

## Fault Detection in Hydroelectric Generating Units with Trigonometric Filters

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**Abstract:** A procedure based on trigonometric filters is proposed for parameter identification in a hydroelectric power generating unit approximated by a Multi Input - Single Output linear continuous-time varying system. First the discrete cosine transform is used as an interpolation filter and secondly an integral transform based on a Fourier kernel is applied to obtain a discrete-time model. Then a recursive least squares algorithm is performed to estimate abrupt changes of parameters and thus detect faults in the power plant. The whole method is successfully applied to a scenario fed with real sampled data featured by a poor excitation.

**Keywords:** System identification, power generation, filtering, discrete cosine transform, discretization, fault detection.

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### 1. INTRODUCTION

Hydroelectricity is the first renewable energy in installed capacity and its advantages are well known: clean energy, flexibility and ancillary services. The latter are services for the power grid in order to guarantee the frequency and voltage stability of the power system.

Concerning the frequency control service, in order to quickly compensate supply-demand imbalances, large hydroelectric power plants are required to perform two types of power generation control: Primary Frequency Control and Secondary Frequency Control. The issue for the producer is to early detect both deviations in power control performance and faults in the process.

Fault diagnosis permits predictive maintenance in order to optimize maintenance planning and to anticipate failures. Different types of faults can occur: abnormal vibrations in the turbine, damages in the generator, mechanical/hydraulic faults in the actuator, sensor faults, detuning of the controller, overload of the PLC... All of these faults have an impact on the pattern of the active power output (power oscillations, saturations, slowdown of the response...). Two approaches are possible to monitor the power output. The first one is a signal processing approach by pattern recognition and comparison with failure signatures (Isermann, 2006), (Schölkopf & Smola, 2002), (Van der Heijden & Duin, 2004). The second one is a system identification approach by estimating parameters associated to a physical model (Landau & Gianluca, 2010), (Söderström & Stoica, 1989), (Padilla, 2017). This latter method will be considered in this paper.

In order to develop a diagnosis tool, an off-line system identification algorithm is developed in an industrial context, with real noisy signals featured by poor excitation. This

problem translates into estimating parameters of a Multi Input - Single Output (MISO) linear continuous time-varying (LTV) model from sampled input-output data.

Two approaches exist in the literature to identify a continuous-time (CT) system from sampled data. The indirect approach uses a discrete-time (DT) model equivalent to the CT model, estimate the discrete parameters and transpose them to the CT model. The direct approach used by (Garnier, 2015) and (Padilla, 2017) estimates directly the physical parameters from the CT model. In the present paper, the direct approach is combined with a tailored trigonometric integral transform to both filter the measurement noise and discretize the LTV model.

Recursive parameter estimation problem is made particularly difficult in practice by 3 main factors: firstly the lack of rich excitation which causes ill-conditioning or stability problems, secondly input and output measurement noise leading to a bias when the least squares estimator is used, and finally varying parameters yielding convergence issues. Hydro generating unit are concerned by these 3 issues: the excitation is not persistent, input and output signals are noisy and parameters are time varying with the operating point or in case of abnormal event.

The paper goes on as follows: in section 2, the process is presented and described by a MISO linear continuous-time model with varying parameters. A data preprocessing technique based on Discrete Cosine Transform is developed in section 3. A discretization scheme based on a trigonometric filter is then proposed in section 4, so as to perform and enhance a RLS type parameter identification algorithm. An application to a real hydroelectric generating unit is then addressed in section 5, and section 6 finally concludes.

## 2. PROCESS MODELLING

This section is concerned with obtaining a low order model of a hydro power plant consisting of a reservoir, a tunnel, a surge tank, a single penstock, several turbines and an output canal (see Fig. 1). The nonlinear model of a power generation unit is represented hereafter by a LTV model based on a hydraulic model and the structure of the turbine controller.

