH dynamics and prove that the corresponding objective function is continuously differentiable. Thanks to an explicit characterization of its critical points, we propose a tailored and computationally efficient optimization algorithm. Simulations performed in Dymola over a one-year time horizon allow us to demonstrate the merits and performances of the proposed method.

Keywords: Energy Management System, Industrial applications of optimization, Modeling and simulation of power system, Energy Control and Optimization, Smart Grids

1. INTRODUCTION

The rapid growth of worldwide installed capacities of solar power and other Distributed Energy Resources, observed in the past decade, is expected to continue thanks to unprecedented production cost reductions and governments support (SolarPower Europe, 2018). As the number of decentralized photovoltaic producers feeding power into the local distribution grids rises, the revenue potentially generated by flexibilities on the grid increases. Indeed, production and consumption flexibilities have the potential to mitigate both the risk associated to higher variability in supplies and the need for substantial grid investments (International Energy Agency, 2018).

In this context, a combination of monitoring, communication and control technologies enabling the concept of smart grids (Iovine et al., 2017; Jaramillo-Lopez et al., 2013), and Home Energy Management Systems (HEMS) at the residential level are gaining momentum (Ha et al., 2006). HEMS are designed to establish communication and to control domestic appliances in order to effectively make use of the home flexibility resources (Wacks, 1991). Some of this flexibility is exploited by self-consumption installations, which aim at using the locally generated energy instead of exporting it, and thus contribute to the distribution grid stability by avoiding voltage rise during photovoltaic (PV) generation peak periods. These installations are increasingly encouraged through regulatory changes, such as simplified administrative procedures (EU Directive, 2018), and new tariffs making self-consumption of the generated power the most profitable option.

Fully benefiting from this renewed self-consumption framework requires HEMS that can automate the coordinated activation of home appliances. A first class of techniques performing this automation relies on scheduling algorithms (Ha et al., 2005; Paridari et al., 2016) or Mixed Integer Linear Programing (Heleno et al., 2015) taking into account tariffs structure and resource allocation constraints on both usage-level and household-level power consumption. A second class explicitely accounts for the transient dynamics of the house and the various appliances—in particular the electrical water tank (Beeker et al., 2016)—and can rely on optimal control (Malisani, 2012), Model Predictive Control (Oldewurtel et al., 2012; Lefort et al., 2013; Sossan et al., 2013; Pflaum et al., 2014; Parisio et al., 2015; Wytock et al., 2017) or their variants such as stochastic optimal control (Pacaud, 2018).

This paper focuses on a simplified set-up where the only regulated appliance is the electric water heater (EWH), while the rest of them represents an uncontrollable load. We aim at maximizing the self-consumption of the PV arrays (a key evaluation criterion for such installations), and at providing an optimization strategy that can be frequently updated. In this context, we propose to use a specific solution which leads to a much more computationally efficient numerical routine than the aforementioned techniques. Reformulating the EWH dynamics and constraints, we transform the problem under consideration as

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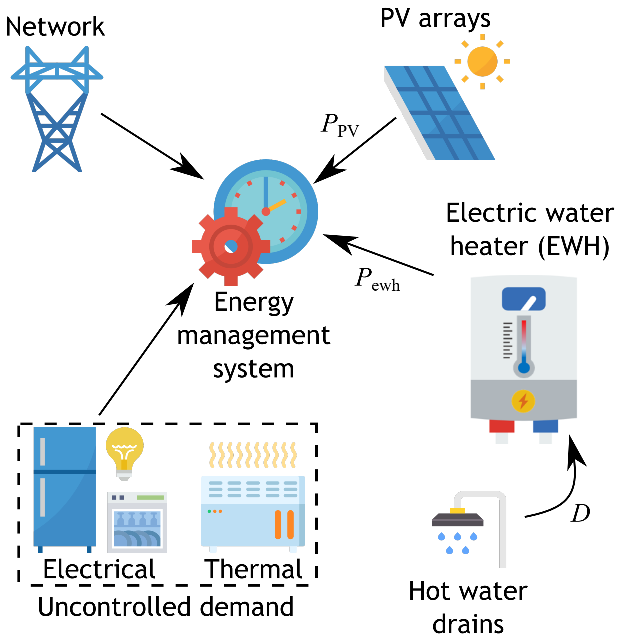

Fig. 1. Setup under consideration: the Energy Management System regulates the EWH.

an unconstrained optimization problem. We prove that the corresponding objective function is continuously differentiable and formulate an explicit expression of its critical points, from which we deduce a numerically effective optimization algorithm. This is the main contribution of the paper. The performance of this optimization methodology is then compared to an industrial heuristic using a high-fidelity simulation platform.