### 2.1 Hydraulic Model

We suppose that the hydroelectric power plant is connected to a large power grid. It is shown in (Kishor & Fraile-Ardanuy, 2017) that in normal operating conditions a 14<sup>th</sup> order linearized model of a hydro generating unit has the following form:

$$\Delta \dot{x} = A\Delta x + B\Delta u \quad (1)$$

$$\Delta P = C\Delta x + D\Delta u \quad (2)$$

with  $x = [Q_a \quad Q \quad Z]^T$

The variables  $P$ ,  $Q$  are respectively the output power and the flow rate provided by the generating unit,  $Q_a$  is the flow rate in the tunnel and  $Z$  is the water level in the surge tank. Parameters depend on the operating point  $(P_0, Q_0, H_0)$  where  $H_0$  represents the net head (gross head  $H_b$  minus head losses) of the power plant. The gate opening  $u$  is the control input computed by the turbine controller.

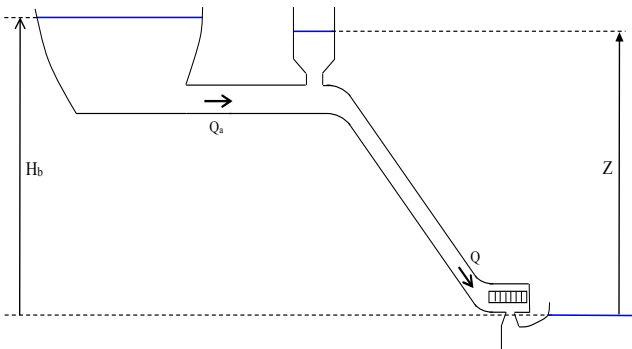


Fig. 1. Hydropower plant with surge tank

### 2.2 Turbine Controller

The process related to power generation consists of a turbine controller (see Fig. 2), an actuator and a generating unit (turbine + power generator). The output is the active power  $P$  and inputs are: the power setpoint  $P_{co}(t)$  including the production program, the signal  $N(t)$  from the grid operator and the network frequency deviation  $\Delta f(t) = f(t) - f_n$  where  $f_n$  is the nominal frequency (50 Hz).

In Fig. 2,  $Q(K,s)$  and  $R(s)$  refer to filters associated to the PI controller  $C(s)$ , with  $s$  for Laplace variable. Parameters  $K$  and

$P_r$  are respectively the gains for the primary and secondary frequency control.

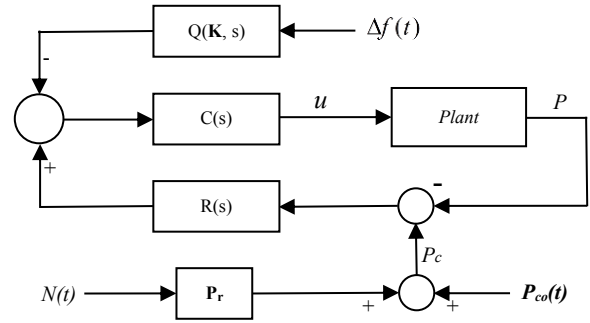


Fig. 2. Power control of a generating unit

### 2.2 Equivalent LTV Model

Closed-loop frequency responses of  $\Delta P$  with respect to  $\Delta f$  and  $\Delta P_c$  were simulated for structure of Fig.2 with realistic 14<sup>th</sup> order models developed from (1)-(2), from which it appears that they can be simplified into 2<sup>nd</sup> order systems in the frequency range of the excitation bandwidth (see Fig. 3):

$$\frac{\Delta P}{\Delta P_c}(s) = \frac{1}{\tau^2 s^2 + 2\xi\tau s + 1} \quad (3)$$

$$\frac{\Delta P}{\Delta f}(s) = \frac{-K}{\tau^2 s^2 + 2\xi\tau s + 1} \quad (4)$$

Since parameters of these transfer functions depend on the operating point, the plant in the present paper is approximated by a continuous-time LTV model (5)-(6) where all parameters  $P_{co}, K, P_r, \tau, \xi$  can vary with time. The aim is to estimate these 5 varying parameters from sampled data  $P, \Delta f, N$ .

$$\tau^2 \ddot{P}(t) + 2\xi\tau \dot{P}(t) + P(t) = P_c(t) - K\Delta f(t) \quad (5)$$

$$P_c(t) = P_{co}(t) + P_r N(t) \quad (6)$$

Varying parameters of this model are gains, time constant and damping ratio which characterize the dynamic behavior of the plant with regard to grid operator requirements.