This paper is organized as follows. Section 2 details the context and problem under consideration. In Section 3, the core of our algorithm is presented with slight modifications to the problem. Section 4 follows with the numerical experiments specifications and results analysis. Finally Section 5 reports conclusions and perspectives.

2. PROBLEM STATEMENT

2.1 Setup

In this section, we describe the electrical system under consideration, depicted in Fig. 1.

The system consists of an individual house equipped with PV arrays, heating and cooling equipments, an EWH controlled by an On/Off heating authorization, and other uncontrollable electrical appliances (such as lights, a fridge, a dishwasher, etc.).

Thanks to the heating and cooling systems maintaining the house at thermal equilibrium, we assume the house temperature to be nearly constant. Therefore these appliances are considered as uncontrollable, and are not considered as decision variables in the remainder of this work.

We assume to have perfect knowledge, present and future, of the following elements:

- Weather conditions: outside temperature, solar flux, wind velocity;
- PV production curve (computed by a PV arrays physical model fed with the weather forecast);
- Inhabitants presence;
- Electrical consumption profile of uncontrollable loads;
- Hot water drains of the EWH.

Table 1. Nomenclature

Notation	Description	Unit
C	Total electrical consumption of the household	W
D	Drained energy during hot water consumption	W h
Δ_{ewh}	Duration of heating period of the EWH	s
E	Thermal energy of the water tank	W h
E_{PV}	Energy produced by the PV arrays	W h
k	Thermal loss coefficient of the water tank	h^{-1}
P_{ewh}	Power consumption of the EWH	W
$\overline{P}_{\mathrm{ewh}}$	Power rating of the EWH	W
$\widehat{P}_{\mathrm{PV}}$	Total power produced by the PV arrays	W
P_{PV}	Surplus of PV power	W
SC	Self-consumption over a time period	W h
T_{ewh}	Temperature of water in the the water tank	°C
t_{lim}	Latest acceptable starting date of the heating range	s
τ_{ewh}	Ending date of the heating period	s

2.2 Modeling of the electric water heater

The EWH is modeled as a volume of homogeneous temperature. Leaving aside the modeling of the temperature stratification inside the tank implies a loss of precision, but simplifies the control design (Beeker-Adda, 2016). Taking into account an initial energy state at time t_0, thermal losses, the power input from resistive heating and the hot water consumption during the given period, the energy stored at time t_{f} in the EWH is given by

$$E(t_{\mathrm{f}}) = E(t_0)\,\mathrm{e}^{-k(t_{\mathrm{f}}-t_0)} + \int_{t_0}^{t_{\mathrm{f}}} \mathrm{e}^{-k(t_{\mathrm{f}}-s)}\, P_{\mathrm{ewh}}(s)ds - D[t_0 : t_{\mathrm{f}}] \tag{1}$$

where

- $E(t)$ is the energy stored in the EWH at time t;
- k is the thermal losses coefficient;
- $P_{\mathrm{ewh}}(t)$ is the power input of the EWH at time t;
- $D[t_0 : t_{\mathrm{f}}]$ is the total hot water drain between times t_0 and t_{f}.

We consider the heating period of the EWH to be a unique and continuous period, of constant power consumption. Its consumption curve is hence modeled by the following boxcar function:

$$P_{\mathrm{ewh}}(t_{\mathrm{ewh}}, \Delta_{\mathrm{ewh}}, t) = \overline{P}_{\mathrm{ewh}}\left(H(t - t_{\mathrm{ewh}}) - H(t - \tau_{\mathrm{ewh}})\right) \tag{2}$$

with H the Heaviside function and $\overline{P}_{\mathrm{ewh}}$ the constant EWH rated power. The heating range starting date is denoted t_{ewh} while its ending date is $\tau_{\mathrm{ewh}} = t_{\mathrm{ewh}} + \Delta_{\mathrm{ewh}}$.

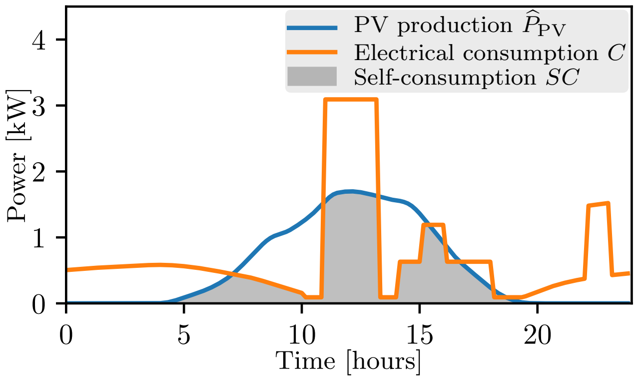

Fig. 2. Schematic representation of the self-consumption $SC(t)$, corresponding to the power that is both locally produced and consumed.