Variables of model (5)-(6) are centred with respect to mean values by setting  $u_1 = \Delta \bar{f} - \Delta f$ ,  $u_2 = N - \bar{N}$  and  $y = P - \bar{P}$ :

$$\tau^2 \ddot{y} + 2\xi\tau \dot{y} + y = u_0 + K u_1 + P_r u_2 \quad (7)$$

where  $u_0$  is the unknown input:

$$u_0 = P_{co} - \bar{P} - K\Delta \bar{f} + P_r \bar{N} \quad (8)$$

The output  $P(t)$  and the frequency  $\Delta f(t)$  are corrupted by zero-mean white measurement noises (see Fig. 4). The signal  $N(t)$  is a noise-free numerical signal (negligible noise) coming

from the grid operator. Hence, taking into account these noises, equation (7) becomes:

$$y(t) = h^T(t)\theta(t) + e(t) \quad (9)$$

with the regressor

$$h^T(t) = [-\ddot{y} \quad -\dot{y} \quad 1 \quad u_1 \quad u_2] \quad (10)$$

and parameters vector  $\theta = [\tau^2 \quad 2\zeta\tau \quad u_0 \quad K \quad P_r]^T$

The variable  $e$  is the so-called *equation error* term containing model error and measurement noise correlated with the regressor. It is well known that this correlation yields a bias in the estimation of  $\theta$  with the least squares method. Yet, we will show in section 5 that with a suitable filtering, it is possible to reduce this bias.

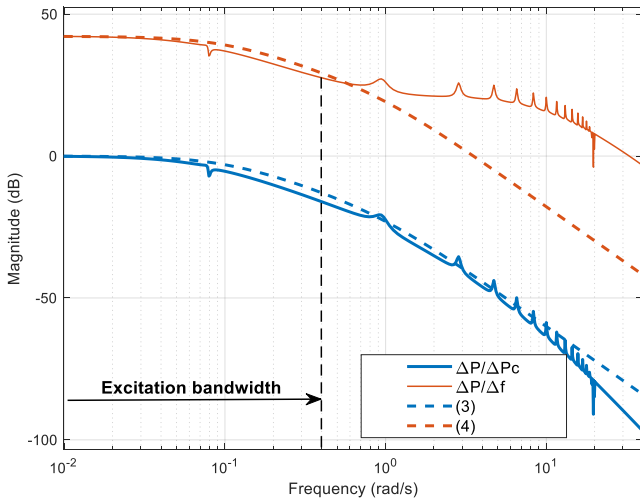


Fig. 3. Bode diagram of  $P$

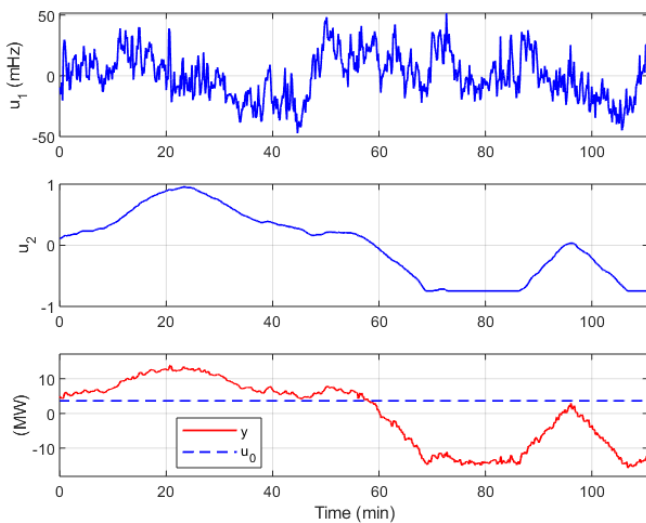


Fig. 4. Measurements signals (inputs  $u_1$ ,  $u_2$  and output  $y$ )

### 3. DATA PREPROCESSING

Preprocessing of measurement signals is an essential step to obtain good performance in system identification (Söderström & Stoica, 1989). It is well known that the accuracy of any estimation method depends greatly on the quality of measurement data expected to be featured by a rich and persistent excitation with some uncorrelated equation error  $e$ .

Unfortunately these conditions are not met in hydroelectric power plants: production demand ( $P_{co}$ ,  $N$ ) varies slowly and measurement noise makes  $e$  dependent of the regressor  $h$ .