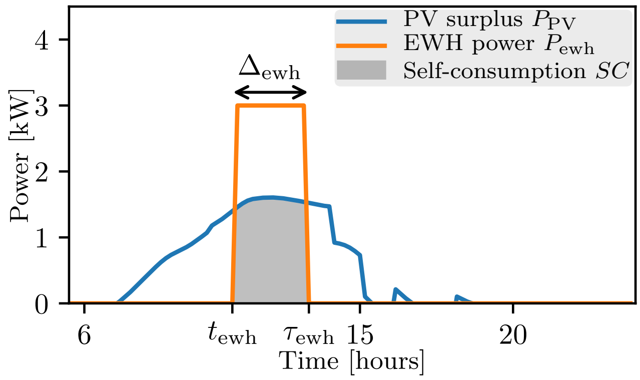

Fig. 3. Schematic representation of the controllable part of the self-consumption $SC(t)$.

This formulation corresponds to a function $P_{\rm ewh}$ null from t_0 to $t_{\rm ewh}$ and from $\tau_{\rm ewh}$ to t_f, and equal to the constant value $\overline{P}_{\rm ewh}$ from $t_{\rm ewh}$ to $\tau_{\rm ewh}$. Hence the energy balance (1) simplifies as

$$E(t_{\rm ewh}, \Delta_{\rm ewh}, t_{\rm f}) = E(t_0)\,{\rm e}^{-k(t_{\rm f}-t_0)} + \frac{\overline{P}_{\rm ewh}}{k}\left({\rm e}^{-k(t_{\rm f}-\tau_{\rm ewh})} - {\rm e}^{-k(t_{\rm f}-t_{\rm ewh})}\right) - D[t_0 : t_{\rm f}] \quad (3)$$

for $[t_{\rm ewh}, \tau_{\rm ewh}] \subset [t_0, t_{\rm f}]$.

Note for later use that, as the tank is modeled with a homogeneous temperature, to any temperature $T_{\rm ewh}$ corresponds a unique specific energy E, related by the following equation:

$$T_{\rm ewh} = T_{\rm amb} + \frac{E}{\rho V c_p} \quad (4)$$

where $T_{\rm amb}$ is the ambient temperature, ρ is the density of water, V is the volume of water in the tank, and c_p is the specific heat capacity of liquid water.

2.3 Optimization problem

Objective function and decision variables The decision variables of our problem are the starting date $t_{\rm ewh}$ and the duration $\Delta_{\rm ewh}$ of the heating range of the EWH. These decision variables are referred to as the followed control strategy: $s = (t_{\rm ewh}, \Delta_{\rm ewh})$.

Our objective is to maximize the self-consumption (SC) of the installation, defined as the part of the photovoltaic production that is locally consumed in order to meet electric consumption (see Fig. 2). Mathematically, it is defined as the integral over the period $[0, \tau]$ of the minimum between the local PV production and the total electric consumption:

$$SC(s) = \int_0^\tau \min(C(s,t), \widehat{P}_{\rm PV}(t))dt \quad (5)$$

with

- C being the overall electric consumption, depending on s and t;
- $\widehat{P}_{\rm PV}$ being the total PV production.

We rewrite this definition by substracting the uncontrollable electric consumption (heating and cooling, lights, white goods) from the PV production, considering only the positive part of the remaining PV production, and eliminating the integral parts where $P_{\rm ewh}(s,t) = 0$:

$$SC(s) = \int_{t_{\rm ewh}}^{\tau_{\rm ewh}} \min(P_{\rm ewh}(s,t), P_{\rm PV}(t))dt + \text{Constant} \quad (6)$$

where $P_{\rm PV}$ is the positive part of the surplus of local production, over which we can optimally place the controllable load.

For the sake of simplicity, we will keep on using the notation SC to refer to the controllable part of (6) (illustrated in Fig. 3).

Constraints The objective function is subject to boundary, physical, and control constraints:

$$E(t_0) = E_0 \quad (7)$$
$$\forall\, t, T_{\rm ewh}(s,t) \le T_{\max} = 65\,°\mathrm{C} \quad (8)$$
$$E(s, t_{\rm f}) = E_{\rm f} \quad (9)$$
$$\tau_{\rm ewh} \le t_{\rm f} \quad (10)$$

The constraint (8) corresponds to the maximal temperature the EWH is allowed to reach; (9) is a comfort constraint: the control shall guarantee that the EWH reaches a specific energy level $E_{\rm f}$ at a final time $t_{\rm f}$ (for instance, to cover hot water consumption at the end of the day). Naturally, the heating period has to be over by the final date $t_{\rm f}$, hence the control constraint (10).

We will refer to the constraints over s as

$$C_{\rm eq}(s) = E(s, t_{\rm f}) - E_{\rm f} \quad (11)$$
$$C_{\rm ineq}(s) = \begin{pmatrix} T_{\rm ewh}(s,t) - T_{\max} \\ \tau_{\rm ewh} - t_{\rm f} \end{pmatrix} \quad (12)$$

where $E(s,t)$ is given in (3) and $T_{\rm ewh}(s,t)$ in (4).

Optimization problem We can now formulate the problem at stake.

Problem 1. Given a PV production curve $P_{\rm PV}$, solve for each day

$$\max_{\substack{s\,=\,(t_{\rm ewh},\Delta_{\rm ewh})\\ E(t_0)\,=\,E_0 \\ C_{\rm eq}(s)\,=\,0 \\ C_{\rm ineq}(s)\,\le\,0}} \int_{t_{\rm ewh}}^{t_{\rm ewh}+\Delta_{\rm ewh}} \min(P_{\rm ewh}(s,t), P_{\rm PV}(t))dt \quad (13)$$

with $P_{\rm ewh}(s,t)$, $C_{\rm eq}$ and $C_{\rm ineq}$ respectively defined in (2), (11) and (12).

3. PROBLEM REFORMULATION AND SOLUTION

In the sequel, we assume that the maximal temperature constraint (8) is non-limiting, and is thereby ignored. However, it is included in the numerical simulations used for evaluation in Section 4.

3.1 Reformulation of the constraints

Two calculations are conducted in order to transform the constraints formulation.

The first transformation shows that Δ_{ewh} is completely defined by t_{ewh} and the boundary constraints (7) and (9).

From (3) we directly get

$$\Delta_{\text{ewh}} = t_f + \frac{1}{k}\log\left(\frac{k}{\overline{P}_{\text{ewh}}}\left[E_{\text{f}} + D[t_0 : t_{\text{f}}] + \frac{\overline{P}_{\text{ewh}}}{k}\,\mathrm{e}^{-k(t_{\text{f}}-t_{\text{ewh}})} - E_0\,\mathrm{e}^{-k(t_{\text{f}}-t_0)}\right]\right) - t_{\text{ewh}} \tag{14}$$

which presents no singularity for the range of numerical values of interest. Hence, equation (14) replaces equation (9), and most importantly, Δ_{ewh} is no longer a decision variable as it is entirely defined by t_{ewh} through (14). This allows us to replace s by only the starting date t_{ewh} thereafter.

The second calculation gives *the explicit last departure date* behind constraints (9) and (10). Consider that the EWH is activated at the latest acceptable date t_{ewh} in order to end the heating period at $t_{\text{ewh}} + \Delta_{\text{ewh}} = t_{\text{f}}$. This specific starting time t_{ewh} is referred to as t_{lim} and can be obtained from (3) as

$$t_{\text{lim}} = t_{\text{f}} + \frac{1}{k}\left[\log(k) + \log\left(E_0\,\mathrm{e}^{-k(t_{\text{f}}-t_0)} - E_{\text{f}} + \frac{\overline{P}_{\text{ewh}}}{k} - D[t_0 : t_{\text{f}}]\right) - \log(\overline{P}_{\text{ewh}})\right] \tag{15}$$

We can now reformulate the problem at hand, under the previous assumptions and after the conducted reformulations:

Problem 2. (Problem 1 simplified). Given a PV production curve $P_{\text{PV}}(t)$, solve for each day

$$\max_{t_{\text{ewh}} \le t_{\text{lim}}} \int_{t_{\text{ewh}}}^{t_{\text{ewh}}+\Delta_{\text{ewh}}(t_{\text{ewh}})} \min(P_{\text{ewh}}(t_{\text{ewh}}, t), P_{\text{PV}}(t))dt \tag{16}$$

where $P_{\text{ewh}}(t_{\text{ewh}}, t) = P_{\text{ewh}}(t_{\text{ewh}}, \Delta_{\text{ewh}}(t_{\text{ewh}}), t)$ is defined in (2) and $\Delta_{\text{ewh}}(t_{\text{ewh}})$ is defined in (14).

3.2 Smoothness analysis of the objective function

To provide a suitable optimization solution, we analyze the nature of the objective function.

Proposition 1. Assume that P_{PV} is a continuous function. Consider $SC = \int_{t_{\text{ewh}}}^{\tau_{\text{ewh}}} \min(P_{\text{ewh}}(t_{\text{ewh}}, t), P_{\text{PV}}(t))dt$ with $P_{\text{ewh}}(t_{\text{ewh}}, t)$ defined in (2) and $\tau_{\text{ewh}} = t_{\text{ewh}} + \Delta_{\text{ewh}}(t_{\text{ewh}})$ with $\Delta_{\text{ewh}}(t_{\text{ewh}})$ defined in (14). Then SC is continuously differentiable with respect to t_{ewh}, and its derivative is given by

$$\frac{dSC}{dt_{\text{ewh}}} = -\min(P_{\text{PV}}(t_{\text{ewh}}), P_{\text{ewh}}(t_{\text{ewh}}, t_{\text{ewh}})) + e^{-k(\tau_{ewh}-t_{ewh})}\min(P_{\text{PV}}(\tau_{\text{ewh}}), P_{\text{ewh}}(t_{\text{ewh}}, \tau_{\text{ewh}})) \tag{17}$$

with $\tau_{ewh} = t_{ewh} + \Delta_{ewh}$ and Δ_{ewh} defined in (14).