To cope with measurement noise and prepare data to transform (9) into a discrete-time model, a procedure based on the Discrete Cosine Transform (DCT) is proposed: firstly, data are upsampled thanks to a DCT trigonometric interpolation, and secondly a denoising is carried out with a DCT smoothing filter.

#### 3.1 Discrete Cosine Transform

Like the Discrete Fourier Transform (DFT), DCT is a unitary transform, that is an orthogonal transform preserving the length of the input vector and hence its energy. This real (not complex, thus easy to implement in an industrial computer) transform is currently used in audio and image compression (JPEG for example) in preference to the DFT because of its property of “energy compaction” meaning that its coefficients are less numerous (concentrated in the low indices) than in the DFT (Rao & Yip, 1990). We can also notice that High Efficiency Video Coding (HEVC) uses an interpolation filter based on the DCT type II (DCT-II) in order to predict pixels close to neighbours. This technique is chosen in the present paper for upsampling and smoothing purpose.

The DCT-II applied to a discrete-time signal  $x(k)$  with the integer  $k = 0, 1, \dots, N-1$  is defined by (Rao & Yip, 1990):

$$X(n) = \sum_{k=0}^{N-1} c_n x(k) \cos \frac{n\pi(2k+1)}{2N} \quad (11)$$

$$x(k) = \sum_{n=0}^{N-1} c_n X(n) \cos \frac{n\pi(2k+1)}{2N} \quad (12)$$

where  $c_0 = 1/\sqrt{N}$  and  $c_n = \sqrt{2/N}$  for  $n > 0$ .

Here  $X(n)$  are the coefficients for the discrete cosine transform of  $x(k)$  with the frequency index  $n = 0, 1, \dots, N-1$ .

#### 3.2 Upsampling and Smoothing with DCT

The sampling period of measured signals is  $T_s = 1$  s and the time constant of the process is about 5 s, thus the sampling frequency is suited for identification purpose (Aström & Wittenmark, 2011). Nevertheless, upsampling the signals may be of interest for increasing the number of points in the computation of integral transforms used in the identification

procedure and hence improve the fault estimation performance (see section 4).

The basic idea of the DCT interpolation is to calculate coefficients  $X(n)$  from (11) with the known integer indices  $k$ ,  $n$ , and then use the inverse DCT transform (12) at desired fractional indices  $\ell$  (for instance  $\ell = 0.1, 0.2, \dots$ ) instead of integer indices  $k$ . Hence, a trigonometric interpolation polynomial  $x_p$  associated to  $x$  is given by (Sauer, 2018):

$$x_p(\ell) = \sum_{n=0}^{N-1} c_n X(n) \cos \frac{n\pi(2\ell+1)}{2N} \quad (13)$$

with  $0 \leq \ell \leq N-1$ ,  $\ell \in \mathbb{Q}$ .

It is a cosine series of the signal  $x$  constrained to pass through the points  $x(kT_s)$ . To filter the noise, it is convenient to relax this last constraint by using only the first  $M$  frequency indices ( $M \leq N$ ) of the DCT transform (high frequencies are removed). Thus, a trigonometric interpolation filter is obtained by substituting the sum of  $N$  cosine in (13) by a sum of  $M$  cosine:

$$x_p(\ell) = \sum_{n=0}^{M-1} c_n X(n) \cos \frac{n\pi(2\ell+1)}{2N} \quad (14)$$

Equation (14) performs an efficient smoothing filter which is a DCT least squares approximation of the signal  $x$  (see Sauer, 2018). It is used in our application for upsampling and smoothing  $u_1$ ,  $u_2$  and  $y$ . An illustration is given in Fig. 5 for the active power signal  $y(t)$ .