Interestingly, note that the objective function smoothness property would still hold if Δ_{ewh} were independent of t_{ewh}, only with a simpler derivative expression. That is, the following proof principle would also hold for a fixed duration appliance.

Proof. Consider an EWH of rated power $\overline{P}_{\text{ewh}}$ and of variable duration $\Delta_{\text{ewh}}(t_{\text{ewh}})$.

First, observe that, $\forall\, t \in [t_{\text{ewh}}, \tau_{\text{ewh}}]$, the function $t \mapsto \min(P_{\text{ewh}}(t_{\text{ewh}}, t), P_{\text{PV}}(t)) = \min(\overline{P}_{\text{ewh}}, P_{\text{PV}}(t))$ is continuous, P_{PV} being at least $\mathcal{C}^0$.

Hence one can apply Leibniz integral rules for differentiation under the integral sign on SC which gives

$$\frac{dSC}{dt_{\text{ewh}}} = \min(\overline{P}_{\text{ewh}}, P_{\text{PV}}(\tau_{\text{ewh}}))\frac{d\tau_{\text{ewh}}}{dt_{\text{ewh}}} - \min(\overline{P}_{\text{ewh}}, P_{\text{PV}}(t_{\text{ewh}})) \tag{18}$$

We know from (14) that τ_{ewh} is continuous with respect to t_{ewh}. Taking the derivative with respect to t_{ewh} of (3) and using the fact that $E(t_{\text{ewh}}, \Delta_{\text{ewh}}, t_{\text{f}}) = E_{\text{f}}$ is constant, one gets the derivative of the ending date:

$$\frac{d\tau_{ewh}(t_{ewh})}{dt_{ewh}} = \frac{\mathrm{e}^{-k(t_{\text{f}}-t_{ewh})}\,\overline{P}_{\text{ewh}}}{\mathrm{e}^{-k(t_{\text{f}}-\tau_{ewh}(t_{ewh}))}\,\overline{P}_{\text{ewh}}} = \mathrm{e}^{-k(\tau_{ewh}(t_{ewh})-t_{ewh})} \tag{19}$$

which is obviously continuous.

This yields the conclusion.

3.3 Stationary condition and resolution algorithm

We follow the classic idea of optimization methods for continuously differentiable objective function (Nocedal and Wright, 1999), that is to identify the points where the objective function derivative is null.

Following Proposition 1, the stationary condition for our problem writes as

$$\min(P_{\text{PV}}(t), \overline{P}_{\text{ewh}}) = e^{-k(\tau_{ewh}(t)-t)}\min(P_{\text{PV}}(\tau_{\text{ewh}}(t)), \overline{P}_{\text{ewh}}) \tag{20}$$

Algorithm 1 Calculate $t^\star_{\text{ewh}}$ solution of Problem 2

Requires: $P_{\text{PV}}(\cdot), \overline{P}_{\text{ewh}}, t_0, E_0, t_{\text{f}}, E_{\text{f}}, D(\cdot), I = [0, \tau]$
1: **for all** $t \in I$ **do**
2:   **if** t is solution of (20) with $t_{\text{ewh}} = t$ **then**
3:     Compute $SC(t)$
4:     Update Buffer = [Buffer, $SC(t)$]
5:   **end if**
6: **end for**
7: Identify $t^\star_{\text{ewh}} = \underset{t}{\operatorname{argmax}}\,\text{Buffer}$

We propose to determine exhaustively the solutions to the stationary condition and then compare the corresponding values of the objective function in order to solve Problem 2. This procedure is summarized in Algorithm 1. Note that this algorithm is of course intended to be used in discrete time. In particular, steps 1 and 7 only consider a finite number of elements, and a solution of (20) is identified

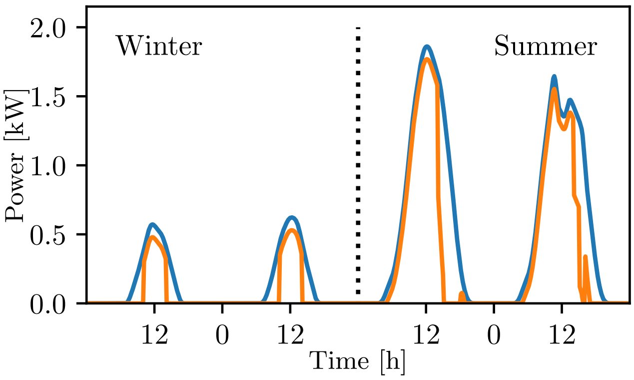

Fig. 4. Total PV production (blue) and PV surplus (orange) for 2 winter days and 2 summer days.

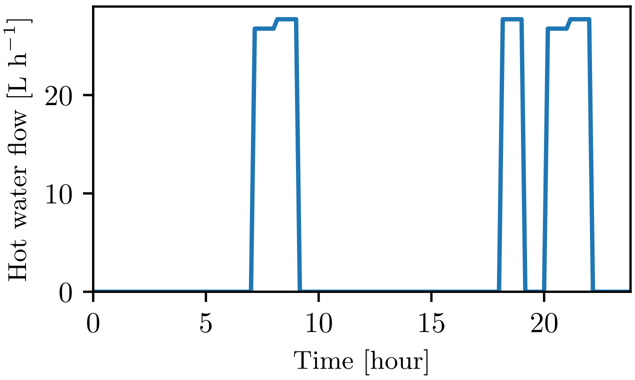

Fig. 5. Hot water drains for a winter day.

in step 2 when the objective function derivative (17) goes from a positive to a non-positive value between two consecutive dates. We now apply this algorithm in a high-fidelity environment.

4. NUMERICAL EXPERIMENTS

4.1 Test scenario and hardware specifications

We consider two crystalline silicon PV arrays of 1.5 $\mathrm{kW_p}$ each, both inclined of 15° relative to the horizontal and of respective azimuth 15° and 105° relative to the south. An EWH of volume $V = 200$ L and power rating $\overline{P}_{\mathrm{ewh}} = 3$ kW is considered, to cover the consumption of 2 inhabitants. The scenario of outdoor temperature, input cold water, and solar irradiance correspond to a house located in Clermont-Ferrand, in the center of France. Fig. 4 displays the total PV production curve and the corresponding PV surplus curve, after substraction of the uncontrolled electric consumption, for a few days.

The hot water drains scenario is perfectly known and is repeated each day with the same pattern reported in Fig. 5, with drains in the morning (from 7 a.m. to 9 a.m.) and in the evening (from 6 p.m. to 7 p.m. and from 8 p.m. to 10 p.m), ranging between 26.8 $\mathrm{L\,h^{-1}}$ and 27.7 $\mathrm{L\,h^{-1}}$, for a total consumption of 956 L per week at 60 °C. Only slight quantity variations occur seasonally, with the same pattern tuned down in summer.

Table 2. Hardware specifications

PV capacity	3 $\mathrm{kW_p}$
EWH Power rating	3 kW
EWH volume	200 L
Weekly hot water consumption at 60 °C	956 L

The final time t_{f} is set as 6 p.m., in order to satisfy the main hot water drains of the day.

The overall simulation is conducted with the modeling and simulation software Dymola (based on the object-oriented modeling language Modelica, see Wiström (2013)), including models for the house, the PV arrays, and the uncontrolled electric loads.

The proposed method is validated through comparison with two other EWH control strategies:

- a passive heuristic strategy, triggering the heating authorization consistently during [1 a.m.-7 a.m.] and [12 p.m.-2 p.m.];
- an industrial reference heuristic control.

This industrial reference control was developed for commercial use in residential housing equipped with PV arrays with the double aim of maximizing the consumption of local PV production and ensuring a reduced electricity bill. It follows priority rules to choose each day whether or not to start the heating authorization, according to the following principles:

- determine when to start the EWH according to a PV surplus threshold set up in advance depending on various criteria;
- figure out if heating the tank without PV production is necessary, according to whether or not the daily heating duration target has been met.

Note that the reference control does not require the hot water drains sequence to perform an estimation of the next heating duration target, whereas our proposed method requires exact knowledge of them.

4.2 Numerical results

Simulations consist of a daily optimization over one year, starting January 1st, using Algorithm 1 with I restricted to the interval of start dates such that the heating authorization would cover the valuable times of high PV production. The initial and target energy levels E_0 and E_{f} for the proposed method correspond to the energy levels reached at these dates by the EWH under the reference control. Results are extracted with a 10-minute time-step.

The evolution of the cumulative self-consumption rate for each strategy over a one-year simulation is shown in Fig. 6. This rate is the measure usually referred to, for self-consumption installations. It is the ratio of the total self-consumption (controllable and uncontrollable consumptions considered) to E_{PV}, the total local energy production over the period.

$$SC_{\%}(s) = \frac{SC(s)}{E_{\mathrm{PV}}} \tag{21}$$

E_{PV} being independent of the control strategy s, the optimization previously performed over SC also maximizes $SC_{\%}$.