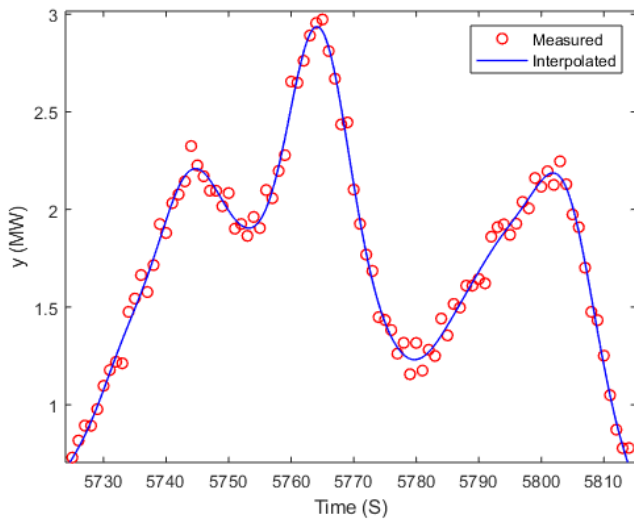


Fig. 5. DCT interpolation and smoothing

Furthermore, by setting  $t = \ell T_s$ , analytical expression (13) or (14) gives access to the continuous-time signal  $x_p(t)$  and hence to all time derivative terms which can be calculated easily from (15):

$$x_p(t) = \sum_{n=0}^{M-1} c_n X(n) \cos \frac{n\pi(2t + T_s)}{2NT_s} \quad (15)$$

#### 4. PARAMETER IDENTIFICATION

In a first section, the continuous-time model (9) will be transformed into a discrete-time model. Then, in the second section a review of RLS algorithm will be presented.

##### 4.1 Discretization

To estimate parameters of the LTV model (9), the first step is to discretize the model. This is done by using a trigonometric integral transform developed in Robert, G., & Besançon, G. (2019a, b). This approach uses an integral kernel based on a Fourier series  $\varphi$ . If  $n$  is the order of the dynamical system (here  $n = 2$ ), the output  $x_f$  of the filter (input  $x$ ) is given by:

$$x_f(k) = \frac{1}{T} \int_{kT}^{(k+1)T} x(t) \varphi(t - kT) dt \quad (16)$$

$$x_f^i(k) = \frac{1}{T} \int_{kT}^{(k+1)T} x^{(i)}(t) \varphi(t - kT) dt \quad (17)$$

where  $(i)$  is the  $i^{\text{th}}$  derivative with respect to  $t$ ,  $i = 1, \dots, n$ .

The discretization period  $T$  is a multiple of the sampling  $T_s$ . It constitutes a tuning parameter for which some guideline is given in section 4.2. Thanks to integral transform, this discretization scheme can be applied for regular or irregular sampling.

The kernel  $\varphi$  verifies the following equality:

$$\varphi_n^{(j)}(0) = \varphi_n^{(j)}(T) = 0, \quad \text{for } j = 0, 1, \dots, n-1 \quad (18)$$

For a 2<sup>nd</sup> order system, by choosing  $\varphi(t) = 1 - \cos \omega t$ , computation of derivative terms are avoided thanks to integration by parts yielding:

$$\int_{kT}^{(k+1)T} x^{(i)}(t) \varphi(t - kT) dt = (-1)^i \int_{kT}^{(k+1)T} x(t) \varphi^{(i)}(t - kT) dt \quad (19)$$

Applying this filter to the continuous-time model (9) and assuming that  $\theta(t)$  is constant in the interval  $[kT, (k+1)T]$ , we obtain:

$$y_f(k) = h_f^T(k) \theta(k) + e_f(k) \quad (20)$$

$$h_f^T(t) = [-y_f^2 \quad -y_f^1 \quad 1 \quad u_{1f} \quad u_{2f}] \quad (21)$$

Applying (16) with a zero-mean error  $e(t)$ , we obtain:

$$e_f(k) = \frac{-1}{T} \int_{kT}^{(k+1)T} e(t) \cos \omega t dt \quad (22)$$

Let  $T_N = NT$  the duration of the considered signals. Hence,  $e(t)$  can be extended with a periodicity  $T_N$  and expanded with a Fourier series where coefficients  $a_n, b_n$  are random variables:

$$e(t) = \sum_{n=1}^{\infty} a_n \cos n\Omega t + b_n \sin n\Omega t, \quad \Omega = 2\pi / T_N \quad (23)$$

Hence,  $e$  will be orthogonal to  $\cos \omega t$  if it exists an integer  $m > 1$  such that  $n\Omega = m\omega = 2\pi m/T$  that is for  $n > N$ . Thus harmonics higher than  $N\Omega$  will be filtered by (22).