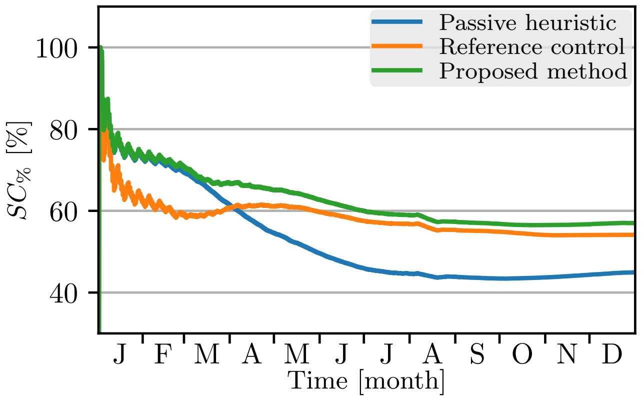

Fig. 6. Cumulative self-consumption rate for each strategy.

Table 3. Final annual $SC_\%$ values for each strategy

Passive heuristic	**Reference control**	**Proposed method**
44.9 %	54.1 %	57.0 %

The final values show that, overall, the proposed method outperforms by a few points the reference control, and that both are largely more efficient than the passive strategy (see Table 3).

A year-long analysis of the daily SC confirms the interest of the proposed method, especially during winter days. Our method outputs a higher SC 249 days of the year and a lower SC for the other 116 days. Over a whole year, the proposed method yields an increase of the daily average SC of + 9.8 % compared to the reference control daily SC. These performances are mitigated by an analysis of the 135 days of the year during which the SC for both the proposed method and the reference control is higher than 6 kWh. For these days of high stakes happening around the summer season, the proposed method outperforms the reference control for only 71 days. This leads to a slight increase of the mean daily SC, by + 0.1 %, compared to the reference control daily SC. The reasons for these seasonal differences are explained further down.

A major advantage of our method is its limited computational load: it achieves optimally scheduling these 365 load curves in an average of 40 seconds with Python 3.7.3 and a Core i3 2.4 GHz processor, with 8 Go RAM. The corresponding time of less than 0.11 seconds per curve is fast enough to consider repeating the procedure several times for various consumption and production scenarios.

Seasonal analysis We now break down the analysis of Fig. 6 into seasonal observations.

First of all, it can be observed that the initial measure points are close to a cumulated SC of 100 % because the small amounts of PV production occurring during these winter days are at first completely used for self-consumption by the uncontrollable consumption.

After these initial days, PV surplus starts to become consistently available, although with very low volumes. The reference control progressively falls behind because

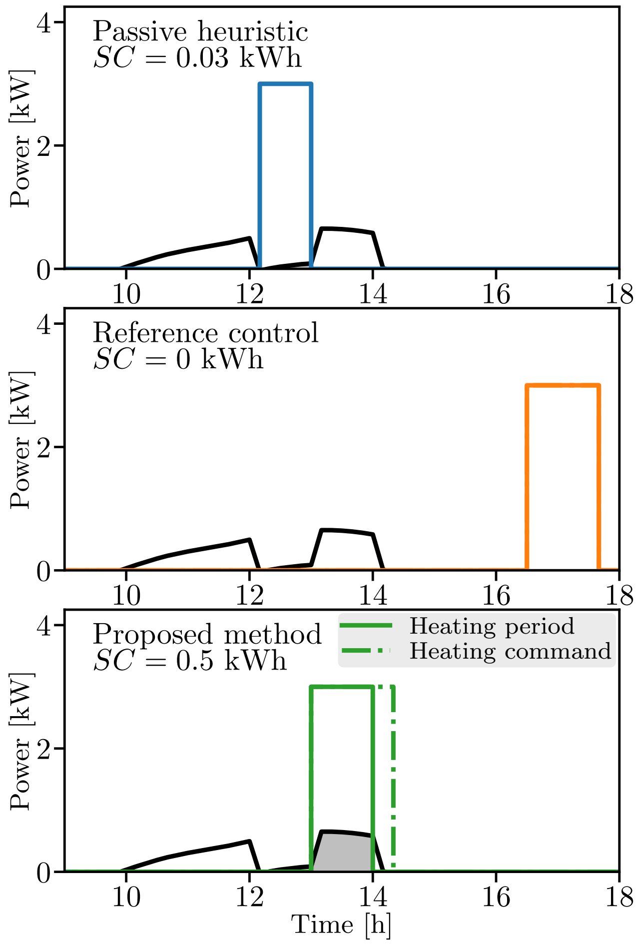

Fig. 7. Comparison of the SC (gray patch) for the three strategies for a winter day PV production (black curves).

it will not try and consume the very low PV production of winter days, as it would result in an extra cost, given the higher grid tariff of daytime, compared to nighttime. Fig. 7 displays the EWH consumption curves resulting from the three strategies and the PV surplus we aim to harness on a winter day, highlighting the differences of approach resulting in a typically better SC with the proposed method.

The reference control then closes the gap with the proposed method during spring time when it starts harnessing the PV surplus available at sufficient levels. It even matches performances with the proposed method on summer days, because sufficient PV production levels extend the heating authorization period of the reference control, leading to On/Off switches and an overall higher energy consumption for the reference control;

Fig. 8 displays an example of the EWH consumption curves resulting from the three strategies for a summer day PV surplus. For this particular day, SC for our method is 7.3 kWh, exceeding the 6.4 kWh of the reference control.

Thermal limitation It is important to note here that the control resulting of the three strategies is not always exactly followed. This is due to the fact that the EWH model of the Dymola simulations involves a thermostat enforcing

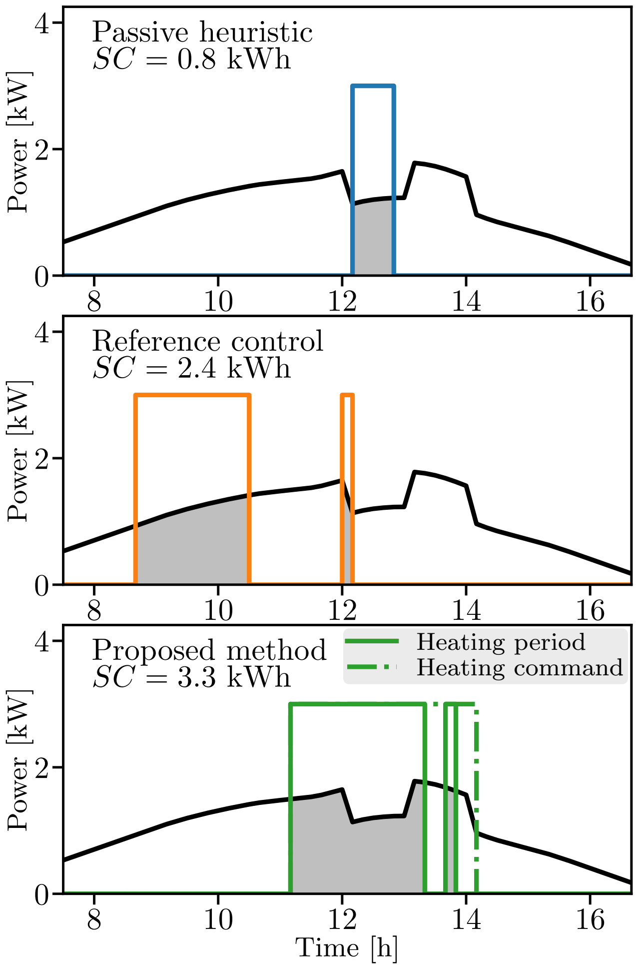

Fig. 8. Comparison of the SC (gray patch) for the three strategies for a summer day PV production (black curves).

the thermal limit (8), thus letting the possibility for the EWH power to oscillate within this heating authorization period, as can be seen in Fig. 7 and 8. We assumed this constraint to be non-limiting for the proposed method but simulations lead us to the opposite conclusion. Certainly, this is a source of suboptimality of our method.

Indeed this fact constitutes a reason why the reference control can perform better than the proposed method during some summer days: the effective heating period given by the proposed method might not be optimally located with regards to the PV production curve, the thermostat stopping the activation during the authorization period.

The impossibility to follow the command of the proposed algorithm causes the energy stored in the tank at $t = t_\mathrm{f}$ to miss the target E_f set for the control, with a median daily relative error of 5.5 %. Part of that error can also be related to the homogeneous temperature EWH model, failing to account for the essential phenomenon of stratification.

5. CONCLUSION AND PERSPECTIVES

This paper details a methodology to optimize the starting date of an EWH in order to maximize the self consumption rate of a residential PV installation. This method is simple and has a very low computational cost. Numerical experiments proved the relevance of the method, achieving an improvement of three absolute points in yearly self-consumption rate over an industrial energy management solution.

Our approach would benefit from several functional enhancements: first, taking the maximum temperature constraint (8) into account in the EWH model; then, implementing an estimation feature for future hot water consumption, to overcome the unrealistic assumption of perfect knowledge on this matter; finally, making use of the lightweight side of the algorithm by incorporating it in a robust optimization method, to overcome uncertainty on consumption an production profiles.

A bigger shift of vision would be to adapt this work towards minimizing the total household electricity bill and including other controllable home appliances. An electrochemical battery to be controlled can also be considered, even if studies often prove them to represent an overall cost as long as they do not provide additional services (e.g., Quoilin et al. (2016), Goebel et al. (2017), Roberts et al. (2019)). These are directions of future works.

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