#### 4.2 RLS algorithm

To deal with varying parameters, the conventional RLS algorithm with forgetting factor  $\lambda$  is used (Landau & Lozano, 2011). Multiplying (20) by  $\lambda^{k-i}h_f(i)$ , it comes:

$$\hat{\theta}(k) = R^{-1}(k)z(k) \quad (24)$$

where 
$$R(k) = \sum_{i=1}^k \lambda^{k-i} h_f(i) h_f^T(i)$$

and 
$$z(k) = \sum_{i=1}^k \lambda^{k-i} h_f(i) y_f(i)$$

The matrix  $R$  is very important since it determines the performance of the RLS estimator (stability, accuracy and convergence rate). It is a symmetric matrix which is positive definite in our application since elements of  $h$  are linearly independent. Thus,  $R$  is invertible and its determinant (product of its eigenvalues) is strictly positive. However, poor excitation of our system renders this matrix ill-conditioned (condition number of order  $10^5$ ) that is the spread of eigenvalues is large and sensitive to noise (Stenlund & Gustafsson, 2002).

As a consequence, numerical instabilities may appear depending on the value of the tuning parameter  $T$ . If  $T$  is large, the filtering of the noise will be high, the estimation bias will be low (attenuation of the correlation between  $h$  and  $e$ ) and the condition number will be high because of the loss of information (leakage effect giving numerical instabilities and slow convergence). On contrary, if  $T$  is small, the filtering of the noise will be low, the estimation bias will be high and the condition number will be low (good numerical stability and fast convergence). Thus a trade-off is to be found between numerical stability, accuracy and convergence rate.

In recursive form the trade-off holds again. From (24), the RLS estimator can be written (Landau & Lozano, 2011):

$$G(k) = \frac{P(k-1)h_f(k)}{\lambda + h_f^T(k)P(k-1)h_f(k)} \quad (25)$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + G(k)(y_f(k) - h_f^T(k)\hat{\theta}(k-1)) \quad (26)$$

$$P(k) = (I - G(k)h_f^T(k))P(k-1) / \lambda \quad (27)$$

Having  $P(k) = R^{-1}(k)$ , Riccati equation (27) suffers from round-off error propagation linked to the ill-conditioning problem discussed hereinbefore.

## 5. INDUSTRIAL APPLICATION

The identification procedure is evaluated for a 240 MW power generating unit (Pelton turbine) installed in France, with real input data injected in a linear simulator yielding the power output.

In order to simulate a fault, abrupt changes of 3 parameters are considered: the gain  $P_r$  is doubled, the time constant  $\tau$  is reduced and the damping coefficient  $\xi$  is lowered from 1.7 to 0.5 in order to identify two different behaviours, from non-oscillatory (real poles) to oscillatory one (complex poles).

Input and output signals are available with a sampling period  $T_s = 1$  s. An upsampling to  $T_s/10$  and a smoothing are carried out with the DCT interpolation filter (14). Then, parameters are estimated with the procedure given in section 4 and with formula given in appendix.

Good results are obtained, as displayed in Fig. 6 and 7: from the estimates indeed, model parameters  $K$ ,  $P_r$ ,  $\tau$ ,  $\zeta$  together with unknown input  $P_{co}$  can be well recovered.

Let us notice that the Least Mean Square (LMS) algorithm given by (28) (Diniz, 2012) was also tested for comparison, but yielding a too slow convergence because of an excitation not persistent enough (see Fig. 8).

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \mu h_f(k)(y_f(k) - h_f^T(k)\hat{\theta}(k-1)) \quad (28)$$

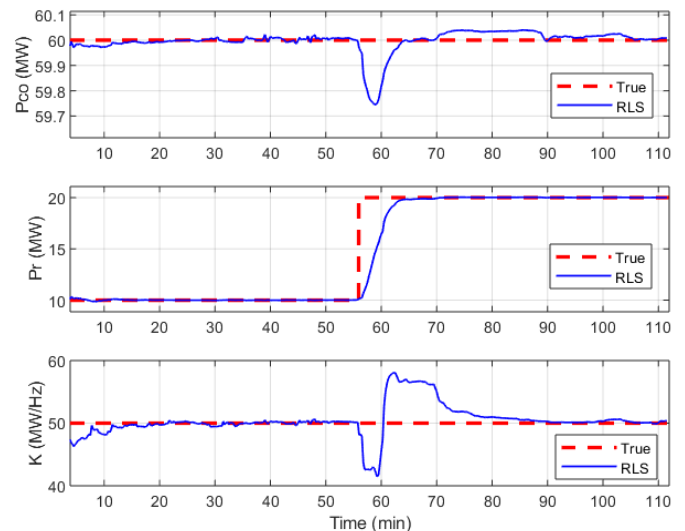


Fig. 6. Tracking performance for a variation of  $P_r$



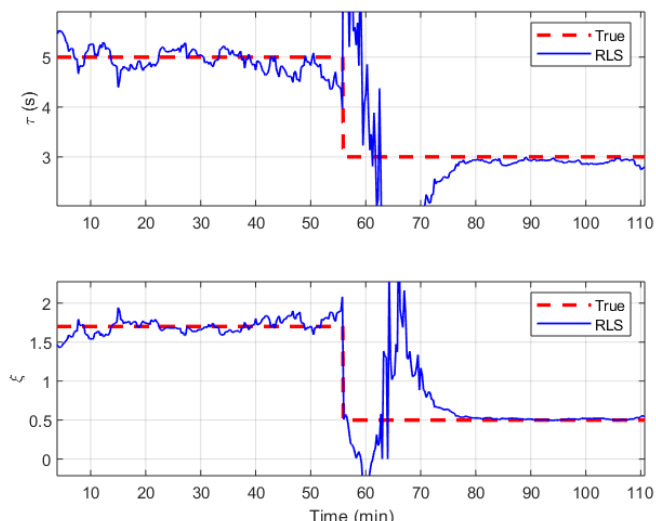


Fig. 7. Tracking performance for a variation of  $\tau, \xi$

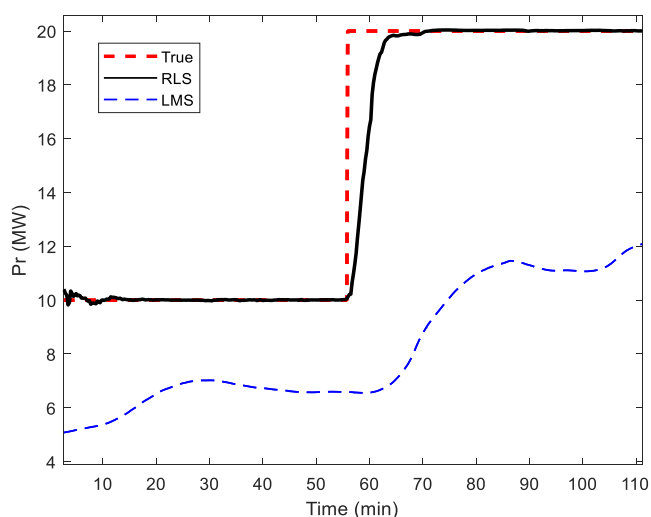


Fig. 8. Tracking of  $P_r$

## 6. CONCLUSION

For fault detection in hydroelectric power plants, an identification procedure based on trigonometric filters and RLS algorithm was developed. These filters enabled to discretize the model and to identify the changes in process parameters from an overdamped to an underdamped 2<sup>nd</sup> order LTV system.

Simulation results indeed show a good matching between expected and estimated parameters despite poor excitation and noisy signals.

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## APPENDIX

Physical parameters and identification parameters are linked with the following relations:

$$\begin{aligned} \theta_1 = \tau^2 ; \theta_2 = 2\xi\tau ; \theta_3 = u_0 ; \\ \theta_4 = K ; \theta_5 = P_r \end{aligned} \quad (29)$$

and reciprocally:

$$\begin{aligned} K = \theta_4 ; P_r = \theta_5 ; u_0 = \theta_3 ; \\ \tau = \sqrt{\theta_1} ; \xi = \frac{\theta_2}{2\sqrt{\theta_1}} ; \\ P_{co} = \theta_3 + \bar{P} + \theta_4 \Delta \bar{f} - P_r \bar{N} \end{aligned} \quad (30)$$